## Combinatorial optimization

 for design and operations of telecommunication systemsTalk given at the Department of Systems and Industrial Engineering University of Florida - Gainesville, Florida
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## Mauricio G. C. Resende <br> Mauricio G.C. Resende <br> 

ATET Labs Research Florham Park, New Jersey mgcr@research.att.com www.research.att.com/ mgcr mgcr@research.att.com www.research.att.com/ ${ }^{\text {mgcr }}$<br>$\qquad$ r<br>AťT Labs Research<br>

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## ATET Research



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## Summary of talk

- Network migration
- Modem pool placement for Internet service provider
- Local access network design
- Traffic routing on a virtual private network
- Internet traffic engineering
- Survivable IP network design


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Application 1: Network migration scheduling

## Network migration scheduling

- Voice service is moving from traditional switch-based networks to modern IP networks.
- Traffic has to be transitioned from old network to new network.
- How traffic transition is done can lead to different costs.


## Network migration scheduling

- Inter-nodal traffic from an outdated network is migrated to a new network.
- All traffic originating or terminating at a given node in the outdated network is moved to a specific node in the new network.
- Routing is predetermined in both networks and therefore capacities are known.


## Network migration scheduling



Traffic between nodes in the same network is routed in that network.

## Network migration scheduling



## Network migration scheduling



Suppose node $y_{0}$ in the old network is migrated to node $y_{n}$ in the new network.

## Network migration scheduling



Suppose node $y_{0}$ in the old network is migrated to node $y_{n}$ in the new network.

## Network migration scheduling

- When node $y_{0}$ is migrated to $y_{n}$ in the new network, one or more temporary links may have to be used, since node y ${ }_{0}$ may be adjacent to one or more still-active nodes in the old network.
old network


## Network migration scheduling

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## Network migration scheduling problem

- Find a migration ordering of the vertices such that the maximum sum of the capacities of the temporary links is minimized.



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min cut linear arrangement problem


## GRASP with Path-relinking for MCLA

repeat \{
$\Pi=$ GreedyRandomizedConstruction( $\Pi$ );
$\Pi=$ LocalSearch $(\Pi) ;$
$\Pi=$ PathRelinking( $\Pi) ;$
save $\Pi$ as $\Pi^{*}$ if best so far;
\}
return $\Pi^{*}$;

## Path-relinking (Glover, 1996)

- Exploration of trajectories that connect high quality (elite) solutions:



## Path-relinking

- Path is generated by selecting moves that introduce in the initial solution attributes of the guiding solution.
- At each step, all moves that incorporate attributes of the guiding solution are evaluated and the best move is selected:
initial
solution
- guiding
solution


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Path-relinking for the MCLA problem


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Path-relinking for the MCLA problem


Path-relinking for the MCLA problem

## Reference

- M.G.C. Resende \& C.C. Ribeiro, "GRASP with pathrelinking: Recent advances and applications," in "Metaheuristics: Progress as Real Problem Solvers," Ibaraki, Nonobe and Yagiura, (Eds.), pp. 29-63, Springer, 2005.


## A real-world migration example

- Old network has 140 switches (nodes) and 9730 trunks (links).
- Traffic between switches is known.
- One switch is "deloaded" at each time period.
- All traffic into (out of) deloaded switch is moved to new network.
- New trunks may have to be temporarily deployed to handle the traffic between the old and new networks.



cut $\quad 1.8 e+07$


## capacity










## Another example: phone migration

- Phone migration occurs when an organization upgrades to a newer phone switch (PBX).
- All phones using the old PBX must be moved to the new PBX.
- Each phone belong to one of more sets of phones that need to be moved together in same time period.
- Given penalties for not moving a pair of phones together and a maximum number of phones that can be moved in a time period, find groupings such that total penalty is minimized.


## Multi-line hunt group



## Multi-line hunt group



If phone does not answer, go on to next phone.
(5 to 100 phones in group)

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## Call pickup (CPU)

Any phone in group can pickup call for any other phone in group.


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## Intercomm (ICOM)

Allows speed dialing between group members.


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Allows speed dialing between group members.


## Series completion



## Series completion

If call not answered ...

## Series completion

If call not answered, it moves to .........................next in series.


## Series completion

If call not answered, it moves to . next in series.


## Series completion

If call not answered, it moves to ................................................................................... next in series.


## Series completion

If call not answered, it moves to next in series ...

... until it is finally answered by voice mail.

## Shared TN

Assistant answers all calls to group.

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Assistant answers all calls to group.


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## Real-world example

- 8 periods, 2855 phone numbers, 397 groups
- At most 375 phones can be moved in a period.
- Penalties:
- MLHG: 10
-CPU : 4
-ICOM: 3
-SC : 2
- STN : 1


## Penalty



Work with Diogo Andrade


Application 2: Modem pool placement for Internet service provider

## Modem pool placement for Internet service provider

- Worldnet: ATET's Internet Service Provider
- Dial-up: hundreds of points of presence (PoPs)
- Telephone numbers customers must call when making an Internet connection.
- Current footprint:
- 1305 PoPs;
- $77.66 \%$ of the telephone numbers in the U.S. can make local calls to Worldnet.


## Footprint Optimization

- In general: more PoPs, better coverage.
- For a fixed coverage, we don't want more PoPs than necessary.
- Not all PoPs are the same:
- Each has an associated network cost:
- Hourly rate paid by Worldnet to network company.
- Between \$0.04 and \$0.14 in the continental US.
- Up to \$0.42 in Hawaii and Alaska.
- No setup cost.


## Worldnet

- When is a call local ("free")?
- Not simply "within same area code".
- Telephone system divided into exchanges:
- Area code + first three digits (973360, for example).
- Each PoP has a coordinate.
- We know which exchanges can make local calls to each coordinate (the coverage).
- Just a big table;
- 69,534 exchanges covered by current footprint.
- Goal: keep only cheaper PoPs, preserve coverage.


## Footprint Optimization

- 270 PoPs could be eliminated by inspection:
- Dominated by cheaper PoPs
- 335 additional PoPs could be eliminated:
- Only 700 PoPs left;
- New footprint covers all exchanges currently covered;
- No exchange has to make a more expensive call.
- How did we do it?
- We solved this as a p-median problem.

n (=11) potential facility locations
$m$ (=15) users

n (=11) potential facility locations
$m$ (=15) users


> Users home into nearest open facility.
$\mathrm{n}(=11)$ potential service locations
m (=15) customers
$d(u, f)=$ cost of servicing user $u$ by facility f

## Footprint Optimization

- In our case:
- each exchange is a p-median user:
- 69,534 in total (all currently covered).
- each coordinate is a p-median facility:
- 1035 in total (all currently open).
- Distances: network cost.
- (PoP rate) • (hours used by exchange)
- With $p=1035$, we get the current network cost.
- We want the smallest p that preserves that cost.
- Solve the $p$-median problem for various values of $p$ to find best.
- 700 was the value we found.


## Expanding the Footprint

- Second problem:
- Increase coverage beyond $77.66 \%$.
- ATET can use UUNet PoPs:
- 1,498 candidate PoPs.
- 568 of those cover at least one new exchange.
- Main question:
- If we want to open p new PoPs, which PoPs do we open?
- Goal: maximize coverage.
- This is the maximum cover problem:
- It can be solved as a p-median problem.


## Expansion



| Coverage | Footprint |
| :---: | :--- |
| $77.66 \%$ | current |
| $78 \%$ | current+3 |
| $79 \%$ | current+19 |
| $80 \%$ | current+41 |
| $81 \%$ | current+72 |
| $82 \%$ | current+113 |
| $83 \%$ | current+177 |
| $84 \%$ | current+301 |
| $84.27 \%$ | current+464 |

## References

- M.G.C. Resende, "Computing approximate solutions of the maximum covering problem using GRASP," J. of Heuristics, vol. 4, pp. 161-171, 1998.
- M.G.C. Resende \& R.F. Werneck, "A hybrid heuristic for the p-median problem," J. of Heuristics, vol. 10, pp. 5988, 2004.
- M.G.C. Resende \& R.F. Werneck, "A fast swap-based local search procedure for location problems," to appear in Annals of Operations Research, 2005.

Application 3: Local access network design

## Local access network design

- We wish to roll out broadband service in different markets.
- We need to determine which markets we should go for.
- For each candidate market, estimate profit (or loss) associated with rolling out service.


## Local access network design

- Build a fiber-optic network for providing broadband connections to business and residential customers.
- Design a local access network taking into account trade-off between:
- cost of network
- revenue potential of network


## Local access network design

- Graph corresponds to local street map
- Edges: street segments
- Edge cost: cost of laying the fiber on the corresponding street segment
- Vertices: street intersections and potential customer premises
- Vertex penalty: estimate of potential loss of revenue if the customer were not to be serviced (intersection nodes have no penalty)


## Local access network design



## Collect all prizes

(Steiner problem in graphs)


## Collect some prizes

(Prize collecting Steiner Problem in Graphs)


## Multi-start heuristic

S. Canuto, M.G.C. Resende, \& C.C. Ribeiro, "Local search with perturbations for the prize-collecting Steiner tree problem in graphs," Networks, vol. 38, pp. 50-58, 2001

- Repeat:
- Perturb problem data and solve using approximation algorithm of Goemans and Williamson (1996);
- If solution is new, perform swap-based local search;
- Attempt to insert solution into POOL;
- Select solution at random from POOL and explore path from current iterate and POOL solution using pathrelinking;
- Starting from best POOL solution, apply variable neighborhood search;


## A cutting planes algorithm: Lower bound

A. Lucena \& M.G.C. Resende, "Strong lower bounds for the prize collecting Steiner tree problem in graphs," Discrete Applied Mathematics, vol. 141, pp. 277-294, 2004.

- Integer programming (IP) formulation
- Cutting planes algorithm to solve linear programming relaxation of IP

Application 4: Traffic routing on a virtual private network

## Traffic routing on a virtual private network

- Frame relay service offers virtual private networks to customers by providing long-term private virtual circuits (PVCs) between customer endpoints on a backbone network.
- Routing is done either automatically by switch or by the network designer without any knowledge of future requests.
- Over time, these decisions cause inefficiencies in the network and occasionally offline rerouting (grooming) of the PVCs is needed:
- integer multicommodity network flow problem: Resende $\mathcal{E}$ Ribeiro (2003)

Traffic routing on a virtual private network


Traffic routing on a virtual private network


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Traffic routing on a virtual private network


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## Reference

- M.G.C. Resende \& C.C. Ribeiro, "A GRASP with pathrelinking for private virtual circuit routing," Networks, vol. 41, pp. 104-114, 2003.

Application 5: Internet traffic engineering

## Internet traffic engineering

- Internet traffic has been doubling each year [Coffman $\varepsilon$ Odlyzko, 2001]
- In the1995-96 period, there was a doubling of traffic each three months!
- Web browsers were introduced.
- Increasingly heavy traffic (due to video, voice, etc.) will raise the requirements of the Internet of tomorrow.


## Internet traffic engineering

- Objective: make more efficient use of existing network resources.
- Routing of traffic can have a major impact on efficiency of network resource utilization.


## Packet routing

Packet's final destination.

When packet arrives at router, router must decide where to


Routing consists in finding a link-path from source to destination.

## OSPF (Open Shortest Path First)

- OSPF is a commonly used intradomain routing protocol (IGP).
- Routers exchange routing information with all other routers in the autonomous system (AS).
- Complete network topology knowledge is available to all routers, i.e. state of all routers and links in the AS.



## OSPF routing

- Assign an integer weight $\in\left[1, w_{\text {max }}\right]$ to each link in AS. In general, $w_{\text {max }}=65535=2^{16}-1$.
- Each router computes tree of shortest weight paths to all other routers in the AS, with itself as the root, using Dijkstra's algorithm.


## OSPF routing



## OSPF routing

## Routing table



Routing table is filled with first hop routers for each possible destination.

## OSPF routing

## Routing table



Routing table is filled with first hop routers for each possible destination.

## OSPF routing



## OSPF routing



## OSPF weight setting

- OSPF weights are assigned by network operator.
- CISCO assigns, by default, a weight proportional to the inverse of the link bandwidth (Inv Cap).
- If all weights are unit, the weight of a path is the number of hops in the path.
- We propose a hybrid genetic algorithm to find good OSPF weights.
- Memetic algorithm
- Genetic algorithm with optimized crossover


## Minimization of congestion

- Consider the directed capacitated network $G=(N, A, c)$, where $N$ are routers, $A$ are links, and $c_{a}$ is the capacity of link $a \in A$.
- We use the measure of Fortz \& Thorup (2000) to compute congestion:

$$
\Phi=\Phi_{1}\left(\mathrm{I}_{1}\right)+\Phi_{2}\left(\mathrm{I}_{2}\right)+\ldots+\Phi_{|\mathrm{A}|}\left(\mathrm{I}_{|\mathrm{A}|}\right)
$$

where $I_{a}$ is the load on link $a \in A$,

$$
\begin{aligned}
& \Phi_{a}\left(I_{a}\right) \text { is piecewise linear and convex, } \\
& \Phi_{a}(0)=0, \text { for all } a \in A .
\end{aligned}
$$

## Piecewise linear and convex $\Phi_{a}\left(l_{a}\right)$ link congestion measure



## OSPF weight setting problem

- Given a directed network $G=(N, A)$ with link capacities $c_{\mathrm{a}} \in A$ and demand matrix $D=\left(d_{\mathrm{s}, \mathrm{t}}\right)$ specifying a demand to be sent from node $s$ to nodet:
- Assign weights $w_{a} \in\left[1, w_{\max }\right]$ to each link $a \in A$, such that the objective function $\Phi$ is minimized when demand is routed according to the OSPF protocol.

ATET Worldnet backbone network (90 routers, 274 links)


## Genetic and hybrid genetic algorithms for OSPF weight setting problem

- Genetic
- M. Ericsson, M.G.C. Resende, E P.M. Pardalos, " A genetic algorithm for the weight setting problem in OSPF routing, J. of Combinatorial Optimization, vol. 6, pp. 299333, 2002.
- Hybrid genetic
- L.S. Buriol, M.G.C. Resende, C.C. Ribeiro, \& M. Thorup, "A hybrid genetic algorithm for the weight setting problem in OSPF/IS-IS routing," Networks, vol. 46, pp. 36-56, 2005.


## Genetic algorithms



## Solution encoding

- A population consists of $n P o p=50$ integer weight arrays: $w=\left(w_{1}, w_{2}, \ldots, w_{|A|}\right)$, where $w_{a} \in\left[1, w_{\text {max }}=20\right]$
- All possible weight arrays correspond to feasible solutions.


## Initial population

- nPop solutions, with each weight randomly generated, uniformly in the interval [ $1, \mathrm{w}_{\max } / 3$ ].


## Solution evaluation

- For each demand pair ( $s, t$ ), route using OSPF, computing demand pair loads $I_{a}^{\text {s.t }}$ on each link $a \in A$.
- Add up demand pair loads on each link $a \in A$, yielding total load $I_{a}$ on link.
- Compute link congestion cost $\Phi_{a}\left(I_{a}\right)$ for each link $a \in A$.
- Add up costs: $\Phi=\Phi_{1}\left(I_{1}\right)+\Phi_{2}\left(I_{2}\right)+\ldots+\Phi_{|A|}\left(I_{|A|}\right)$


## Population partitioning

| Class A |  |
| :---: | :---: |
| Class B |  |
| Class C most fit | Population is sorted according to <br> solution value $\Phi$ and solutions are <br> classified into three categories. |
|  | $5 \%$ least fit |

## Population dynamics

## generation t



## Population dynamics



## Population dynamics



## Population dynamics

## generation $t$ <br> generation $t+1$



Class A

## Class C

Class $C$ is replaced by randomly generated solutions.

## Population dynamics

## generation $t$ <br> generation $\mathrm{t}+1$



Class A

Class C
Class $C$ is replaced by randomly generated solutions.

## Population dynamics



## Parent selection

- Parents are chosen at random:
- one parent from Class A (elite).
- one parent from Class B or C (non-elite).
- Reselection is allowed, i.e. parents can breed more than once per generation.
- Better individuals are more likely to reproduce.


## Crossover with random keys Bean (1994)

Crossover combines elite parent $\mathrm{P}_{1}$ with non-elite parent $\mathrm{P}_{2}$ to produce child c:

With small probability child has single gene mutation.

Child is more likely to inherit gene of elite parent.

```
for all genes i=1,2,..., |A | do
    if rrandom[0,1]<0.01 then
        c[i] = irandom[1, W max ]
    else if rrandom[0,1] < 0.7 then
        c[i]= p, [i ]
    else c [i] = p [i ]
    end
```



ATET Worldnet backbone network (90 routers, 274 links)


ATET Worldnet backbone network (90 routers, 274 links)


Rand50a: random graph with 50 nodes and 245 arcs.

RAND50A


## Optimized crossover $=$ crossover + local search



## Fast local search

- Let $A^{*}$ be the set of five arcs $a \in A$ having largest $\Phi_{a}$ values.
- Scan arcs $a \in A^{*}$ from largest to smallest $\Phi_{a}$ :
- Increase arc weight, one unit at a time, in the range

$$
\left[w_{a}, w_{a}+\left\lceil\left(w_{\max }-w_{a}\right) / 4\right\rceil\right]
$$

- If total cost $\Phi$ is reduced, restart local search.


## Dynamic shortest path

- In local search, when arc weight increases, shortest path trees:
- may change completely (rarely do)
- may remain unchanged (e.g. arc not in a tree)
- may change partially
- Few trees change
- Small portion of tree changes



## Dynamic shortest path

Consider one tree at a time.


## Dynamic shortest path

Arc weight is increase by 1 .


## Dynamic shortest path



AT\&T

## Dynamic shortest path



## Dynamic shortest path



Shortest paths from red nodes must traverse blue arc.

## Dynamic shortest path



Test all arcs of type

If $d-d=w$, then

tree.

## Dynamic shortest path



## Dynamic shortest path

L.S. Buriol, M.G.C. Resende, \& M. Thorup, "Speeding up dynamic shortest path algorithms," AT\&T Labs Research Report, 2003.

- Ramalingam \& Reps (1996) allow arbitrary arc weight change.
- We specialized the Ramalingam \& Reps algorithm for unit arc weight change.
- Avoid use of heaps
- Achieve a factor of $2 \sim 5$ speedup w.r.t. Ramalingam E Reps on these test problems

ATET Worldnet backbone network (90 routers, 274 links)



Rand50a: random graph with 50 nodes and 245 arcs.
rand50A

## 1 hour run



## Remark

- Memetic algorithm (MA) improves over pure genetic algorithm (GA) in two ways:
- Finds solutions faster
- Finds better solutions



## Application 6:

## Survivable IP network

 design
## Survivable IP network design

- Given
- $G=(N, A)$, where:
- N is the set of routers
- $A$ is the set of potential arcs where capacity can be installed.
- Demand matrix $D=[d]$, such that for each $(u, v) \in N \times N$
- $\mathrm{d}(\mathrm{u}, \mathrm{v})$ is the traffic demand from router u to router v .
- Single link capacity M


## Survivable IP network design

- Determine, for each arc a
- OSPF weight $w_{a} \in\left[1, w_{\text {max }}\right]$
- Number of links of capacity $M$ installed in arc a (arc multiplicity)
- Such that
- There is sufficient capacity to route all of the demand
- Using OSPF routing with traffic splitting
- Subject to single router or single arc failure


## Traffic splitting



## Traffic splitting



## Traffic splitting



## Traffic splitting



## Genetic algorithm for no-failure case

- Solutions are OSPF weight vectors.
- A OSPF weight vector defines shortest path graphs on which routing is done.
- Assume each arc has unit multiplicity.
- Repeat until feasible capacity/load is achieved:
- Route demand and determine loads on arcs.
- Determine arc multiplicities to insure minimum arc capacities required to flow loads on arcs. Multiplicities are never decreased.


## Genetic algorithm for single-failure case

- Algorithm similar to no-failure case.
- Compute multiplicities for no-failure configuration and for each single-failure configuration.
- For each arc, set its multiplicity to be the maximum multiplicity over all simulated configurations.

Network cost: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs No router or arc failure, single-router failure, single-arc failure, and single-router or single-arc failure.


Relative error of network cost: 74-router, 278-arc, 18-terminal nodes, 306 demand pairs.
No router or arc failure, single-router failure, single-arc failure, and single-router or single-arc failure.


Average network costs for random weights, unit weights, GA weights compared to lower bound. Network has 74 routers, 278 arcs, 18 terminal nodes, and 306 demand pairs.

NET 3


## Refence

L.S. Buriol, M.G.C. Resende, and M. Thorup, "Survivable IP network design with OSPF routing," ATET Labs Technical Report TD-64KUAW, September 2004. To appear in Networks.

## Concluding Remarks

- we have seen a small sample of applications of optimization in telecommunications
- opportunities for optimization arise in practice all the time
- our profession call have a major impact in telecommunications


## Concluding remarks

- These slides, and papers about GRASP, path-relinking, and their telecom applications available at: http://www.research.att.com/~mgcr http://graspheuristic.org

Handbook of Optimization in Telecommunications (HOT), M.G.C. Resende and P.M. Pardalos, eds. Springer, forthcoming in 2006.

37 chapters
79 authors
1162 pages

- Part I: Optimization algorithms
- Part II: Planning and design
- Part III: Routing
- Part IV: Reliability, restoration, and grooming
- Part V: Wireless
- Part VI: The web and beyond


## Part I: Optimization algorithms

- Interior point methods for large scale linear programming
- Nonlinear programming in telecommunications
- Integer programming for telecommunications
- Metaheuristics and applications to problems in telecommunications
- Minimum cost network flow algorithms
- Multicommodity network flow models and algorithms
- Shortest path algorithms


## Part II: Planning and design

- Network planning
- Multicommodity flow problems and decomposition in telecom
- Telecom network design
- Ring network design
- Telecom access network design
- Optimization in distribution network design
- Design of survivable networks
- Design of survivable networks based on p-cycles
- Optimization issues in quality of service
- Steiner tree problems in telecom
- Formulations and methods for hopconstrained min spanning tree problem
- Location problems in telecom
- Pricing and equilibrium in communication networks


## Part III: Routing

- Optimization of dynamic routing networks
- ILP formulations for the routing and wavelength assignment problems: Symmetric systems
- Route optimization in IP networks
- Optimization problems in multicast tree construction


## Part IV: Reliability, restoration, and grooming

- Network reliability optimization
- Stochastic optimization in telecom
- Network restoration
- Telecom network grooming


## Part V: Wireless

- Graph domination, coloring, and cliques in telecom
- Optimization in wireless networks
- Optimization for planning cellular networks
- Load balancing in cellular wireless networks


## Part VI: The web and beyond

- Optimization issues in web search engines
- Optimization in e-commerce
- Optimization issues in combinatorial auctions
- Supernetworks


