

# Otimização Combinatória nas Telecomunicações



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XXXVI SBPO  
São João del Rei, MG  
23 a 26 de novembro de 2004

# Combinatorial Optimization in Telecommunications



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# Summary of talk

- Network migration
- Modem pool placement for Internet service provider
- Local access network design
- Traffic routing on a virtual private network
- Internet traffic engineering
- Survivable IP network design

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# Application 1:

## Network migration scheduling

# Network migration scheduling

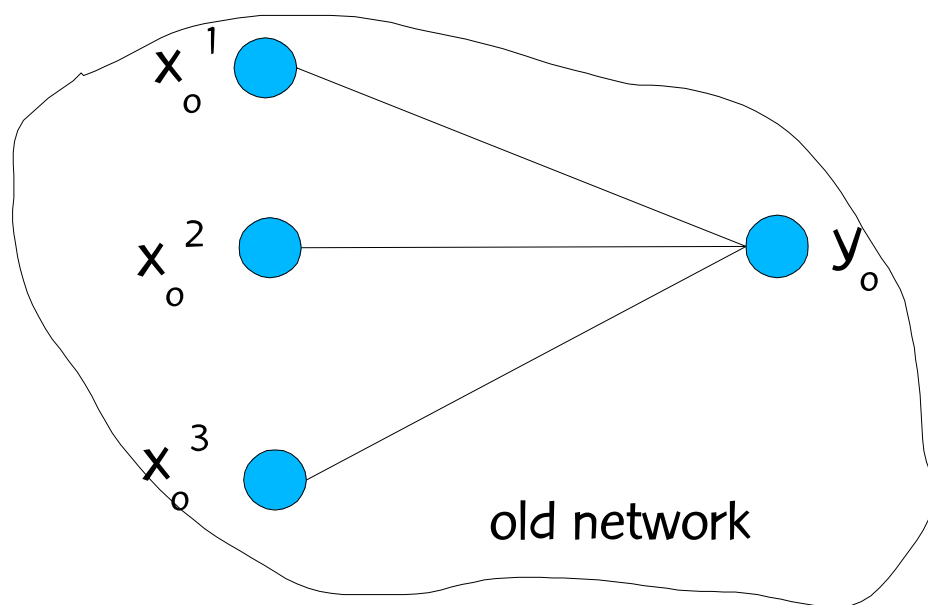
- Inter-nodal traffic from an outdated network is migrated to a new network.
- All traffic originating or terminating at a given node in the outdated network is moved to a specific node in the new network.
- Routing is predetermined in both networks and therefore capacities are known.

# Network migration scheduling

- Traffic between nodes in the same network is routed in that network.
- Suppose node  $y_o$  in the old network is migrated to node  $y_n$  in the new network.
- Let link  $(x_o, y_o)$  have capacity  $c_o$ .
- Traffic from  $x_o$  to  $y_n$  must use a temporary link  $(x_o, y_n)$  with capacity  $c_o$ .

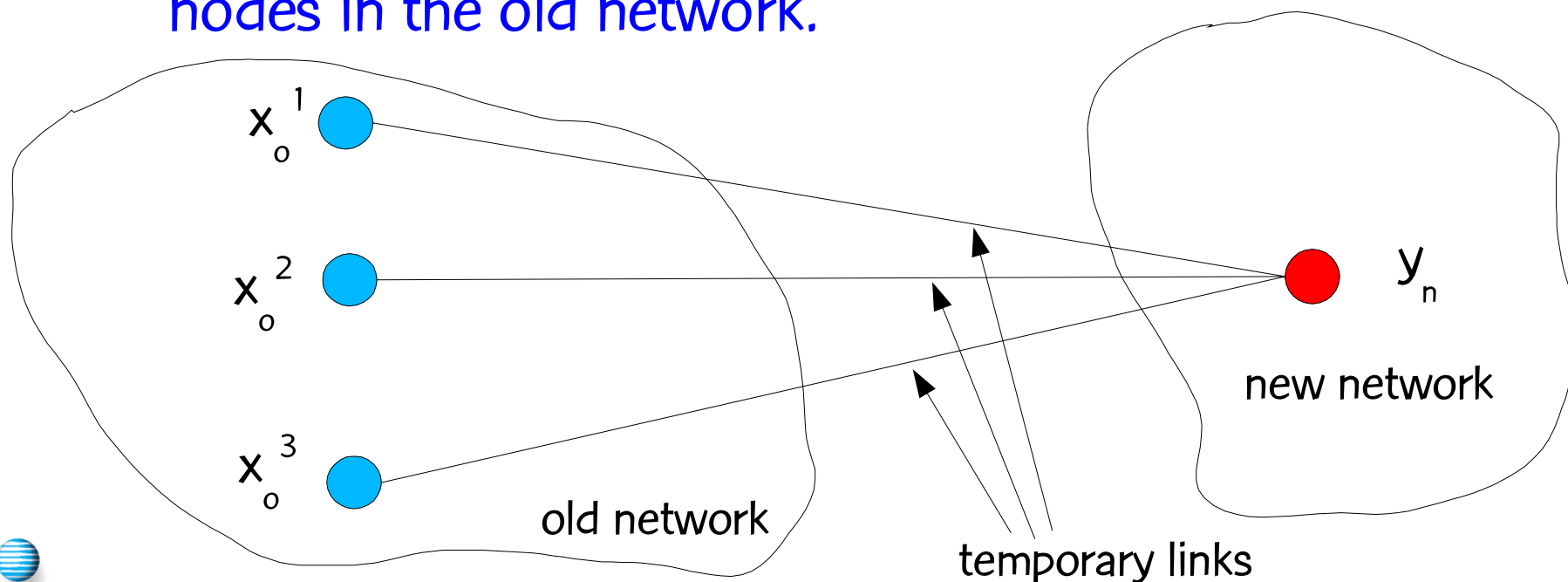
# Network migration scheduling

- When node  $y_o$  is migrated to  $y_n$  in the new network, one or more temporary links may have to be used, since node  $y_o$  may be adjacent to one or more still-active nodes in the old network.



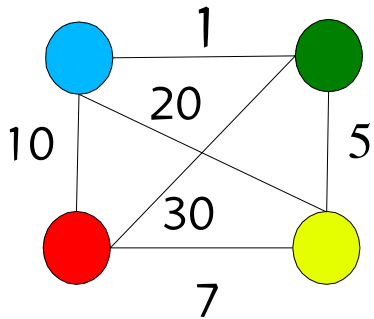
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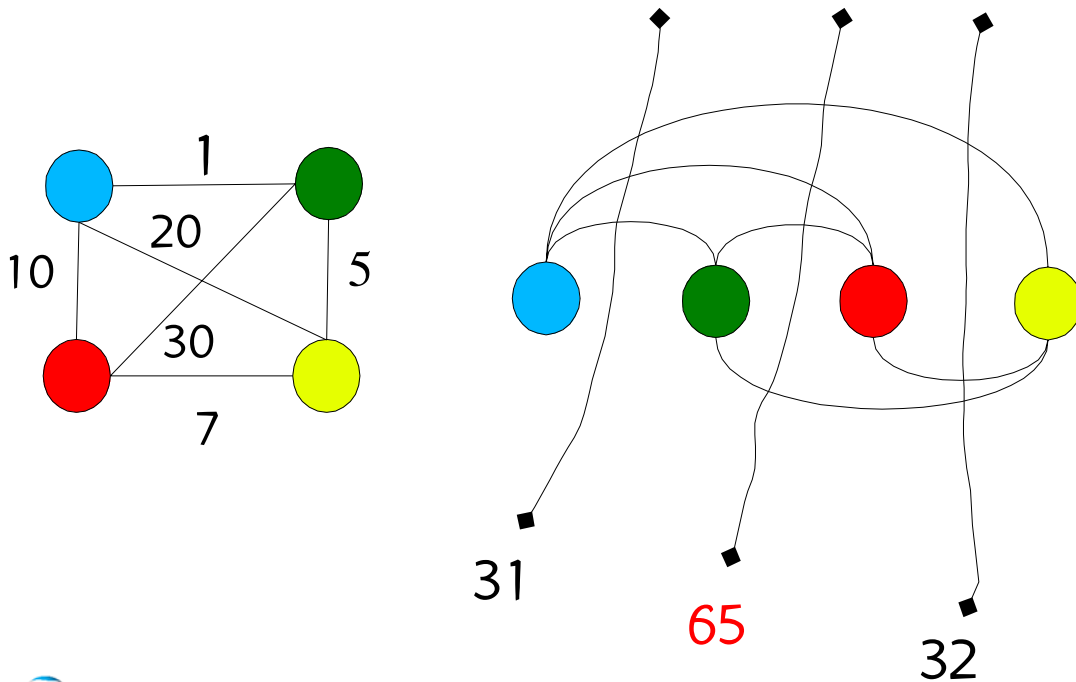
# Network migration scheduling problem

- Find a migration ordering of the vertices such that the maximum sum of the capacities of the temporary links is minimized.



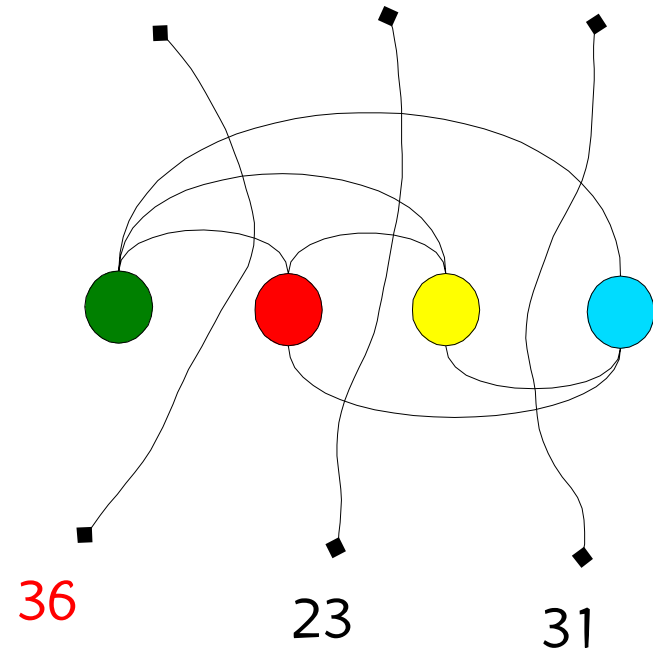
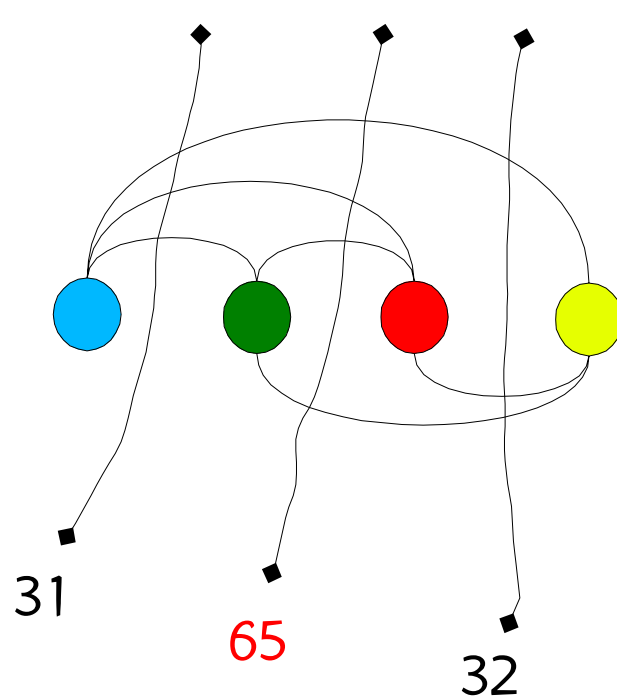
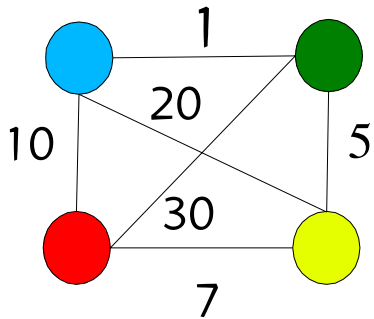
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# GRASP for MCLA

- Given  $G = (V, E)$  and weights  $w_{uv}$  on links  $(u, v)$ . Find permutation of nodes  $\pi_1, \pi_2, \dots, \pi_n$  defining the schedule.
- Suppose nodes  $\pi_1, \pi_2, \dots, \pi_{k-1}$  have been already scheduled and let  $\Omega = V \setminus \{ \pi_1, \pi_2, \dots, \pi_{k-1} \}$  be the set of yet to be scheduled nodes.

# GRASP for MCLA

- Let  $f(u)$  be the sum of link weights from node  $u$  to all nodes in  $\Omega \setminus \{u\}$ : choose  $u$  with small  $f(u)$
- Likewise, let  $b(u)$  be the sum of link weights from  $u$  to all nodes in  $V \setminus \Omega \setminus \{u\}$ : choose  $u$  with largest  $b(u)$
- Greedy choice: choose  $u$  with smallest  $f(u) - b(u)$
- Greedy randomized choice: choose  $u$  from set of nodes with small  $f(u) - b(u)$  value.
- Local search: swap order of node pairs (observe only cuts between nodes are potentially affected by swap)

# GRASP for MCLA

```
repeat {  
     $\pi$  = GreedyRandomizedConstruction(■);  
     $\pi$  = Local Search( $\pi$ );  
    save  $\pi$  as  $\pi^*$  if best so far;  
}  
return  $\pi^*$ ;
```

# Path-relinking

- Path-relinking:
  - Intensification strategy exploring trajectories connecting elite solutions: Glover (1996)
  - Originally proposed in the context of tabu search and scatter search.
  - Paths in the solution space leading to other elite solutions are explored in the search for better solutions:
    - selection of moves that introduce attributes of the guiding solution into the current solution

# Path-relinking

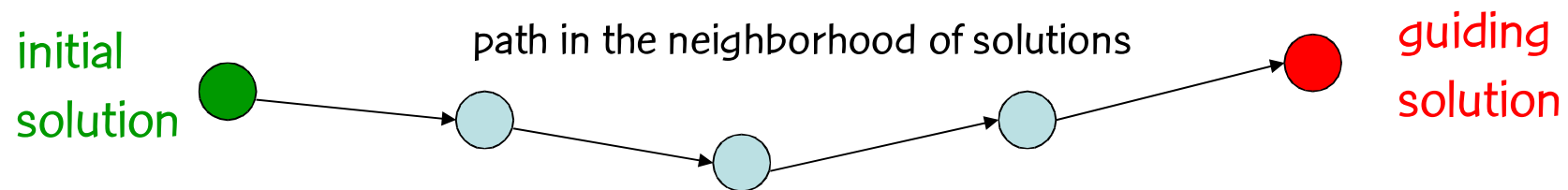
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# Path-relinking

- Exploration of trajectories that connect high quality (elite) solutions:



# Path-relinking

- Path is generated by selecting moves that introduce in the initial solution attributes of the guiding solution.
- At each step, all moves that incorporate attributes of the guiding solution are evaluated and the best move is selected:

initial  
solution



● guiding  
solution



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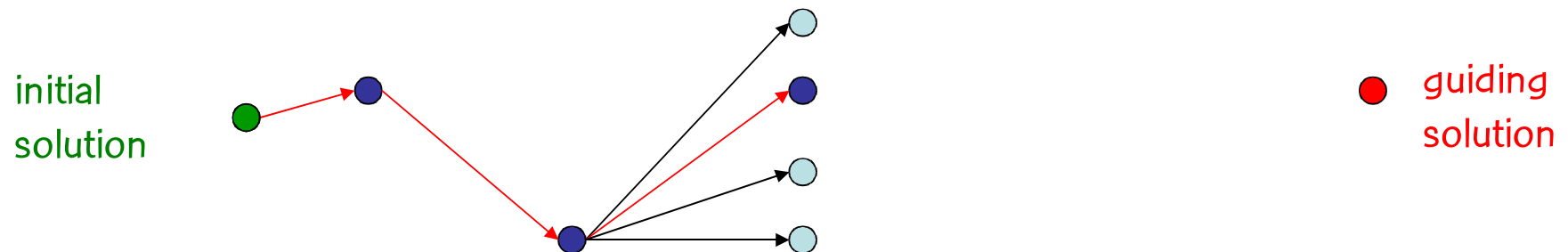
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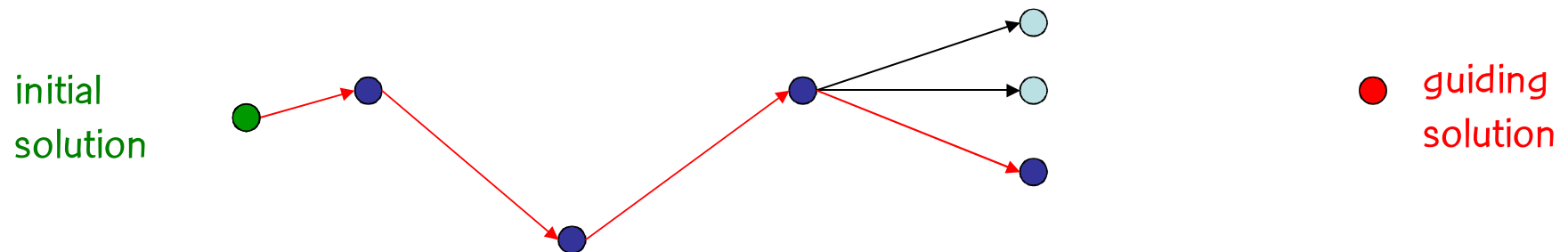
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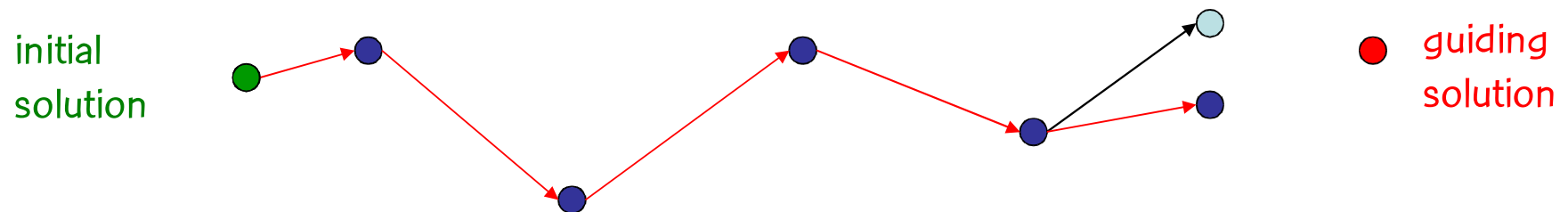
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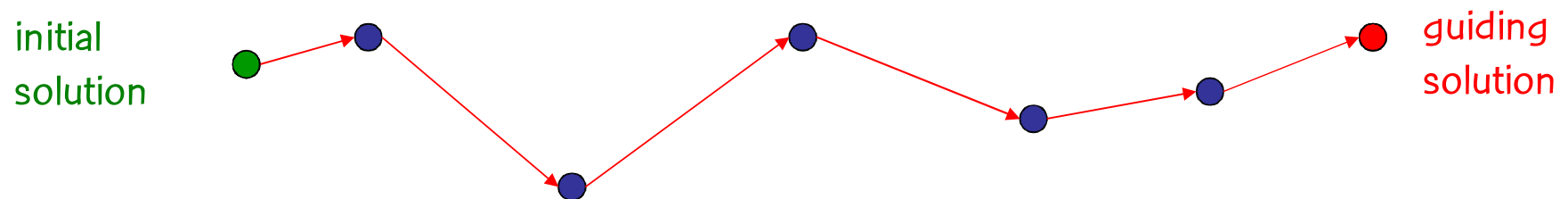
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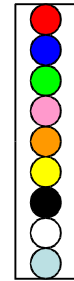
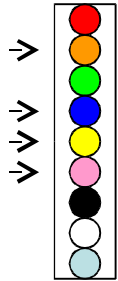
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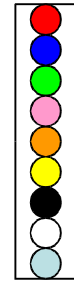
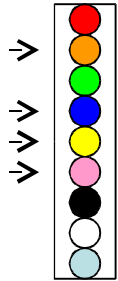
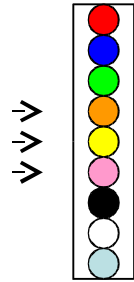


# Path-relinking

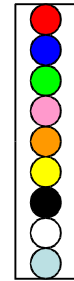
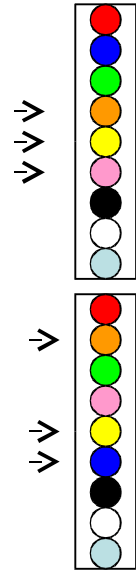
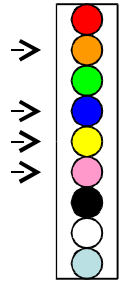
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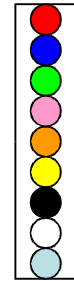
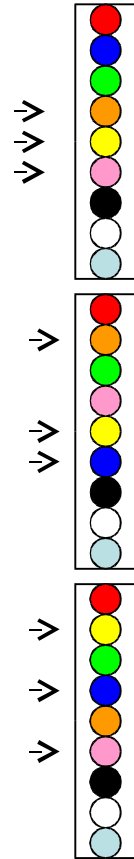
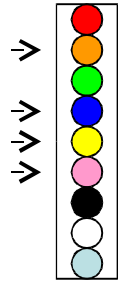


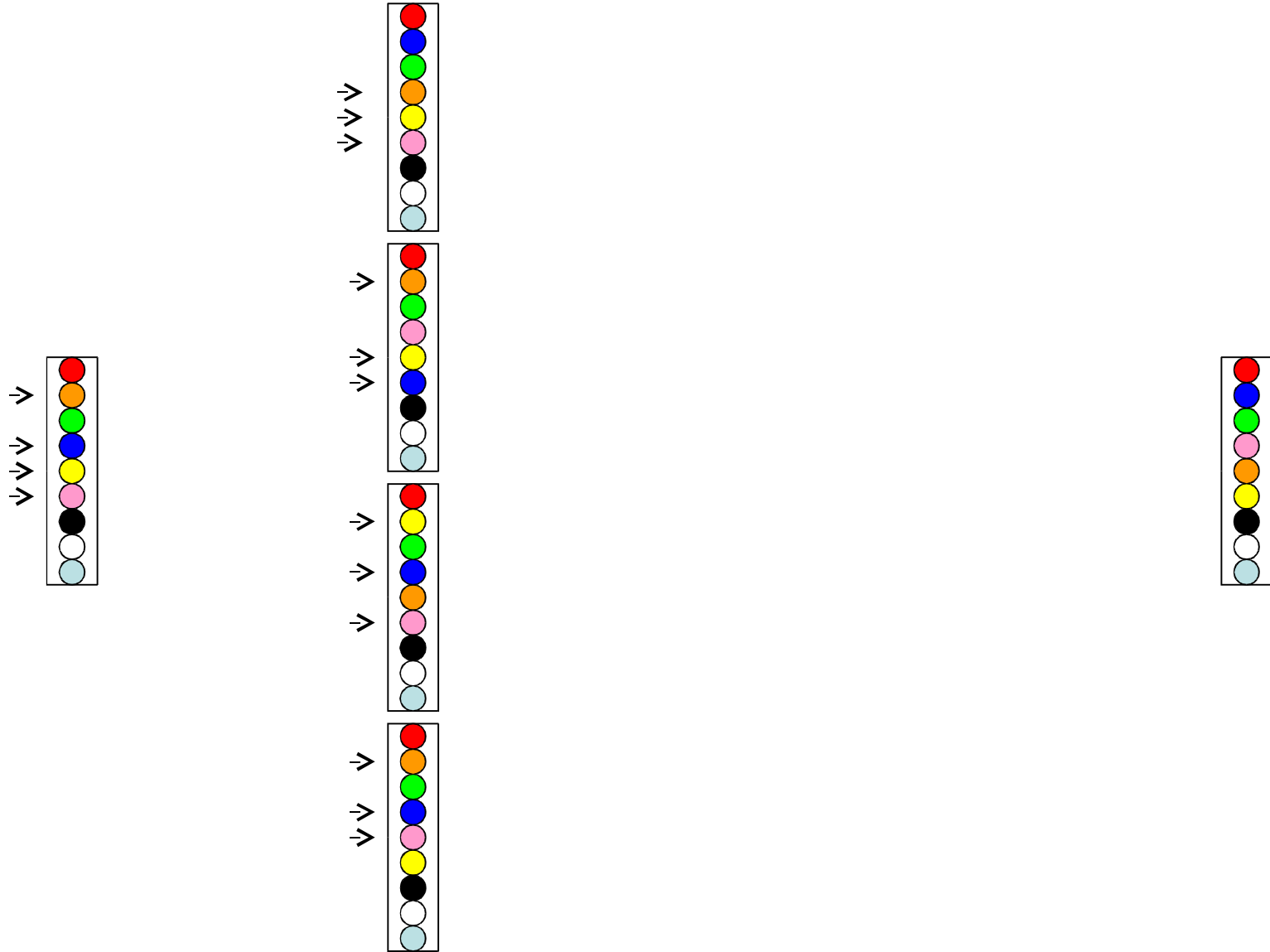


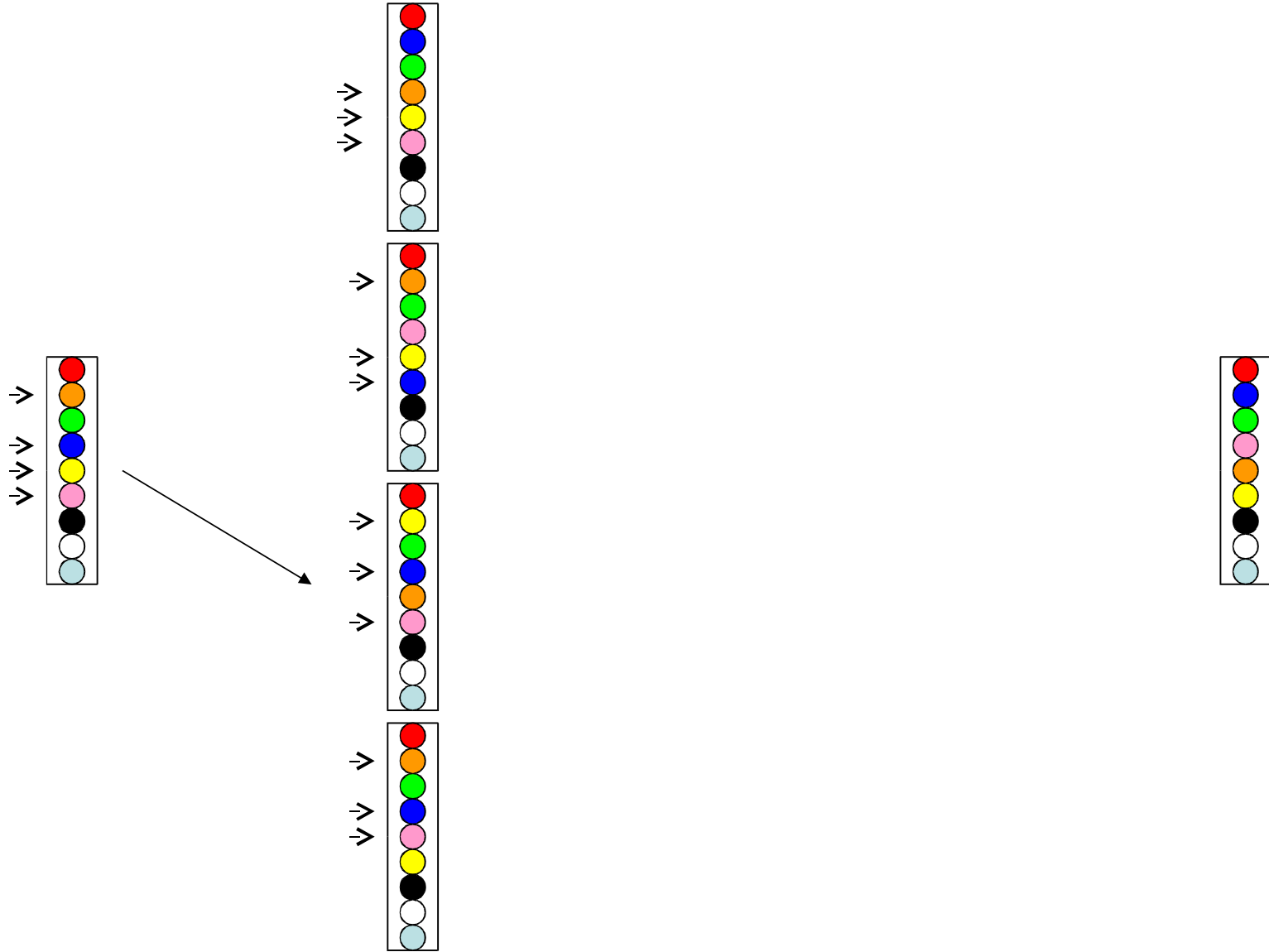


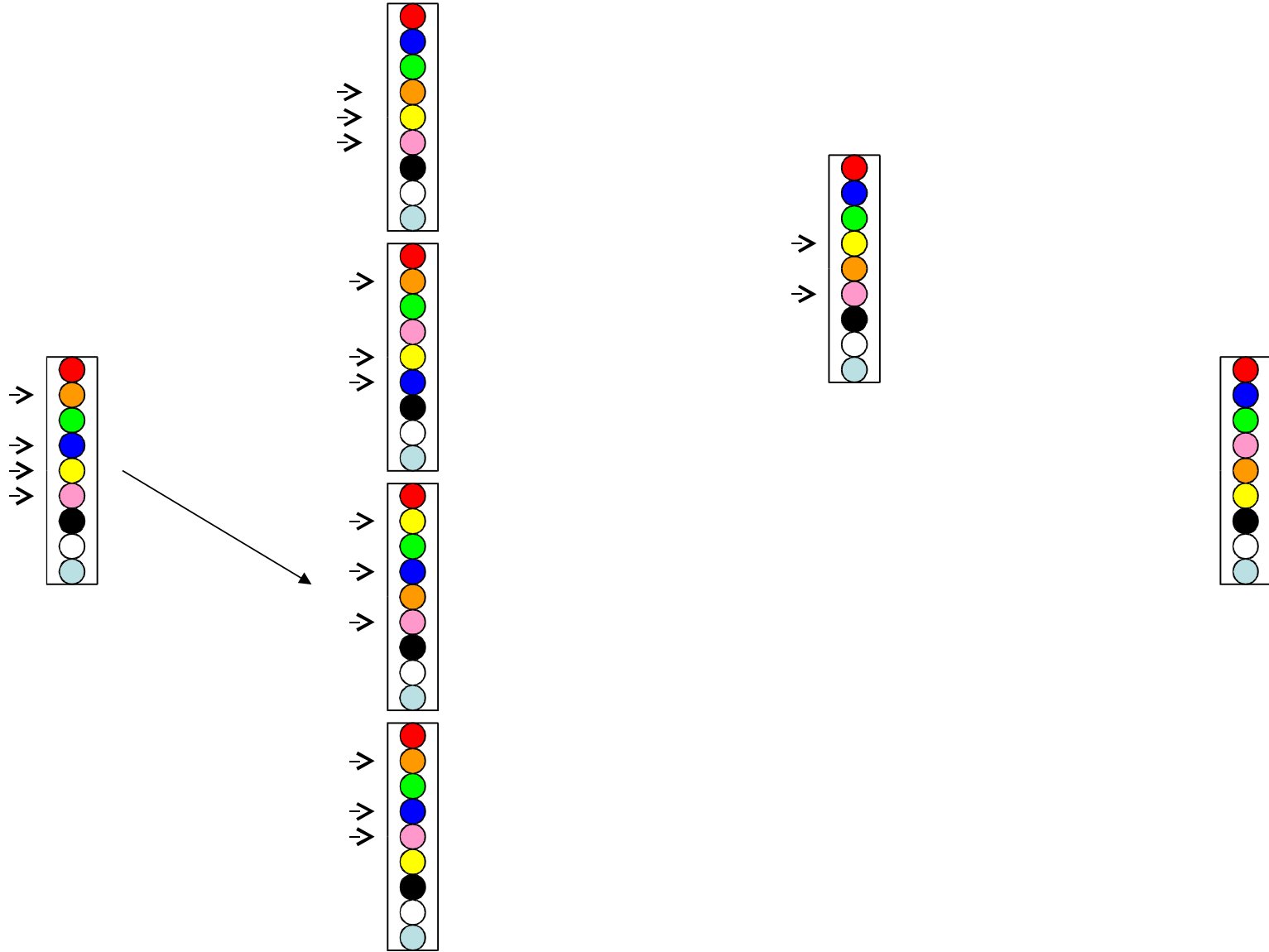


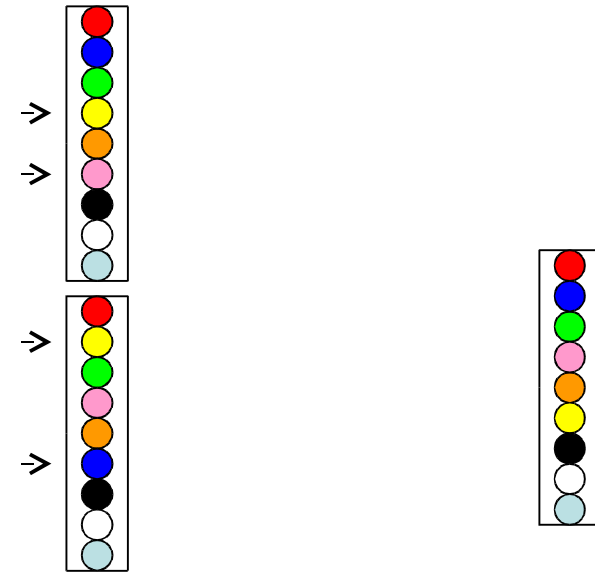
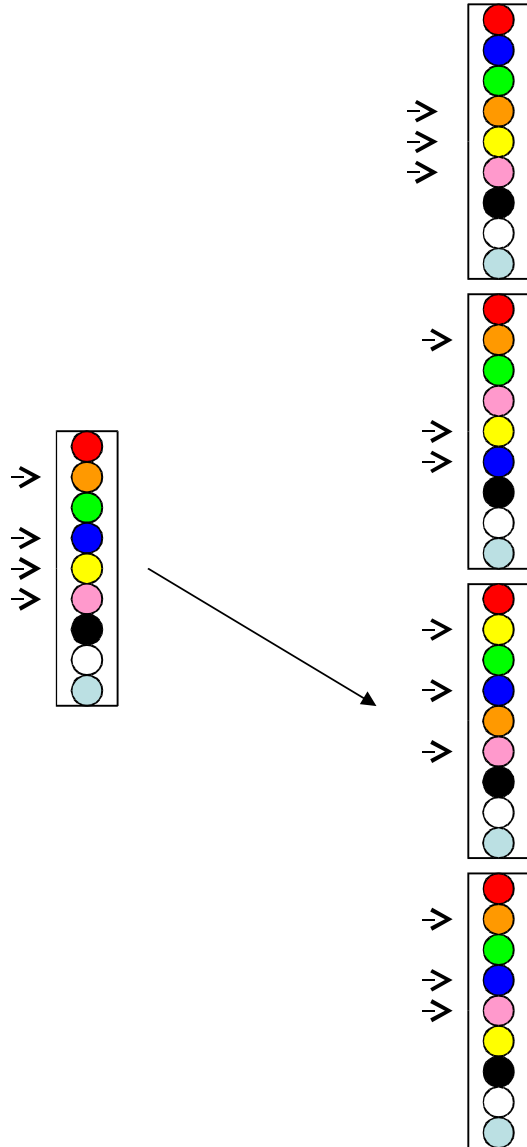


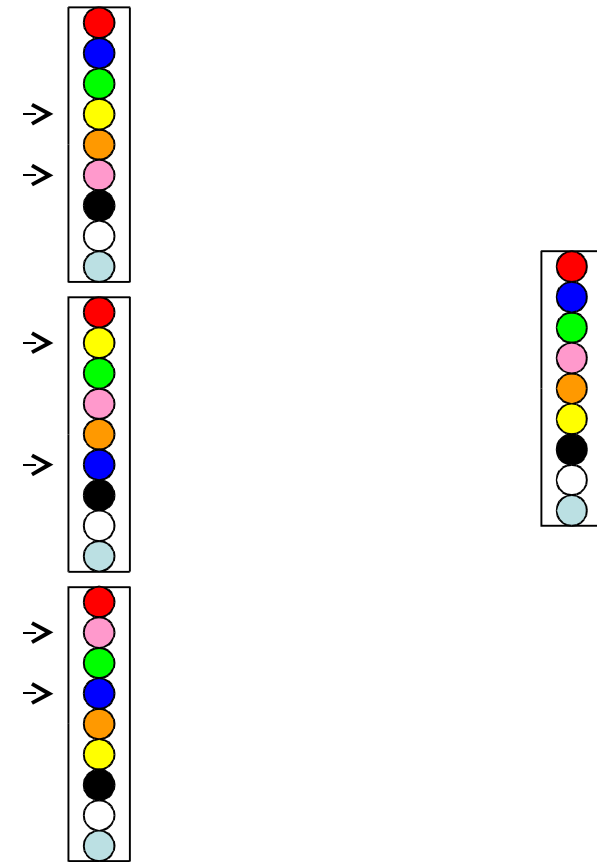
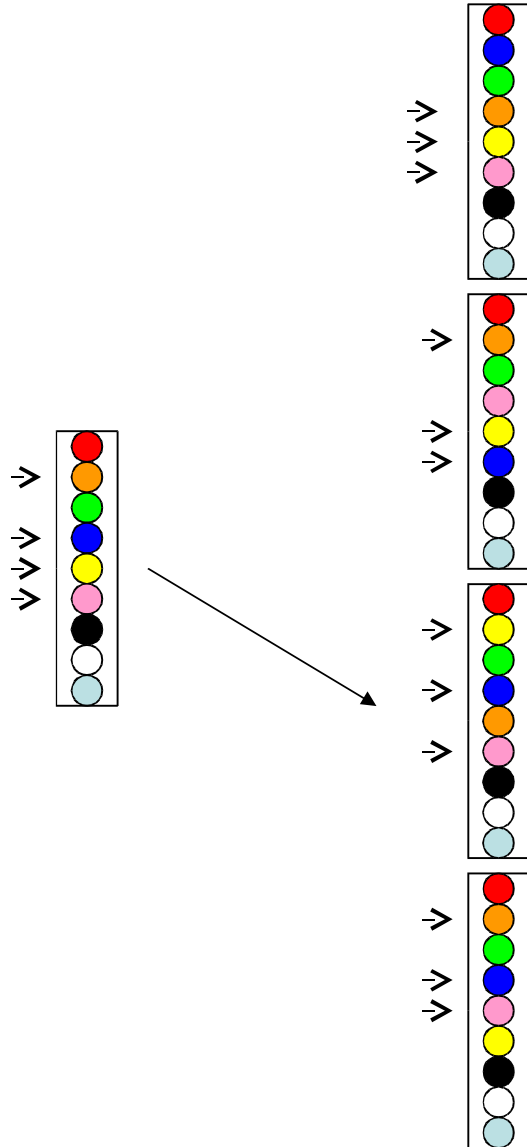


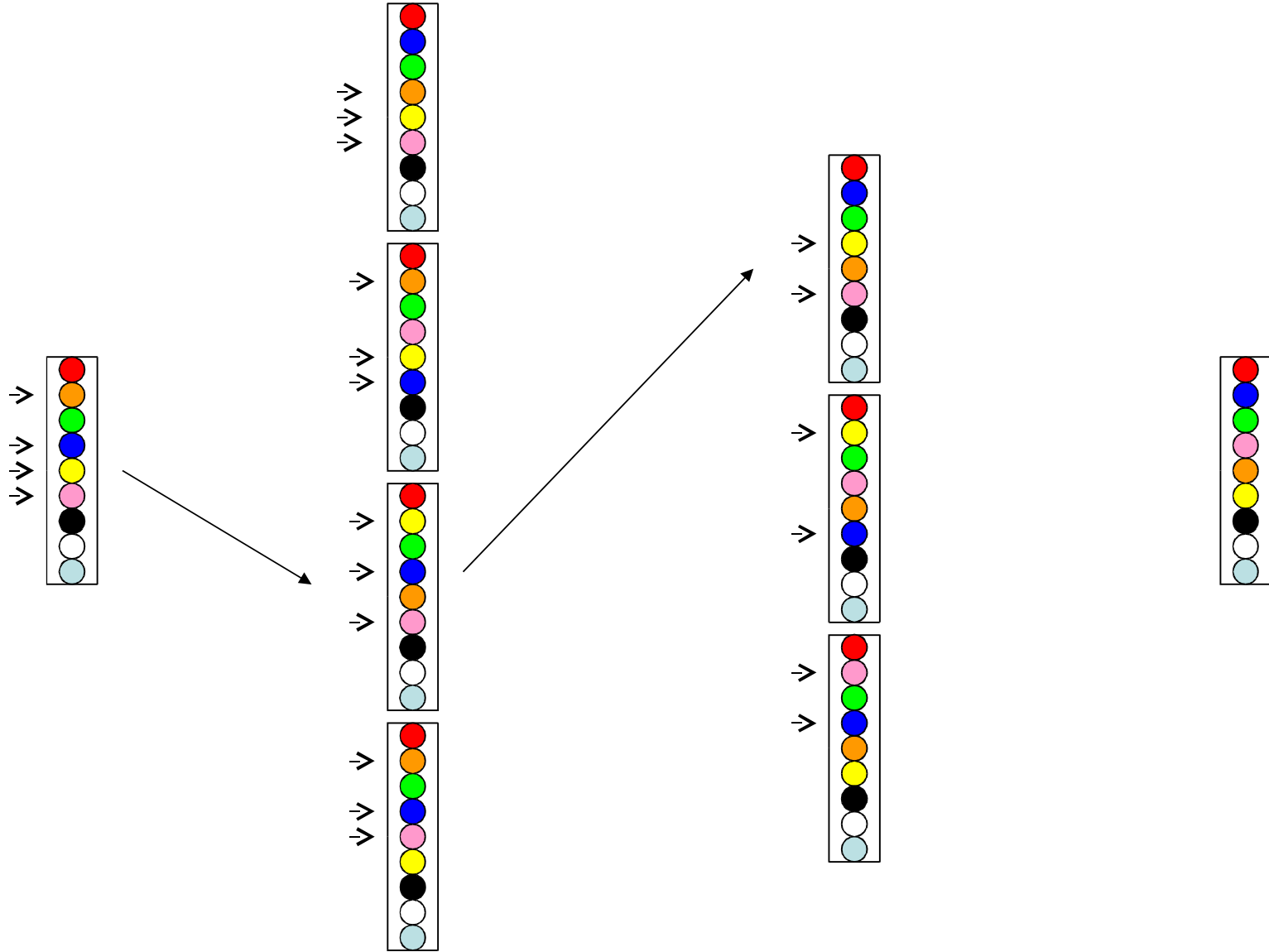




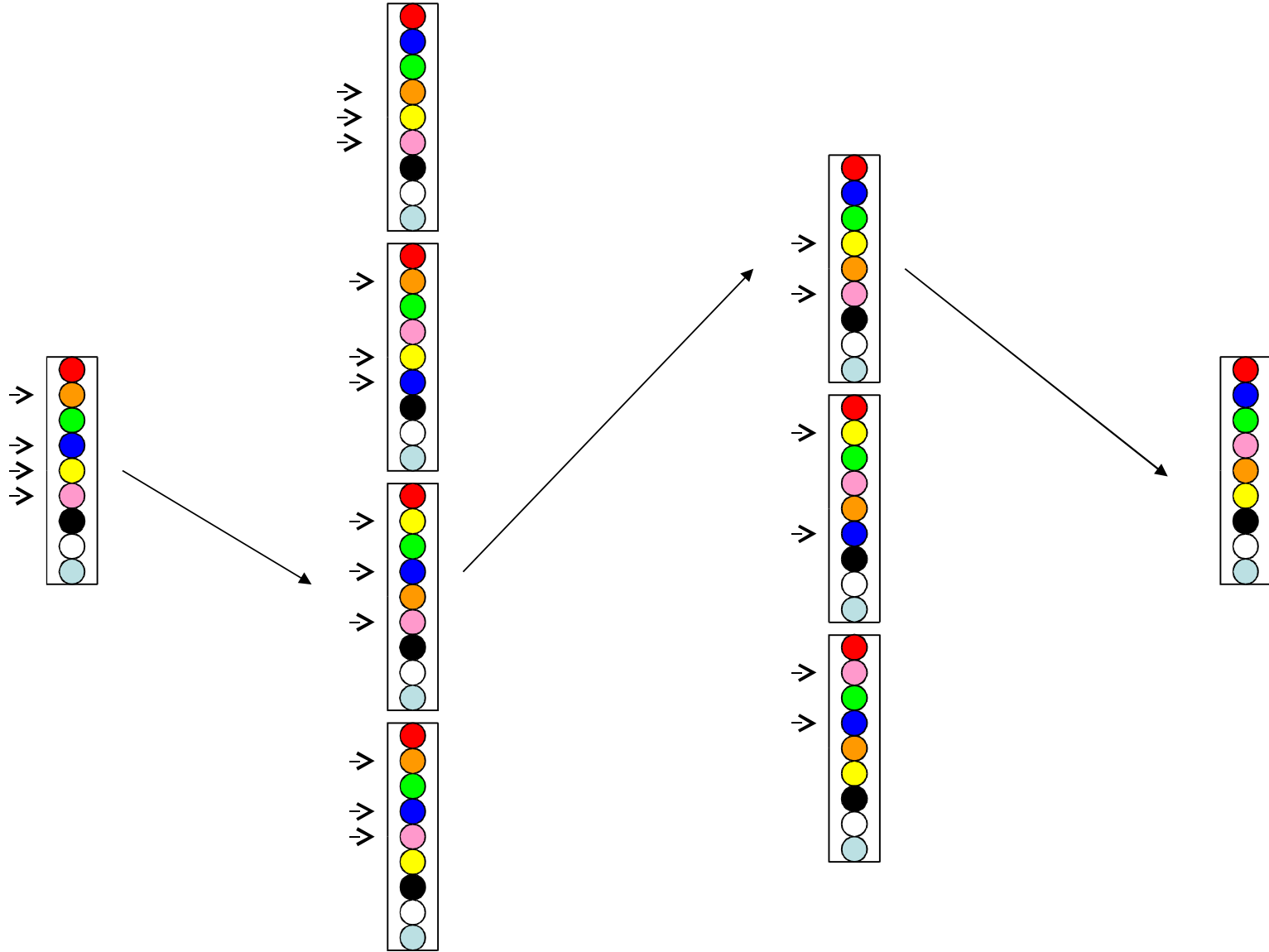








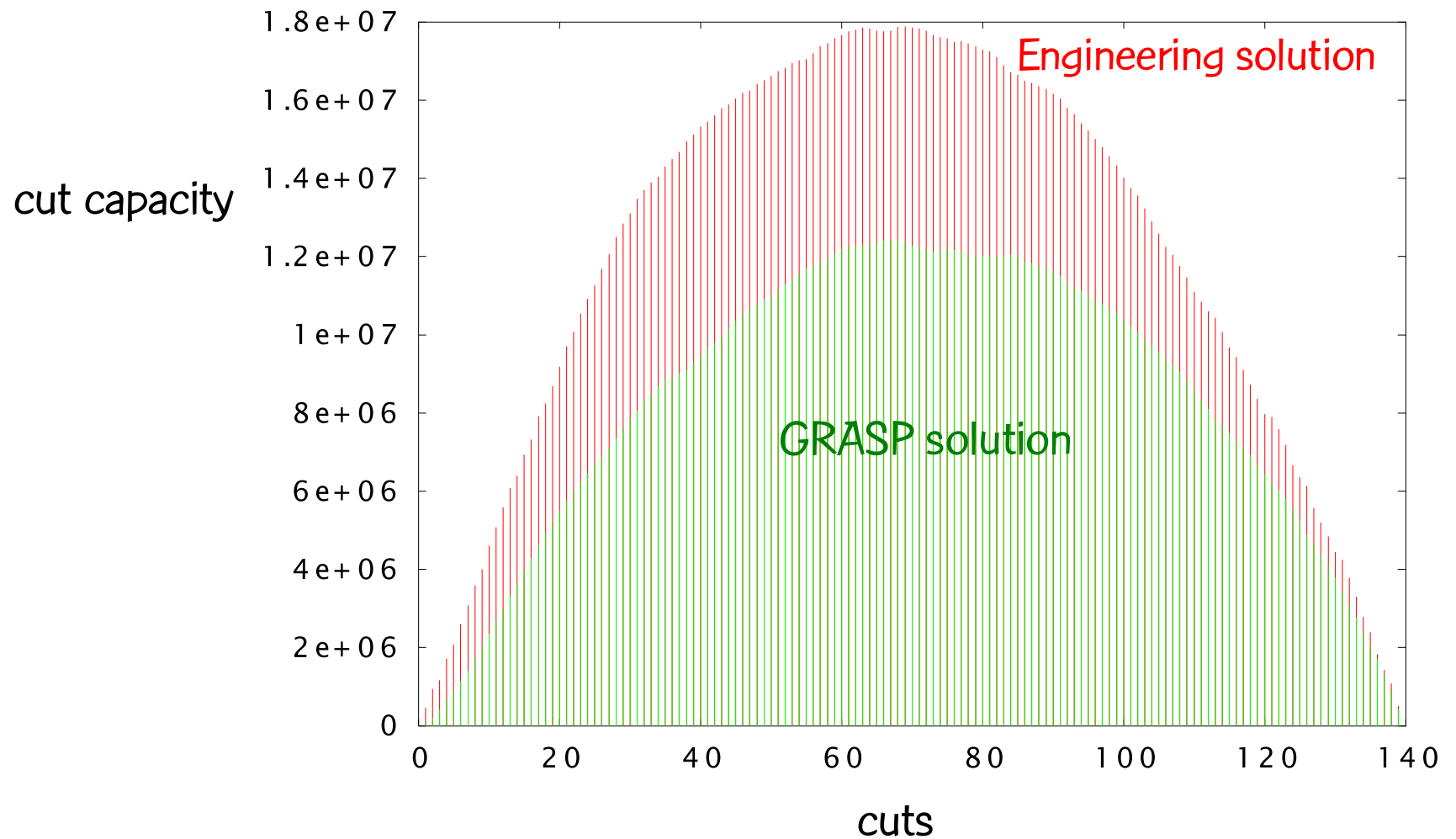




A real-world migration example: 140 nodes, 9730 links

Engineering solution:  $1.78 \times 10^7$

GRASP solution :  $1.23 \times 10^7$  (31% reduction)



# Application 2:

## Modem pool placement for Internet service provider

# Modem pool placement for Internet service provider

- Worldnet: AT&T's Internet Service Provider
- Dial-up: hundreds of points of presence (PoPs)
  - Telephone numbers customers must call when making an Internet connection.
- Current footprint:
  - 1305 PoPs;
  - 77.66% of the telephone numbers in the U.S. can make local calls to Worldnet.

# Footprint Optimization

- In general: more PoPs, better coverage.
- For a fixed coverage, we don't want more PoPs than necessary.
- Not all PoPs are the same:
  - Each has an associated **network cost**:
    - Hourly rate paid by Worldnet to network company.
    - Between \$0.04 and \$0.14 in the continental US.
    - Up to \$0.42 in Hawaii and Alaska.
  - No setup cost.

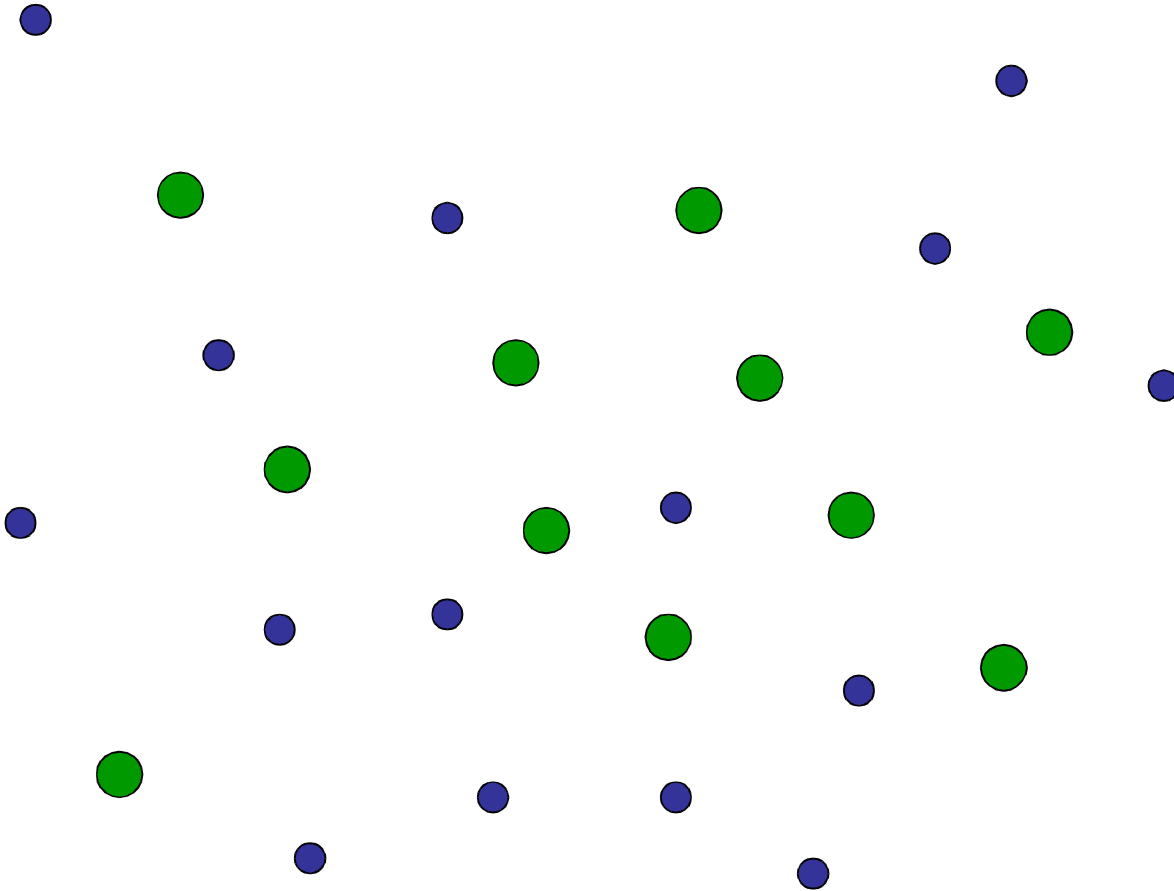
# Worldnet

- When is a call local?
  - Not simply “within same area code”.
  - Telephone system divided into exchanges:
    - Area code + first three digits (973360, for example).
- Each PoP has a coordinate.
- We know which exchanges can make local calls to each coordinate (the coverage).
  - Just a big table;
  - 69,534 exchanges covered by current footprint.
- Goal: keep only cheaper PoPs, preserve coverage.

# Footprint Optimization

- Further improvement:
  - 335 additional coordinates could be eliminated:
    - Only 700 PoPs left;
    - New footprint covers all exchanges currently covered;
    - No exchange has to make a more expensive call.
- How did we do it?
  - We solved this as the p-median problem.

## p-median problem

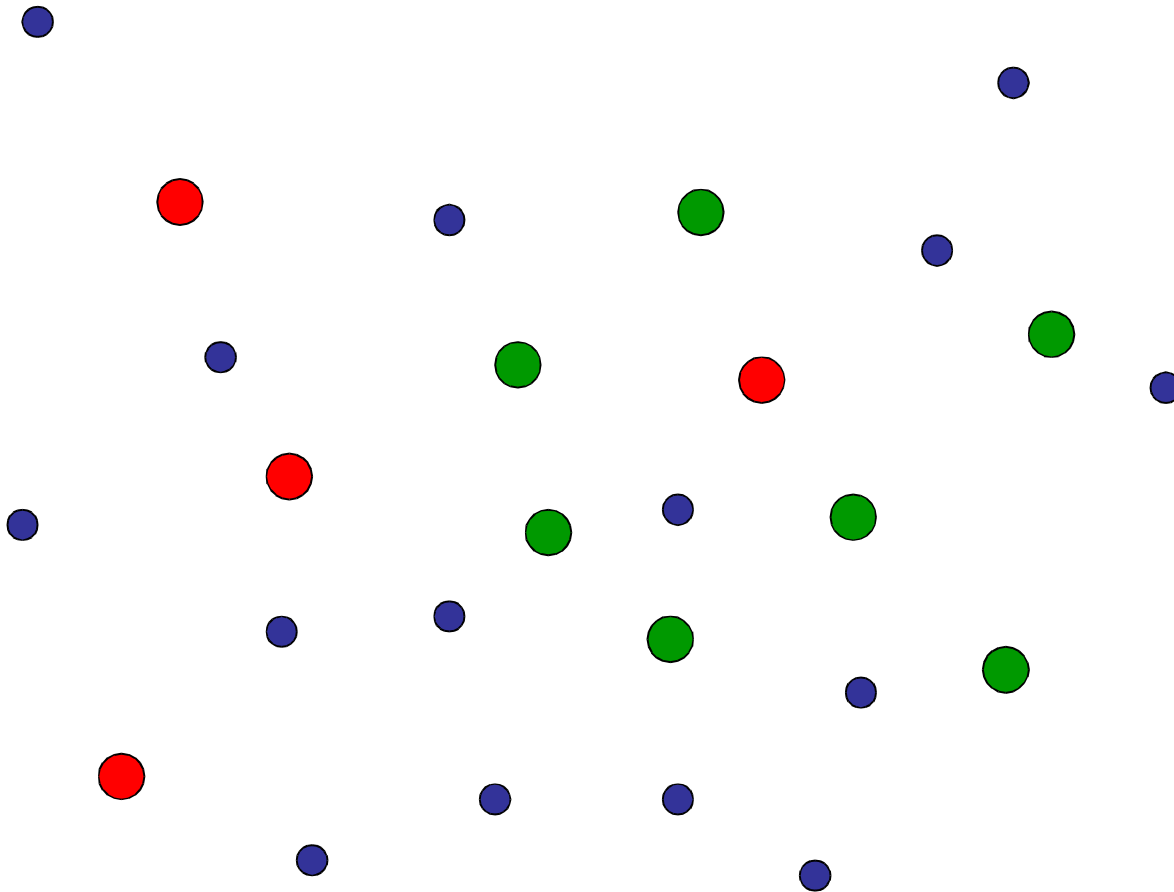


$n (=11)$  potential facility locations

$m (=15)$  users



# p-median problem

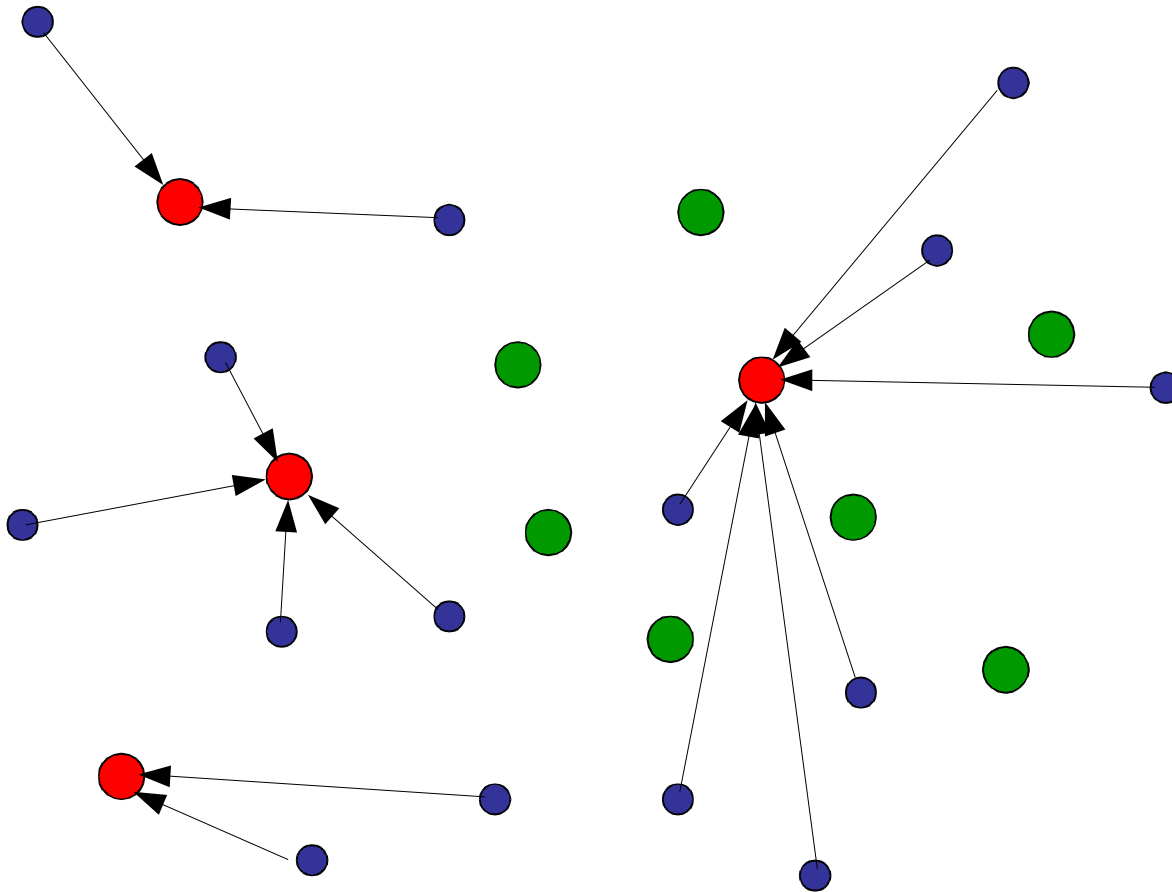


$p = 4$  facilities to be opened

$n (=11)$  potential facility locations

$m (=15)$  users

## p-median problem



Users home into nearest open facility.

$d(u, f)$  = cost of servicing user  $u$   
by facility  $f$

$n$  (=11) potential service locations

$m$  (=15) customers

# Footprint Optimization

- In our case:
  - each exchange is a  $p$ -median user:
    - 69,534 in total (all currently covered).
  - each coordinate is a  $p$ -median facility:
    - 1035 in total (all currently open).
  - Distances: network cost.
    - $(\text{PoP rate}) \cdot (\text{hours used by exchange})$
- With  $p=1035$ , we get the current network cost.
- We want the smallest  $p$  that preserves that cost.
  - Solve the  $p$ -median problem for various values of  $p$  to find best.
  - 700 was the value we found.

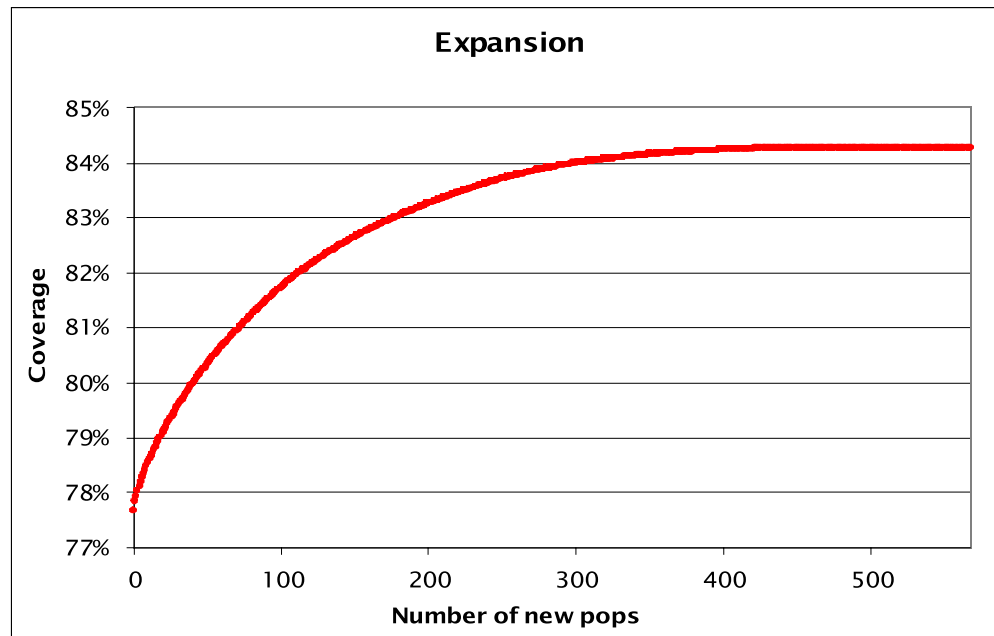
# Expanding the Footprint

- Second problem:
  - Increase coverage beyond 77.66%.
- AT&T can use UUNet PoPs:
  - 1,498 candidate PoPs.
  - 568 of those cover at least one new exchange.
- Main question:
  - If we want to open  $p$  new PoPs, which PoPs do we open?
    - Goal: maximize coverage.
- This is the maximum cover problem:
  - It can be solved as a  $p$ -median problem.

# From maximum cover to p-median

- Idea: minimize number of customers not covered.
  - Users:
    - exchanges not currently covered.
  - Facilities:
    - all candidate UUNet PoPs;
    - dummy facility  $f_0$ .
  - Distances:
    - $d(u, f_i) = 0$ , if PoP  $i$  covers exchange  $u$ .
      - if  $u$  is covered, does not contribute to solution.
    - $d(u, f_0) = (\# \text{ of customers in exchange } u)$ ;
    - $d(u, f_i) = \infty$ , if PoP  $i$  does not cover  $u$ .
      - $u$  not covered: assigned to  $f_0$ , contributes to solution.
  - A dummy user can be used to ensure that  $f_0$  will always belong to the solution.

# Expansion



Coverage	Footprint
77.66%	current
78%	current+3
79%	current+19
80%	current+41
81%	current+72
82%	current+113
83%	current+177
84%	current+301
84.27%	current+464

# Papers

- M.G.C. Resende, "Computing approximate solutions of the maximum covering problem using GRASP," *J. of Heuristics*, vol. 4, pp. 161-171, 1998.
- M.G.C. Resende & R.F. Werneck, "A hybrid heuristic for the p-median problem," *J. of Heuristics*, vol. 10, pp. 59-88, 2004.
- M.G.C. Resende & R.F. Werneck, "A fast swap-based local search procedure for location problems," to appear in *Annals of Operations Research*, 2005.

# Application 3:

## Local access network design



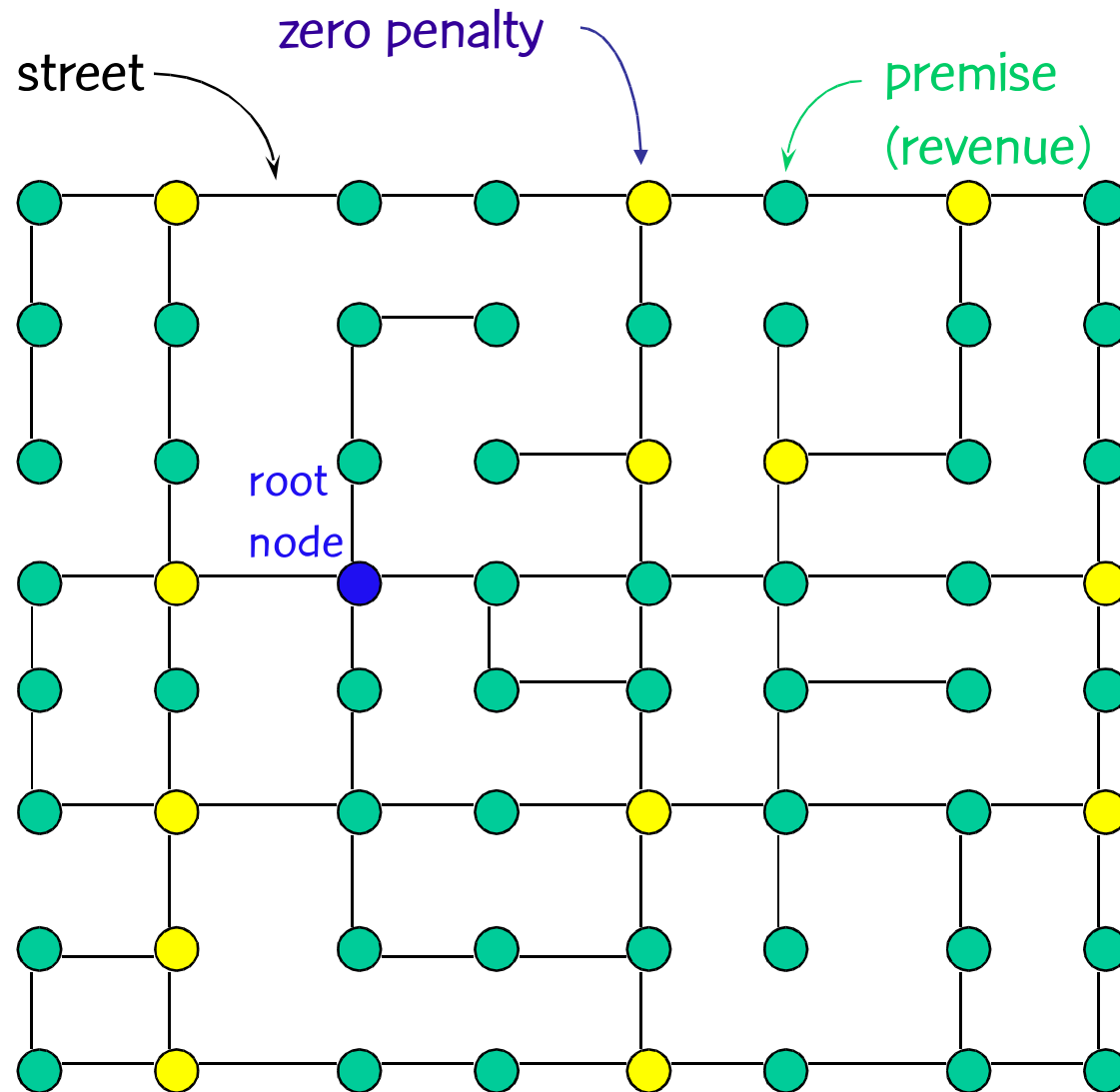
# Local access network design

- Build a fiber-optic network for providing broadband connections to business and residential customers.
- Design a local access network taking into account tradeoff between:
  - cost of network
  - revenue potential of network

# Local access network design

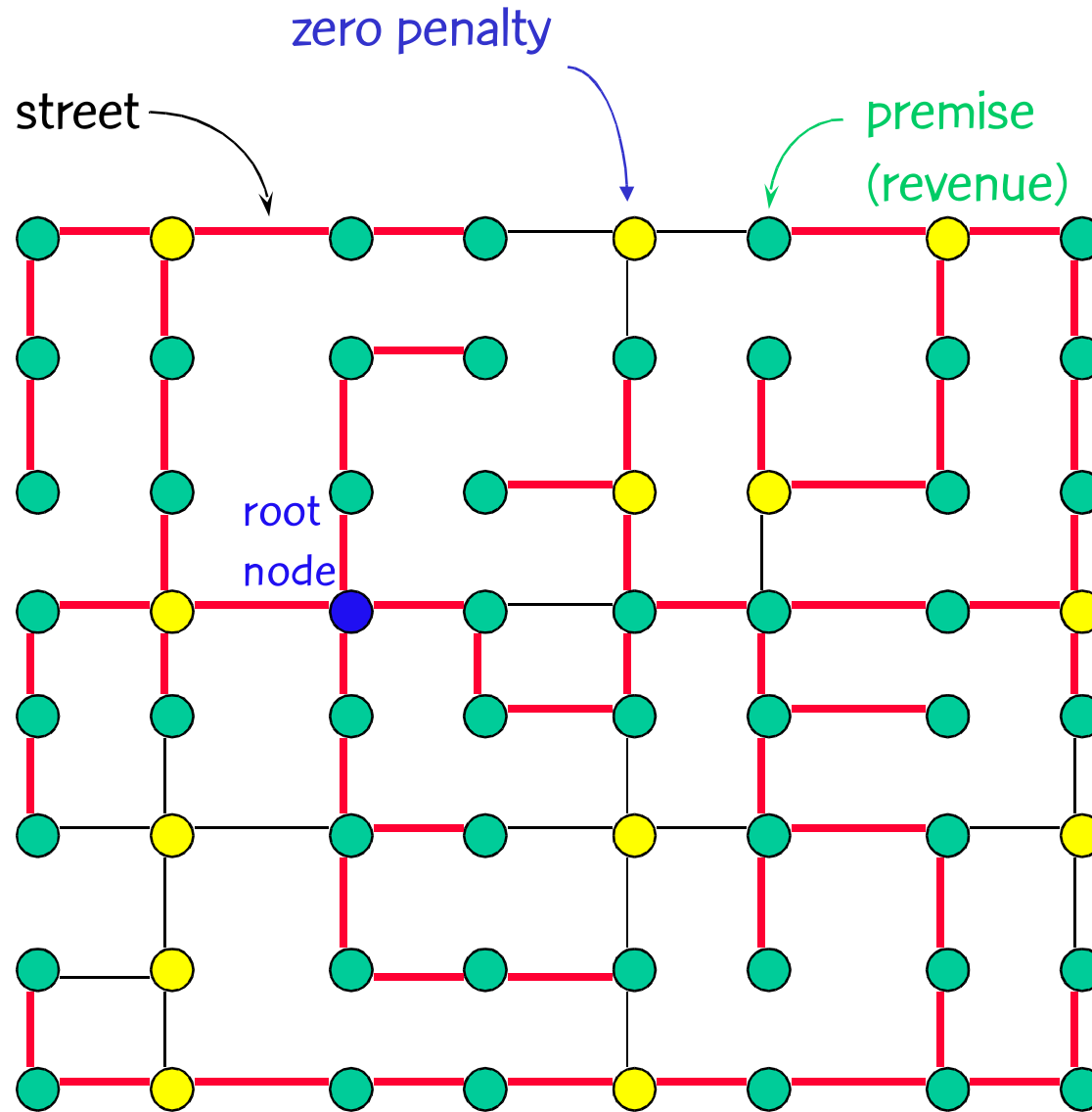
- Graph corresponds to local street map
  - Edges: street segments
    - Edge cost: cost of laying the fiber on the corresponding street segment
  - Vertices: street intersections and potential customer premises
    - Vertex penalty: estimate of potential loss of revenue if the customer were not to be serviced (intersection nodes have no penalty)

# Local access network design



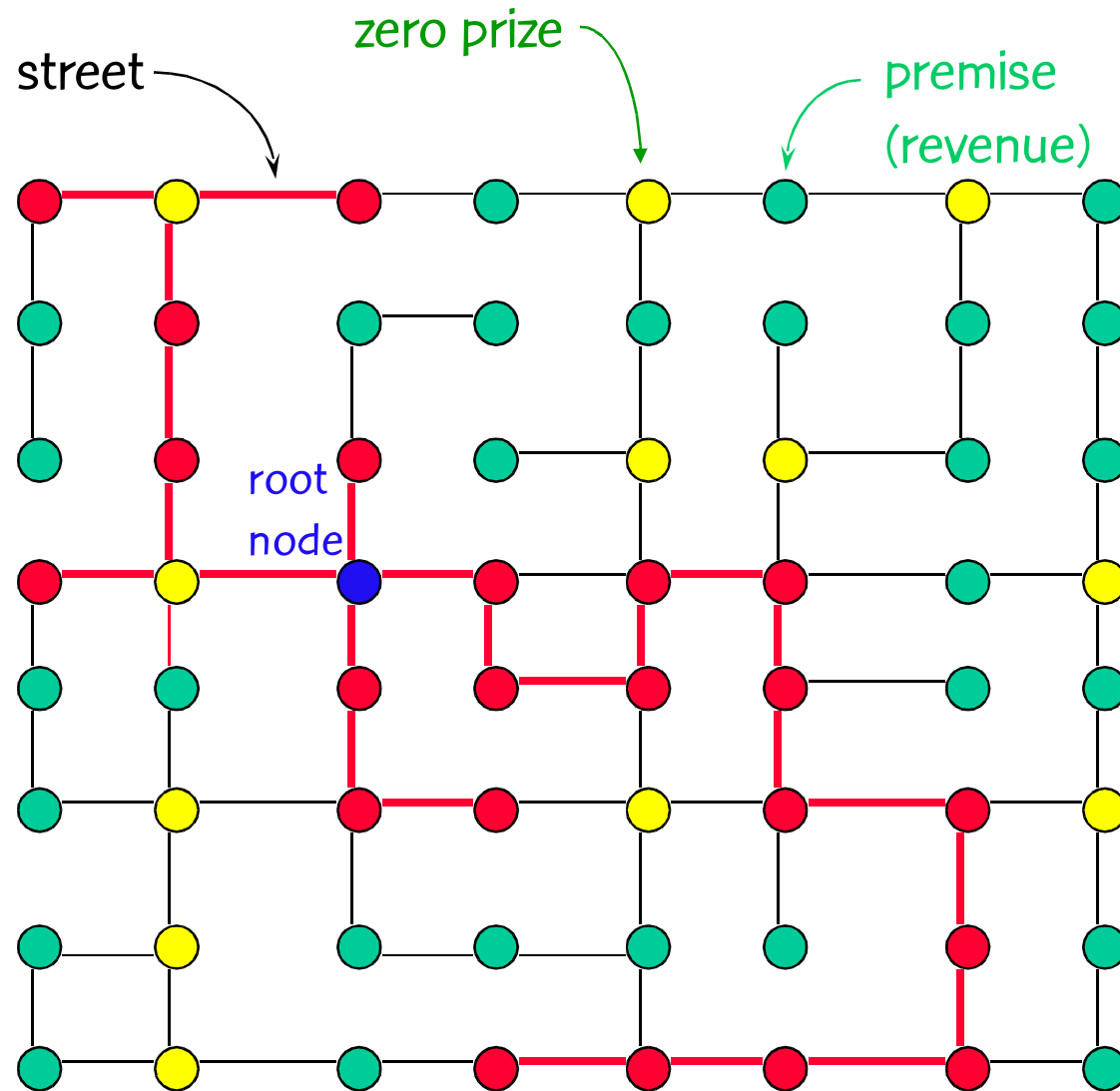
# Collect all prizes

(Steiner problem in graphs)



# Collect some prizes

(Prize collecting Steiner Problem in Graphs)



# Multi-start heuristic

S. Canuto, M.G.C. Resende, & C.C. Ribeiro, "Local search with perturbations for the prize-collecting Steiner tree problem in graphs," Networks, vol. 38, pp. 50-58, 2001

- Repeat:
  - Perturb problem data and solve using approximation algorithm of Goemans and Williamson (1996);
  - If solution is new, perform swap-based local search;
  - Attempt to insert solution into POOL;
  - Select solution at random from POOL and explore path from current iterate and POOL solution using path-relinking;
- Starting from best POOL solution, apply variable neighborhood search;

# A cutting planes algorithm: Lower bound

A. Lucena & M.G.C. Resende, "Strong lower bounds for the prize collecting Steiner tree problem in graphs," Discrete Applied Mathematics, vol. 141, pp. 277-294, 2004.

- Integer programming (IP) formulation
- Cutting planes algorithm to solve linear programming relaxation of IP

# Computational results

- 114 test problems
  - Smallest instance: 100 nodes & 284 edges
  - Largest instance: 1000 nodes & 25,000 edges
  - Three classes:
    - Johnson, Minkoff, & Phillips (1999) P & K problems
    - Steiner C problems (derived from SPG Steiner C test problems in OR-Library)
    - Steiner D problems (derived from SPG Steiner D test problems in OR-Library)



# Computational results:

## Cutting planes algorithm

- Found optimal LP solutions in 97 of the 114 test problems (85%)
- Found tight lower bounds (equal to best known upper bounds) in 104 instances (91%)
- Of the 97 optimal LP solutions, 94 were integral. Each of the 3 fractional solutions was off of the best known upper bound by less than  $\frac{1}{2}$
- On the 12 instances for which tight lower bounds were not produced, the bounds produced had at most a 1.3% deviation from the best known upper bounds
- In 13 of the 114 instances, single vertex optima were found
- In 7 instances the algorithm took over 100,000 seconds to converge to a lower bound. The longest run took over 10 CPU days.

## Computational results:

heuristic upper bounds

- Heuristic found
  - 89 of 104 known optimal values (86%)
  - solution within 1% of lower bound for 104 of 114 problems

## Number of optima found with each additional heuristic

Type	num	GW	+LS	+PR+VNS		tot
C	38	6	2	25	3	36
D	32	5	6	10	4	25
JMP	34	8	6	12	2	28
	104					89

# Computational results:

## heuristic upper bounds

Number of instances with given relative error

heuristic	< 1%	< 5%	< 10%	max (%)
GW	7	22	29	36.4
+LS	17	34	37	11.1
+PR	35	38	40	9.1
+VNS	38	40	40	1.1

Problem type Steiner C

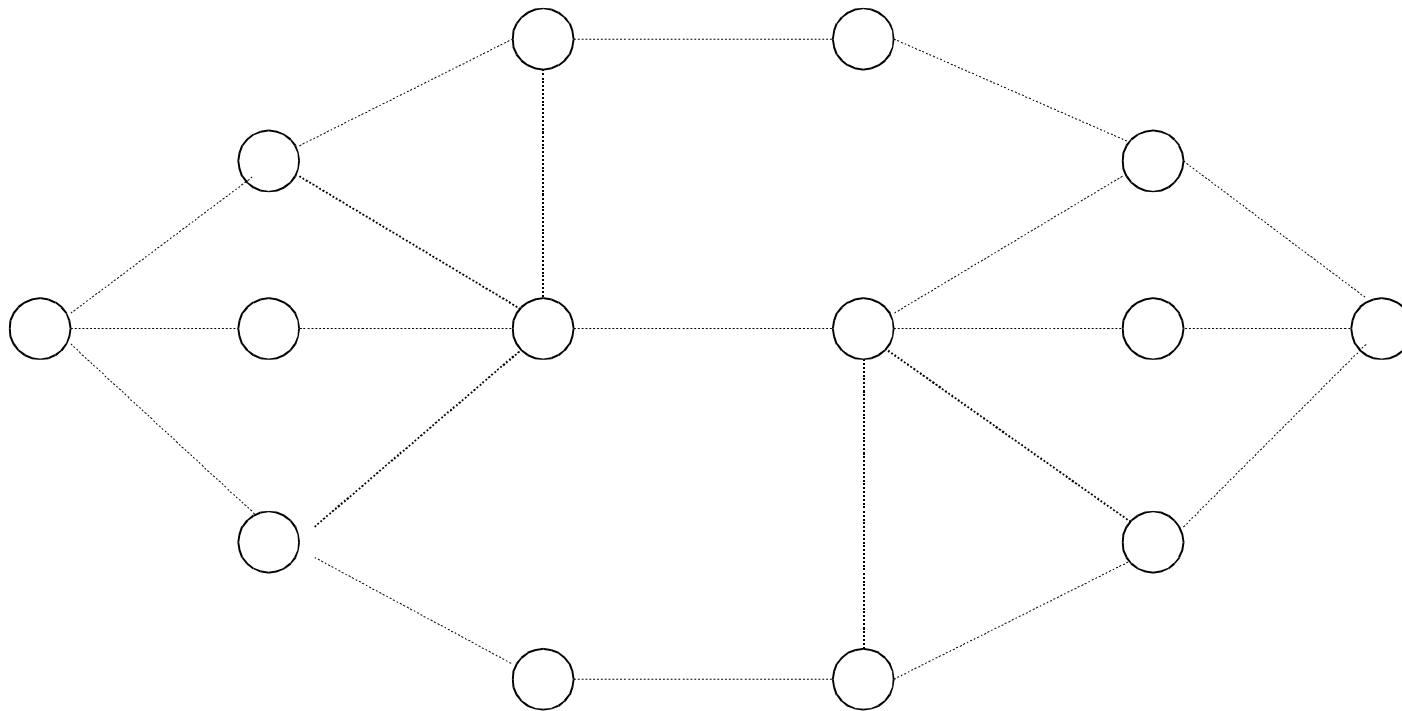
Application 4:

Traffic routing on a virtual  
private network

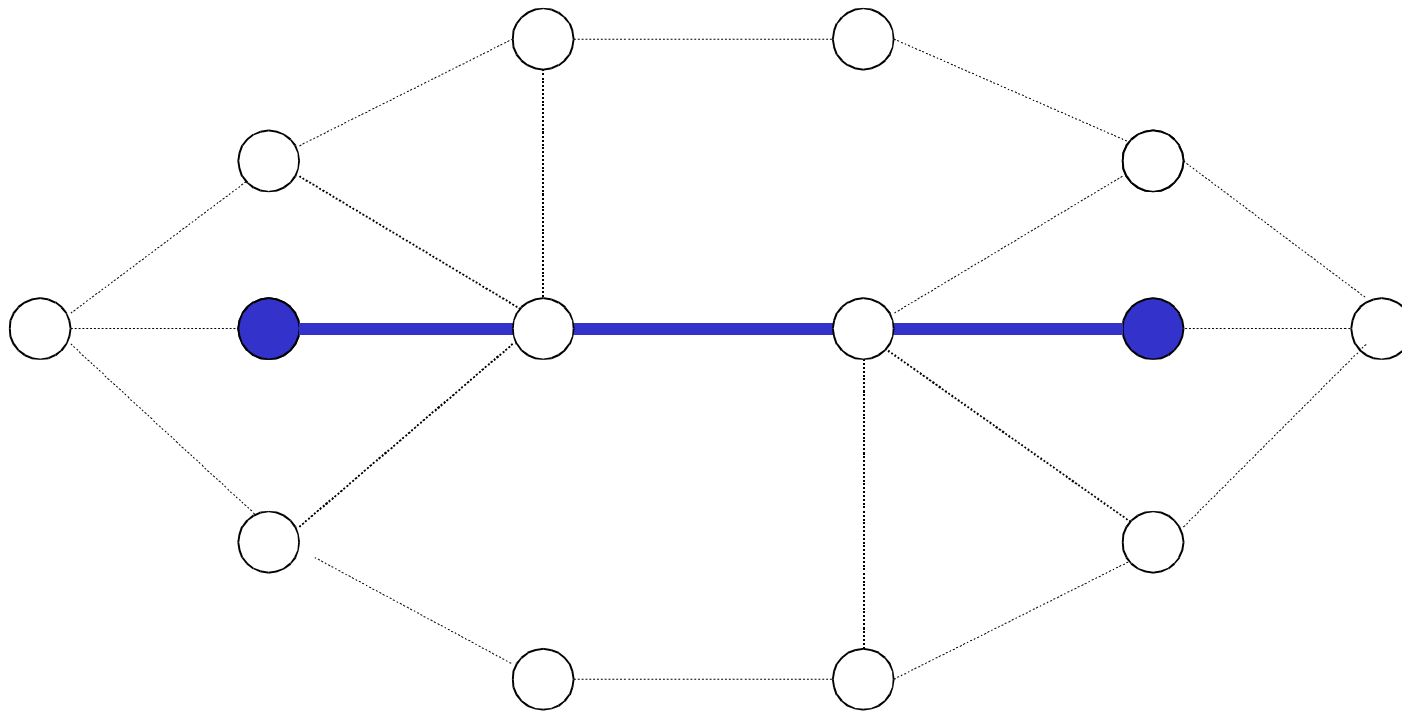
# Traffic routing on a virtual private network

- Frame relay service offers virtual private networks to customers by providing long-term private virtual circuits (PVCs) between customer endpoints on a backbone network.
- Routing is done either automatically by switch or by the network designer without any knowledge of future requests.
- Over time, these decisions cause inefficiencies in the network and occasionally offline rerouting (grooming) of the PVCs is needed:
  - integer multicommodity network flow problem: Resende & Ribeiro (2003)

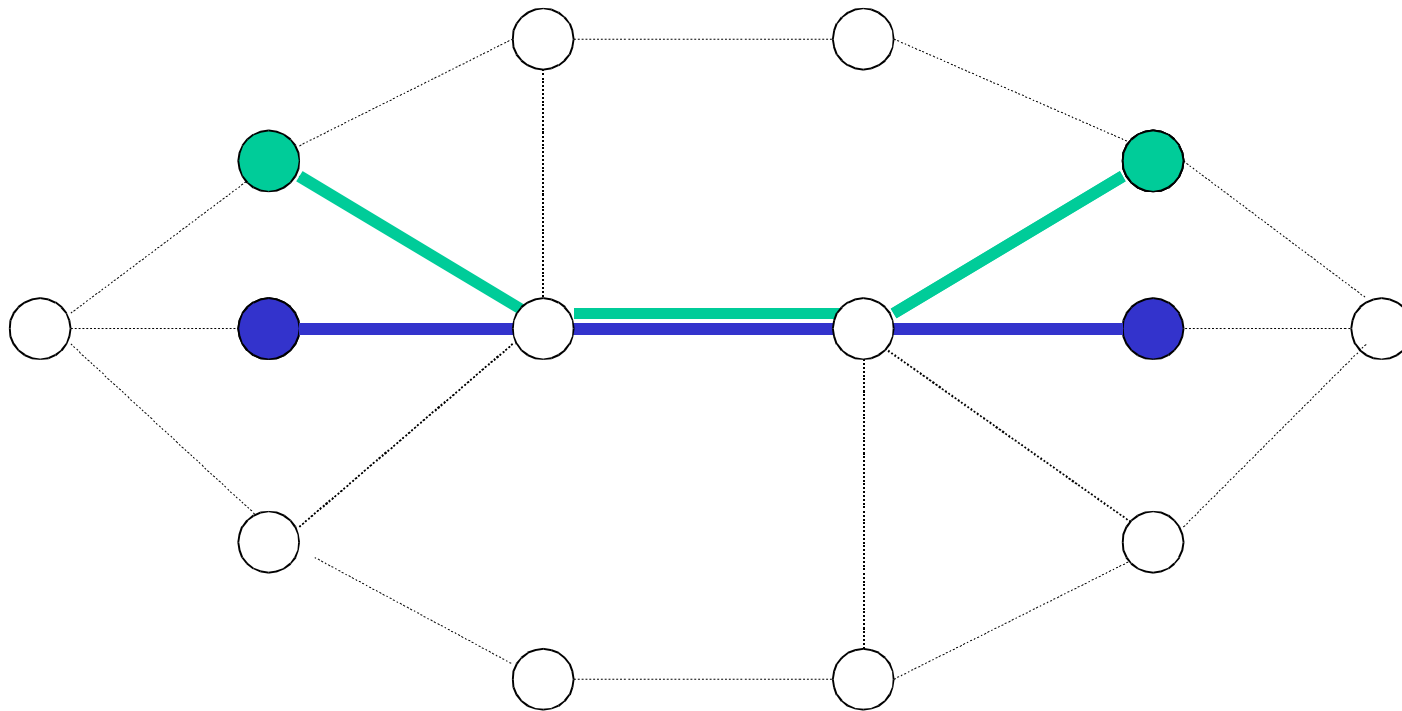
## Traffic routing on a virtual private network



## Traffic routing on a virtual private network

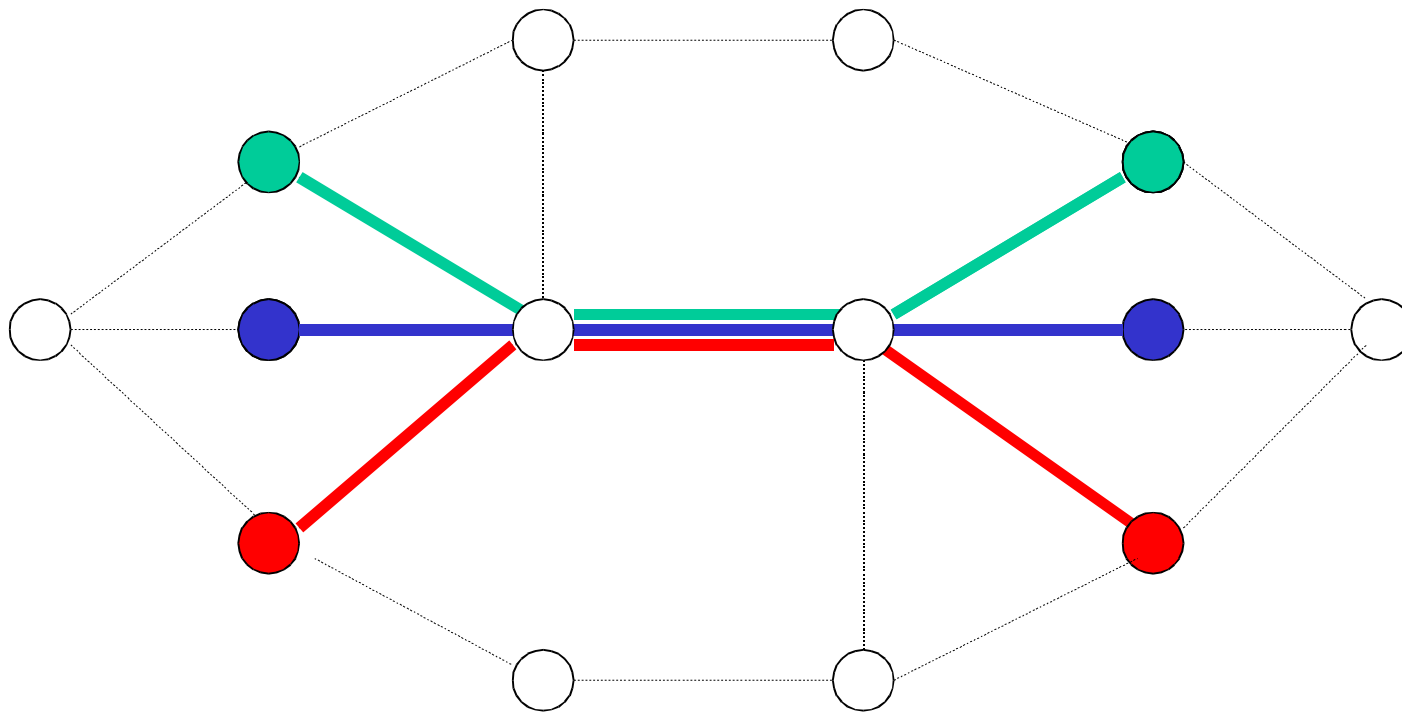


## Traffic routing on a virtual private network

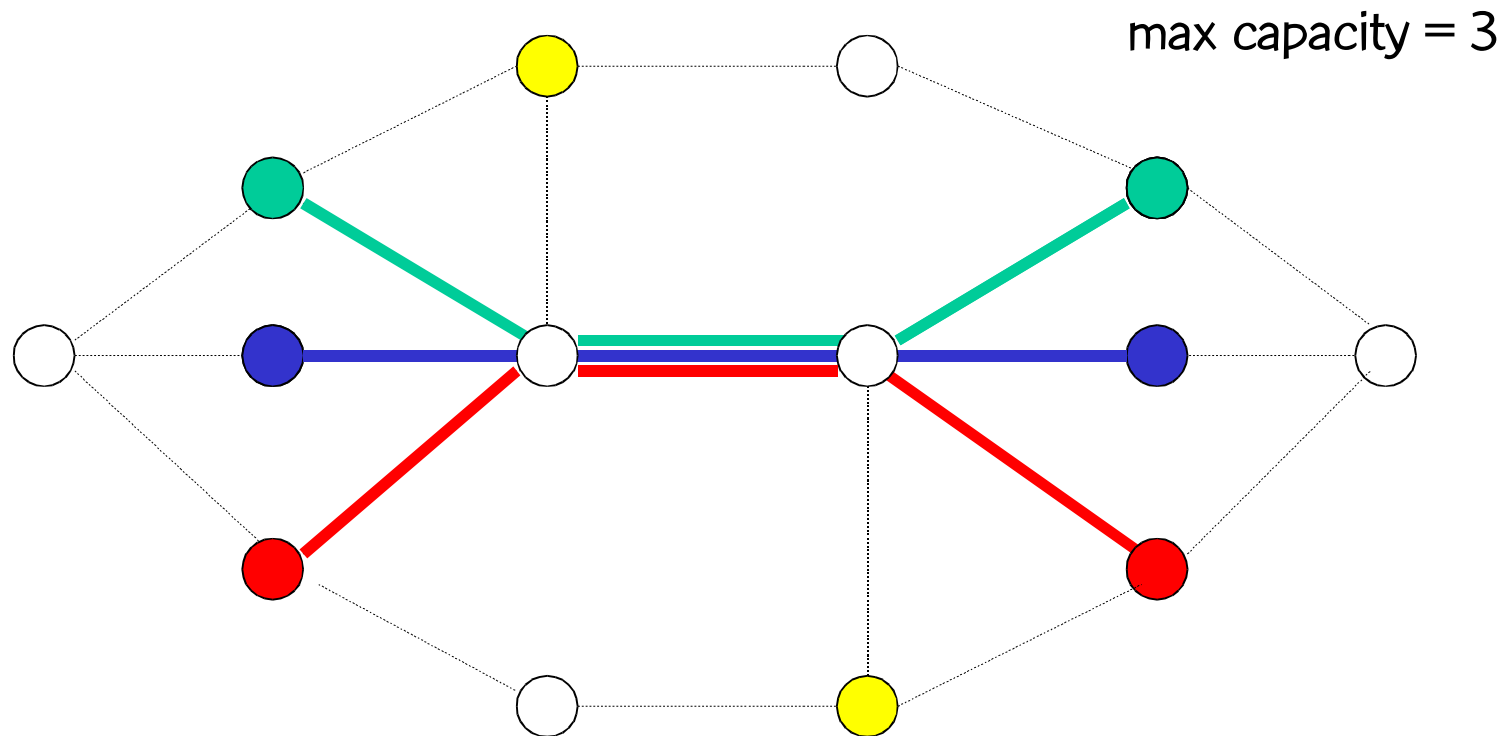




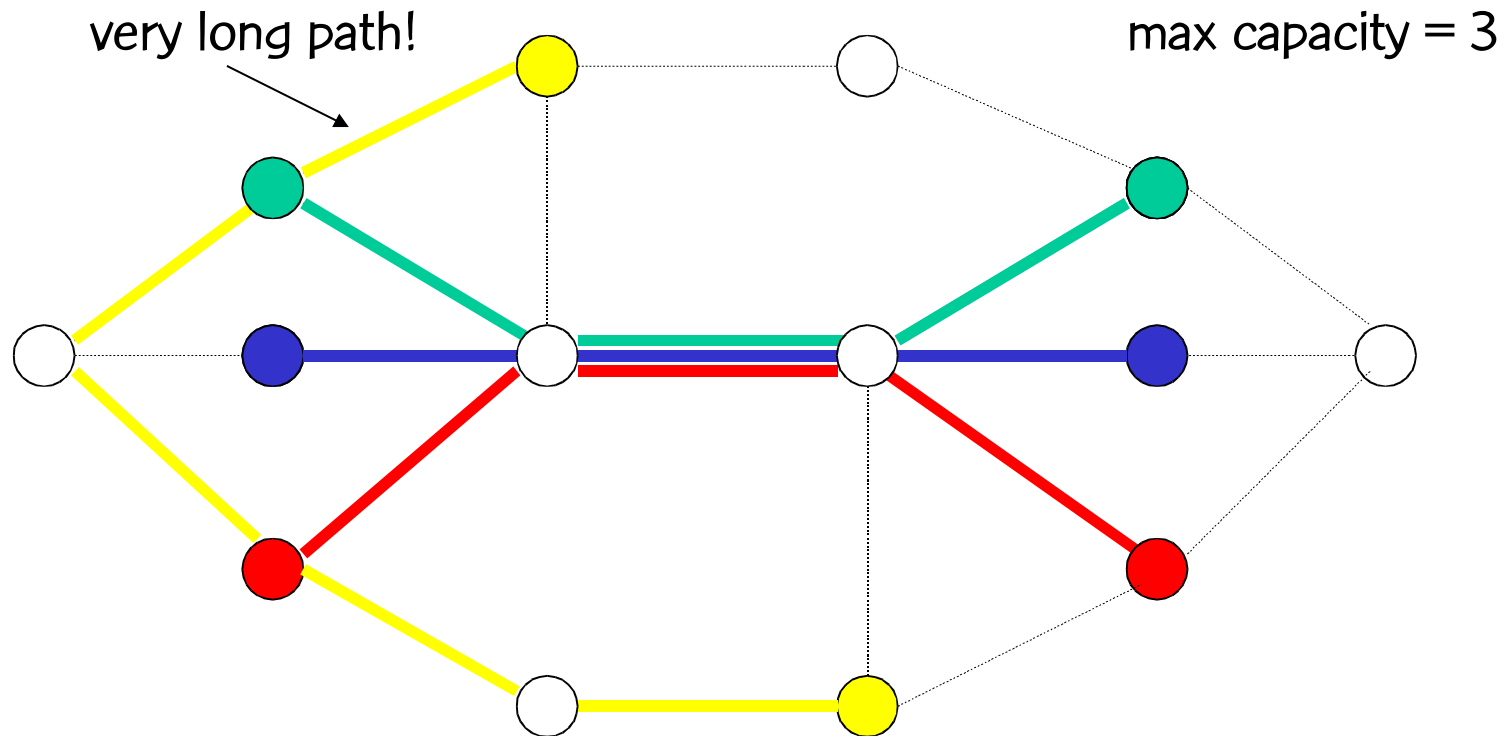
## Traffic routing on a virtual private network



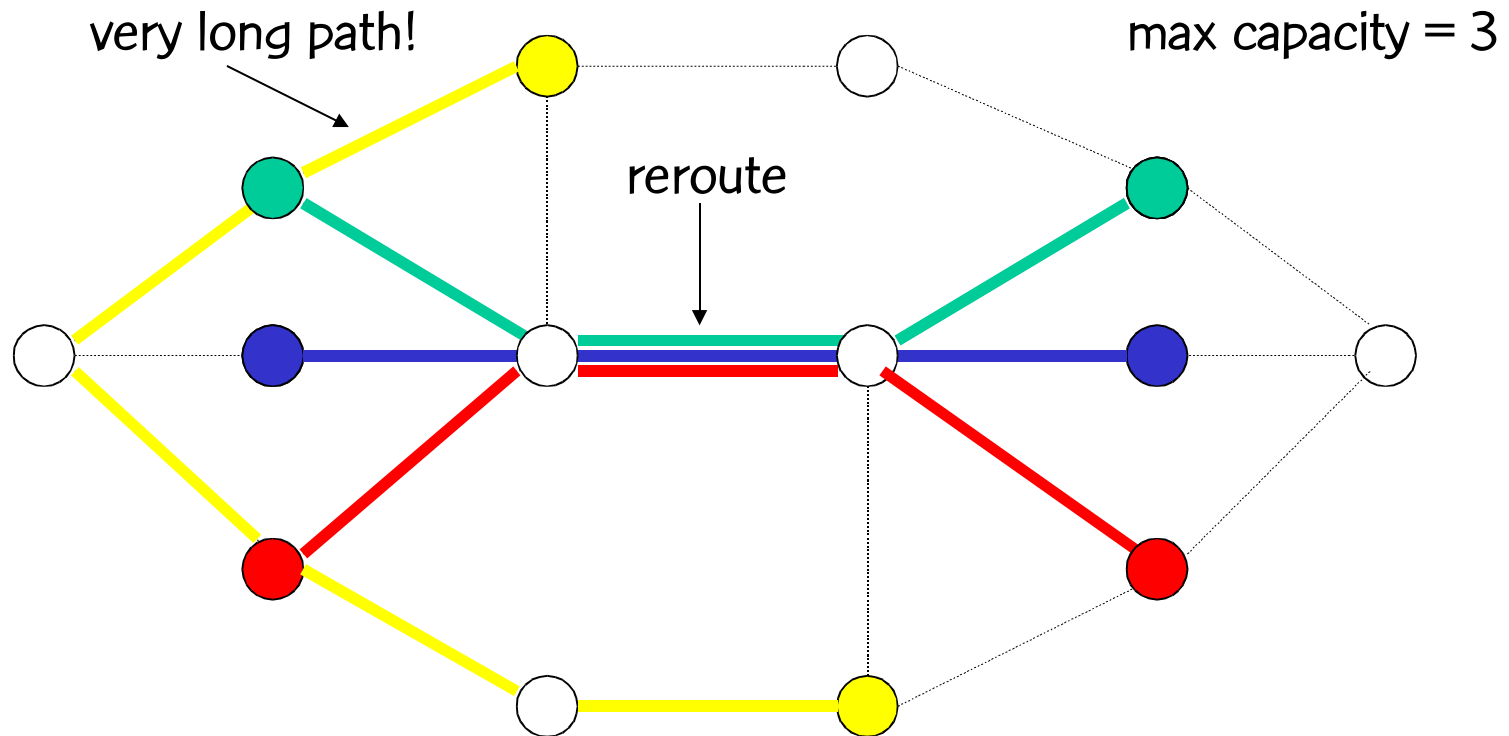
## Traffic routing on a virtual private network



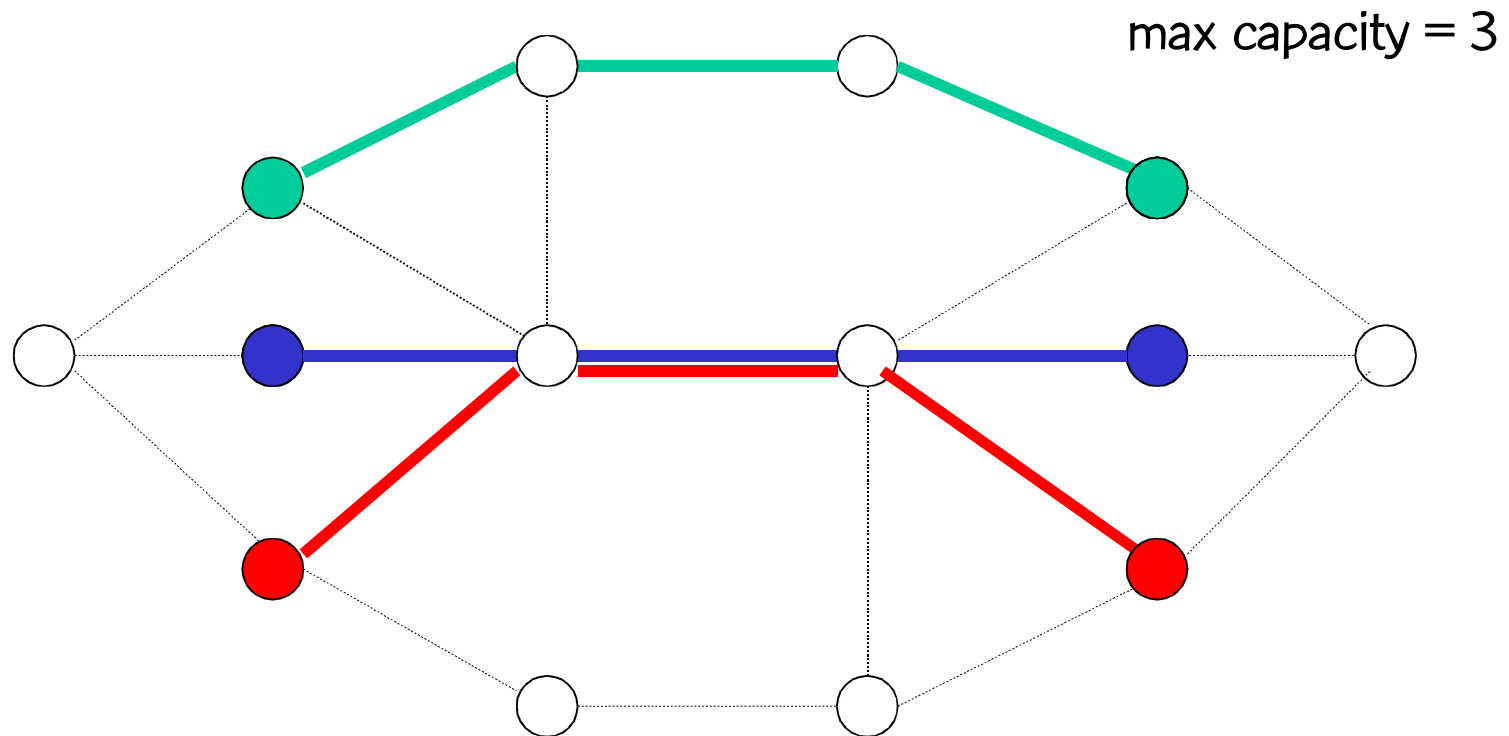
# Traffic routing on a virtual private network



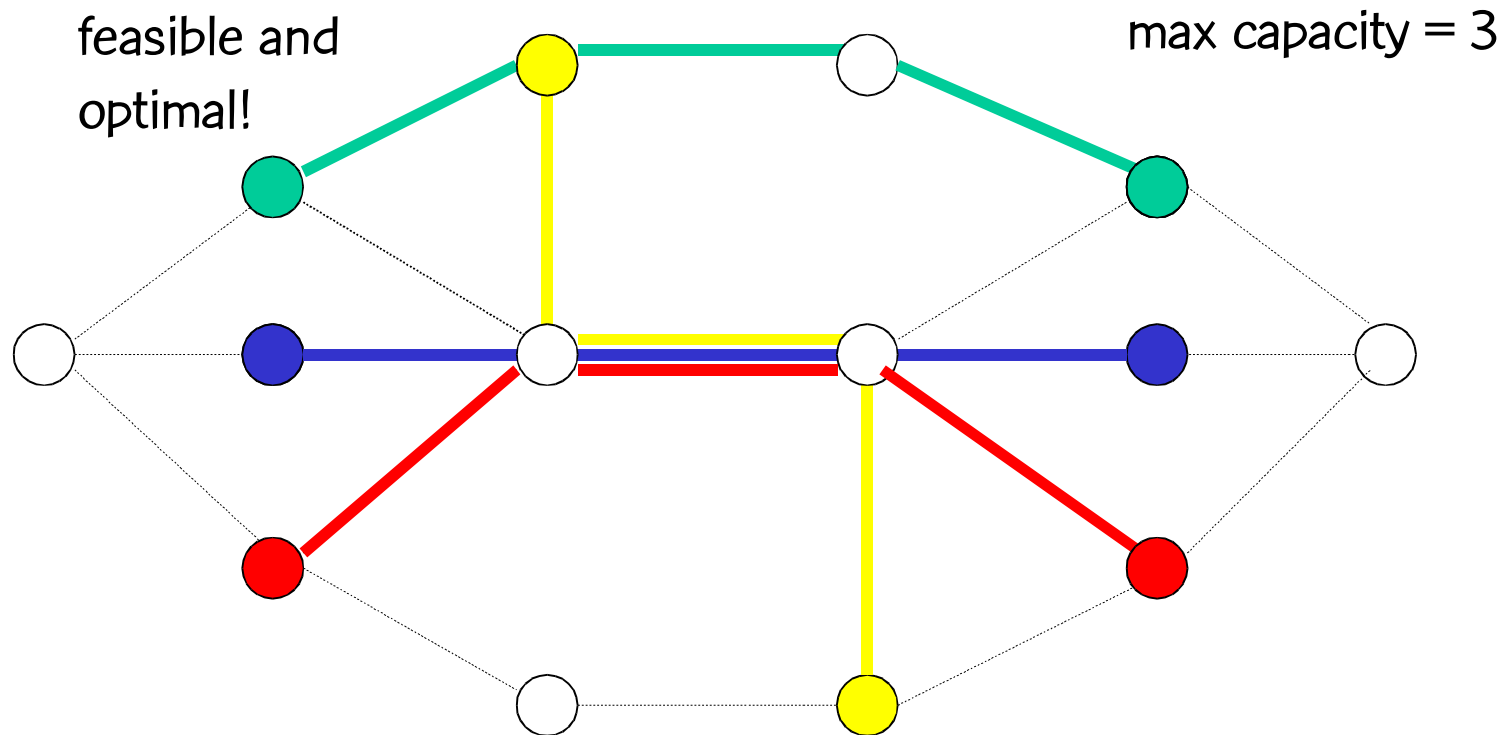
## Traffic routing on a virtual private network



## Traffic routing on a virtual private network



## Traffic routing on a virtual private network



# Papers

- M.G.C. Resende & C.C. Ribeiro, "A GRASP with path-relinking for private virtual circuit routing," *Networks*, vol. 41, pp. 104-114, 2003.
- M.G.C. Resende & C.C. Ribeiro, "GRASP with path-relinking: Recent advances and applications," in "Metaheuristics: Progress as Real Problem Solvers," Ibaraki, Nonobe and Yagiura, (Eds.), Springer, 2005.

# Application 5:

## Internet traffic engineering



# Internet traffic engineering

- Internet traffic has been doubling each year [Coffman & Odlyzko, 2001]
- In the 1995-96 period, there was a doubling of traffic each three months!
  - Web browsers were introduced.
- Increasingly heavy traffic (due to video, voice, etc.) will raise the requirements of the Internet of tomorrow.

# Internet traffic engineering

- **Objective:** make more efficient use of existing network resources.
- **Routing** of traffic can have a major impact on efficiency of network resource utilization.

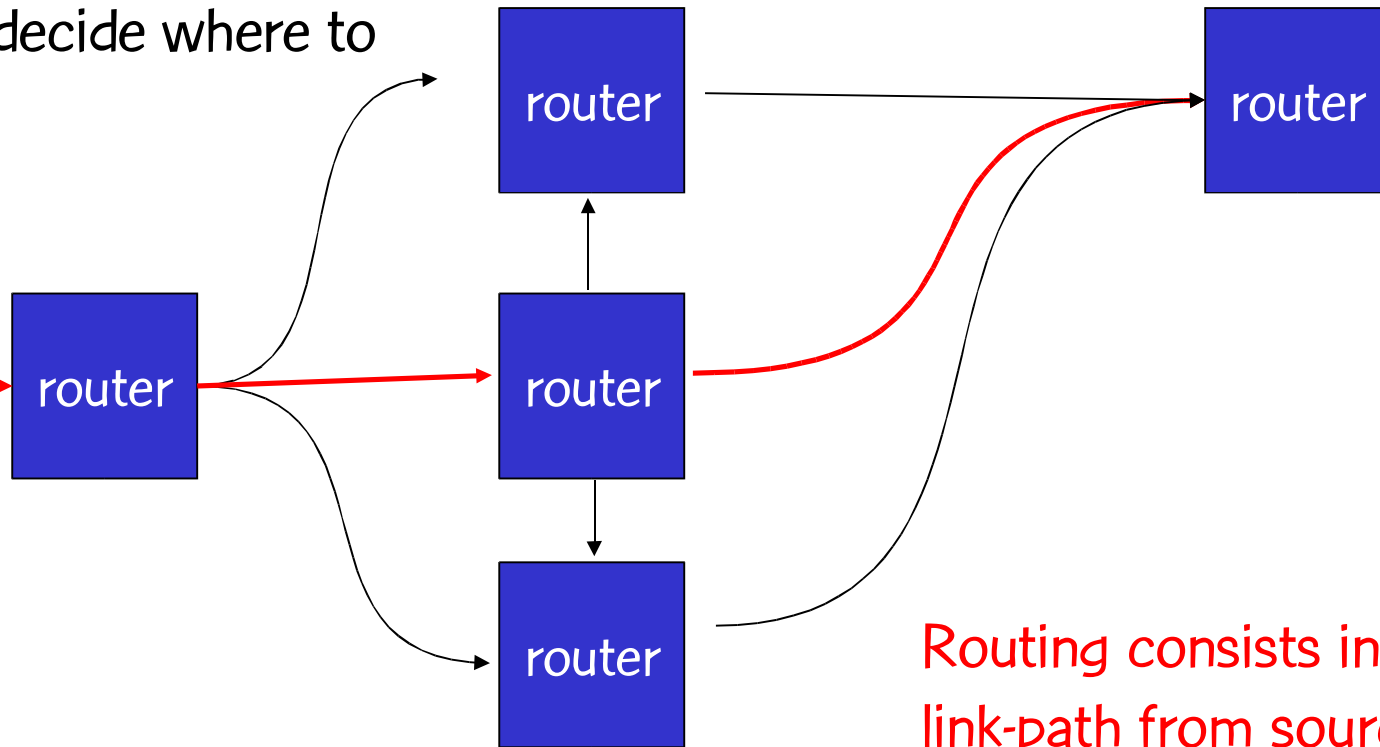
# Packet routing

When packet arrives at router, router must decide where to send it next.



$D_1$	$R_1$
$D_2$	$R_2$
$D_3$	$R_3$
$D_4$	$R_4$

Routing table

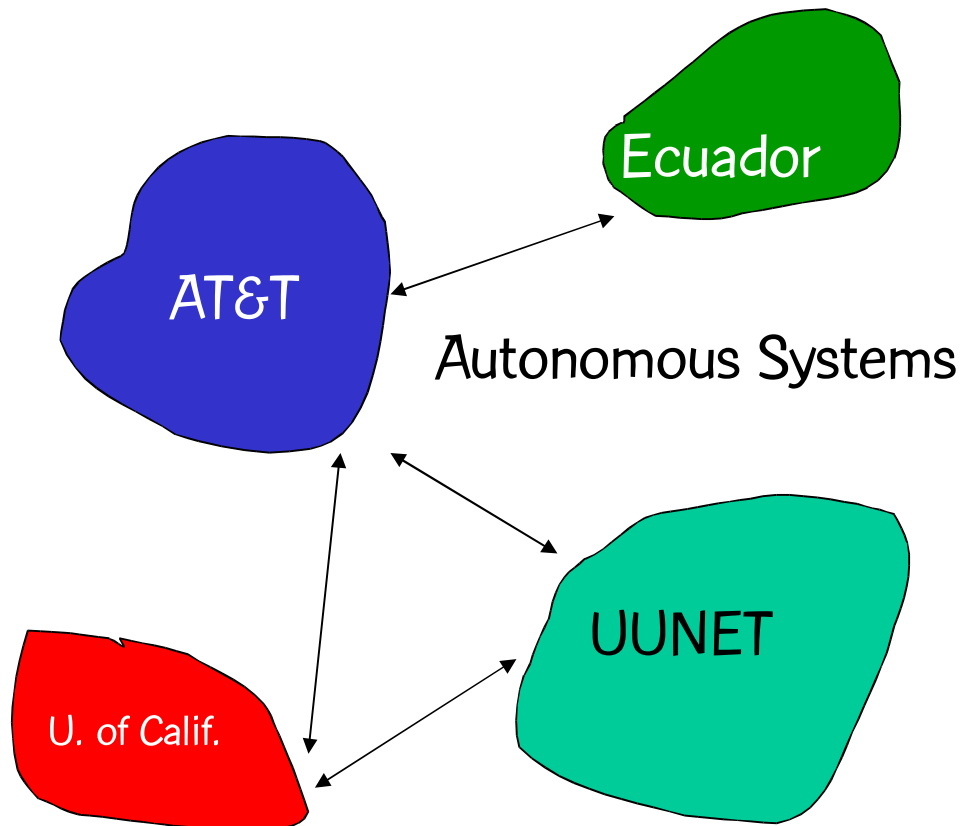


Packet's final destination.

Routing consists in finding a link-path from source to destination.

# OSPF (Open Shortest Path First)

- OSPF is a commonly used intra-domain routing protocol (IGP).
- Routers exchange routing information with all other routers in the autonomous system (AS).
  - Complete network topology knowledge is available to all routers, i.e. state of all routers and links in the AS.



# OSPF routing

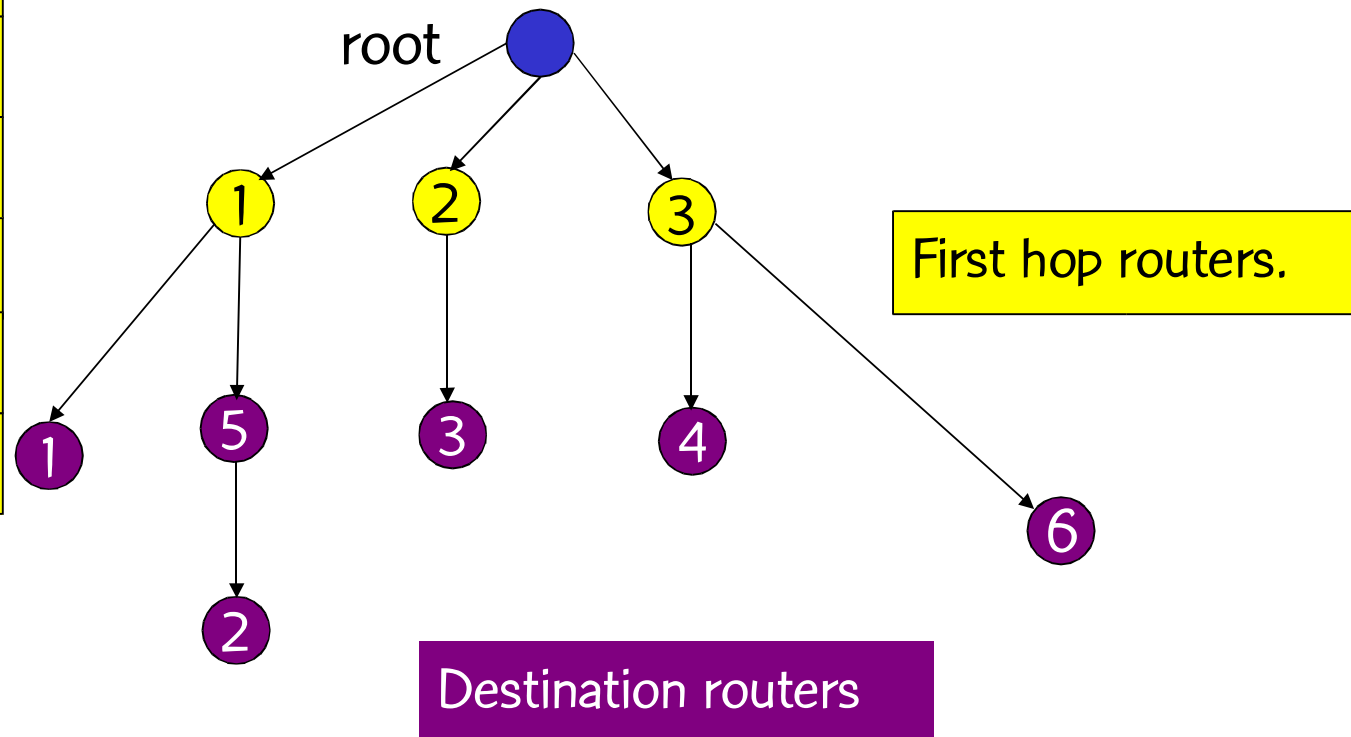
- Assign an integer weight  $| [1, w_{\max}]$  to each link in AS.  
In general,  $w_{\max} = 65535 = 2^{16} - 1$ .
- Each router computes tree of shortest weight paths to all other routers in the AS, with itself as the root, using Dijkstra's algorithm.

# OSPF routing

Routing table

$D_1$	$R_1$
$D_2$	$R_1$
$D_3$	$R_2$
$D_4$	$R_3$
$D_5$	$R_1$
$D_6$	$R_3$

Routing table is filled with first hop routers for each possible destination.

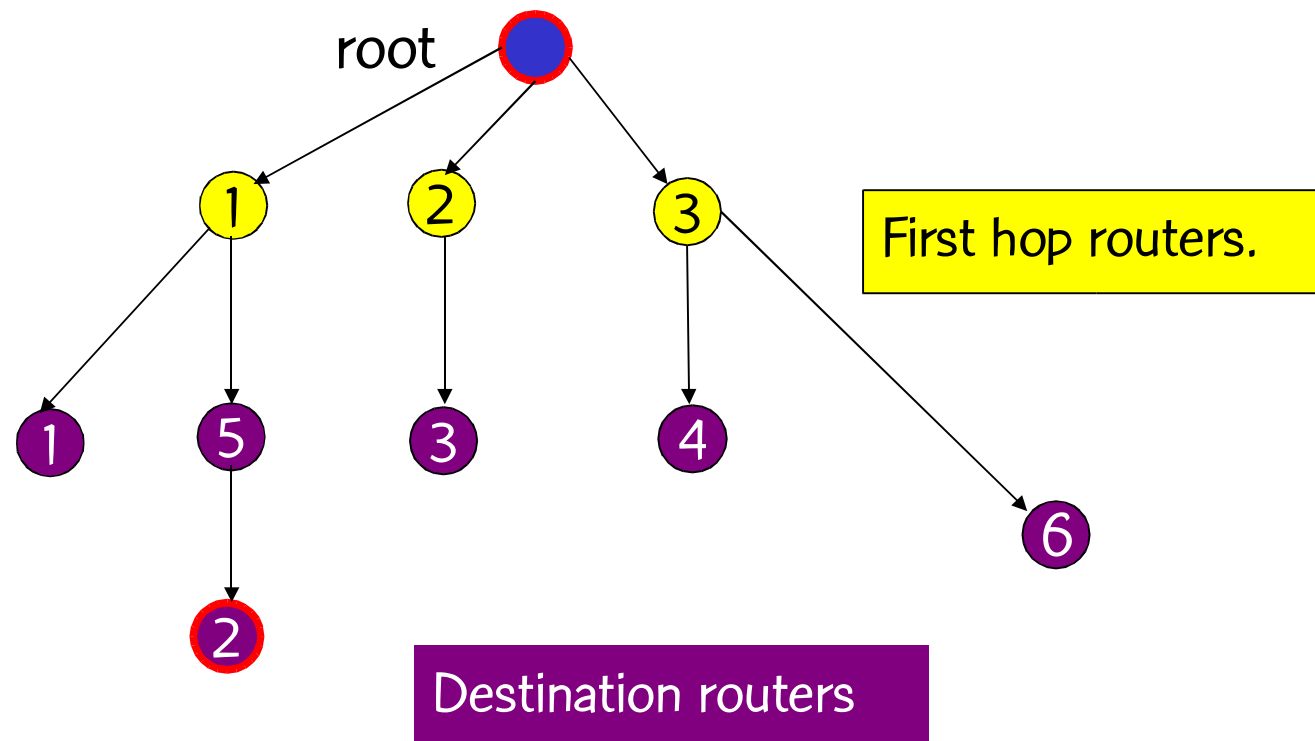


# OSPF routing

Routing table

$D_1$	$R_1$
$D_2$	$R_1$
$D_3$	$R_2$
$D_4$	$R_3$
$D_5$	$R_1$
$D_6$	$R_3$

Routing table is filled with first hop routers for each possible destination.

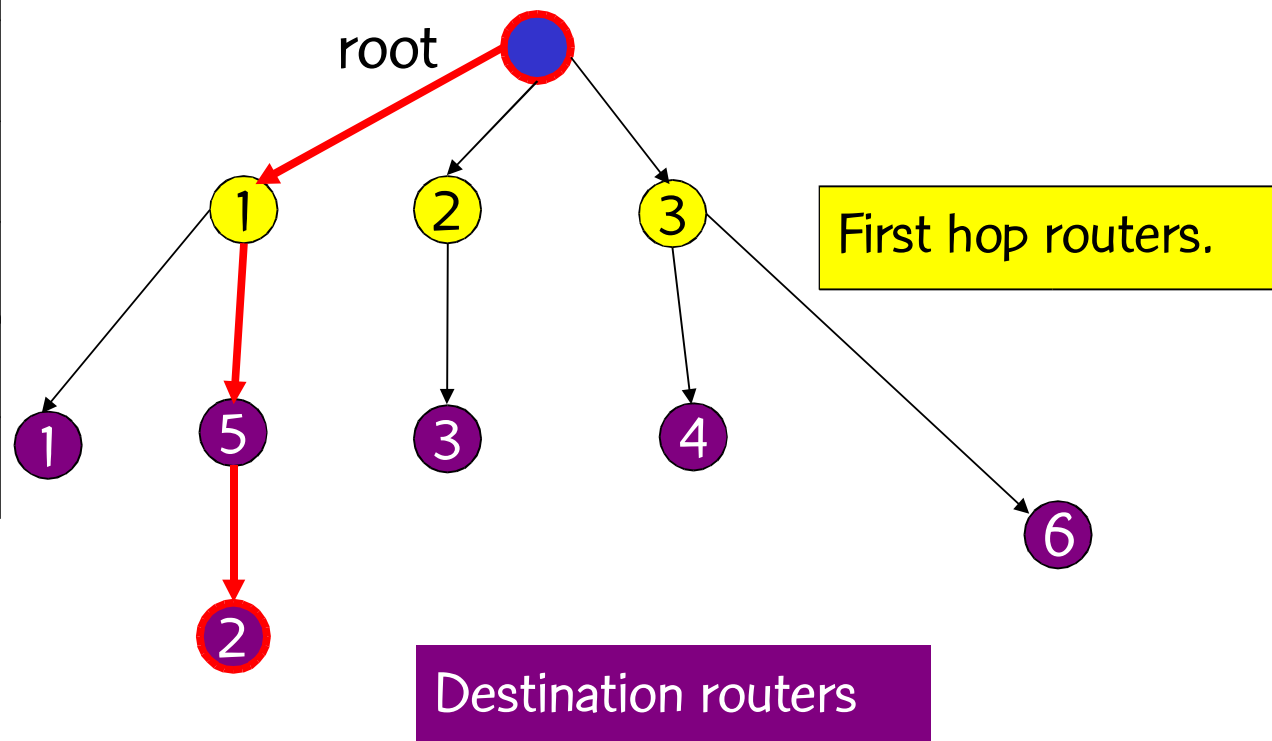


# OSPF routing

Routing table

$D_1$	$R_1$
$D_2$	$R_1$
$D_3$	$R_2$
$D_4$	$R_3$
$D_5$	$R_1$
$D_6$	$R_3$

Routing table is filled with first hop routers for each possible destination.



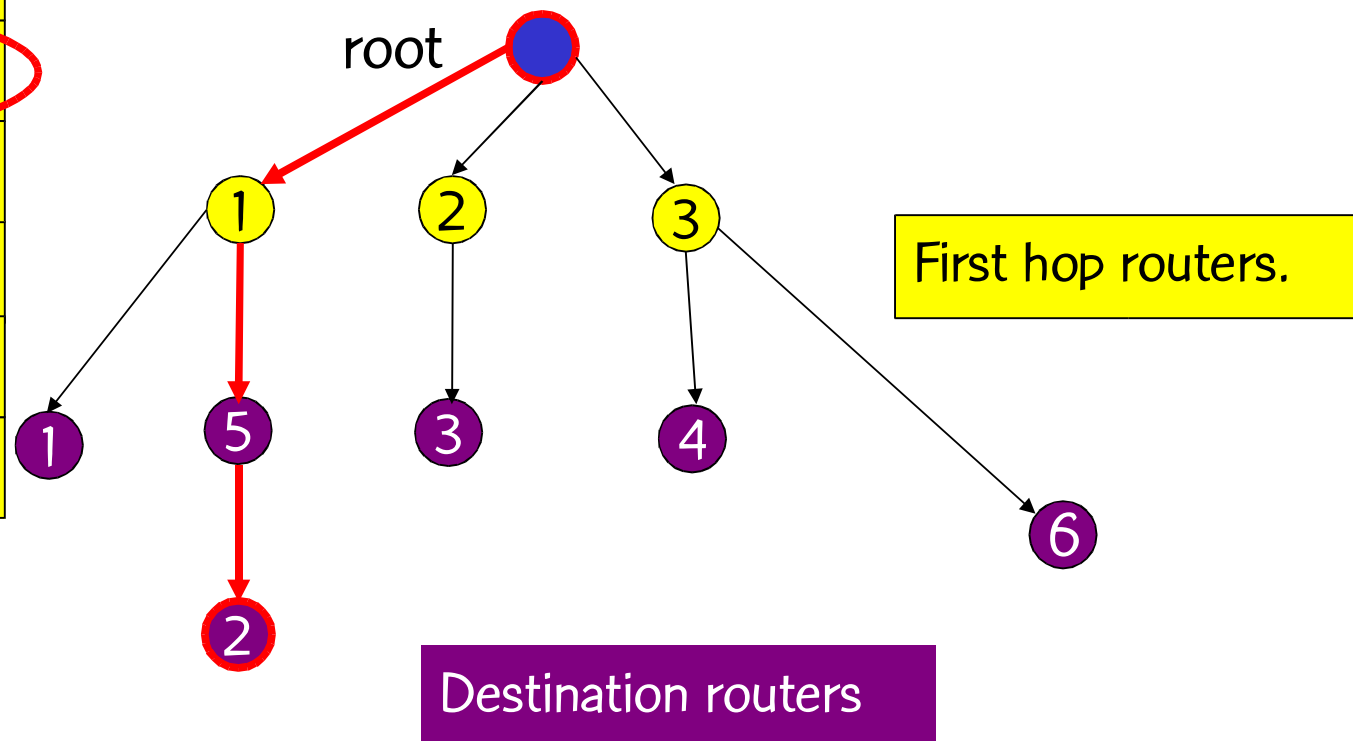


# OSPF routing

Routing table

$D_1$	$R_1$
$D_2$	$R_1$
$D_3$	$R_2$
$D_4$	$R_3$
$D_5$	$R_1$
$D_6$	$R_3$

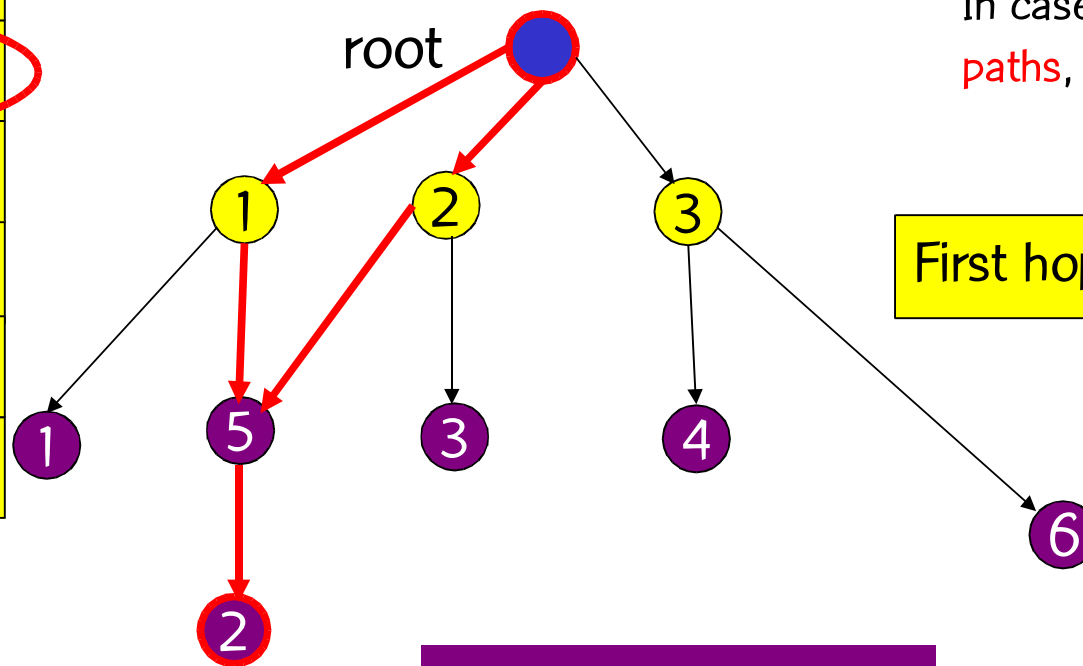
Routing table is filled with first hop routers for each possible destination.



# OSPF routing

Routing table

$D_1$	$R_1$
$D_2$	$R_1, R_2$
$D_3$	$R_2$
$D_4$	$R_3$
$D_5$	$R_1$
$D_6$	$R_3$



Routing table is filled with first hop routers for each possible destination. In case of **multiple shortest paths**, flow is **evenly split**.

First hop routers.

Destination routers

# OSPF weight setting

- OSPF weights are assigned by network operator.
  - CISCO assigns, by default, a weight proportional to the inverse of the link bandwidth (Inv Cap).
  - If all weights are unit, the weight of a path is the number of hops in the path.
- We propose a hybrid genetic algorithm to find good OSPF weights.
  - Memetic algorithm
  - Genetic algorithm with optimized crossover

# Minimization of congestion

- Consider the directed capacitated network  $G = (N, A, c)$ , where  $N$  are routers,  $A$  are links, and  $c_a$  is the capacity of link  $a \in A$ .
- We use the measure of Fortz & Thorup (2000) to compute congestion:

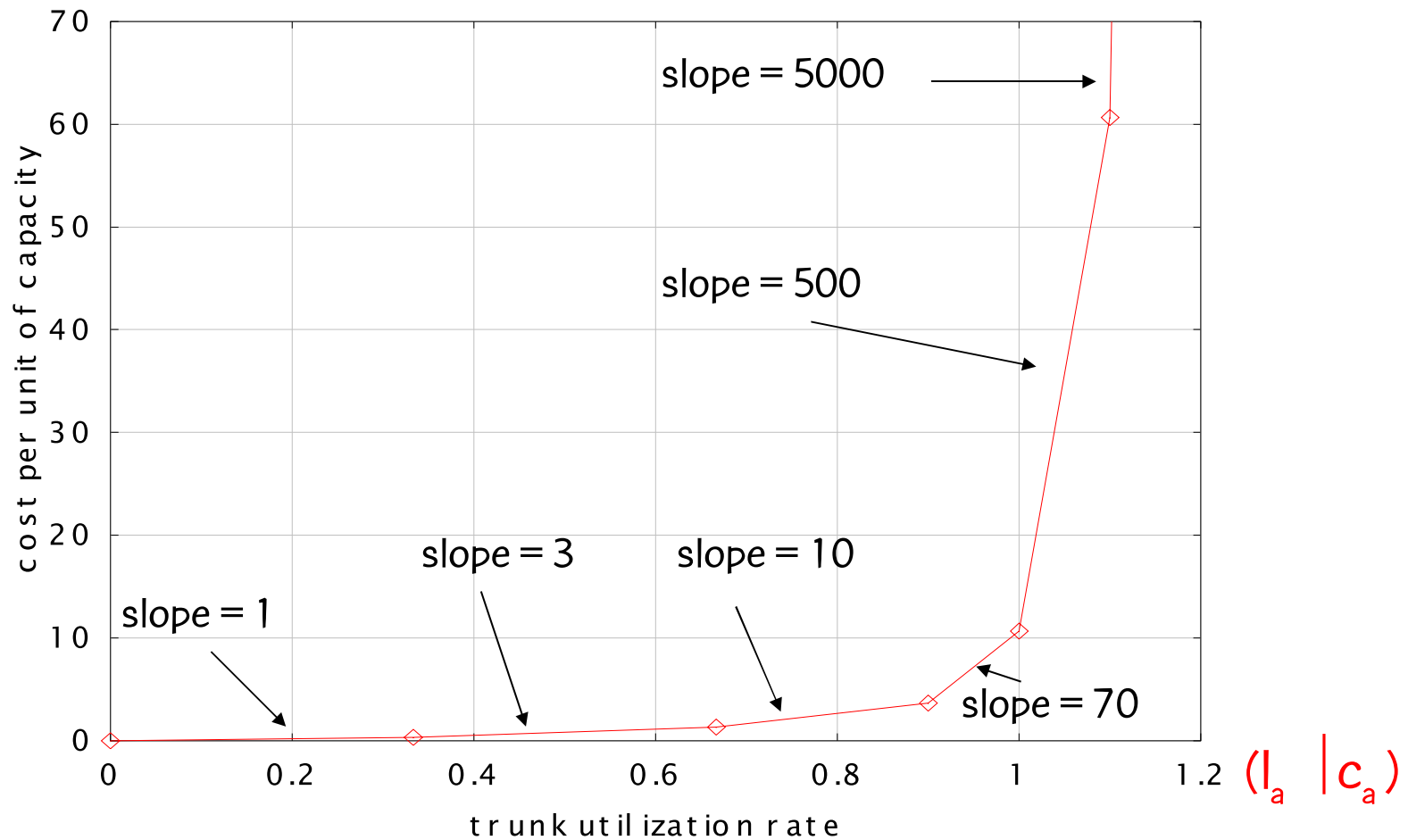
$$C = C_1(I_1) + C_2(I_2) + \dots + C_{|A|}(I_{|A|})$$

where  $I_a$  is the load on link  $a \in A$ ,

$C_a(I_a)$  is piecewise linear and convex,

$C_a(0) = 0$ , for all  $a \in A$ .

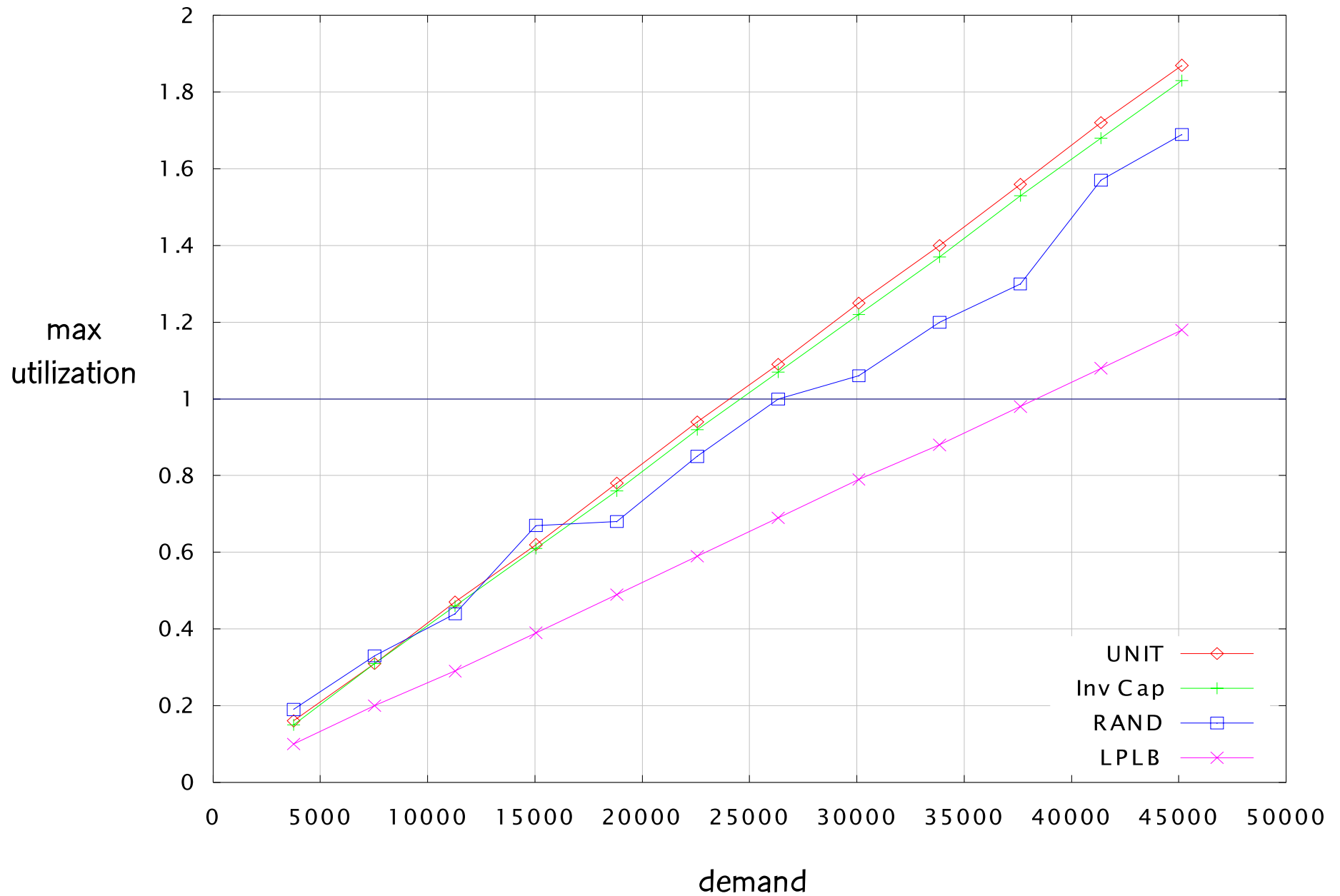
# Piecewise linear and convex $|_a(I_a)$ link congestion measure



# OSPF weight setting problem

- Given a directed network  $G = (N, A)$  with link capacities  $c_a \mid A$  and demand matrix  $D = (d_{s,t})$  specifying a demand to be sent from node  $s$  to node  $t$  :
  - Assign weights  $w_a \mid [1, w_{\max}]$  to each link  $a \mid A$ , such that the objective function  $\mid$  is minimized when demand is routed according to the OSPF protocol.

## AT&T Worldnet backbone network (90 routers, 274 links)

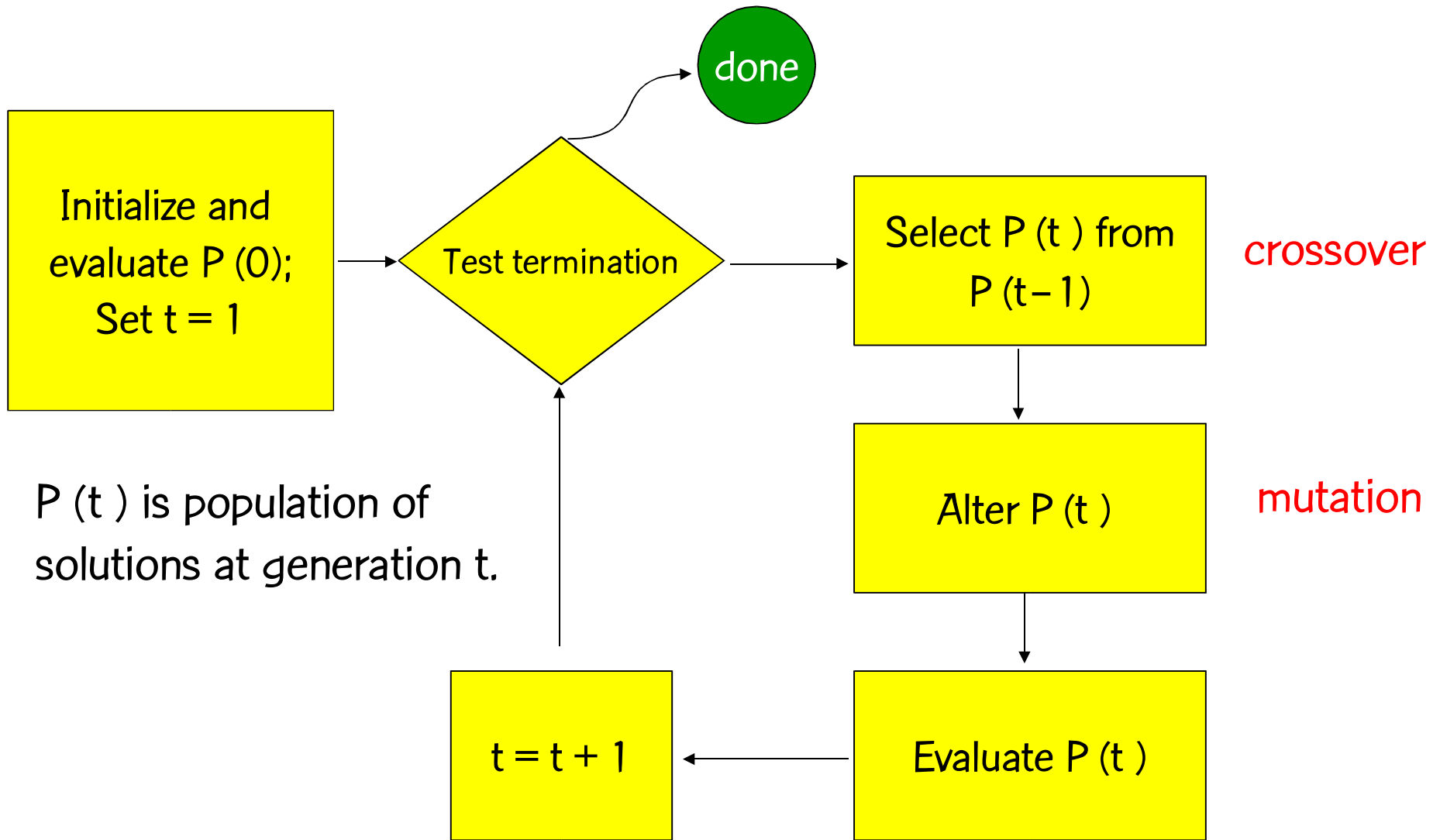


# Genetic and hybrid genetic algorithms for OSPF weight setting problem

- Genetic
  - M. Ericsson, M.G.C. Resende, & P.M. Pardalos, “ A genetic algorithm for the weight setting problem in OSPF routing, J. of Combinatorial Optimization, vol. 6, pp. 299-333, 2002.
- Hybrid genetic
  - L.S. Buriol, M.G.C. Resende, C.C. Ribeiro, & M. Thorup, “A hybrid genetic algorithm for the weight setting problem in OSPF/IS-IS routing,” to appear in Networks, 2005.



# Genetic algorithms



# Solution encoding

- A population consists of  $nPop = 50$  integer weight arrays:  $w = (w_1, w_2, \dots, w_{|A|})$ ,  
where  $w_a \in [1, w_{\max} = 20]$
- All possible weight arrays correspond to feasible solutions.

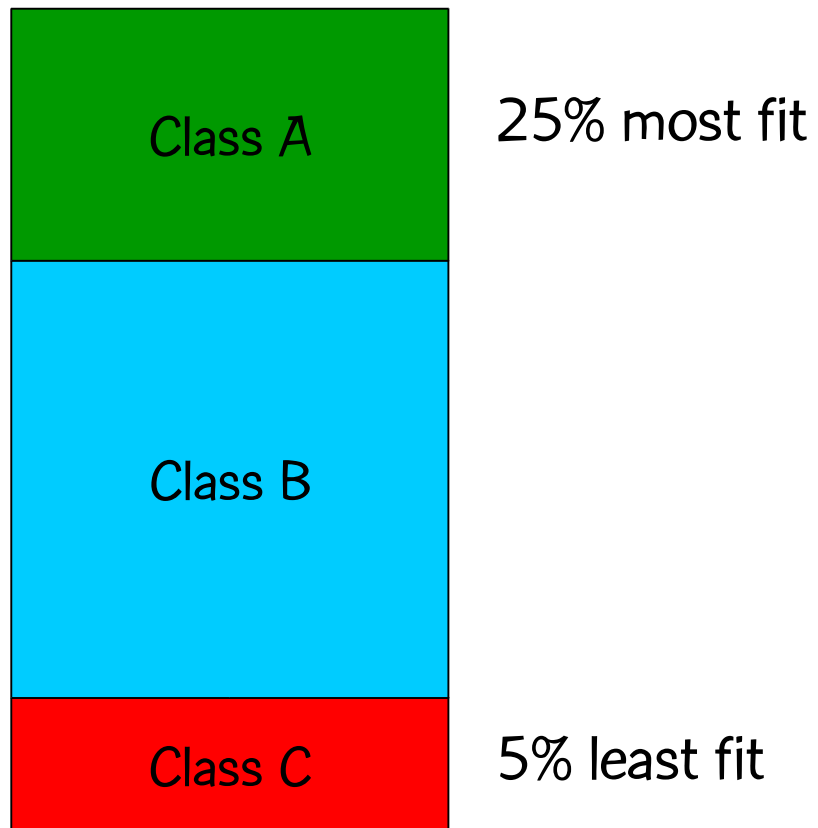
# Initial population

- nPop solutions, with each weight randomly generated, uniformly in the interval  $[1, w_{\max}/3]$ .

# Solution evaluation

- For each demand pair  $(s,t)$ , route using OSPF, computing demand pair loads  $l_a^{s,t}$  on each link  $a \in A$ .
- Add up demand pair loads on each link  $a \in A$ , yielding total load  $l_a$  on link.
- Compute link congestion cost  $c_a(l_a)$  for each link  $a \in A$ .
- Add up costs:  $C = c_1(l_1) + c_2(l_2) + \dots + c_{|A|}(l_{|A|})$

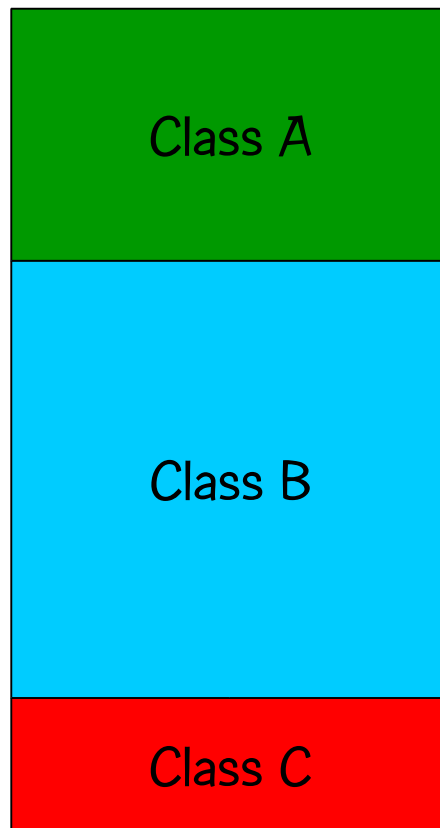
# Population partitioning



Population is sorted according to solution value | and solutions are classified into three categories.

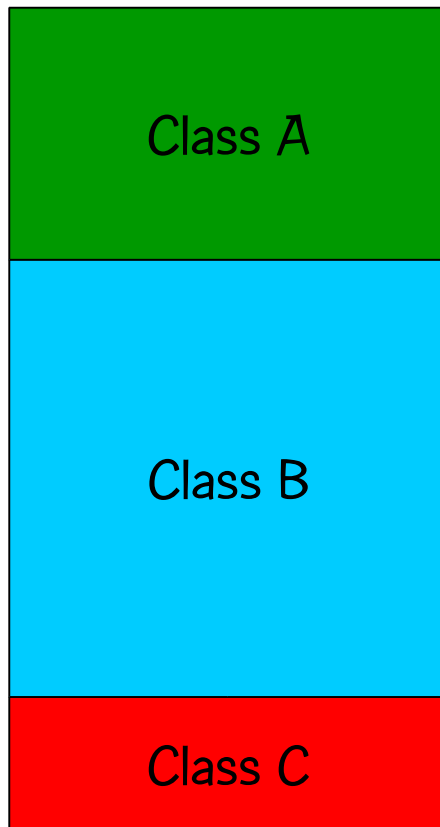
# Population dynamics

generation t



# Population dynamics

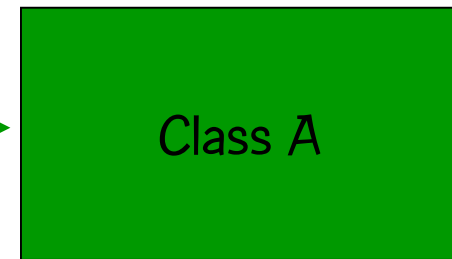
generation t



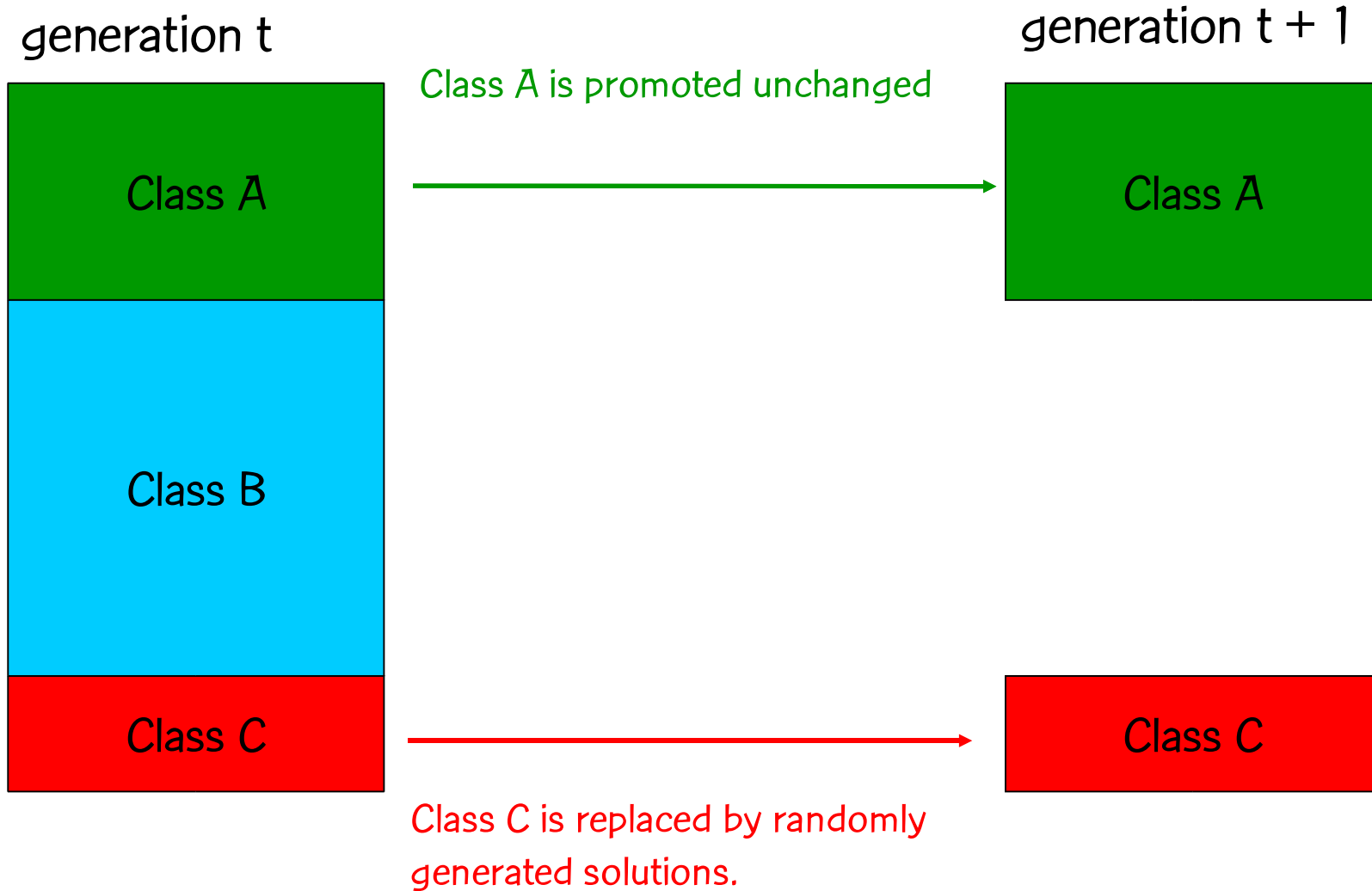
Class A is promoted unchanged



generation t + 1

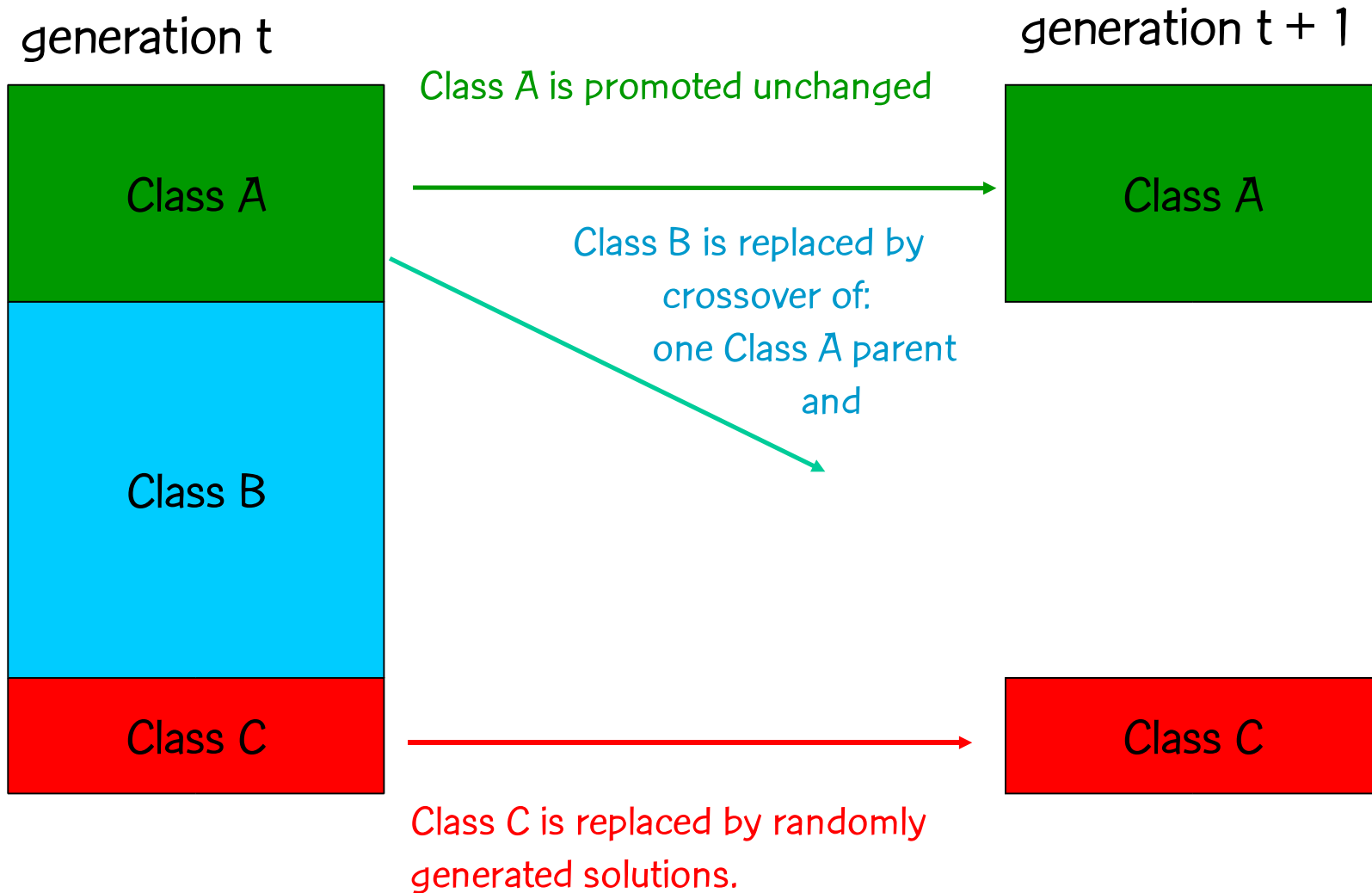


# Population dynamics

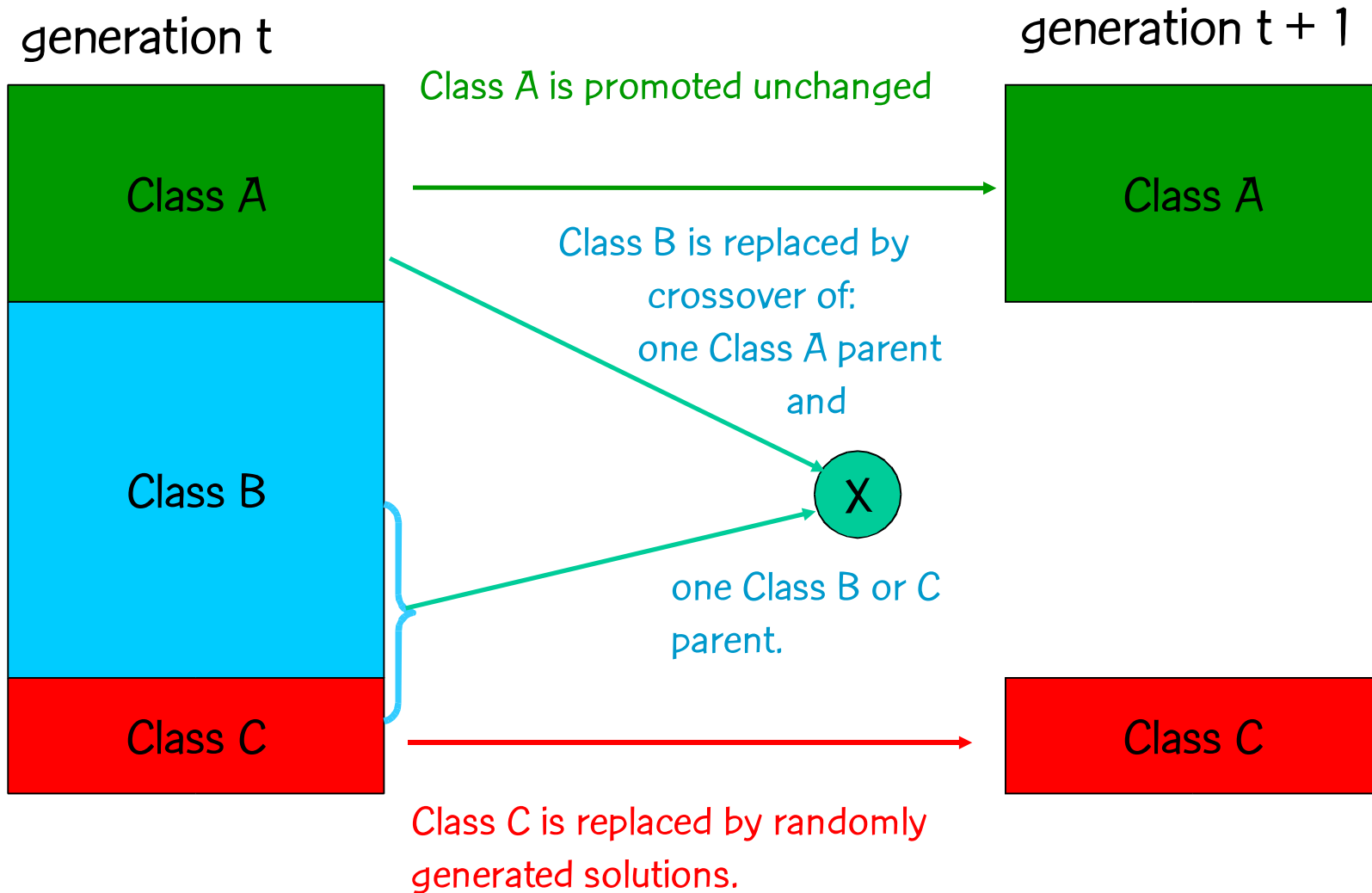




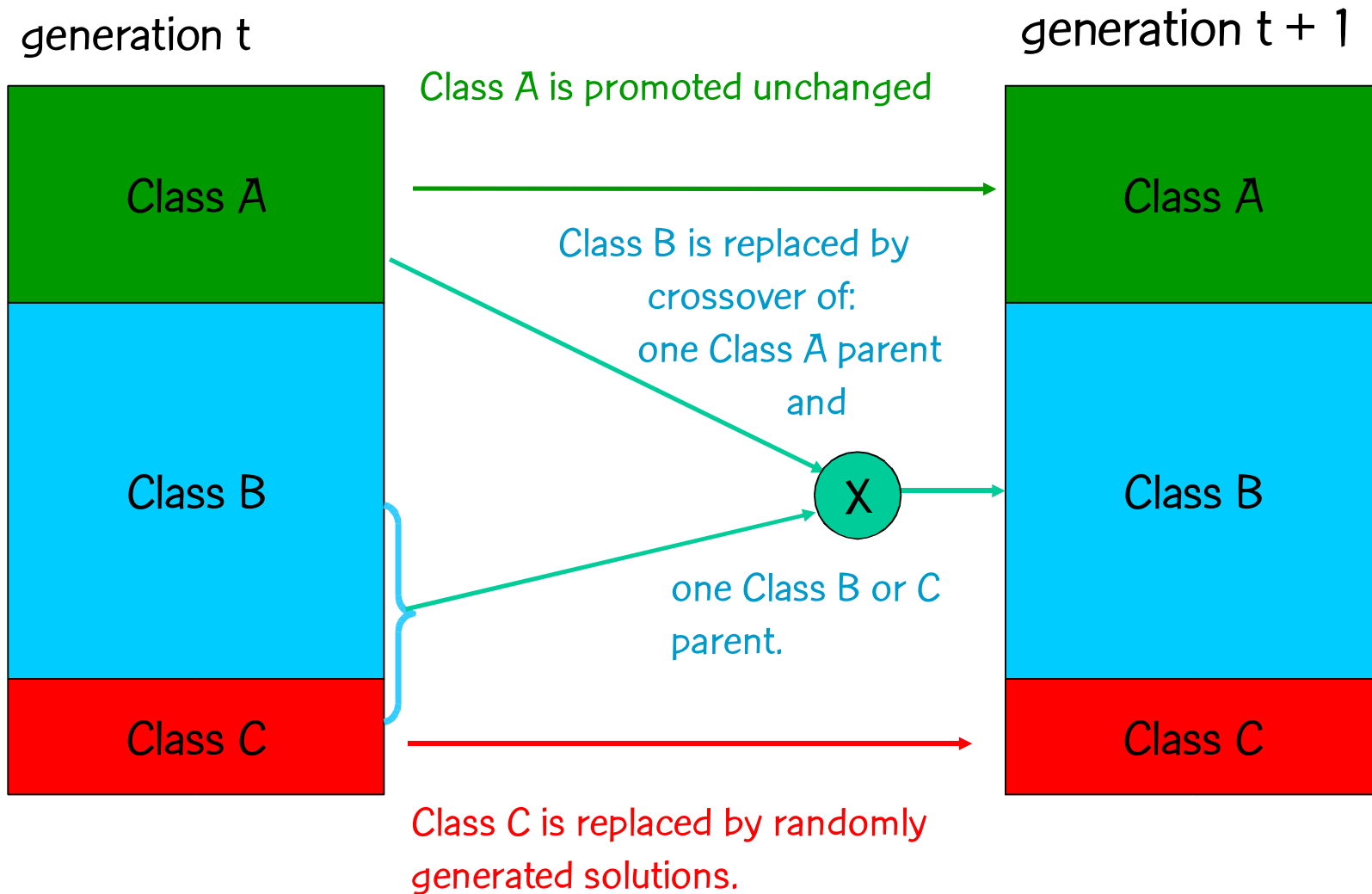
# Population dynamics



# Population dynamics



# Population dynamics



# Parent selection

- Parents are chosen at random:
  - one parent from Class A (elite).
  - one parent from Class B or C (non-elite).
- Reselection is allowed, i.e. parents can breed more than once per generation.
- Better individuals are more likely to reproduce.

# Crossover with random keys

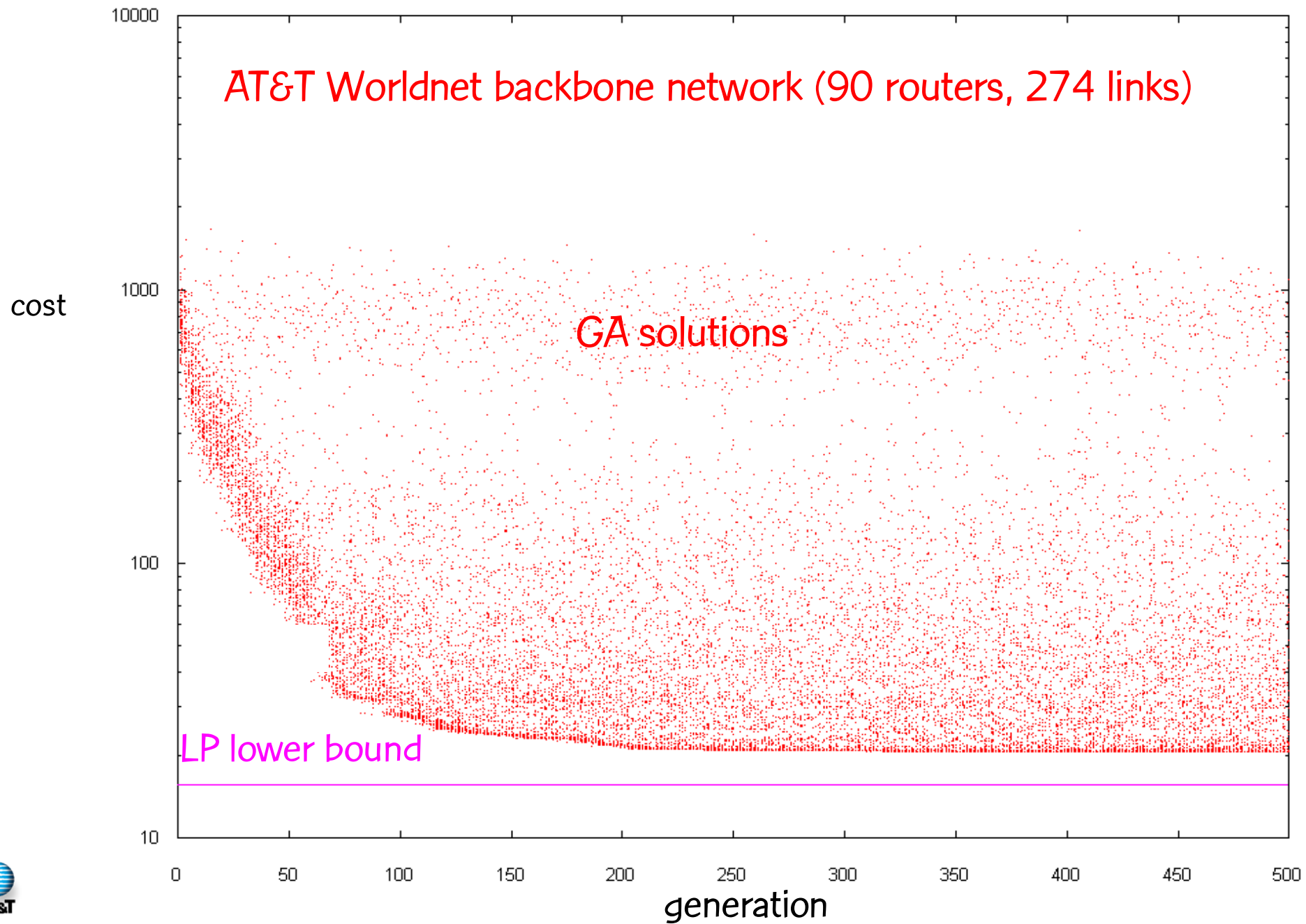
Bean (1994)

Crossover combines elite parent  $p_1$  with non-elite parent  $p_2$  to produce child  $c$  :

With small probability child has single gene mutation.

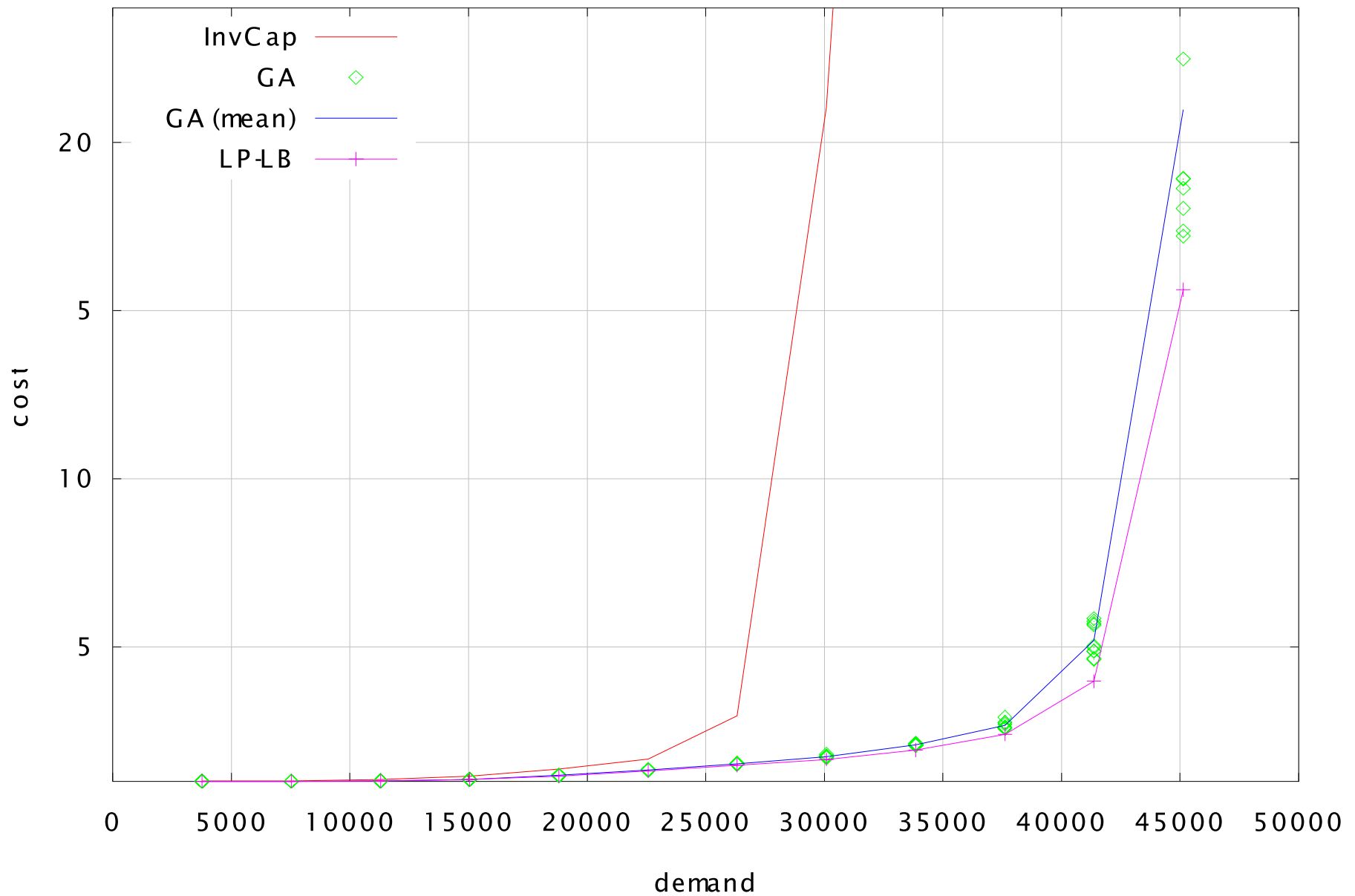
Child is more likely to inherit gene of elite parent.

```
for all genes  $i = 1, 2, \dots, |A|$  do
  if  $\text{rrandom}[0,1] < 0.01$  then
     $c[i] = \text{irandom}[1, w_{\max}]$ 
  else if  $\text{rrandom}[0,1] < 0.7$  then
     $c[i] = p_1[i]$ 
  else  $c[i] = p_2[i]$ 
end
```

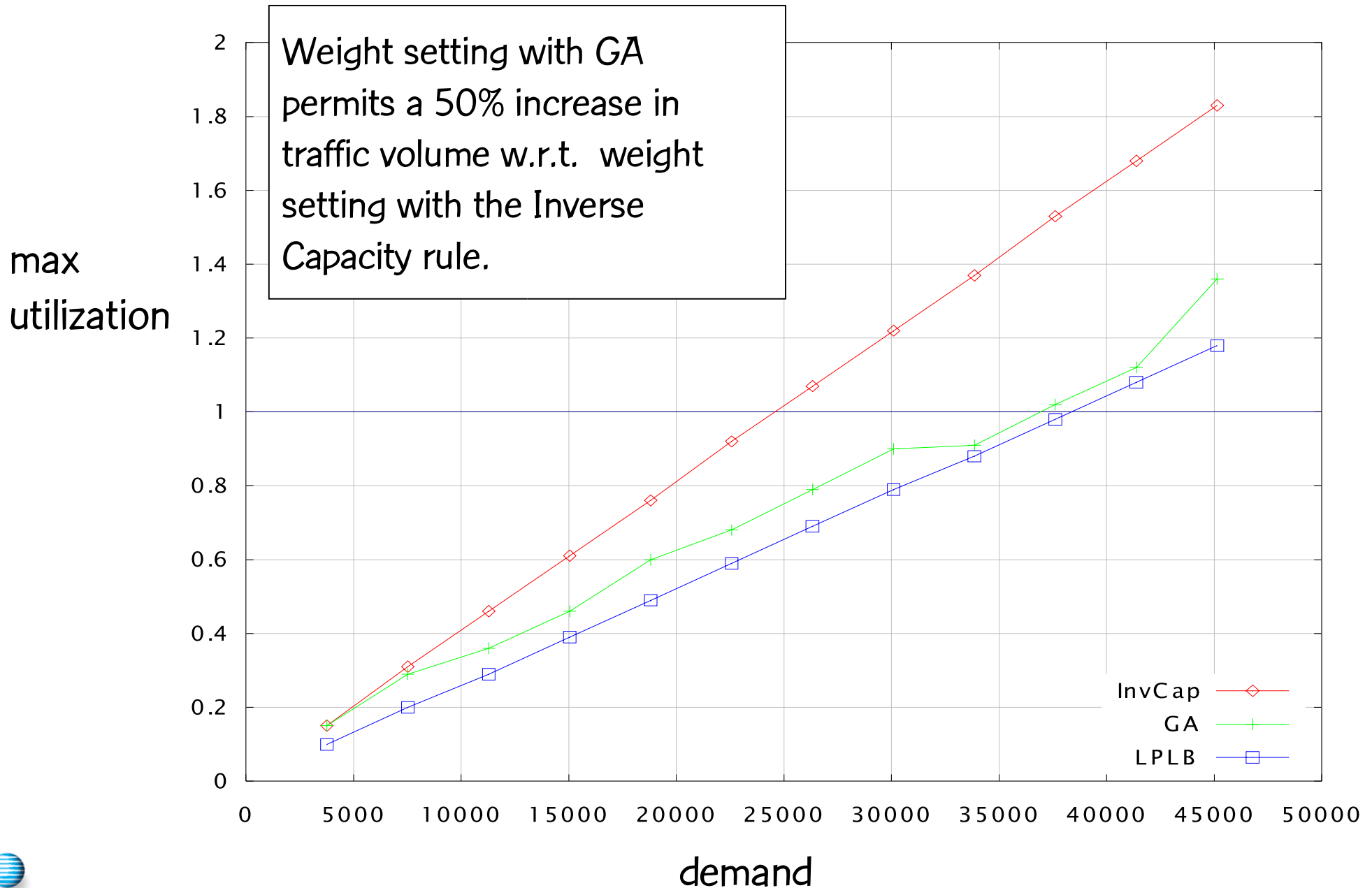


# AT&T Worldnet backbone network (90 routers, 274 links)

att

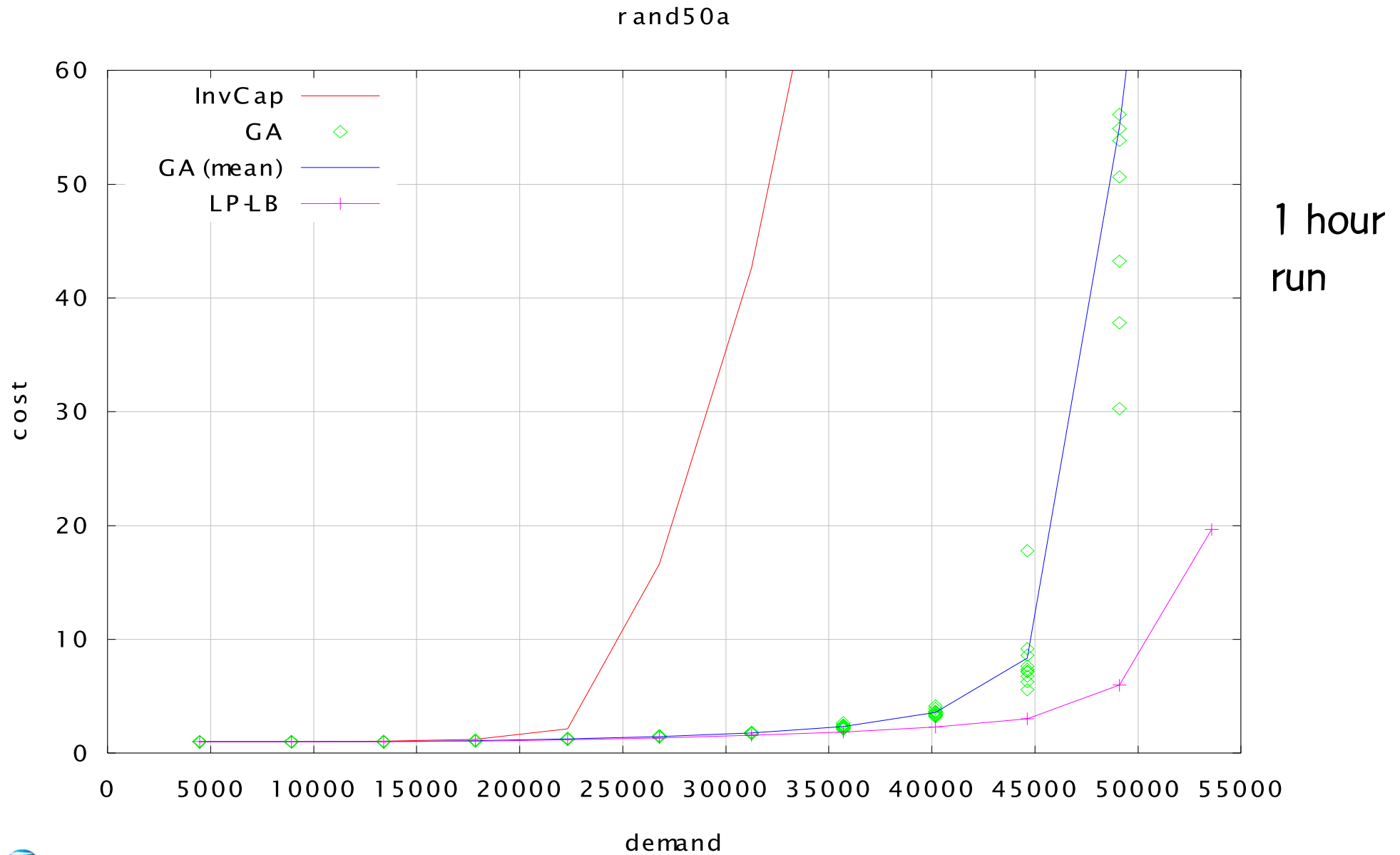


## AT&T Worldnet backbone network (90 routers, 274 links)

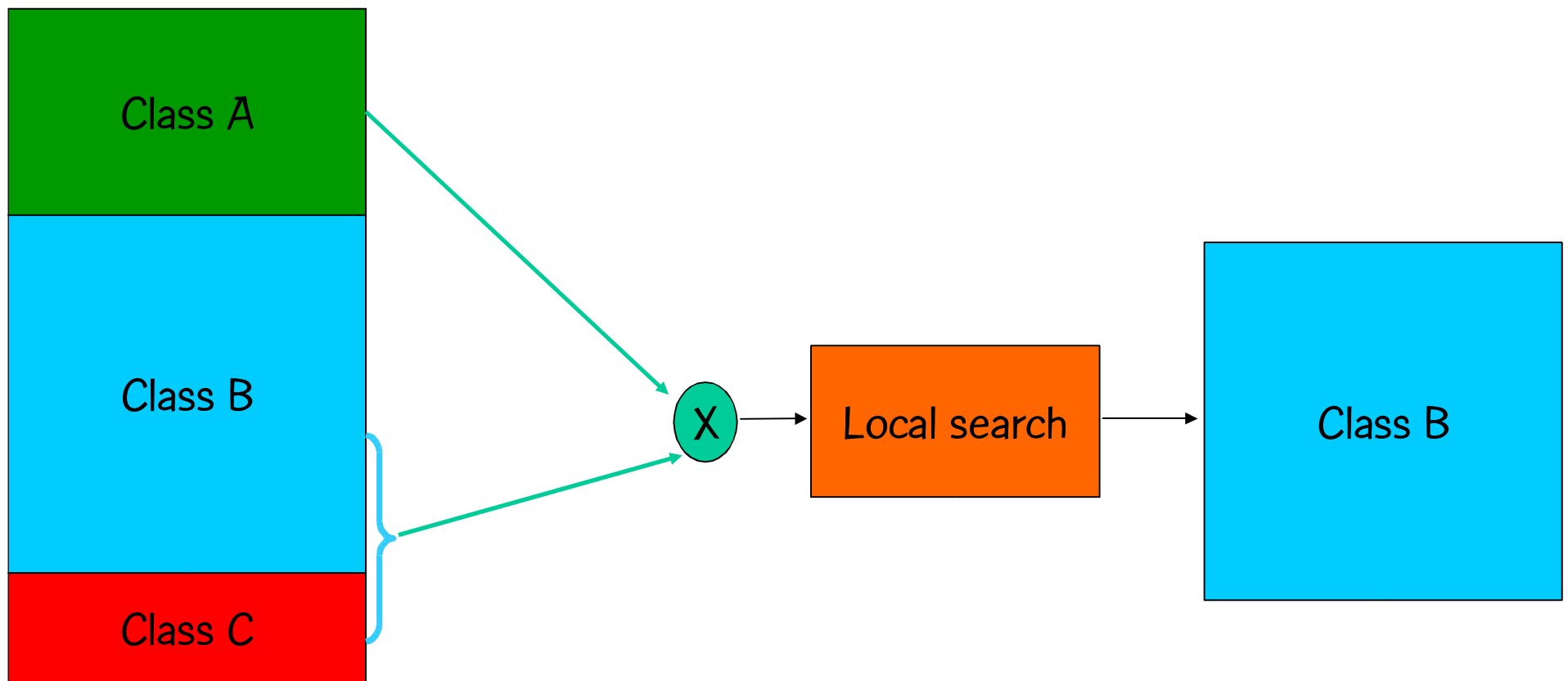




## Rand50a: random graph with 50 nodes and 245 arcs.



Optimized crossover = crossover + local search



# Fast local search

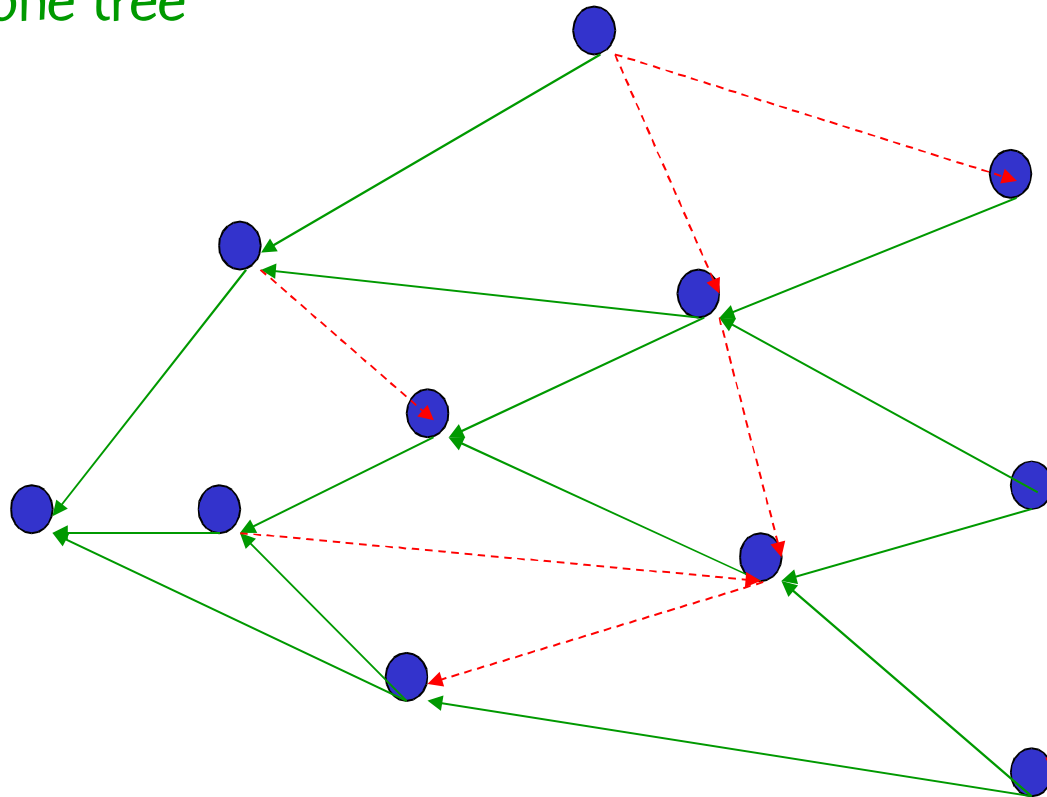
- Let  $\bar{A}^*$  be the set of five arcs  $a \in A$  having largest  $|a|$  values.
- Scan arcs  $a \in \bar{A}^*$  from largest to smallest  $|a|$ :
  - Increase arc weight, one unit at a time, in the range  $[w_a, w_a + \lfloor (w_{\max} - w_a)/4 \rfloor]$
  - If total cost  $|$  is reduced, restart local search.

# Dynamic shortest path

- In local search, when arc weight increases, shortest path trees:
    - may change completely (rarely do)
    - may remain unchanged (e.g. arc not in a tree)
    - may change partially
      - Few trees change
      - Small portion of tree changes
- } Does not make sense to recompute trees from scratch.

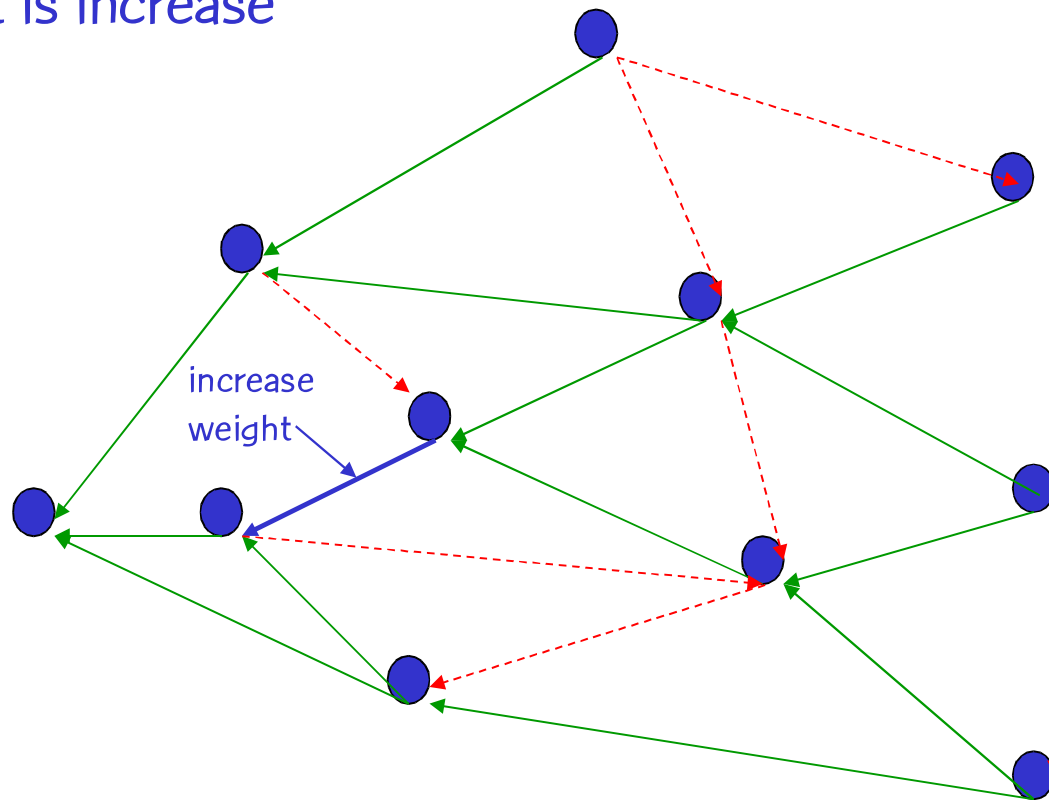
# Dynamic shortest path

Consider one tree  
at a time.



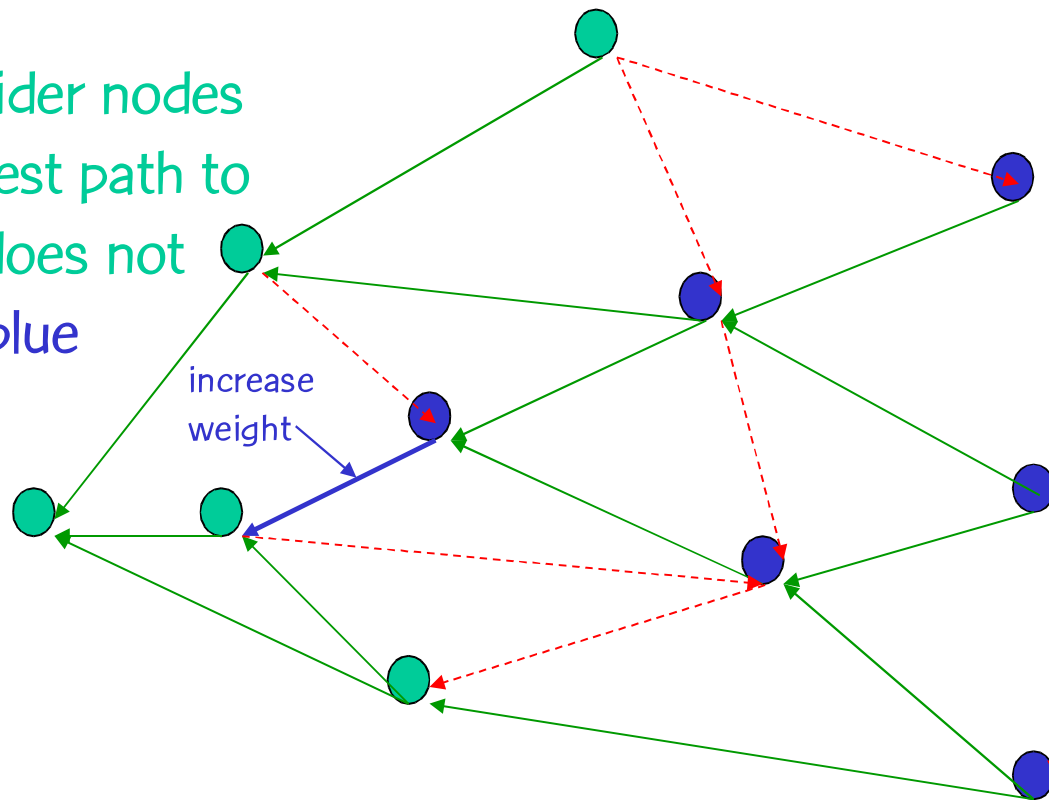
# Dynamic shortest path

Arc weight is increase  
by 1.

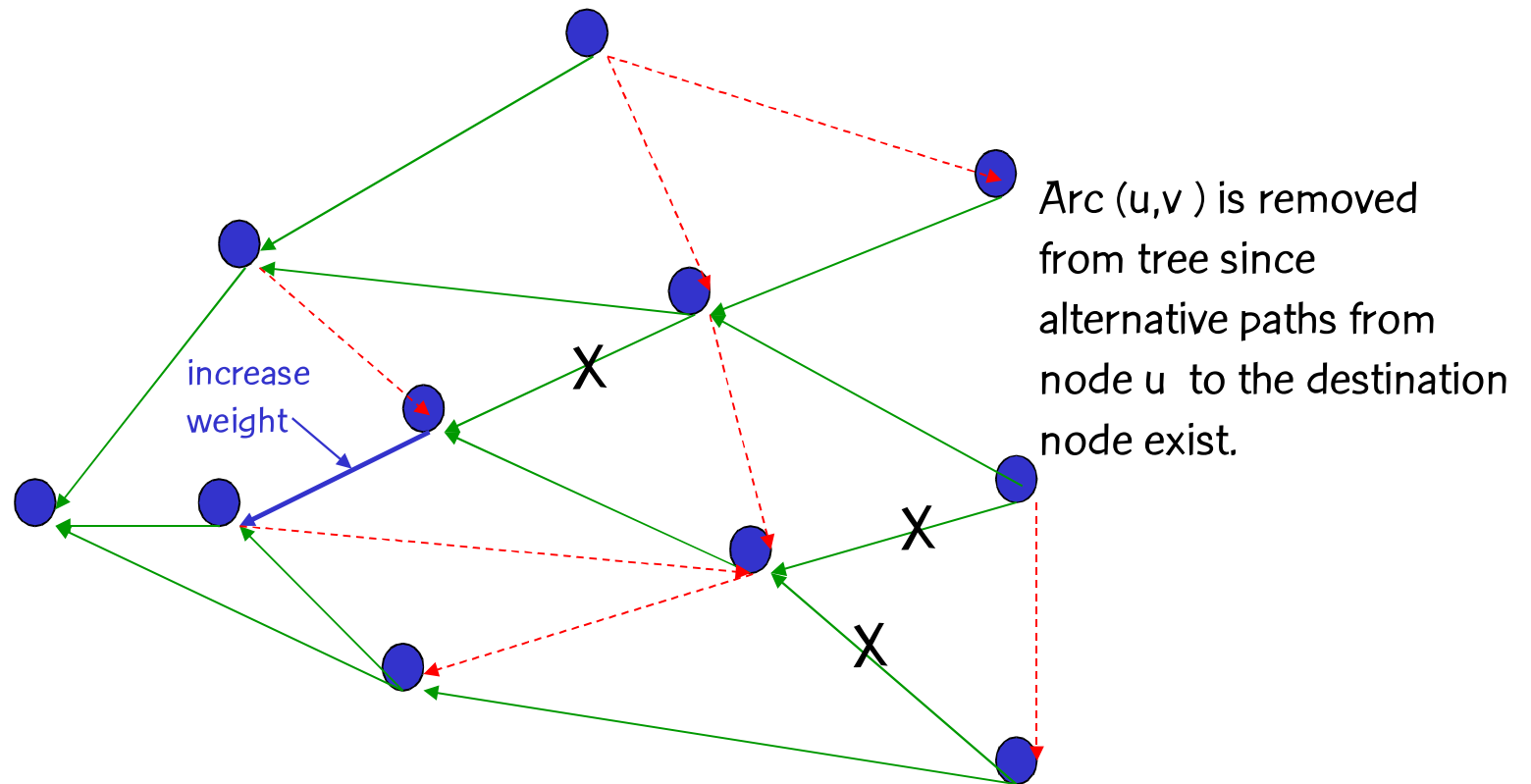


# Dynamic shortest path

Do not consider nodes  
whose shortest path to  
destination does not  
go through blue  
arc.

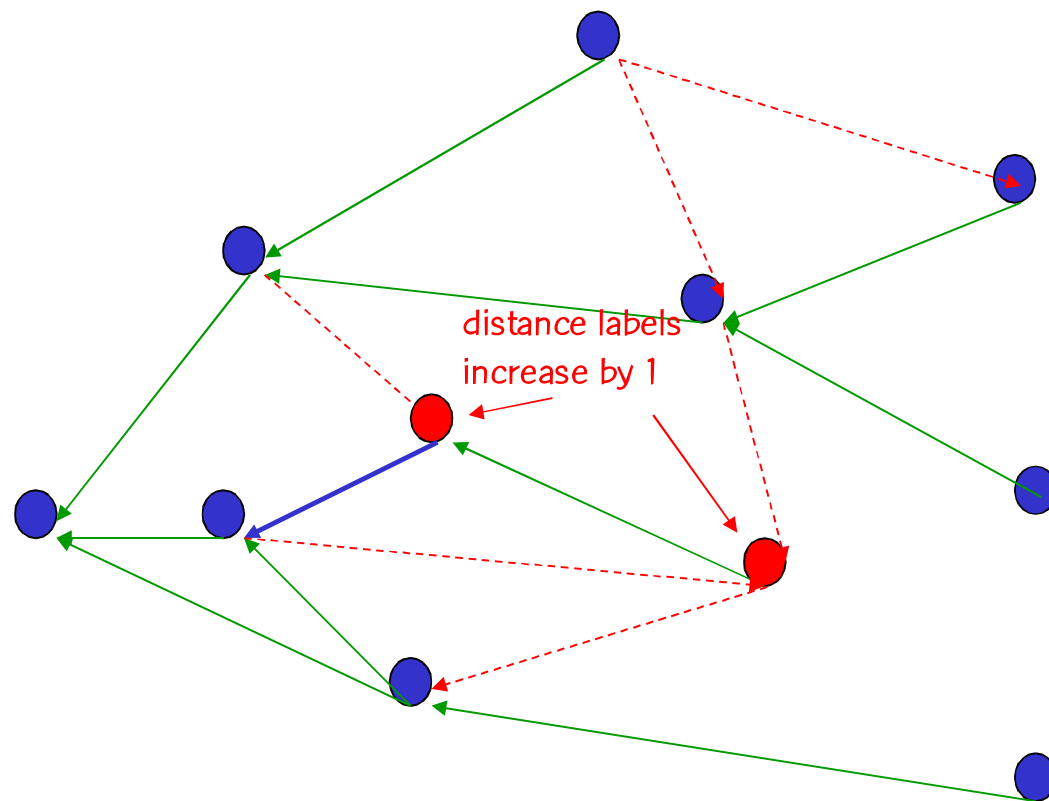


# Dynamic shortest path



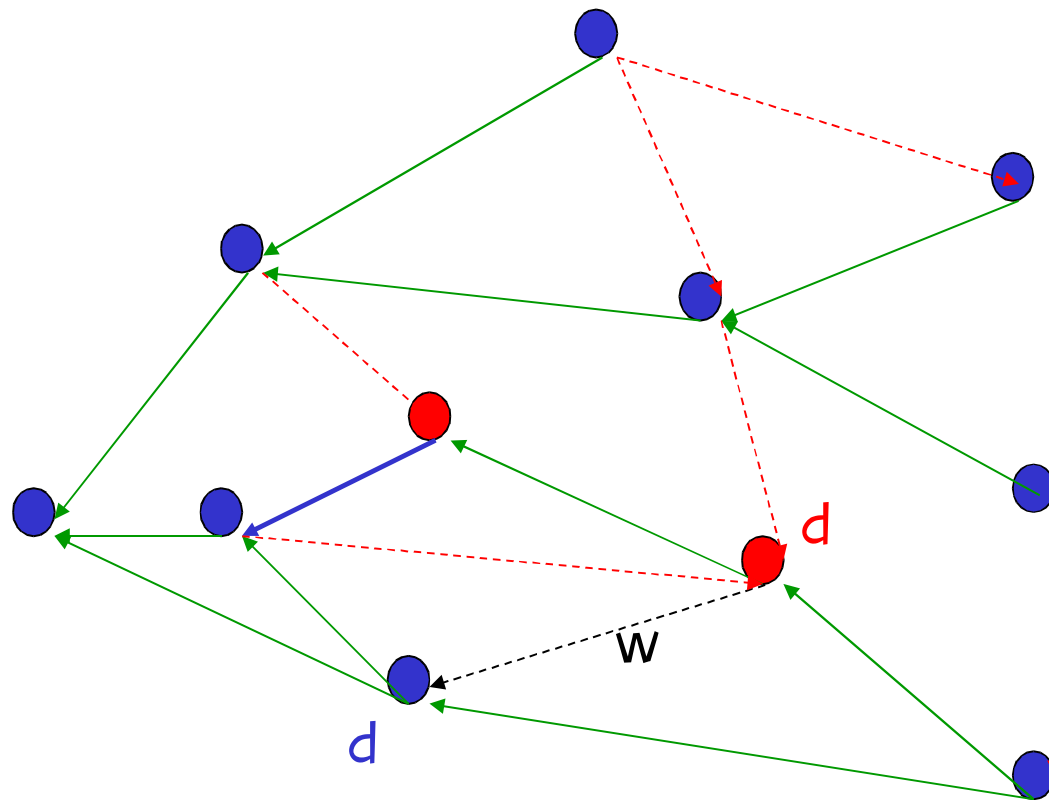


# Dynamic shortest path

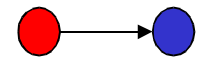


Shortest paths  
from red nodes  
must traverse  
blue arc.

# Dynamic shortest path

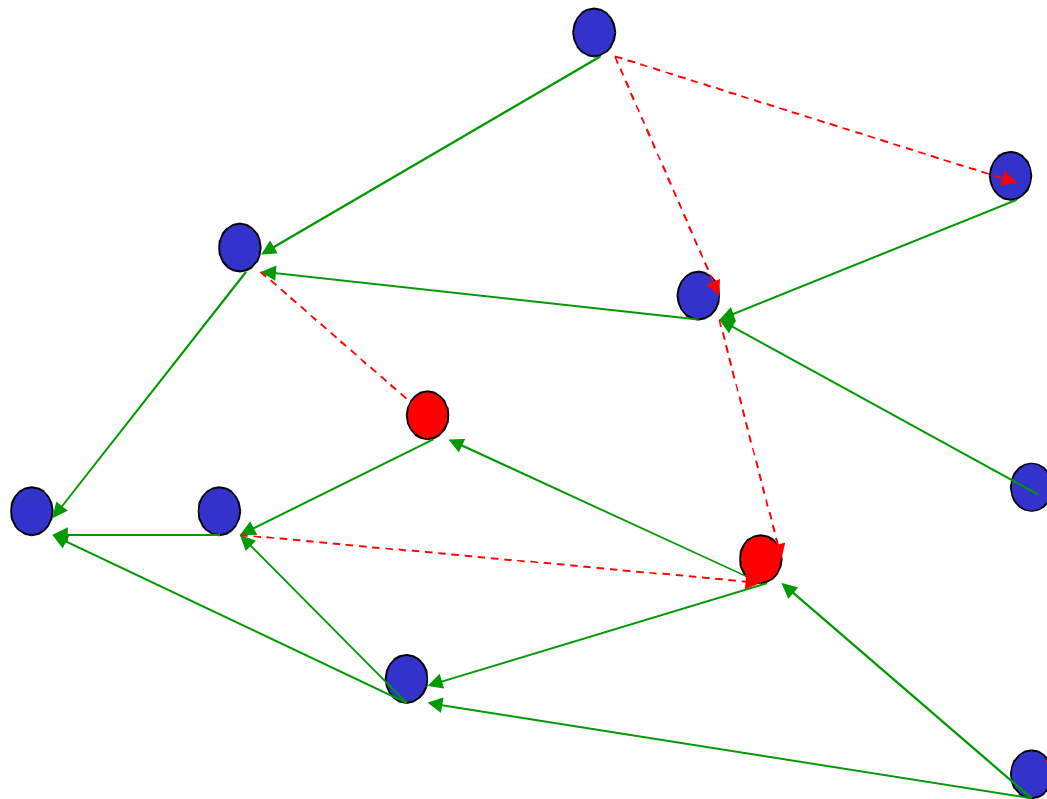


Test all arcs of type



If  $d - d = w$ , then  
red node  $\rightarrow$  blue node enters  
tree.

# Dynamic shortest path



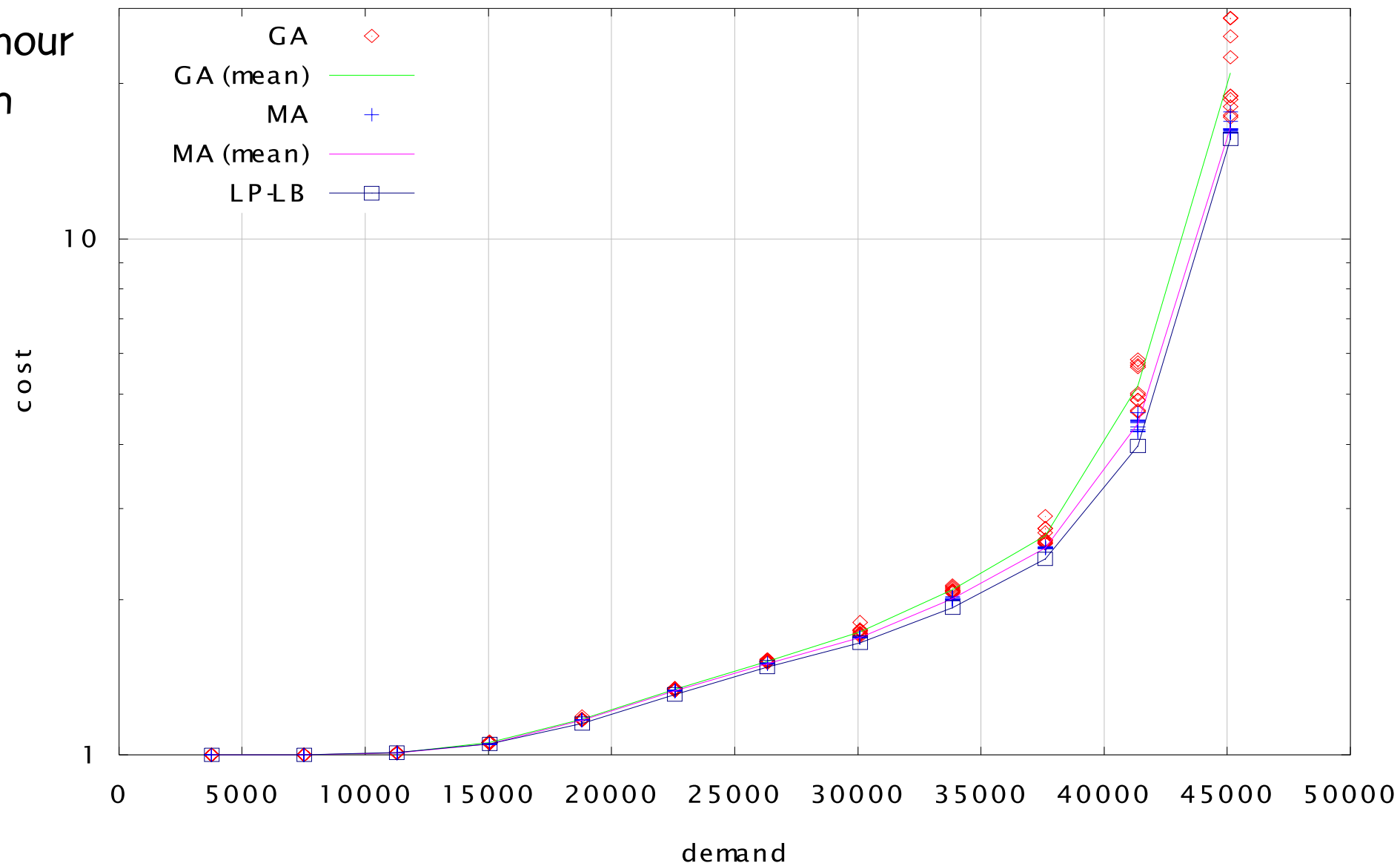
# Dynamic shortest path

L.S. Buriol, M.G.C. Resende, & M. Thorup, "Speeding up dynamic shortest path algorithms," AT&T Labs Research Report, 2003.

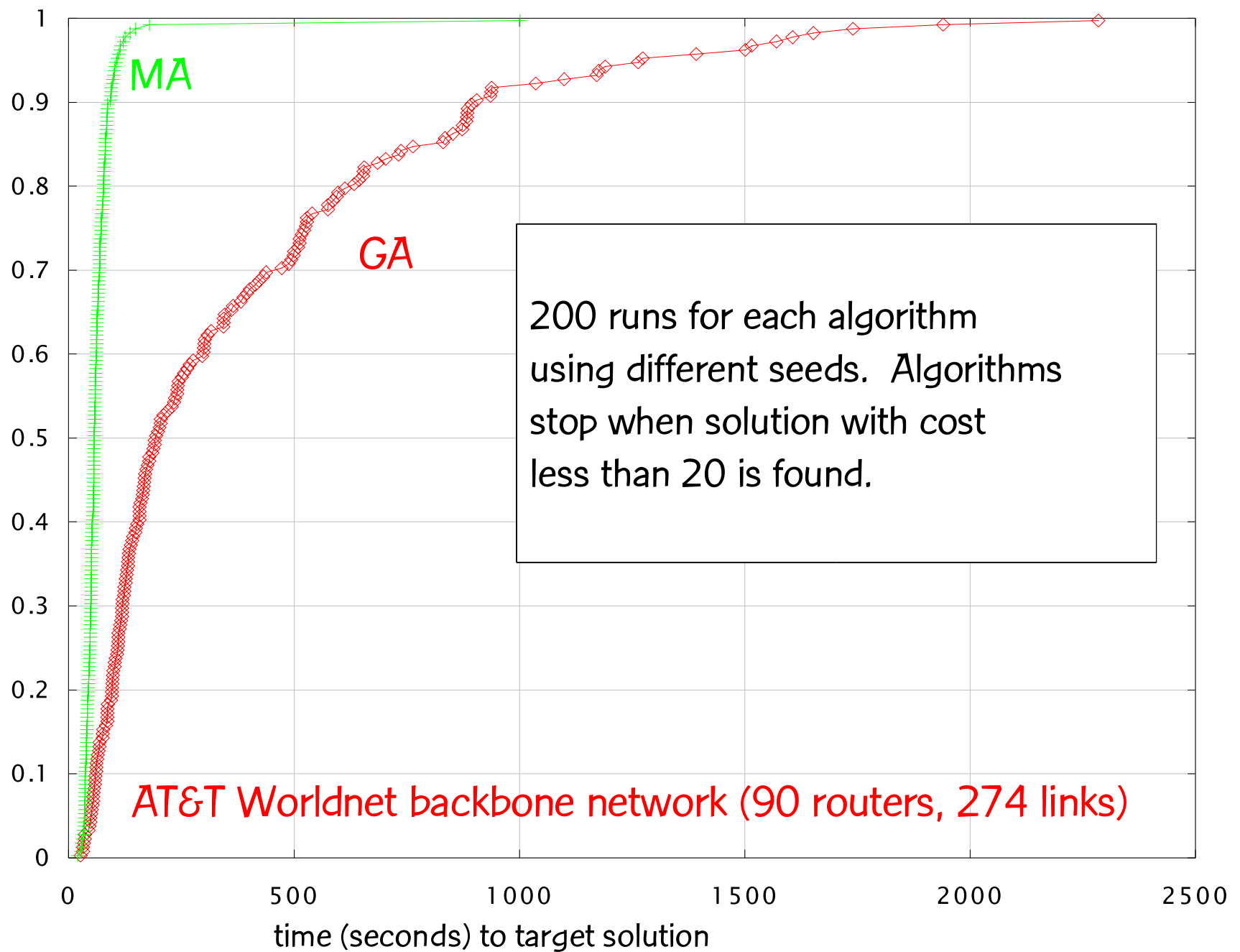
- Ramalingam & Reps (1996) allow arbitrary arc weight change.
- We specialized the Ramalingam & Reps algorithm for unit arc weight change.
  - Avoid use of heaps
  - Achieve a factor of 2 | 5 speedup w.r.t. Ramalingam & Reps on these test problems

## AT&T Worldnet backbone network (90 routers, 274 links)

1 hour  
run



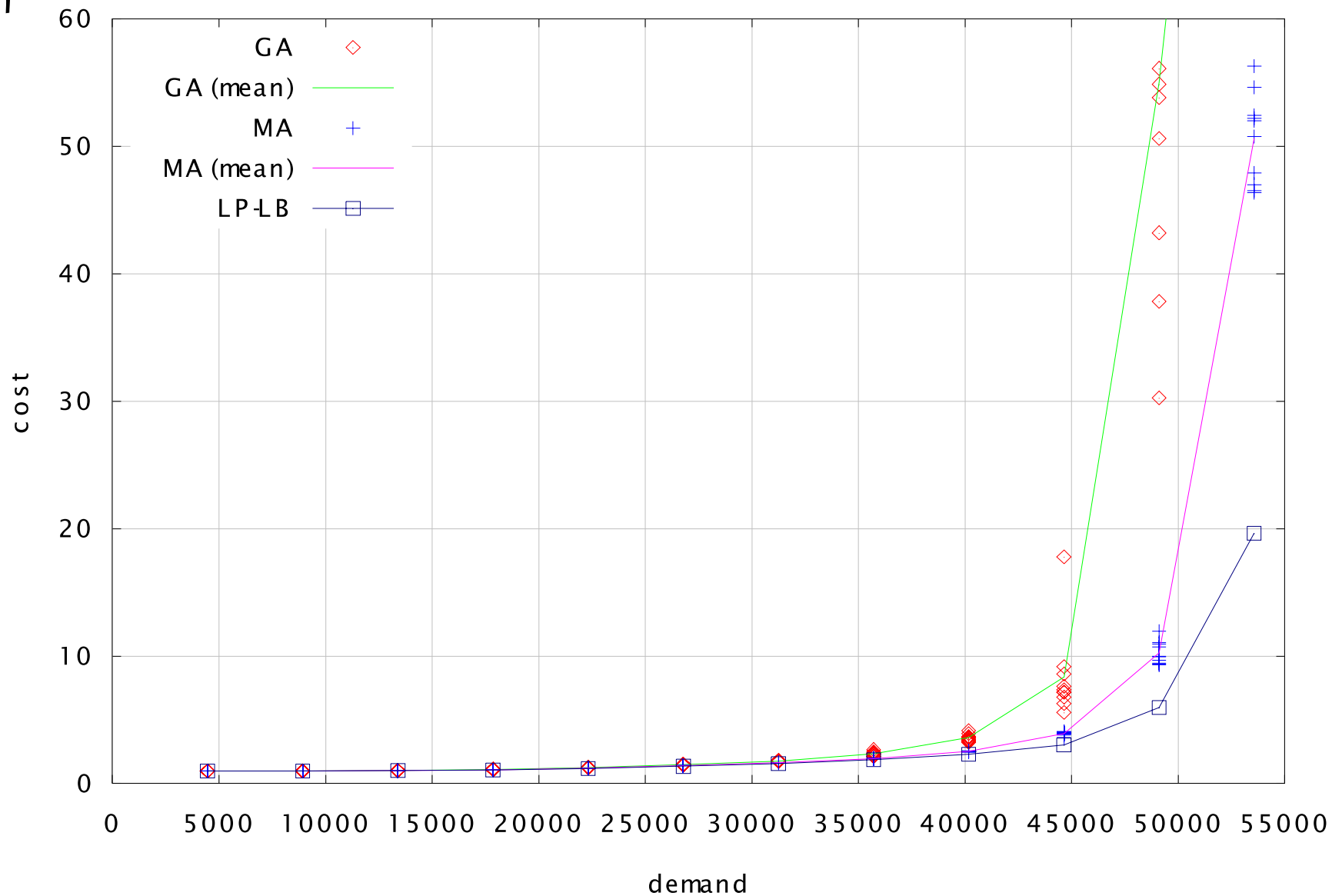
cumulative probability



## Rand50a: random graph with 50 nodes and 245 arcs.

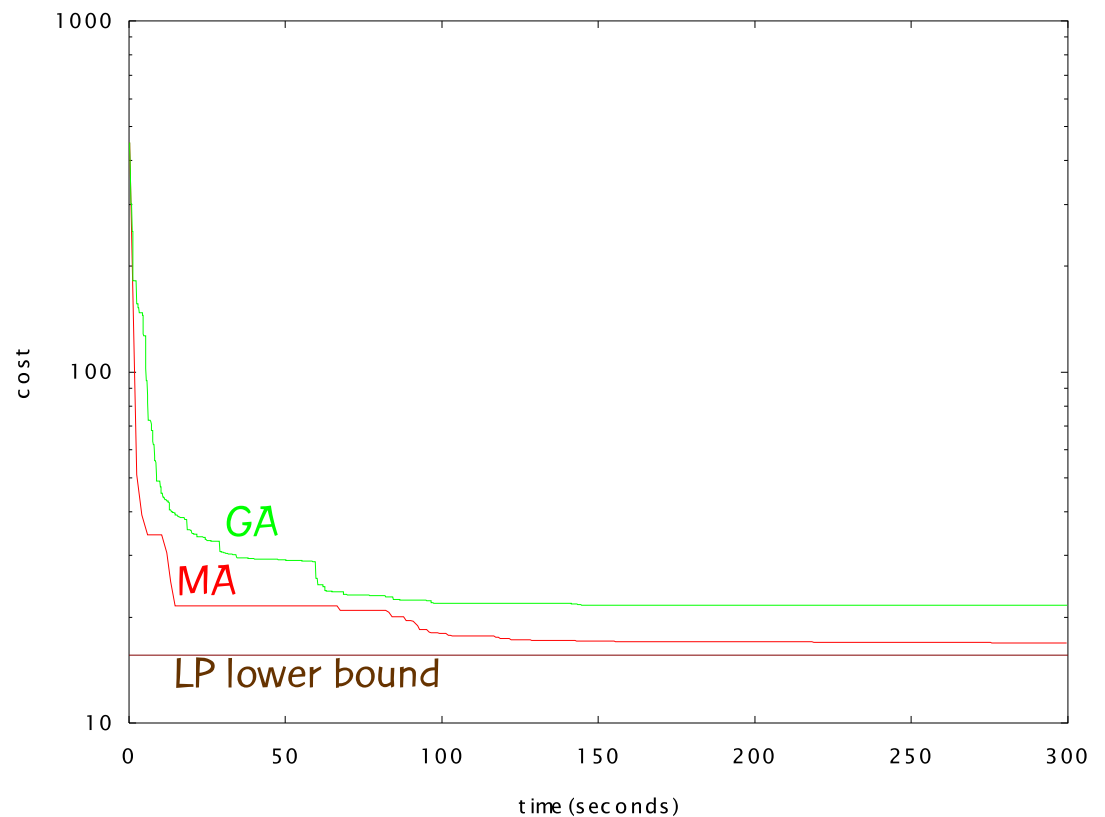
rand50a

1 hour run



# Remark

- Memetic algorithm (MA) improves over pure genetic algorithm (GA) in two ways:
  - Finds solutions faster
  - Finds better solutions





# Application 6:

## Survivable IP network design

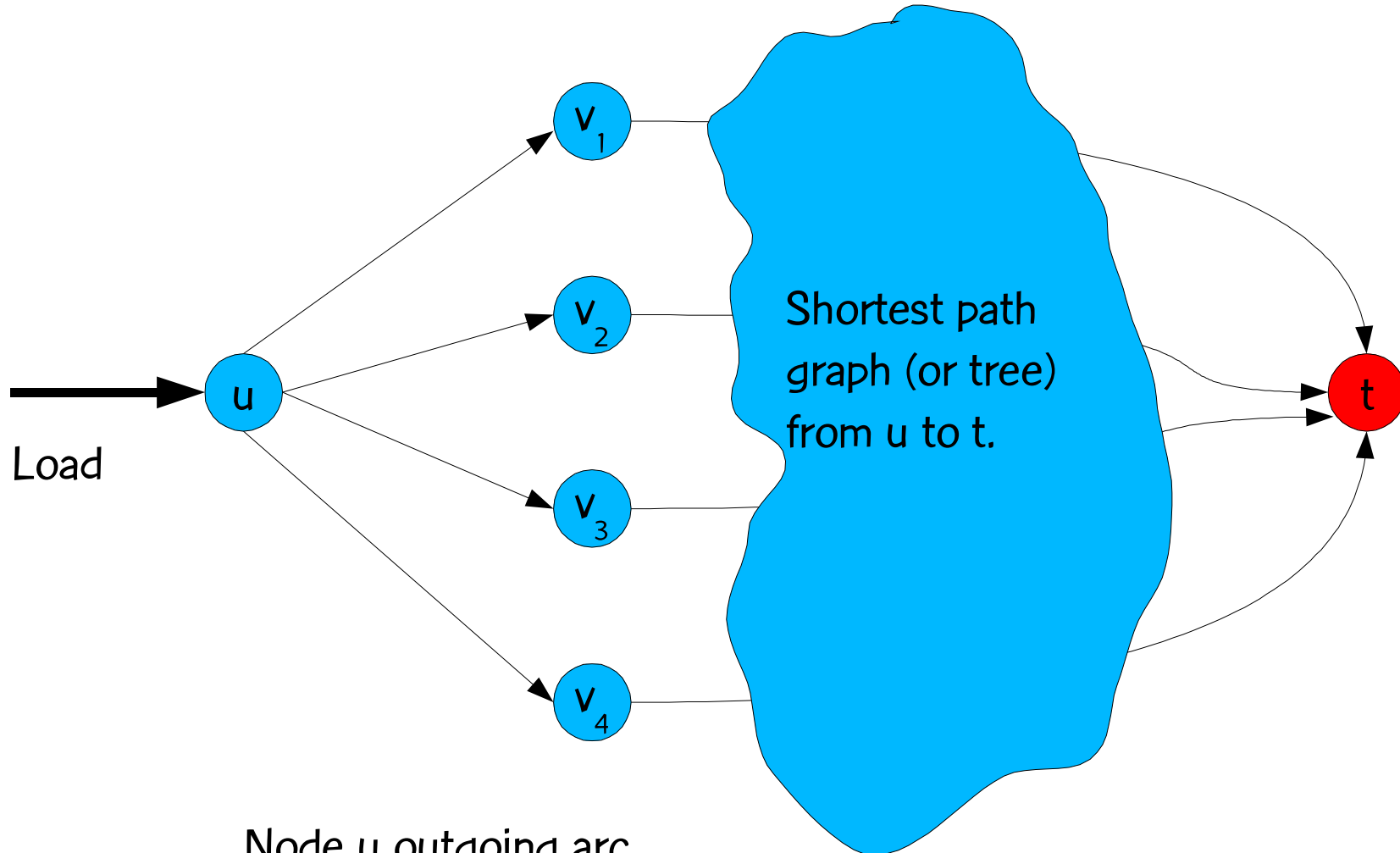
# Survivable IP network design

- Given
  - $G = (N, A)$ , where:
    - $N$  is the set of routers
    - $A$  is the set of potential arcs where capacity can be installed.
  - Demand matrix  $D = [d]$ , such that for each  $(u, v) \in N \cdot N$ 
    - $d(u, v)$  is the traffic demand from router  $u$  to router  $v$ .
  - Single link capacity  $M$

# Survivable IP network design

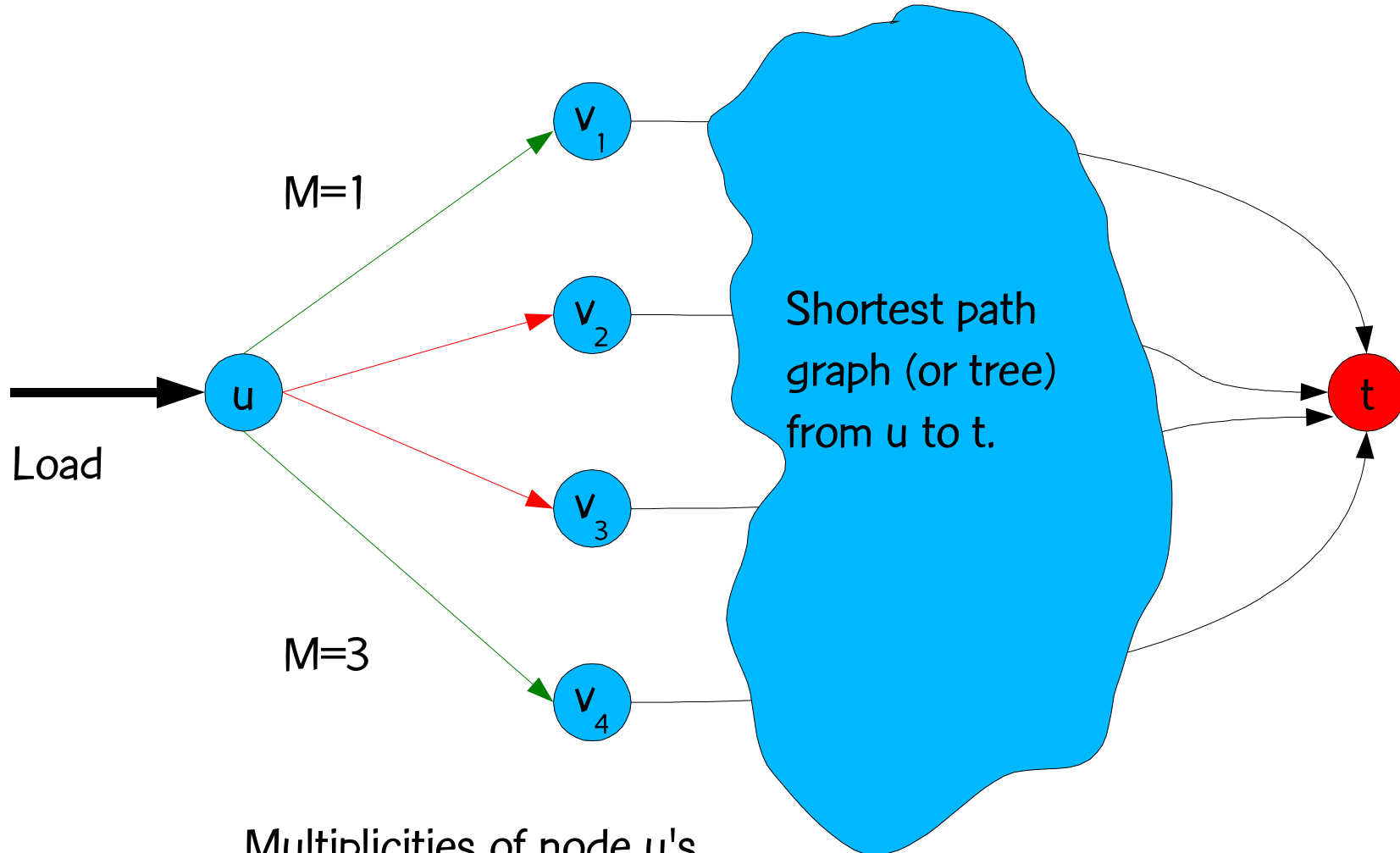
- Determine, for each arc  $a$ 
  - OSPF weight  $w_a \in [1, w_{\max}]$
  - Number of links of capacity  $M$  installed in arc  $a$  (arc multiplicity)
- Such that
  - There is sufficient capacity to route all of the demand
  - Using OSPF routing with traffic splitting
  - Subject to single router or single arc failure

# Traffic splitting



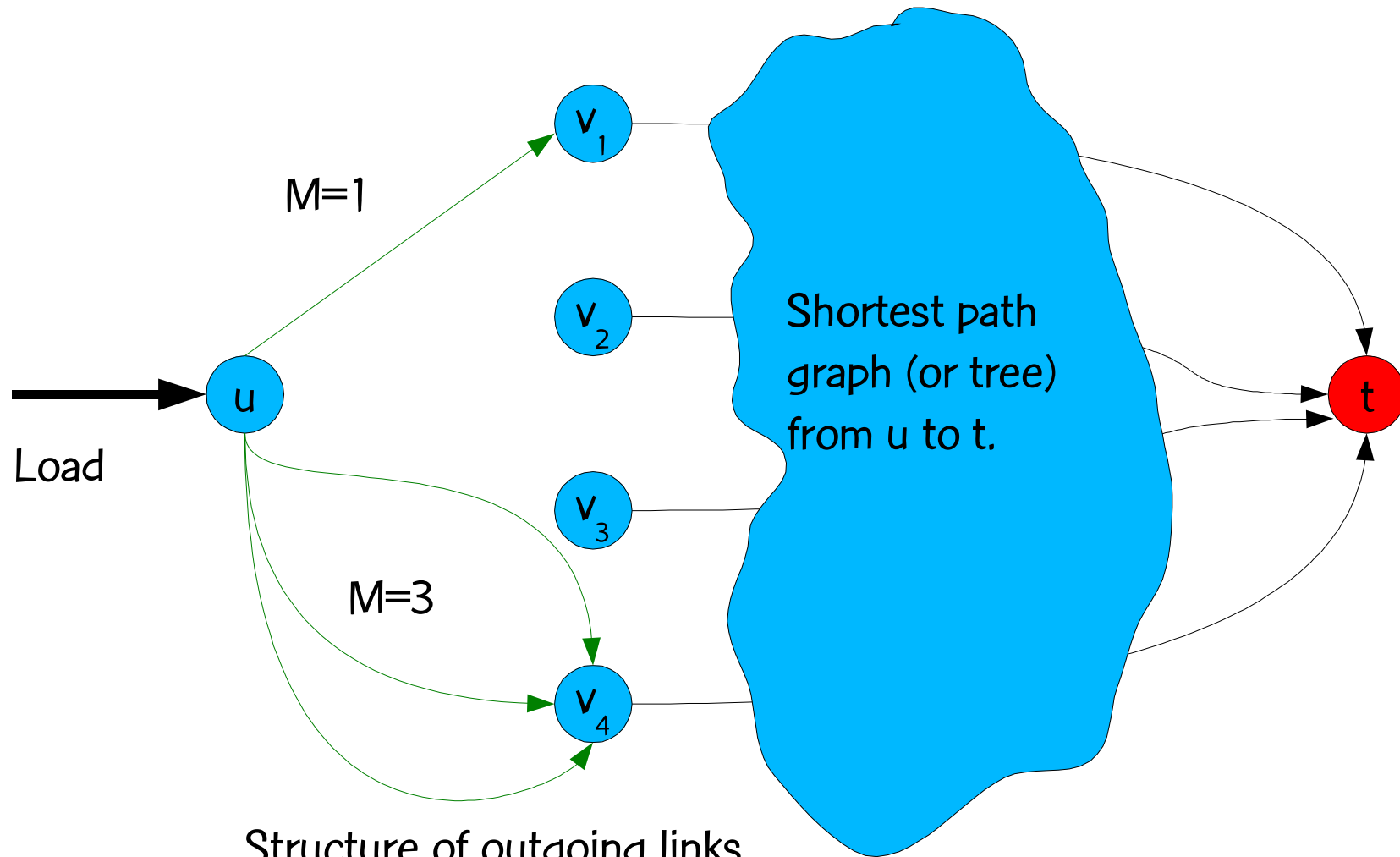
Node  $u$  outgoing arc  
structure

# Traffic splitting



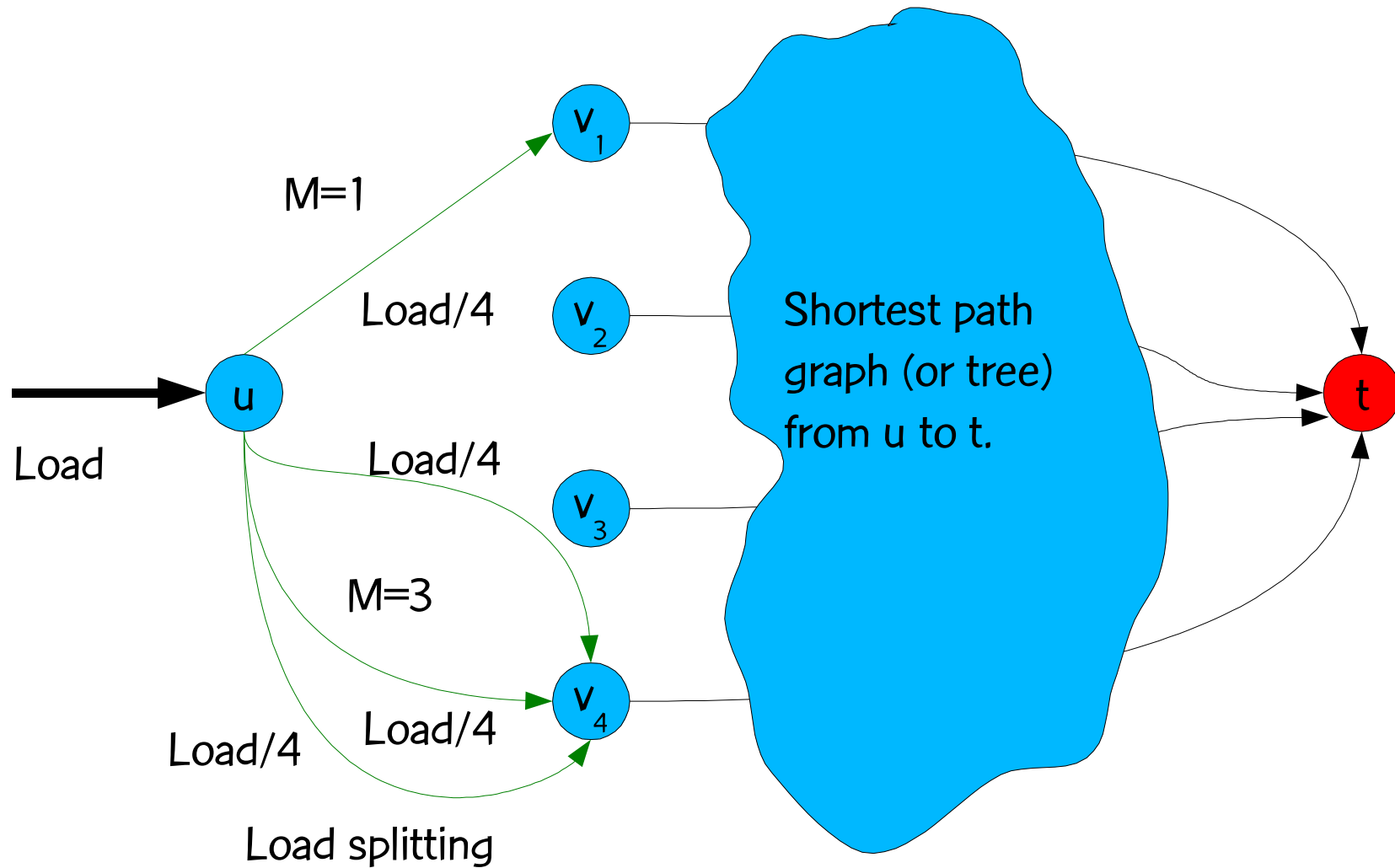
Multiplicities of node  $u$ 's  
outgoing arcs

# Traffic splitting



Structure of outgoing links  
of node  $u$

# Traffic splitting

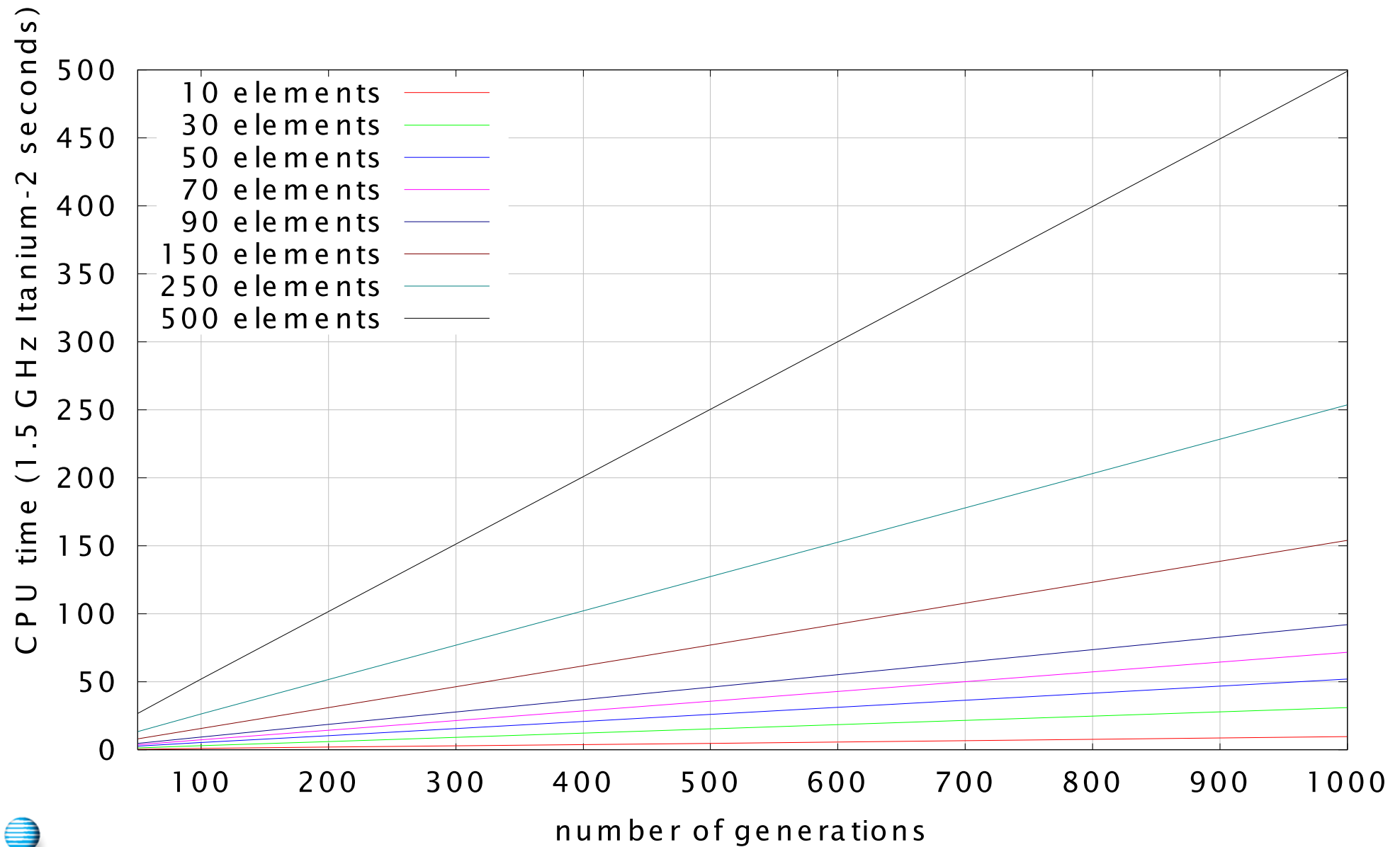


# Genetic algorithm for no-failure case

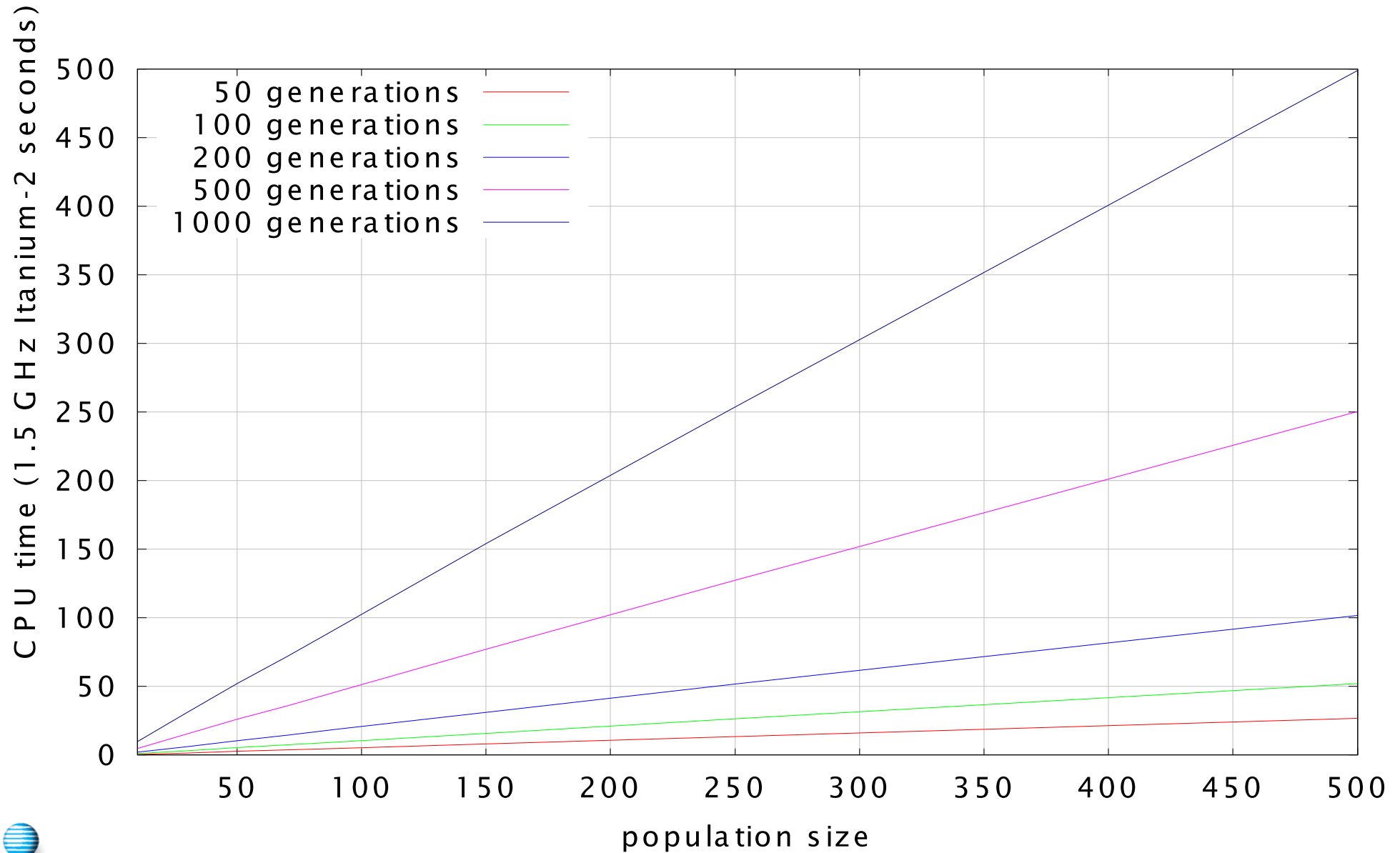
- Solutions are OSPF weight vectors.
- A OSPF weight vector defines shortest path graphs on which routing is done.
- Assume each arc has unit multiplicity.
- Repeat until feasible capacity/load is achieved:
  - Route demand and determine loads on arcs.
  - Determine arc multiplicities to insure minimum arc capacities required to flow loads on arcs. Multiplicities are never decreased.



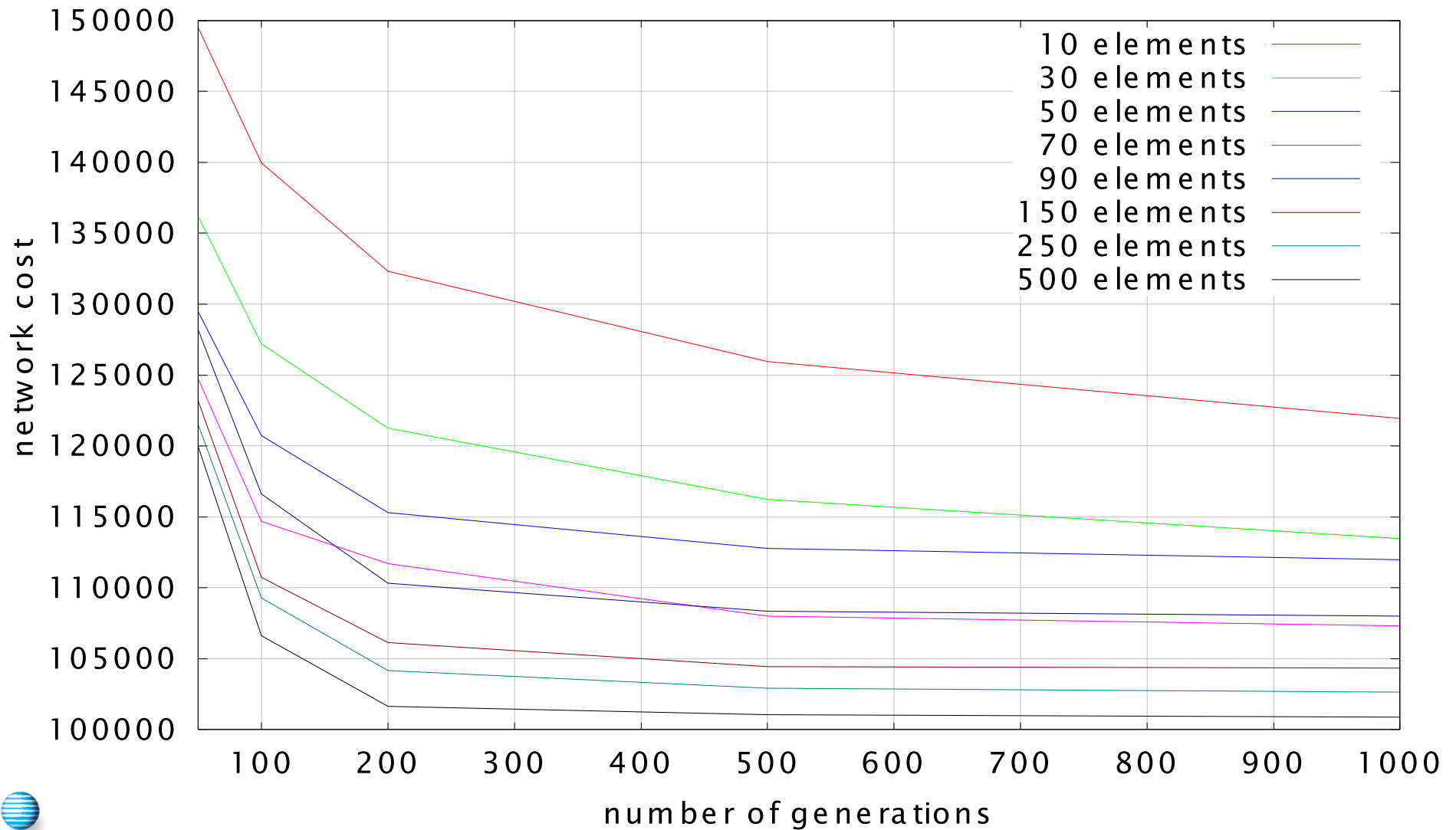
Running time: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs  
No router or arc failure.



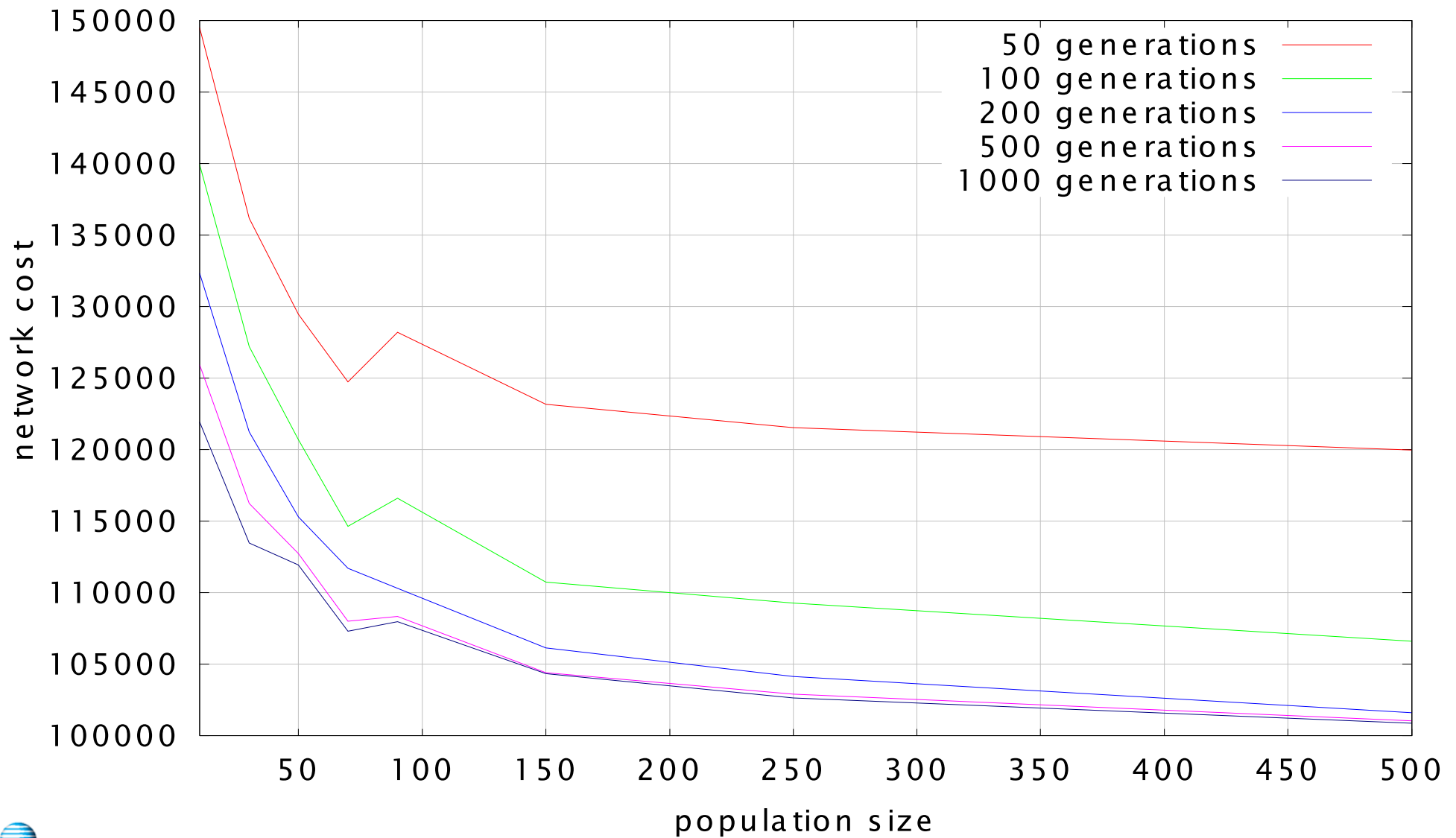
Running time: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs  
No router or arc failure.



Network cost: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs  
No router or arc failure.



Network cost: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs  
No router or arc failure.

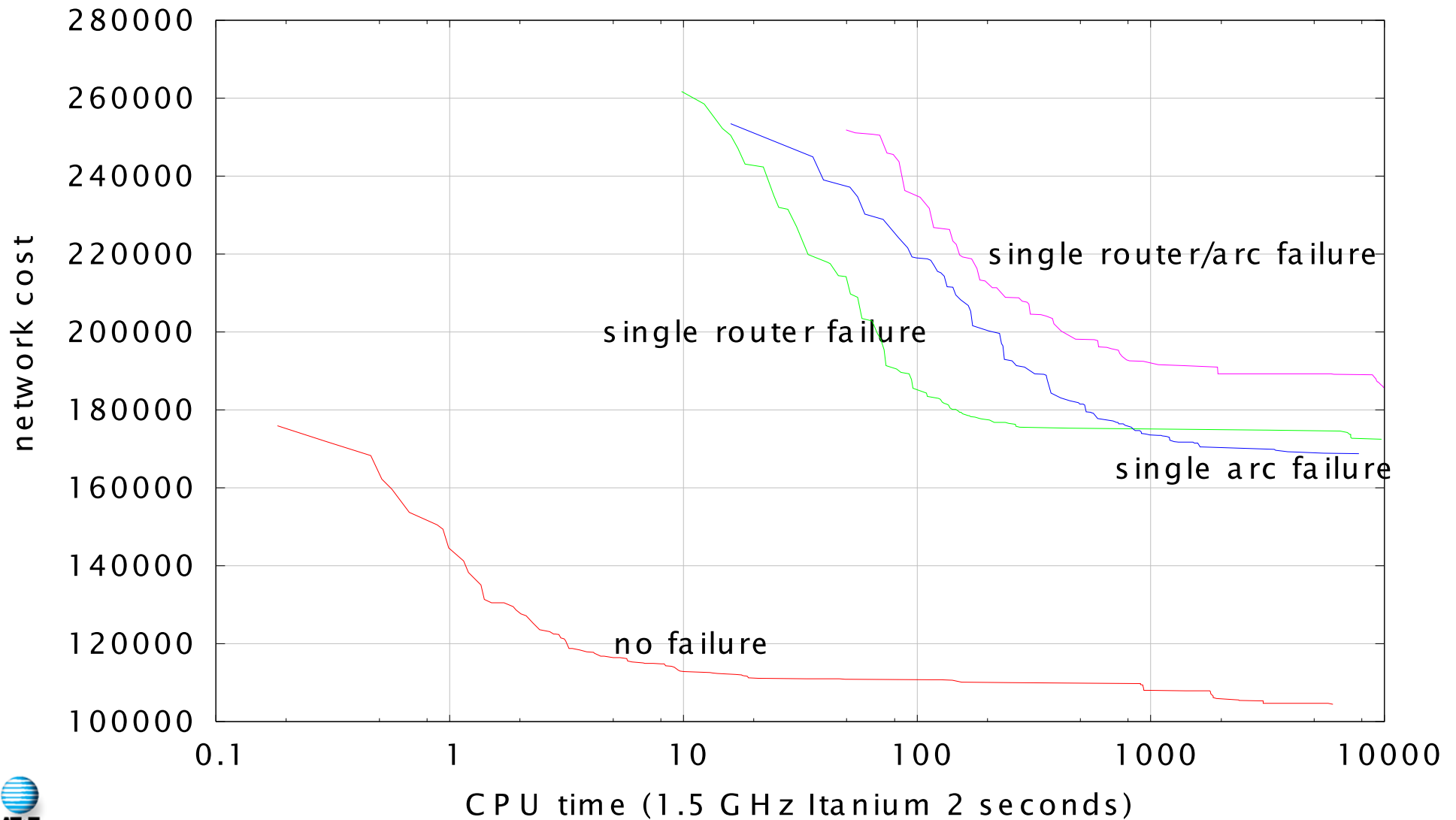


# Genetic algorithm for single-failure case

- Algorithm similar to no-failure case.
- Compute multiplicities for no-failure configuration and for each single-failure configuration.
- For each arc, set its multiplicity to be the maximum multiplicity over all simulated configurations.

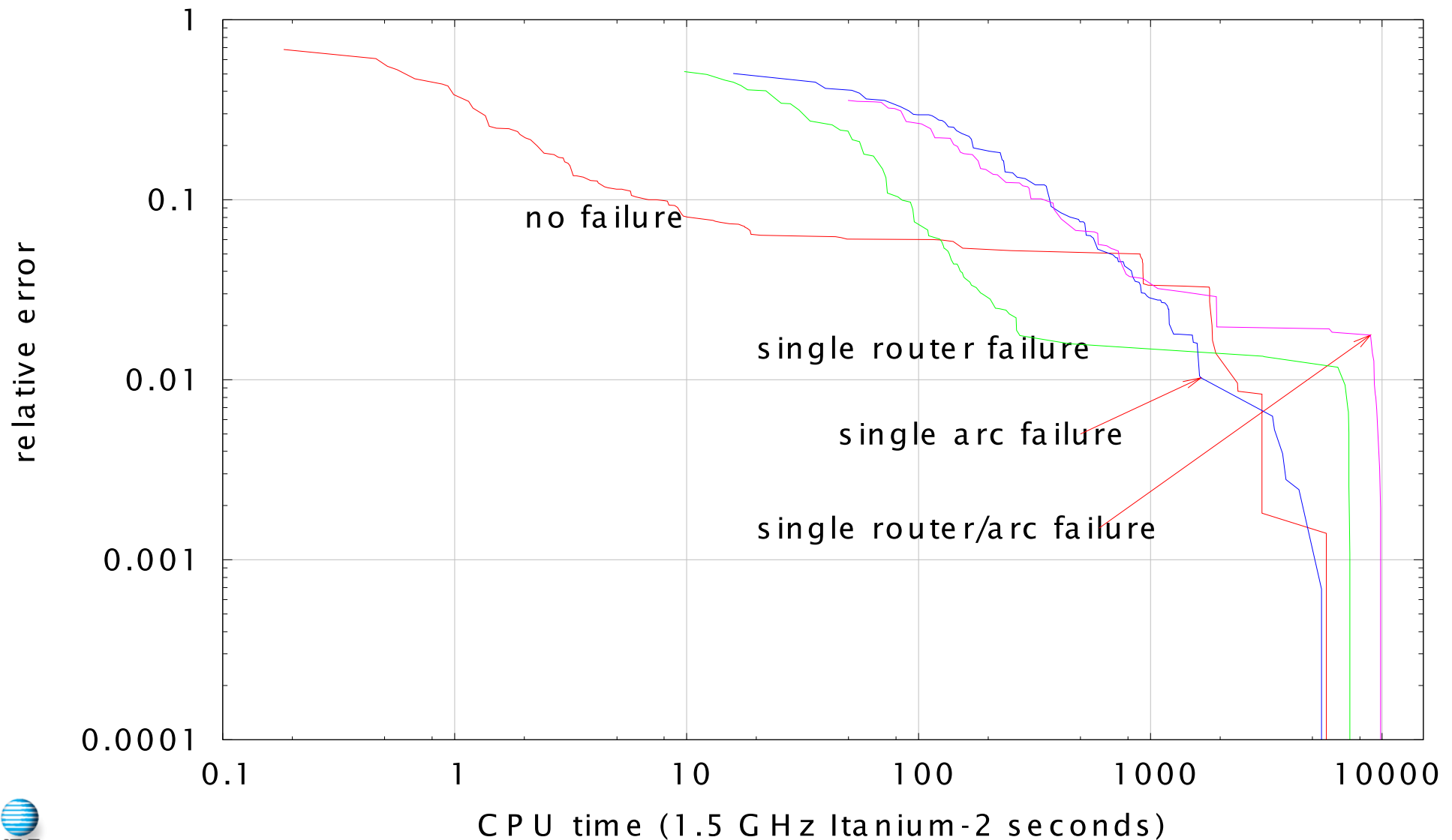
Network cost: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs

No router or arc failure, single-router failure, single-arc failure, and single-router or single-arc failure.

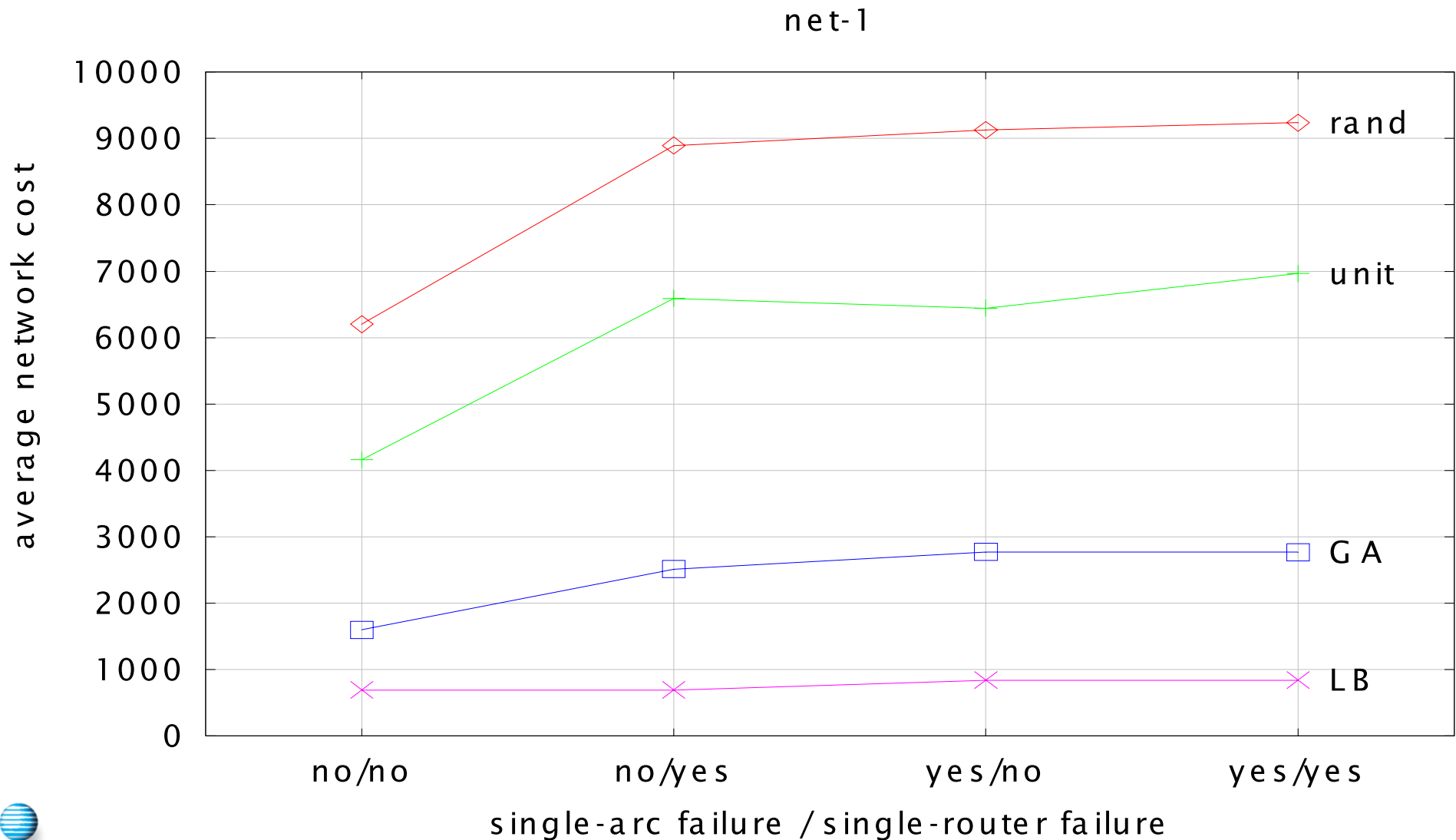


Relative error of network cost: 74-router, 278-arc, 18-terminal nodes,  
306 demand pairs.

No router or arc failure, single-router failure, single-arc failure, and single-router  
or single-arc failure.

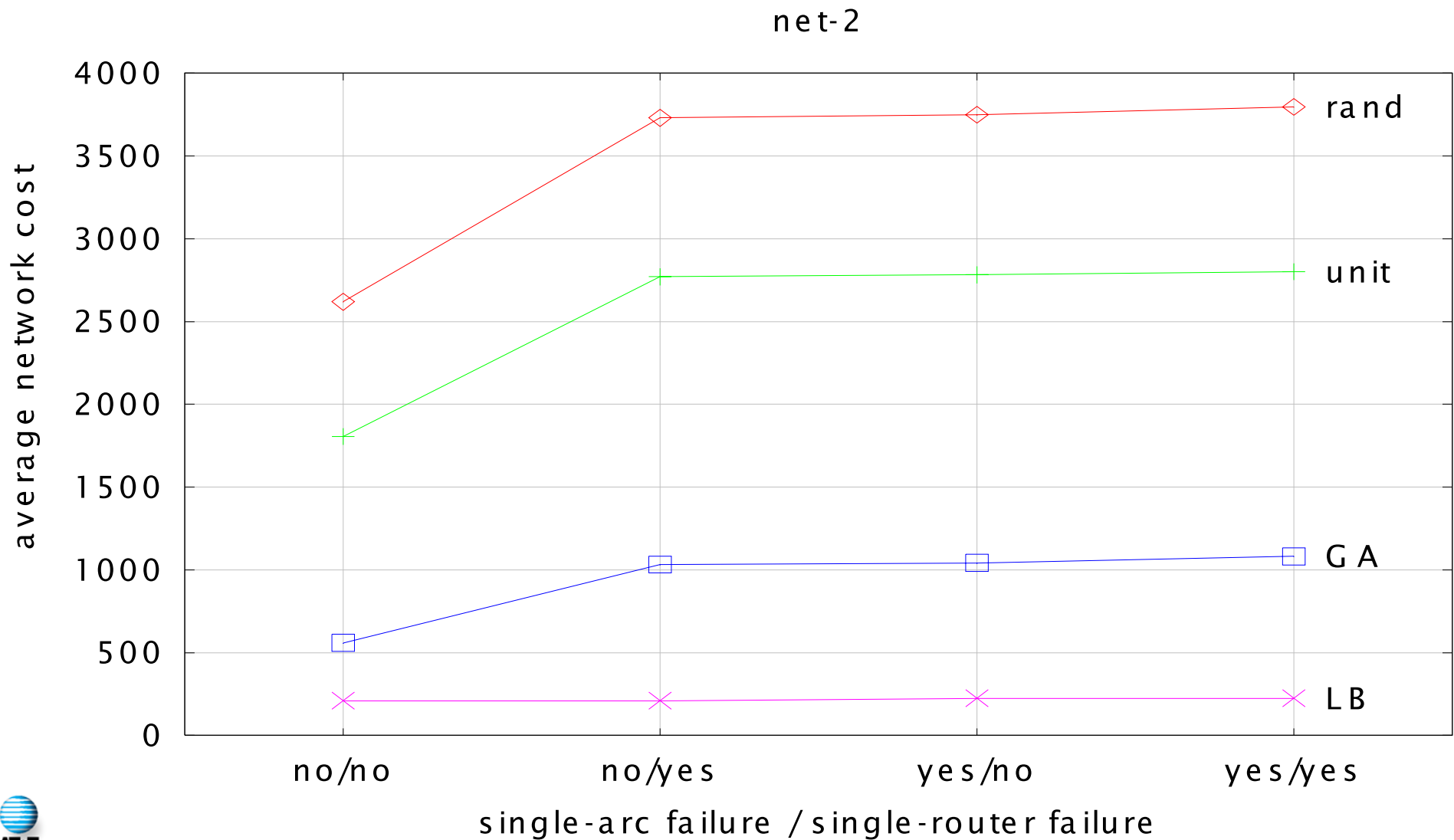


Average network costs for random weights, unit weights, GA weights compared to lower bound. Network has 10 routers, 90 arcs, 10 terminal nodes, and 90 demand pairs.

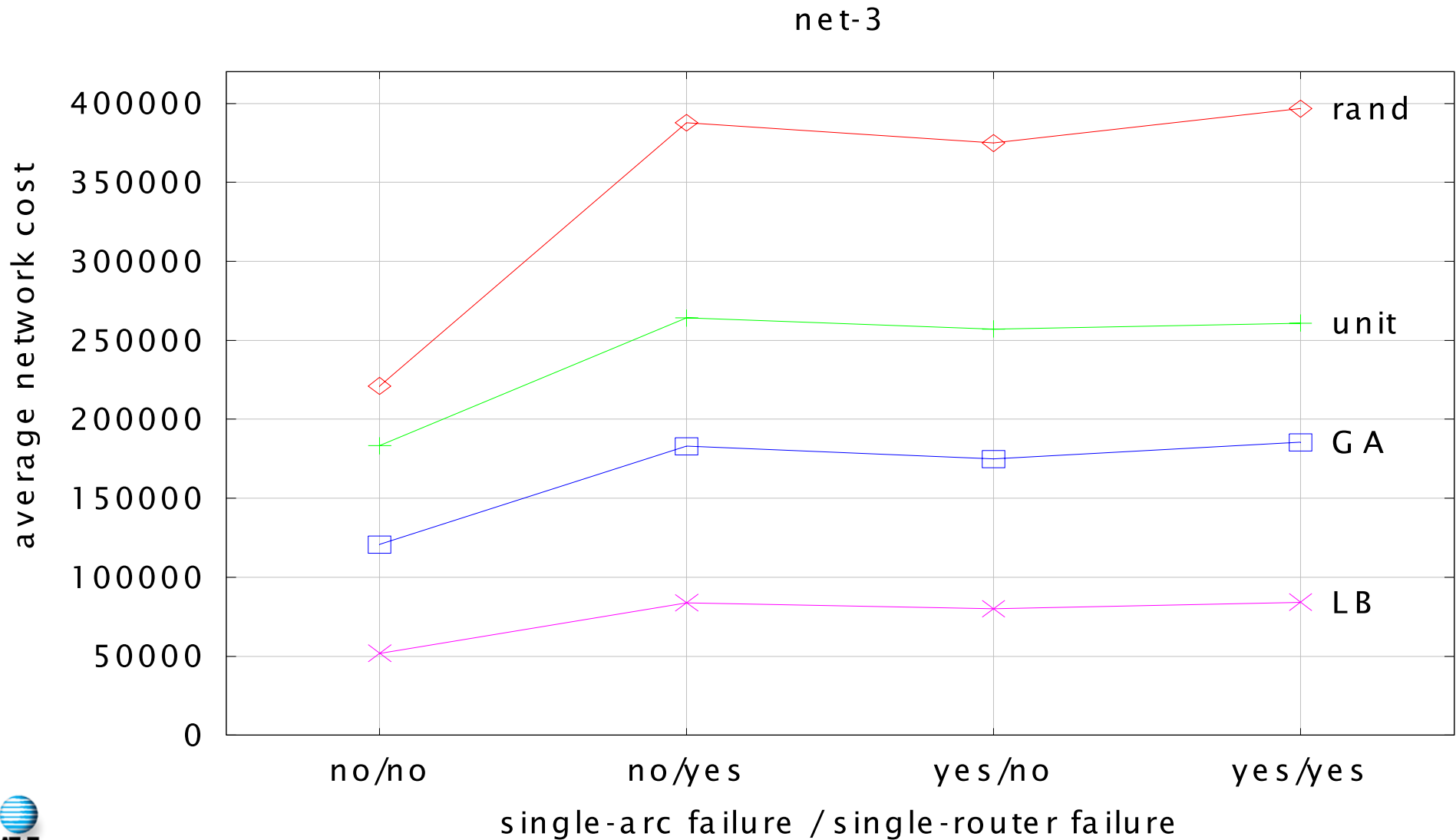




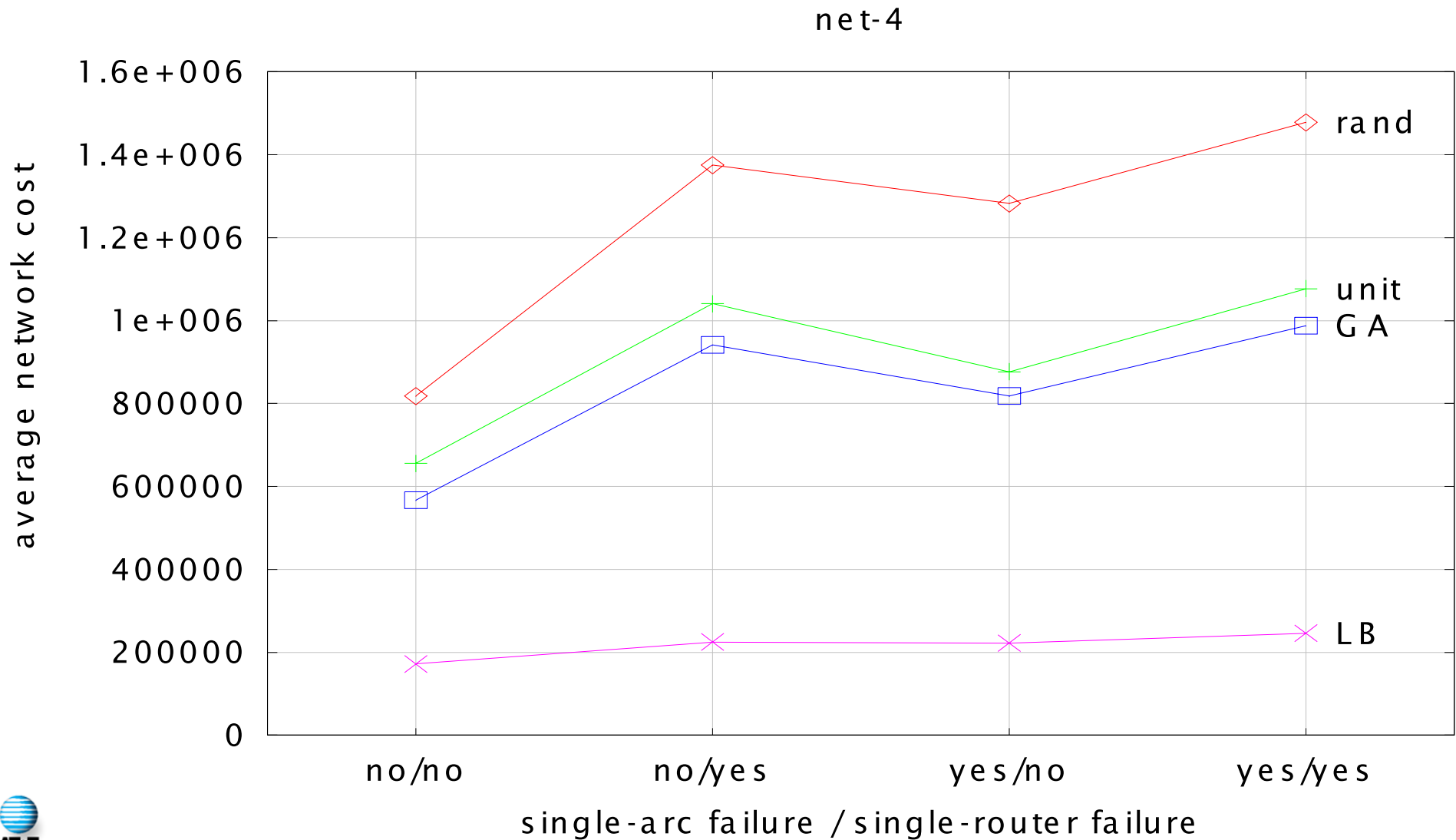
Average network costs for random weights, unit weights, GA weights compared to lower bound. Network has 11 routers, 110 arcs, 11 terminal nodes, and 110 demand pairs.



Average network costs for random weights, unit weights, GA weights compared to lower bound. Network has 74 routers, 278 arcs, 18 terminal nodes, and 306 demand pairs.



Average network costs for random weights, unit weights, GA weights compared to lower bound. Network has 71 routers, 350 arcs, 71 terminal nodes, and 4960 demand pairs.



# Paper

L.S. Buriol, M.G.C. Resende, and M. Thorup, "Survivable IP network design with OSPF routing," AT&T Labs Technical Report TD-64KUAW, September 2004.

# Concluding Remarks

- we have seen a small sample of applications of optimization in telecommunications
- opportunities for optimization arise in practice all the time
- our profession call have a major impact in telecommunications

# Concluding remarks

- These slides, and papers about GRASP, path-relinking, and their telecom applications available at:  
<http://www.research.att.com/~mgcr>  
<http://graspheuristic.org>

# Handbook of Optimization in Telecommunications (HOT), P.M. Pardalos and M.G.C. Resende, eds. Springer, forthcoming in 2005.

- Dynamic programming
- Interior point methods for large-scale LP
- Decomposition methods in telecommunications
- Integer programming
- Lagrangean relaxation
- Minimum cost network flow algorithms
- Shortest path algorithms
- Multi-commodity flow in telecommunications
- Steiner tree problems in telecommunications
- Minimum spanning tree problems
- Metaheuristics
- Nonlinear programming
- Telecommunications network design
- Ring network design
- Computational large-scale linear programming
- Telecommunications access network design
- Network location in telecommunications
- Protection and dynamic optimization of optical networks
- Network location problems in telecommunications
- Optimization in wireless networks
- Optimization issues in combinatorial auctions
- New models for dynamic networks
- Optimization issues in distribution network design
- Optimization issues in network survivability
- Virtual path design
- Network grooming
- Network reliability in telecommunications
- Optimization issues in quality of service
- Frequency assignment problem
- Optimization in cellular phone networks
- Optimization issues in web search engines
- Optimization issues in IP routing
- Network planning in telecommunications
- Pricing and equilibrium in telecommunications
- Discrete multi-commodity network flow problems and applications in telecommunications
- Cliques and graph coloring in telecommunications
- Assignment problems
- Stochastic optimization in telecommunications
- Optimization issues in multicast trees
- Optimization of dynamic routing networks
- Stable paths in interdomain routing
- Network restoration
- Optimization in e-commerce
- Supernetworks

The End