A GRASP for PBX telephone migration scheduling

Talk given at
Eighth INFORMS Telecommunications Conference
Dallas, Texas
March 31, 2006

Mauricio G. C. Resende

AT&T Labs Research
Florham Park, New Jersey
mgcr@research.att.com
www.research.att.com/~mgcr

Joint work with Diogo Andrade

Summary of talk

- Batch scheduling of multi-grouped units
- GRASP for batch scheduling of multi-grouped units
- PBX telephone migration scheduling
- Concluding remarks

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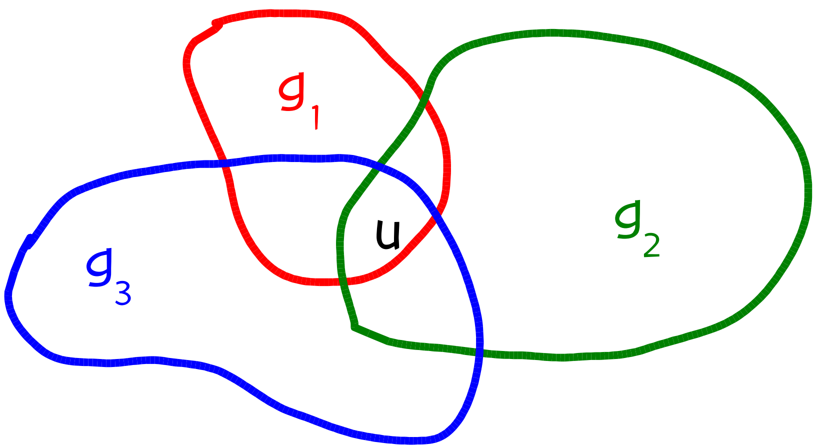
Batch scheduling of multi-grouped units

- Consider a system with
  - A set U of N units
  - A set H of M groups of units

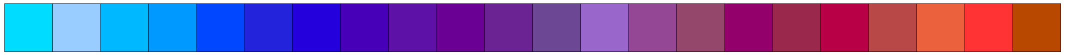
Batch scheduling of multi-grouped units

- Consider a system with
  - A set U of N units
  - A set H of M groups of units
- Each unit $u \in U$ is a member of one or more groups $g_1, g_2, \ldots, g_k \in H$.



Batch scheduling of multi-grouped units

- Given T time periods on which to schedule units.
- No more than C units can be assigned to a single time period.



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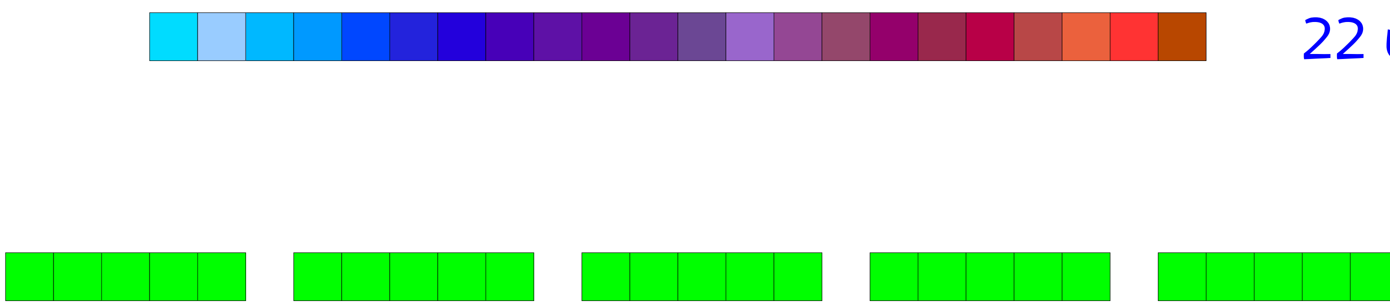

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T=5 time periods

C=5 units/period

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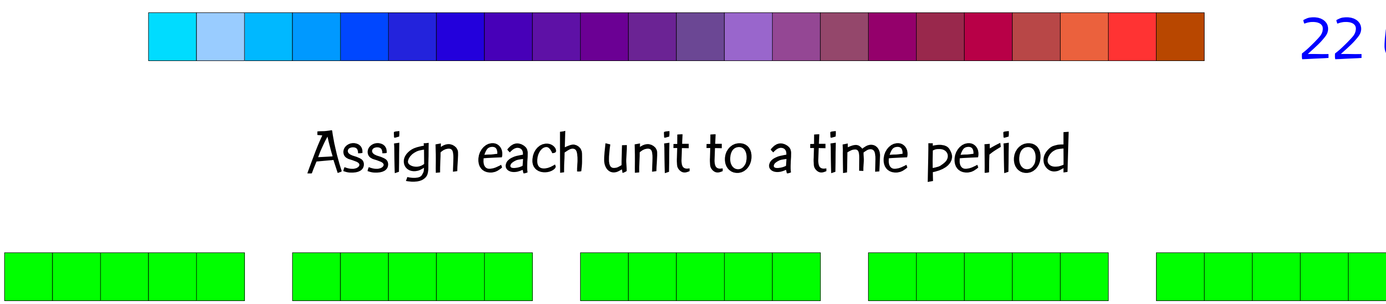

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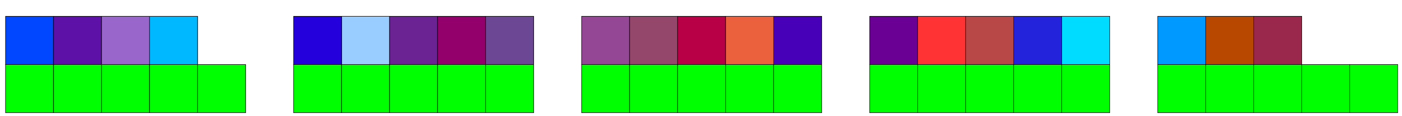

T=5 time periods

C=5 units/period

Batch scheduling of multi-grouped units

- Objective: Schedule two units sharing same group as close together in time as possible.
- Let $w(u,v,g)$ be the per-period penalty associated with assigning a group-g pair u and v to different periods.
- Scheduling penalty: Let $G(u,v) \subseteq H$ be the set of groups shared by units u and v. If units u and v are assigned to periods $\pi(u)$ and $\pi(v)$, respectively, then a penalty

$$p(u,v) = |\pi(u) - \pi(v)| \times \sum_{g \in G(u,v)} w(u,v,g)$$

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Batch scheduling of multi-grouped units

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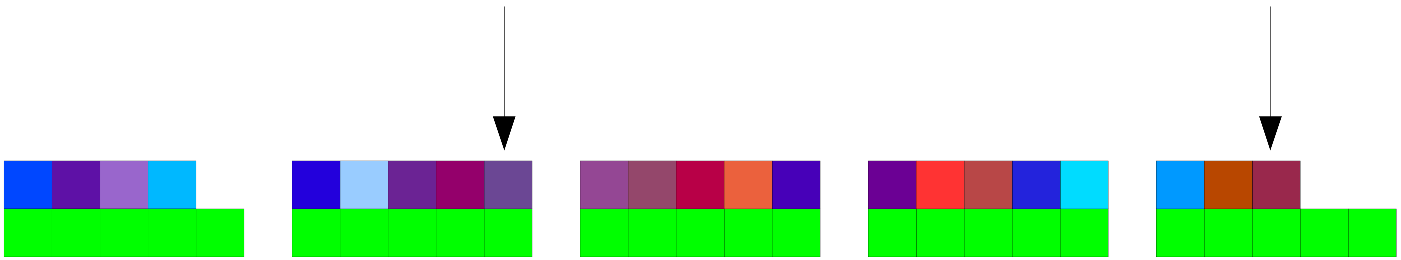
Units 12 and 17 share groups 2, 4, and 8.



T=5 time periods

C=5 units/period

$\pi(12)=2$ $\pi(17)=5$

Batch scheduling of multi-grouped units

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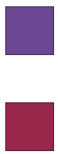

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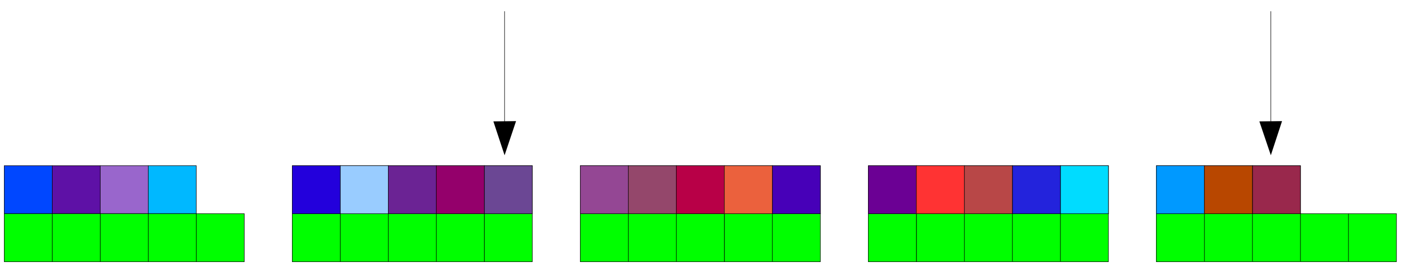
Let w(12,17,2) = 10, w(12,17,4) = 20, w(12,17,8) = 5.



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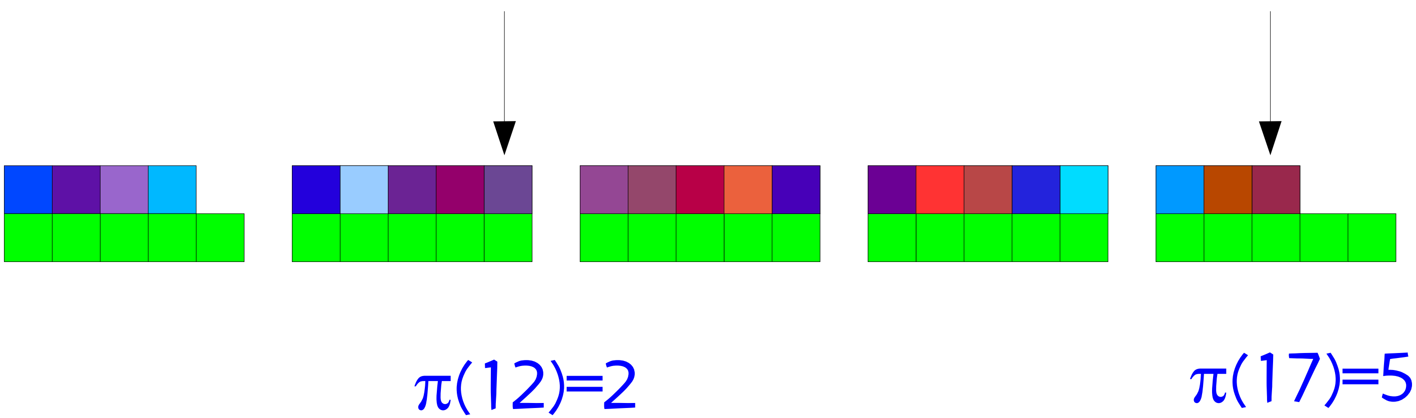

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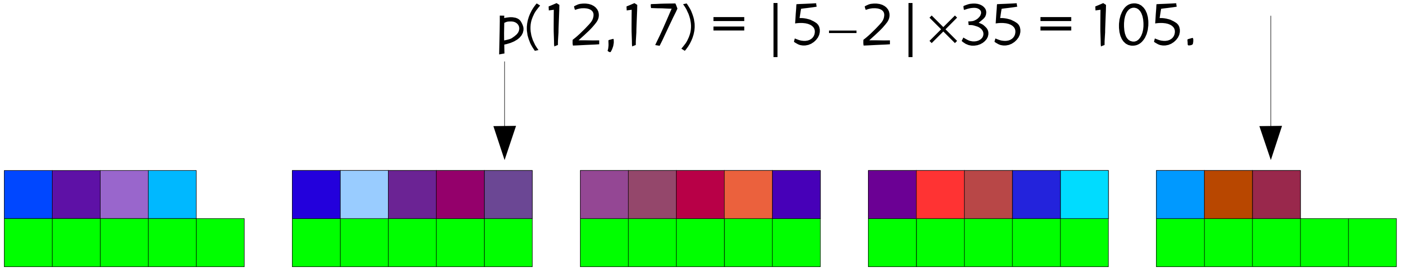
Then $w(12,17,2) + w(12,17,4) + w(12,17,8) = 35$.

Since $\pi(12)=2$ and $\pi(17)=5$, then

$p(12,17) = |5-2| \times 35 = 105$.



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- Given:
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  - a set of units U to be assigned to periods,
  - a set of groups H that units can share,
  - for each pair of units $u,v \in U$, a subset of groups $G(u,v) \subseteq H$ shared by the pair,
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Batch scheduling of multi-grouped units

- Problem: Find assignment π of units to periods that

  minimizes $\sum_{\substack{u,v \in U \times U \\ (u>v)}} |\pi(u) - \pi(v)| \times \sum_{g \in G(u,v)} w(u,v,g)$

  such that
  no more than C units are assigned to any time period.

- Problem is NP-hard. It generalizes the minimum linear arrangement problem: Given a graph $G(V,E)$, find $\pi: V \rightarrow \{1,\ldots,|V|\}$ that

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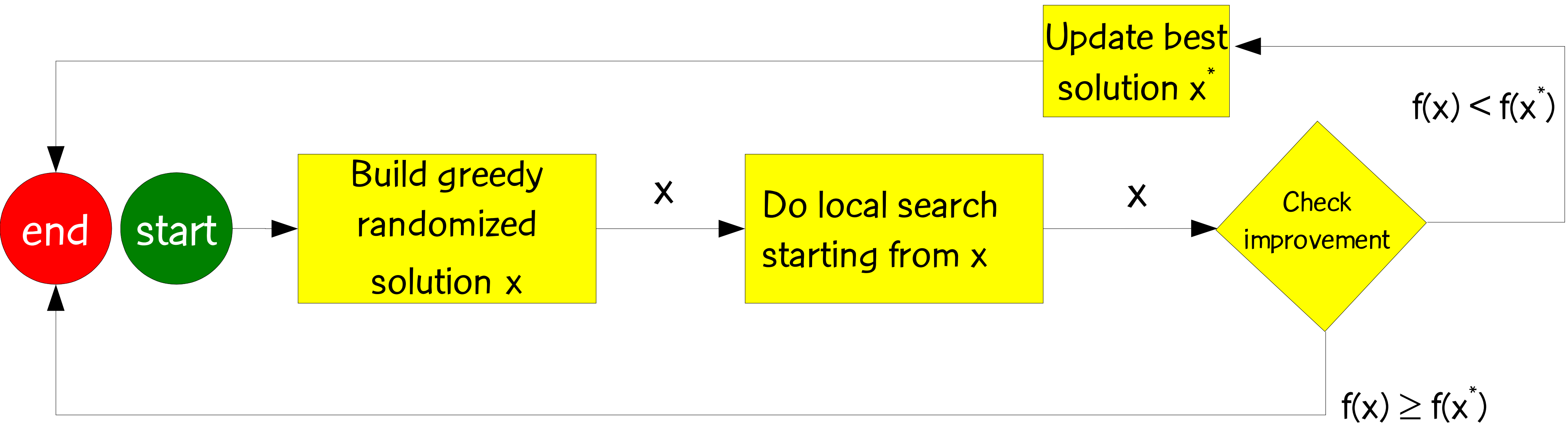
GRASP for batch scheduling of multi-grouped units

- GRASP [Feo & Resende, 1989, 1995] is a multi-start metaheuristic for combinatorial optimization.
- Each GRASP iteration consists of:



- The best solution found, over all iterations, is returned.

GRASP construction

- Construction sequences units and assigns them evenly to time periods.
- Let $\sigma(u)$ be the position of unit u in the sequence.
- Construction seeks to optimize approximate objective function $\sum_{\substack{u,v \in U \times U \\ (u>v)}} |\sigma(u) - \sigma(v)| \times \sum_{g \in G(u,v)} w(u,v,g)$
- Greedy algorithm: next unit k in sequence minimizes $\sum_{v \in \text{unselected}} \sum_{g \in G(k,v)} w(k,v,g) - \sum_{u \in \text{selected}} \sum_{g \in G(u,k)} w(u,k,g)$
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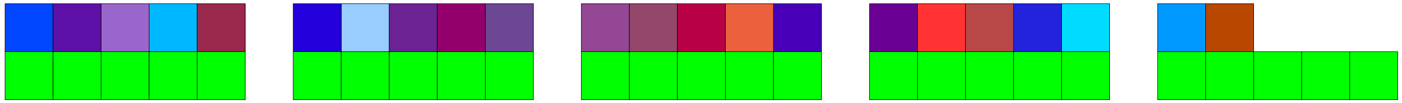
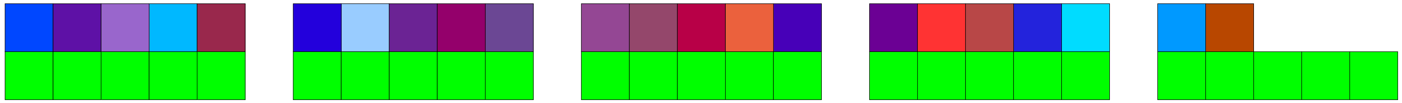
GRASP local search phase

- Input: Assignment of units to periods:



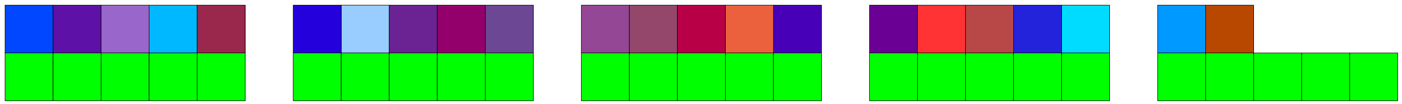
GRASP local search phase

- Local search: Examine neighborhood of current solution. If better solution found, make it current solution.



GRASP local search phase

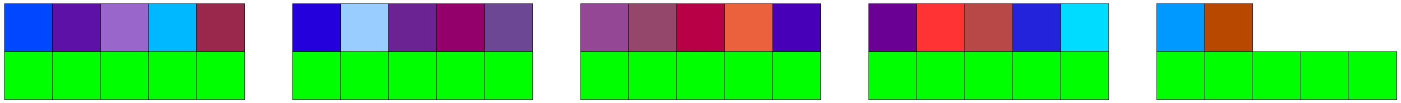
- Three neighborhoods: Swap units, move unit, swap periods.



GRASP local search phase

- Swap units neighborhood: Swaps places of two units assigned to distinct periods.

solution



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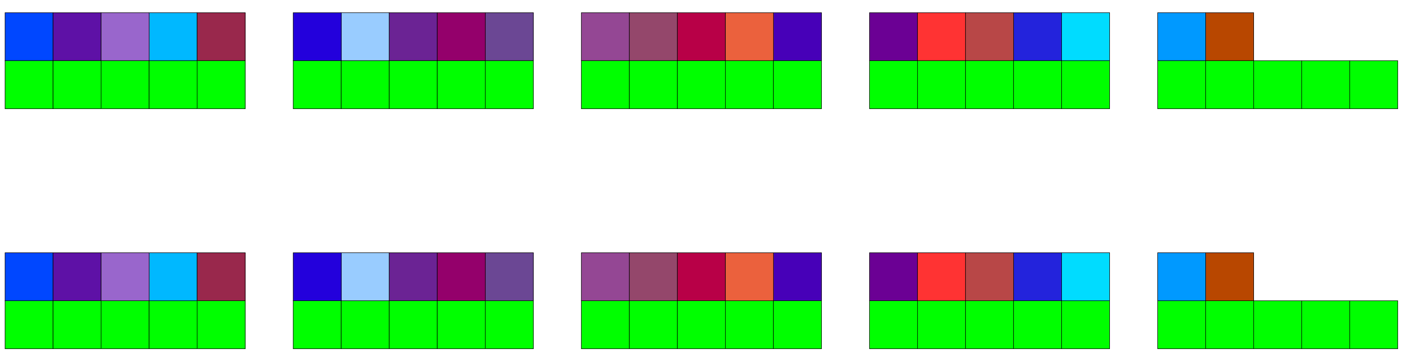

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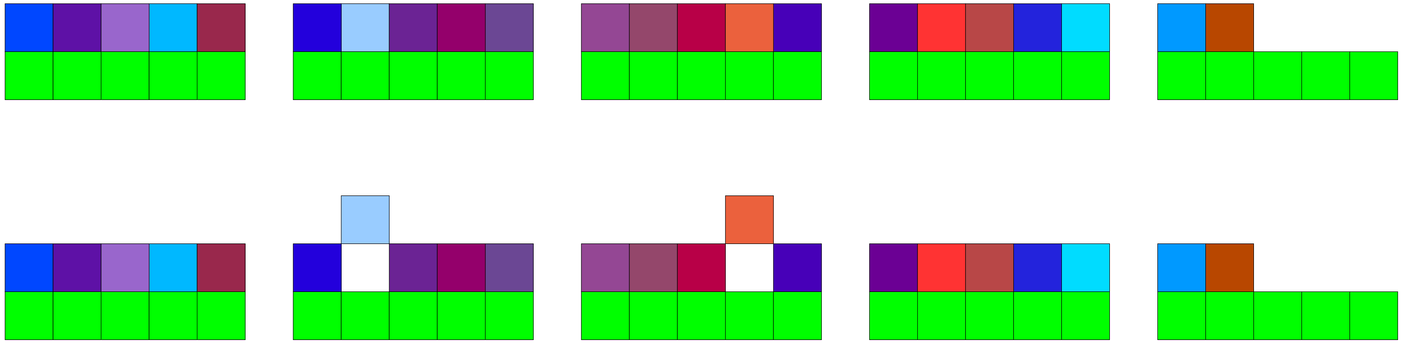

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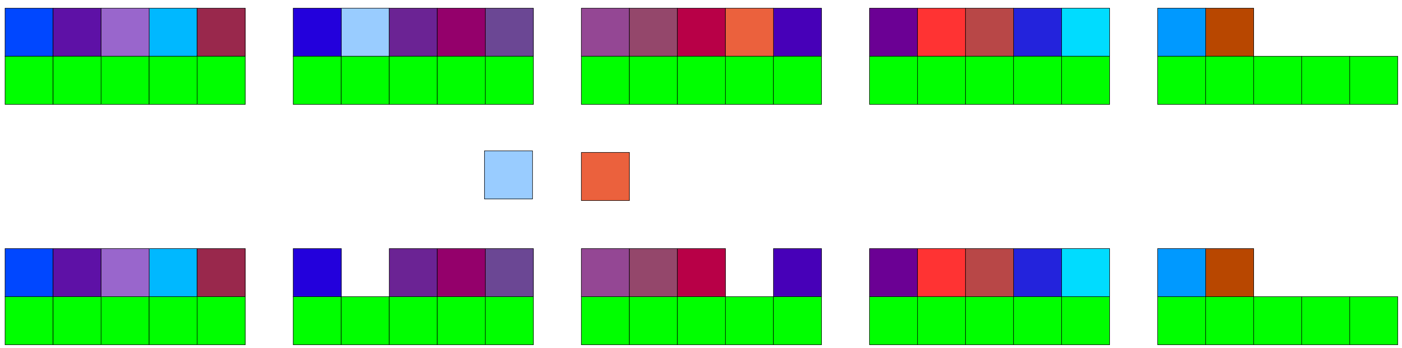

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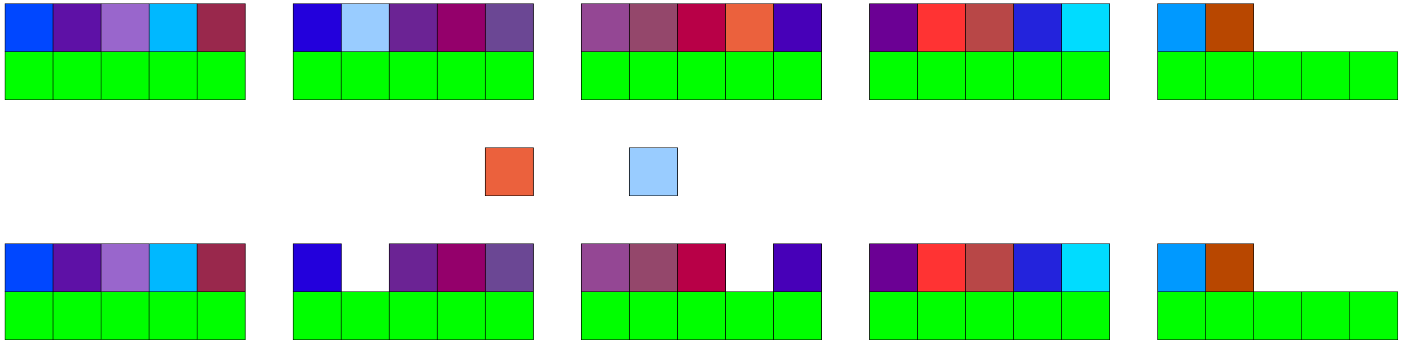

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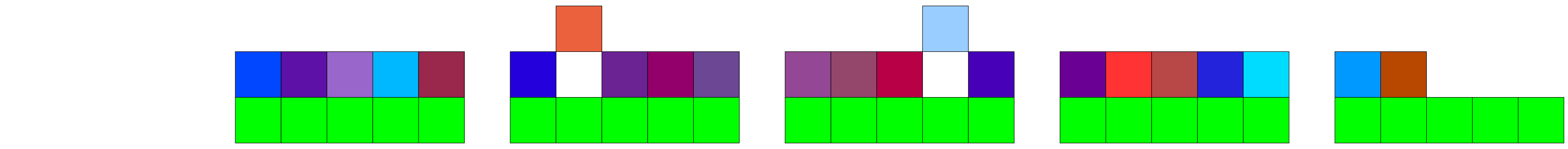

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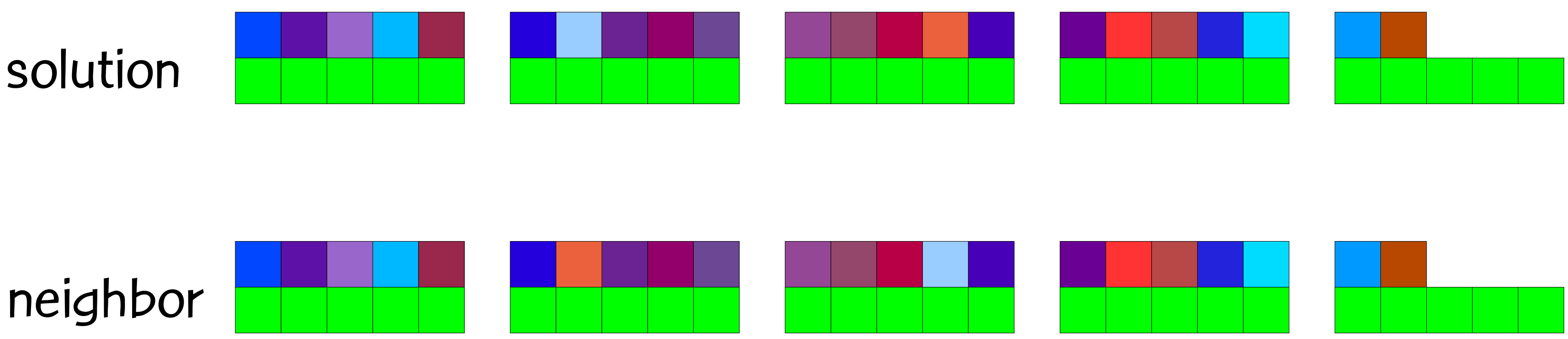

GRASP local search phase

- Move unit neighborhood: Moves unit from current period to available period.

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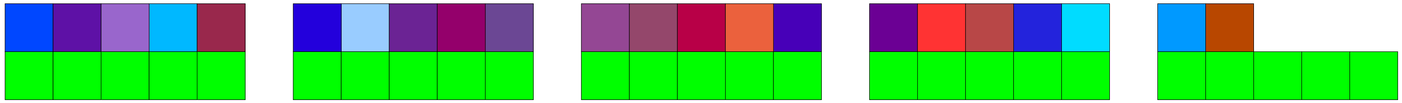

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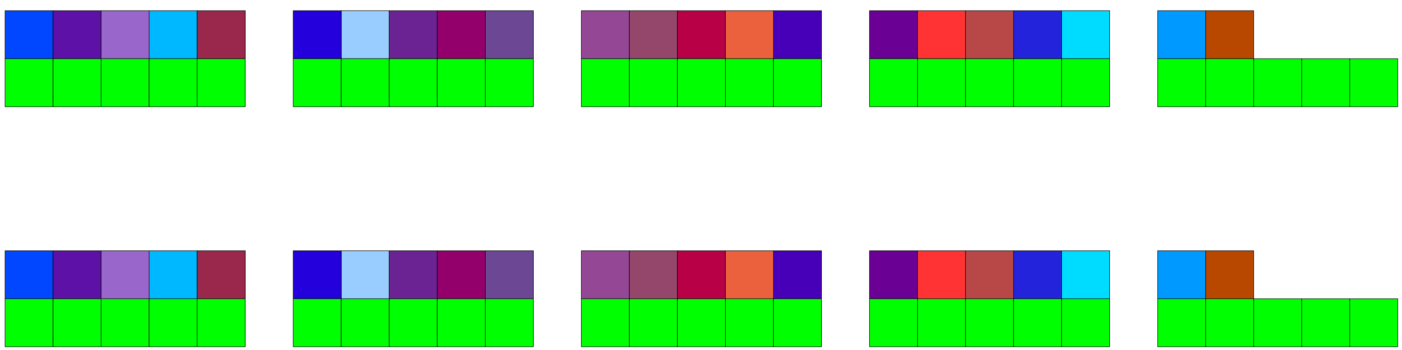

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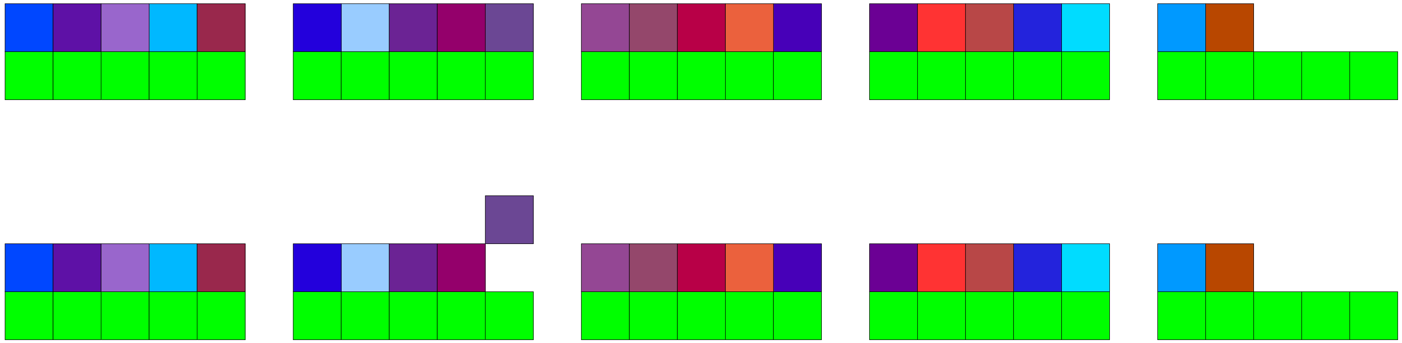

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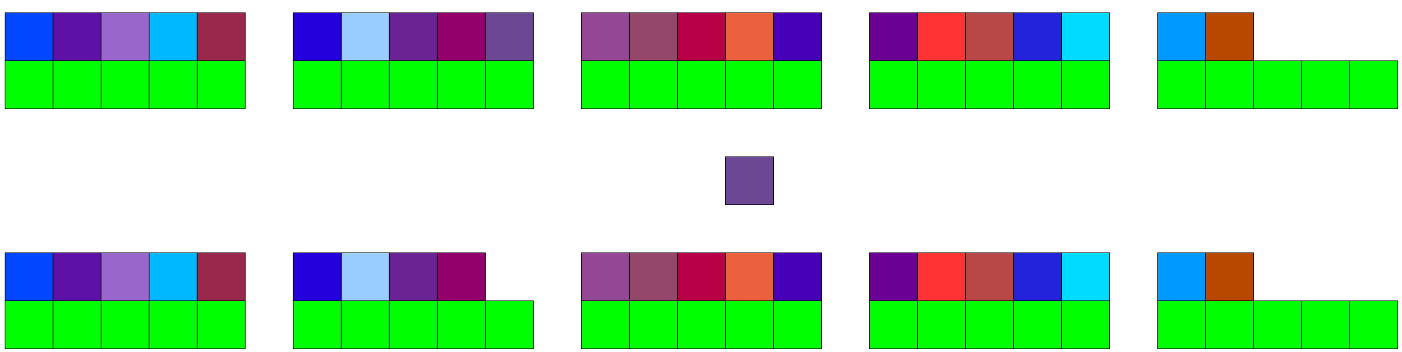

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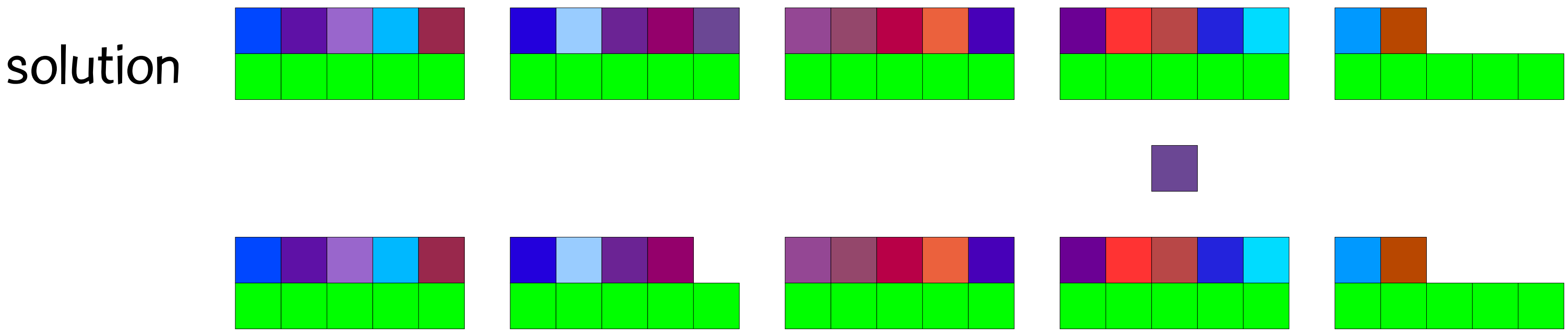

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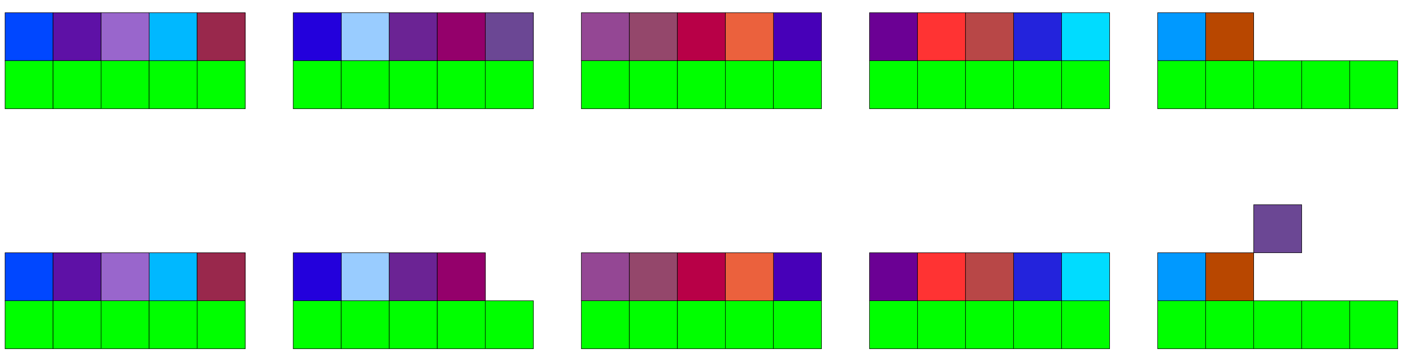

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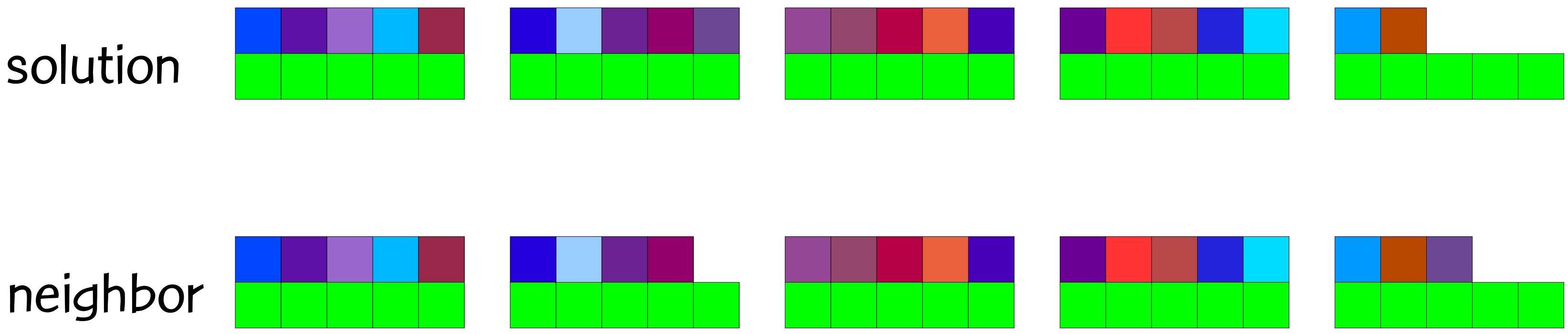

GRASP local search phase

- Swap periods neighborhood: Swap all units in period i with all units in period j.

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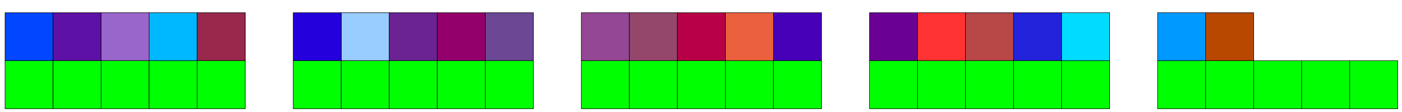

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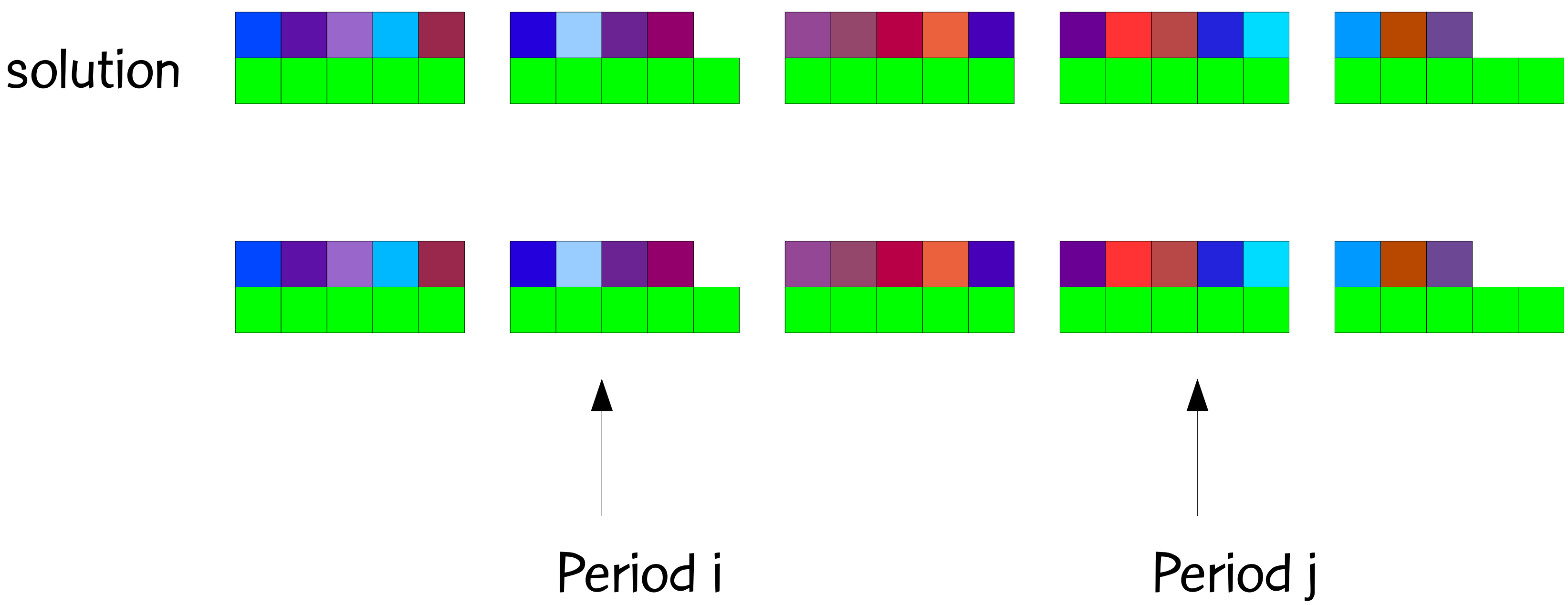

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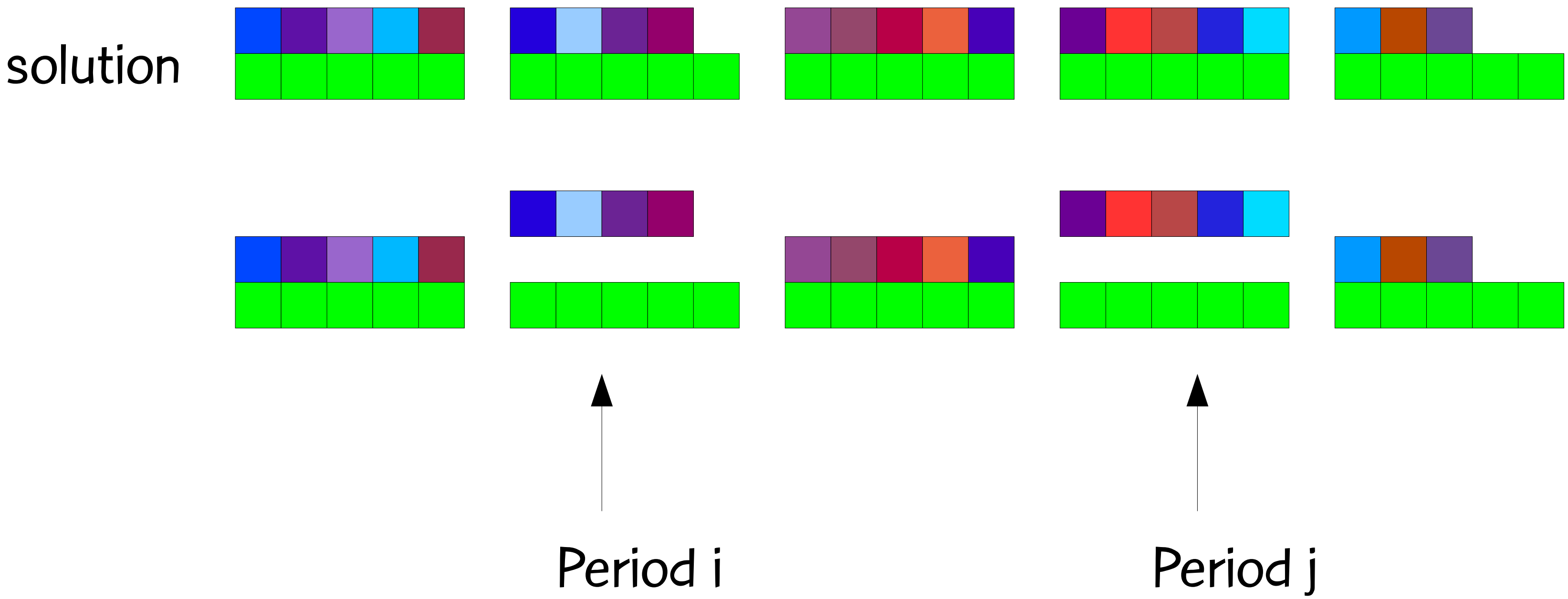

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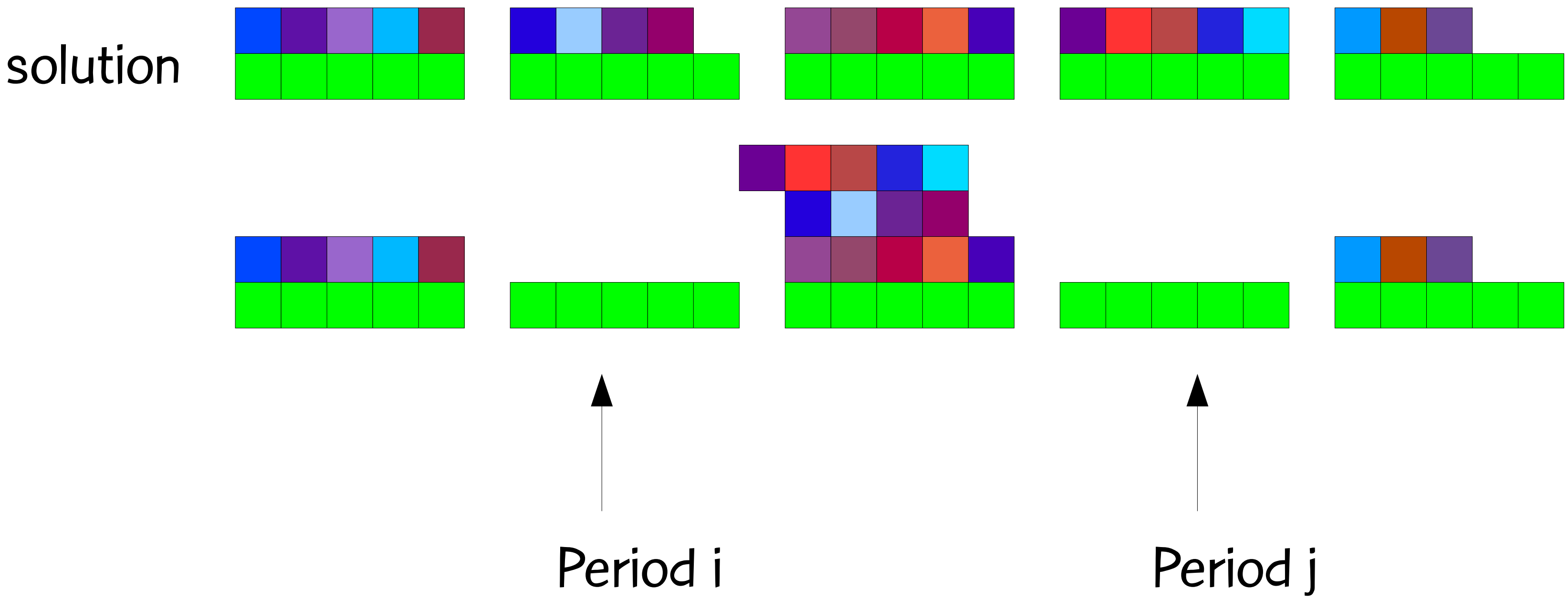

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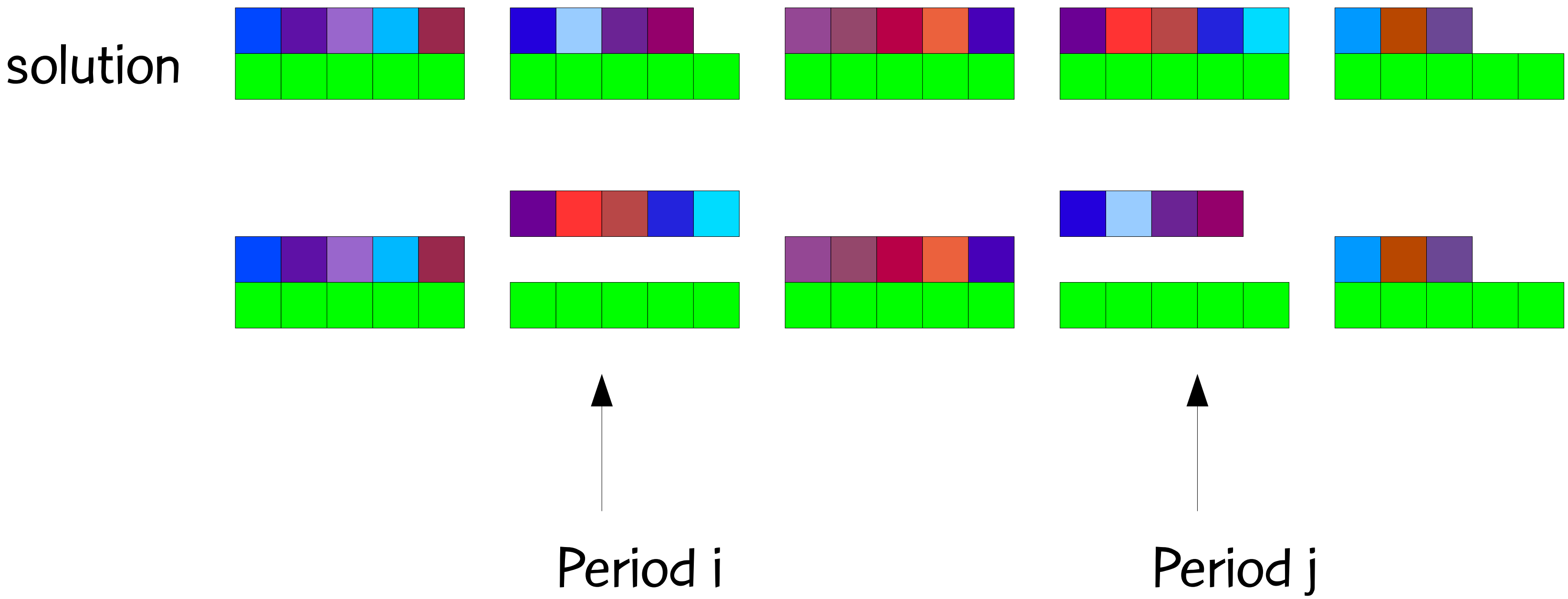

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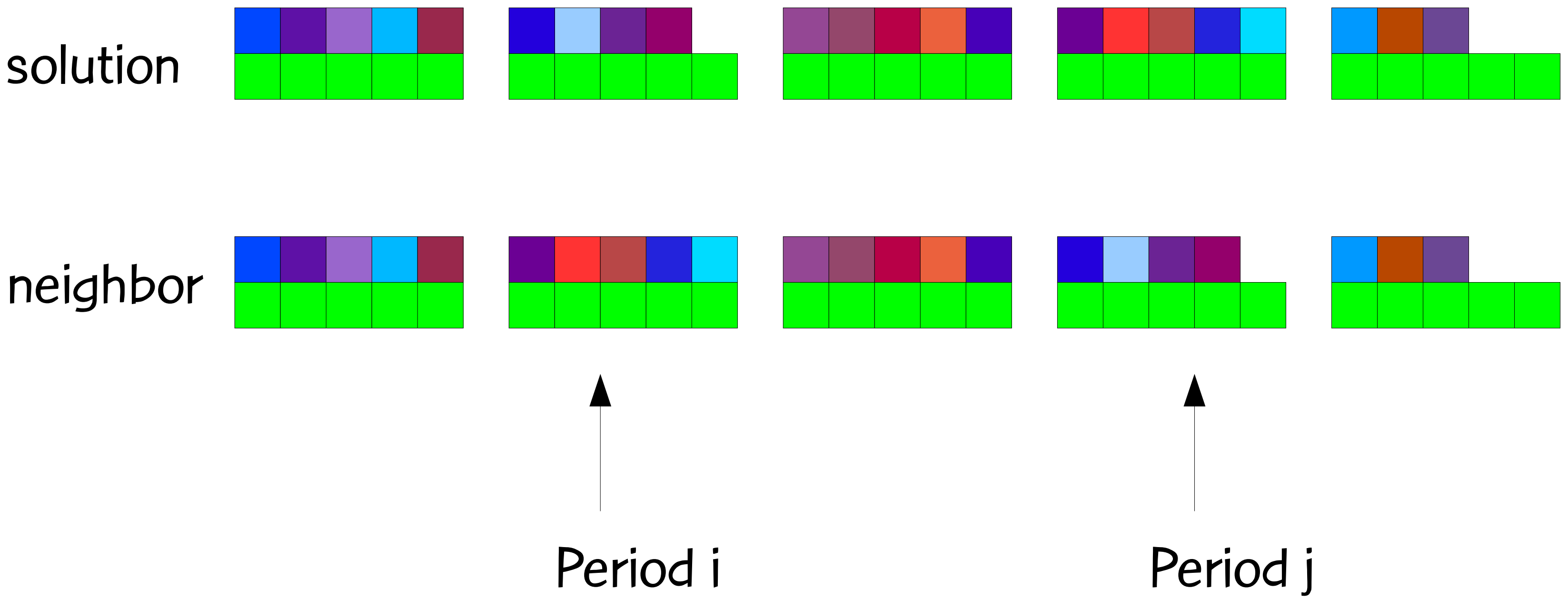
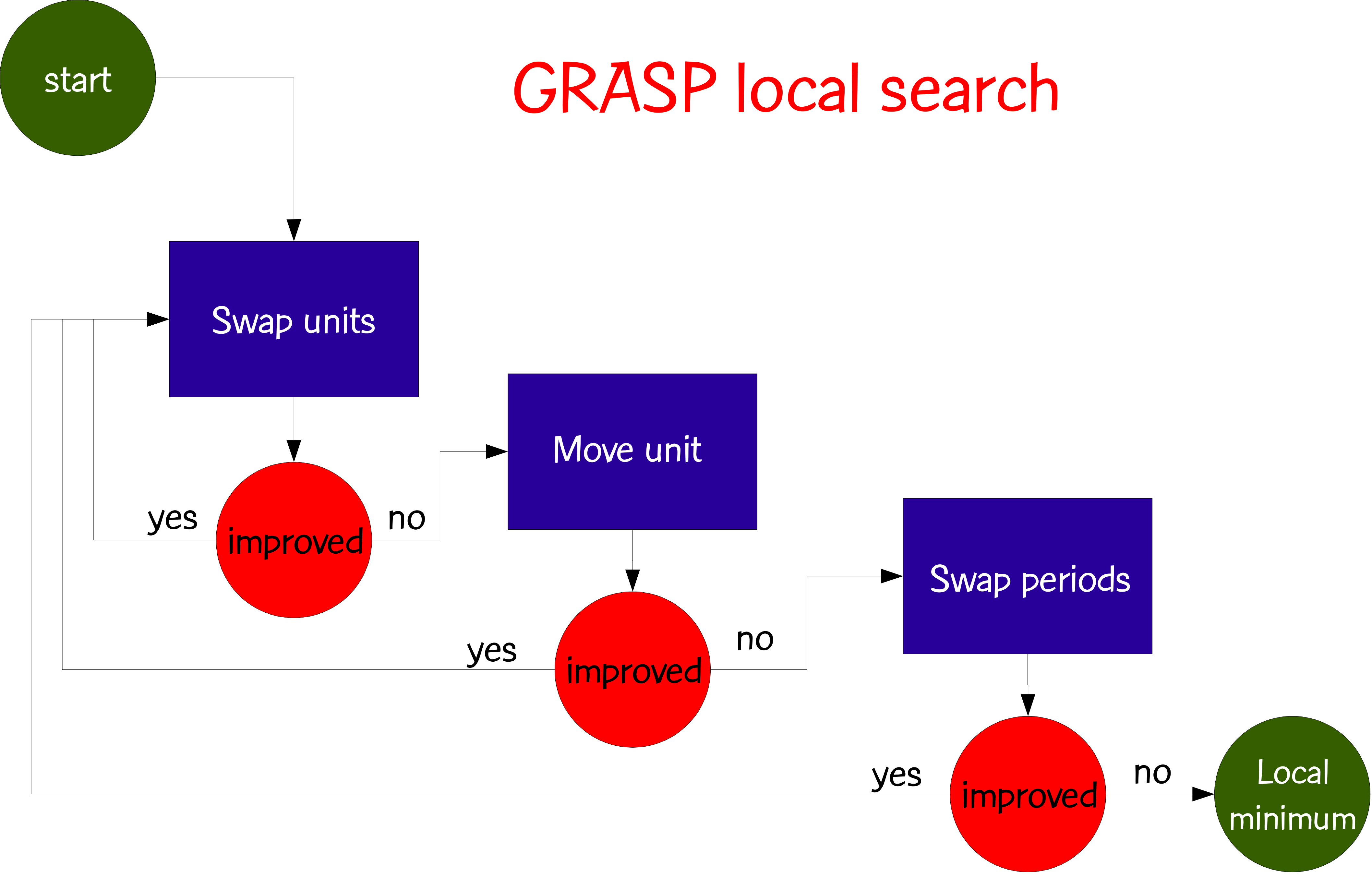


GRASP local search



PBX telephone migration scheduling

- Phone migration occurs when an organization upgrades to a newer phone switch (PBX).
- All phones using the old PBX must be moved to the new PBX.
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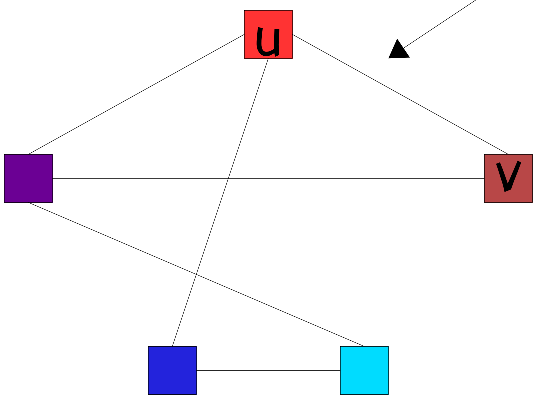
$$\sum_{g \in G(u,v)} w(u,v,g)$$



Each phone is a unit.
Edge weight between phones is penalty associated with phone pair.

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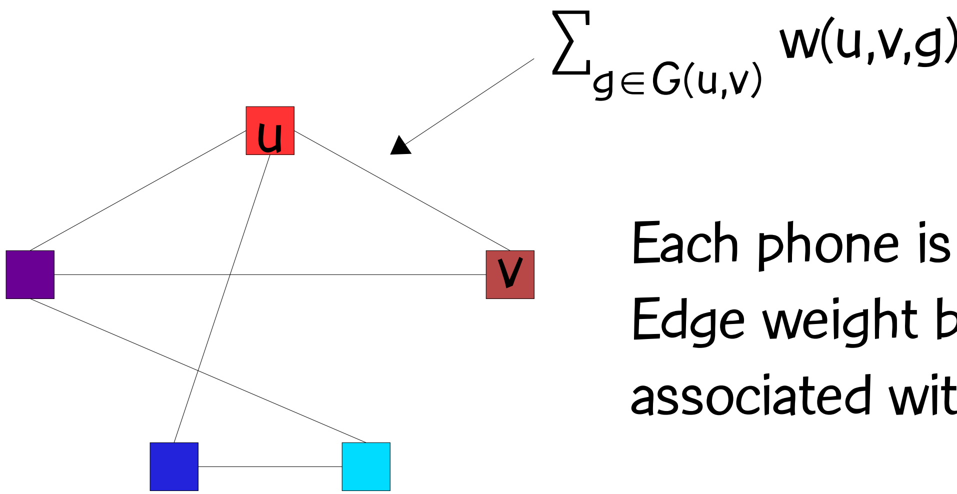

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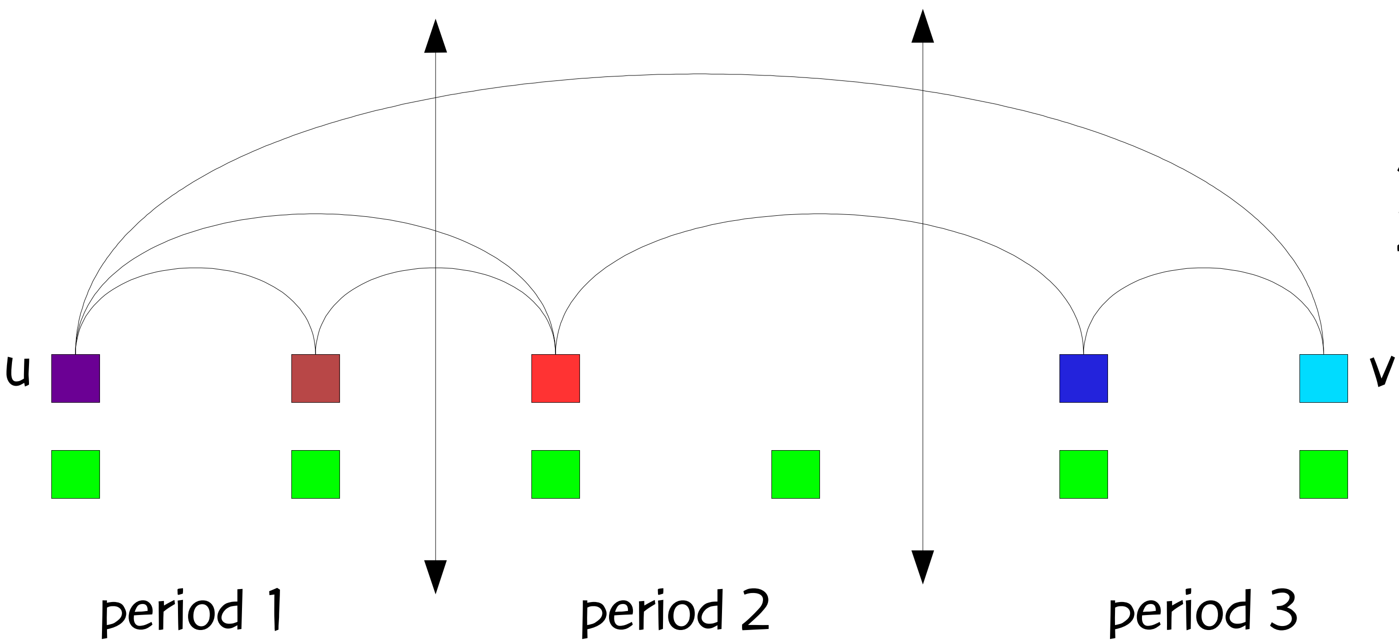
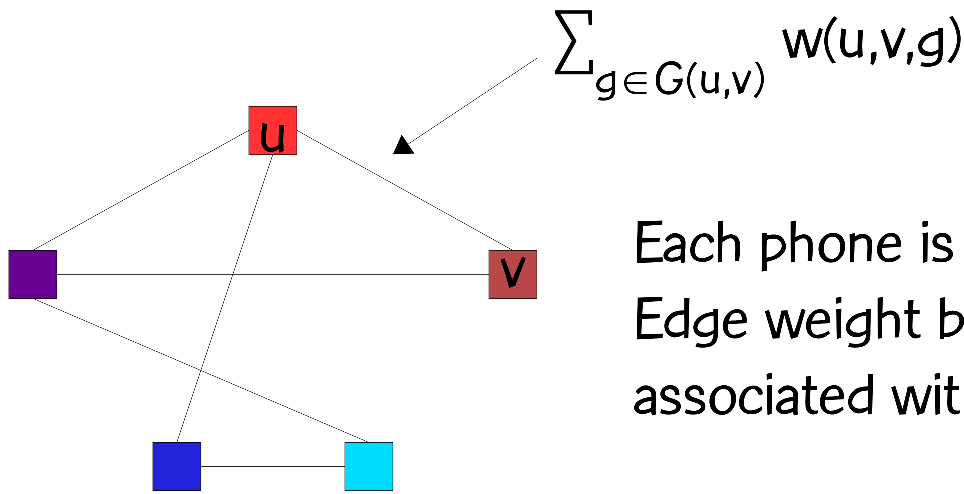


3 periods
2 units/period max

PBX telephone migration scheduling



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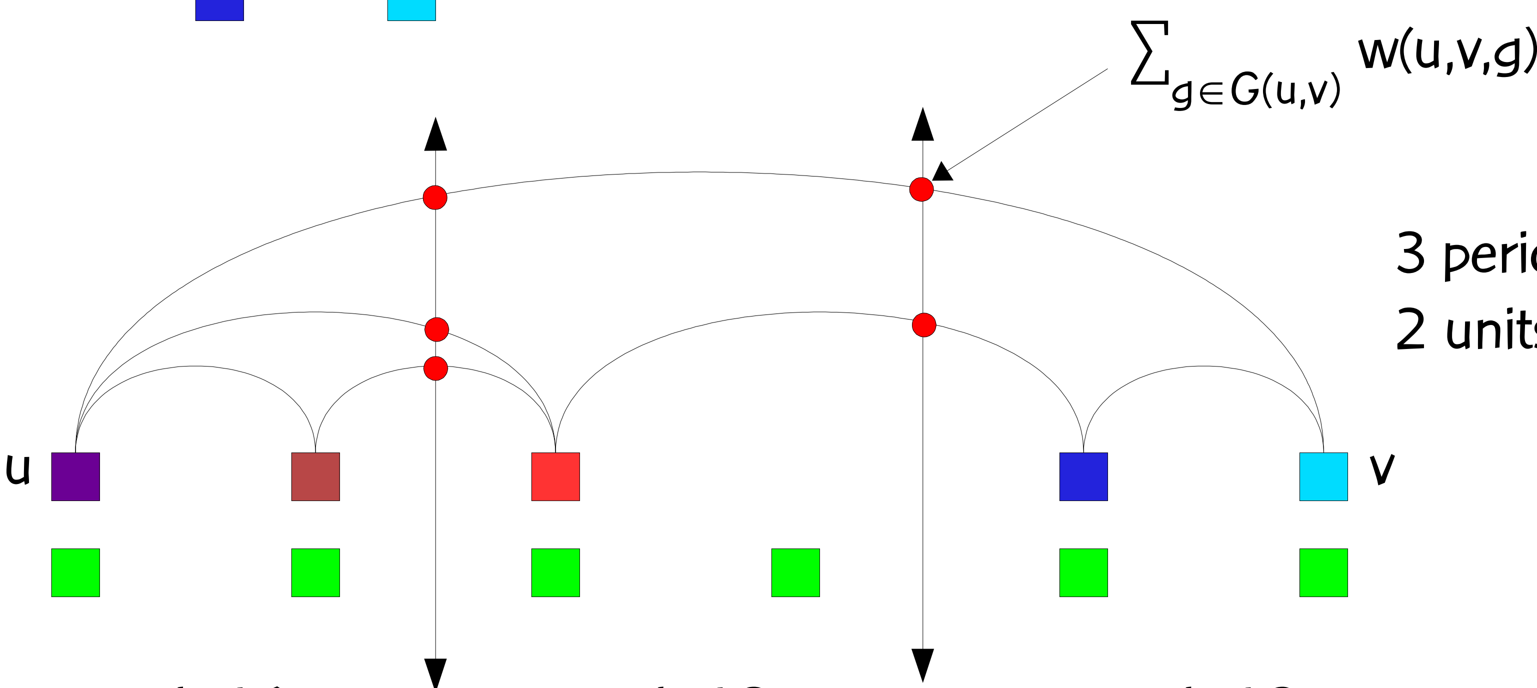

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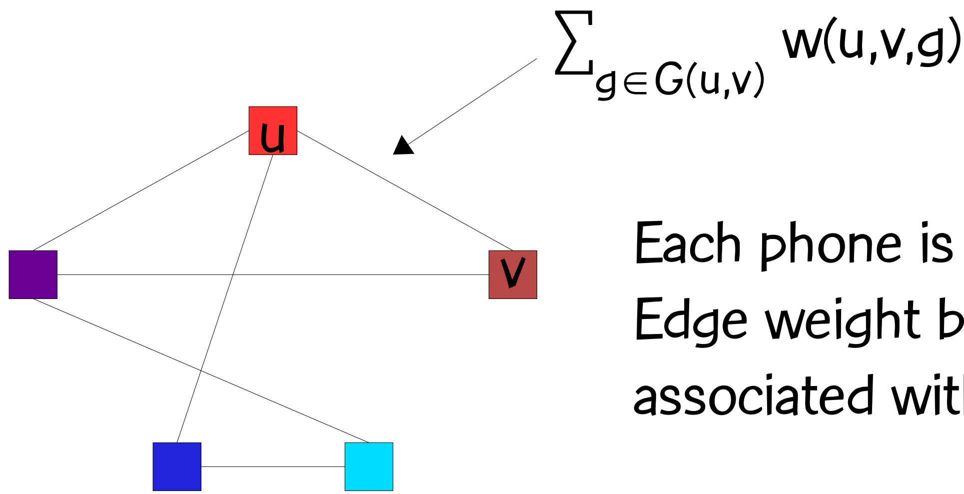

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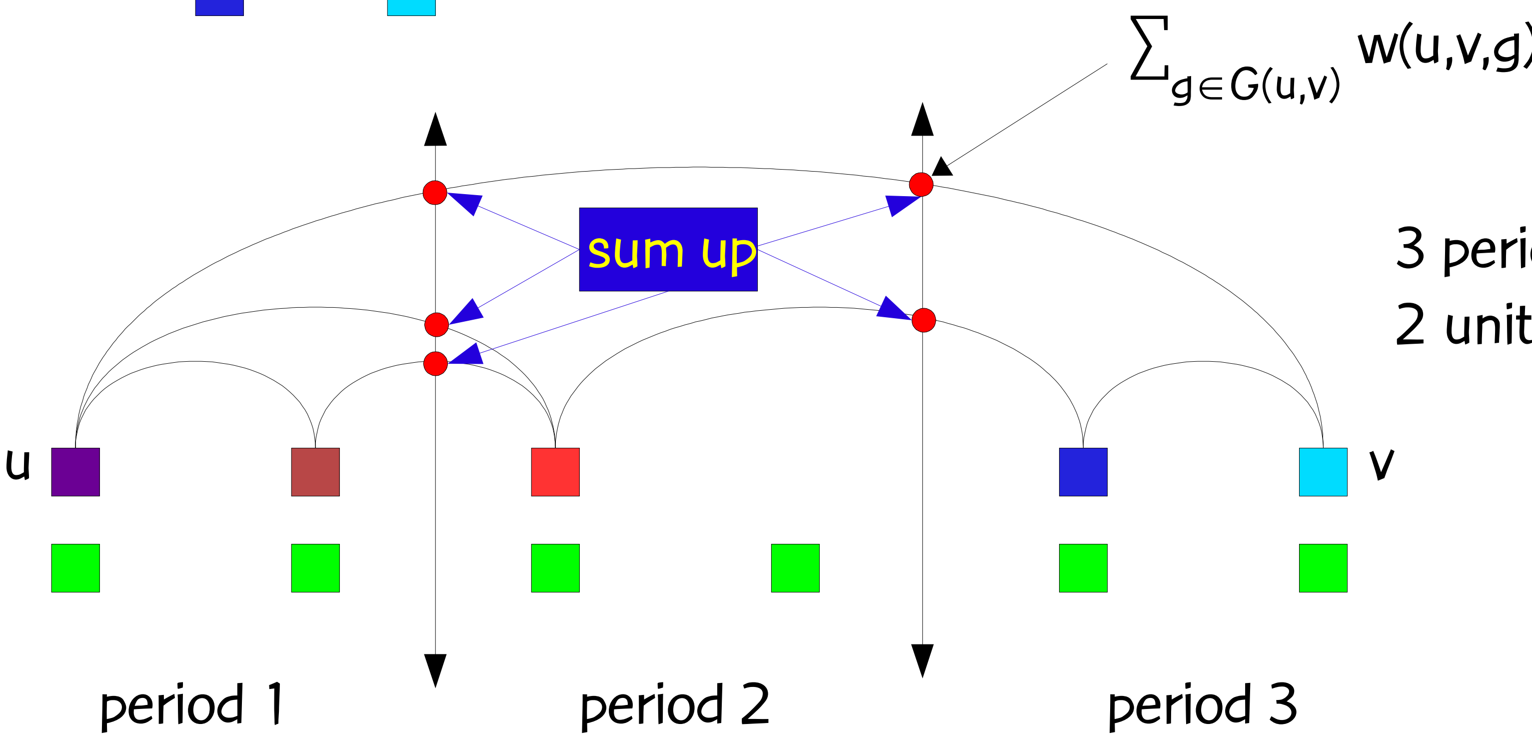

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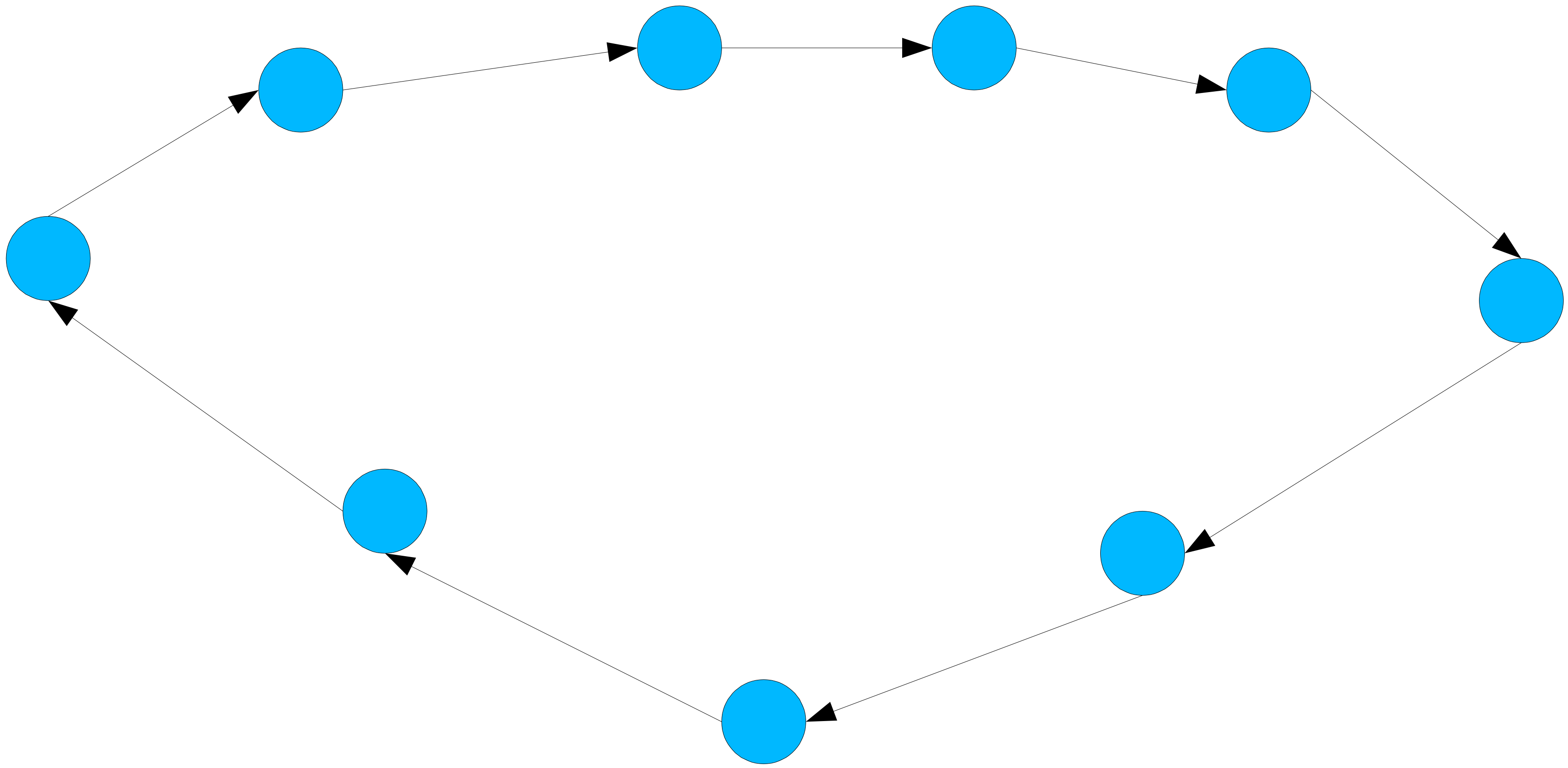
Multi-line hunt group



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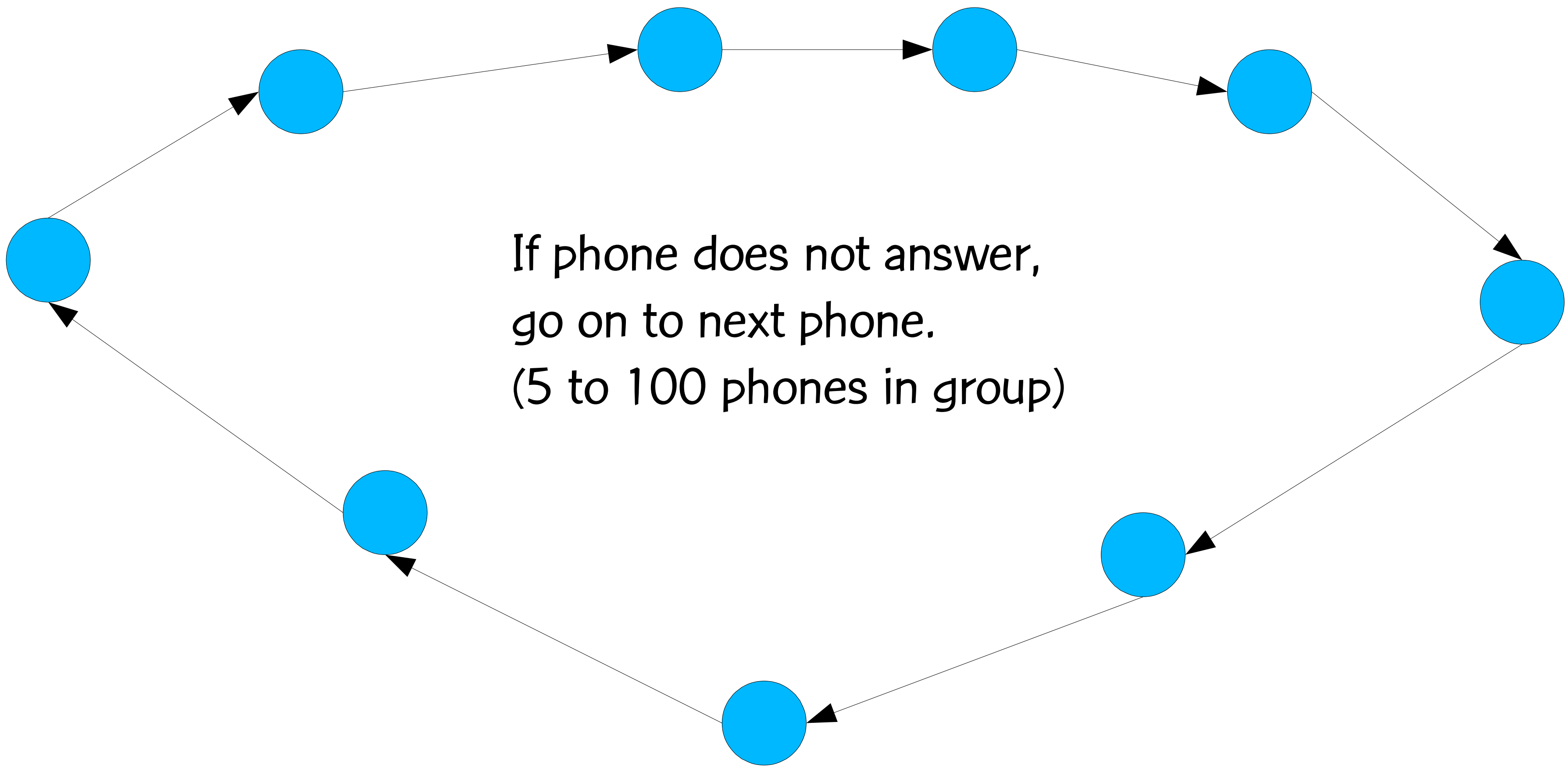

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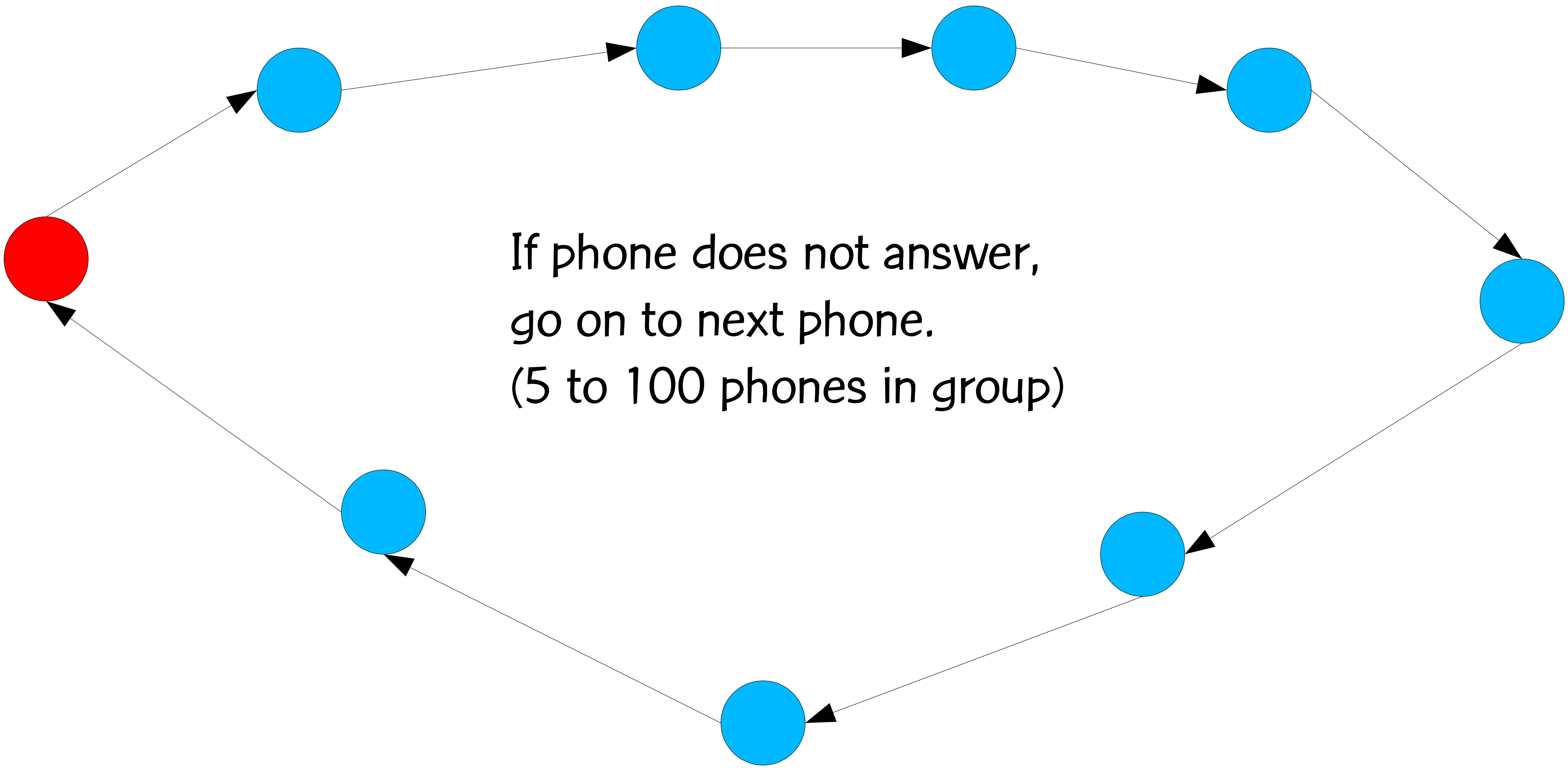
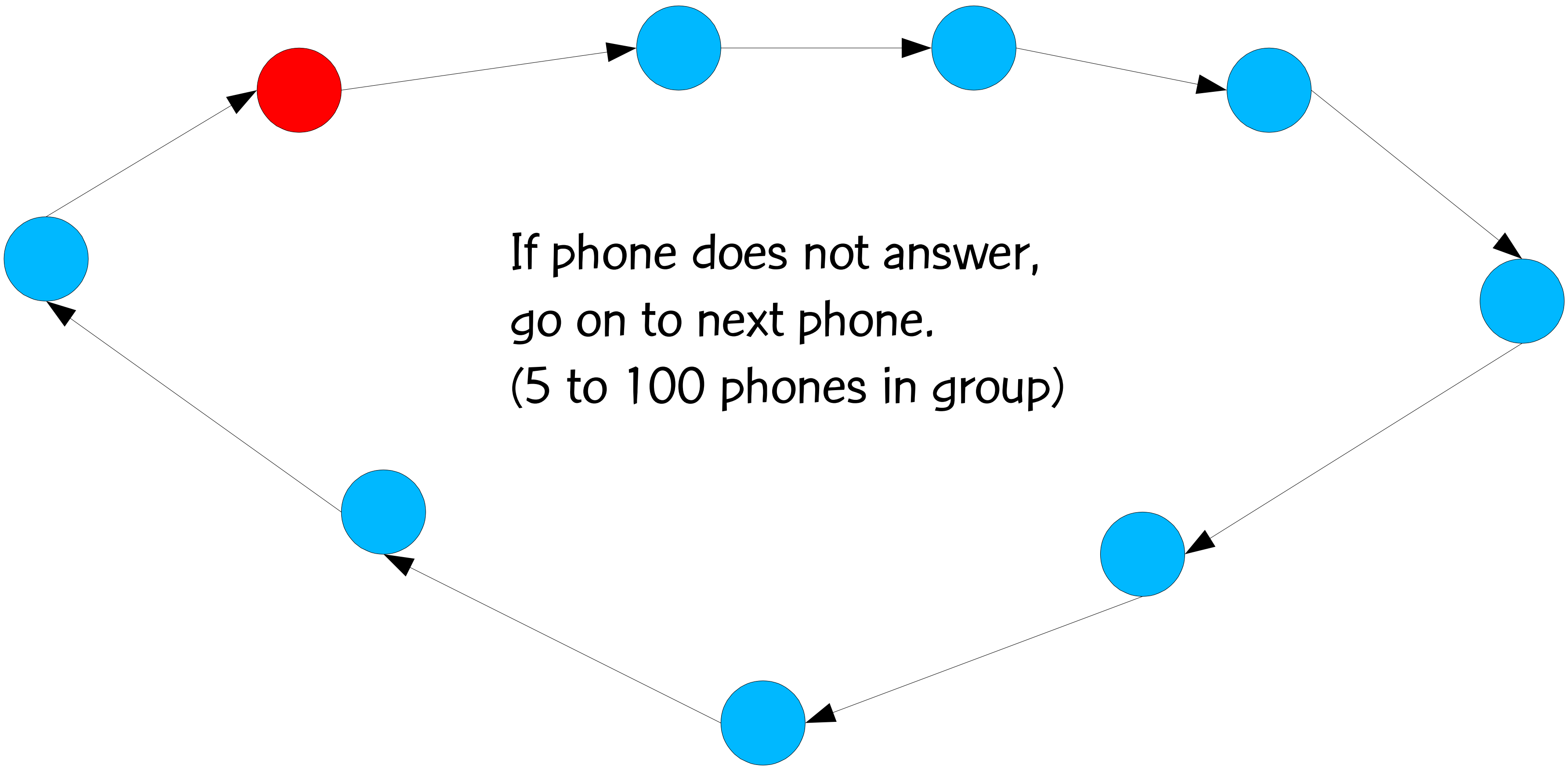


If phone does not answer,
go on to next phone.
(5 to 100 phones in group)

Multi-line hunt group



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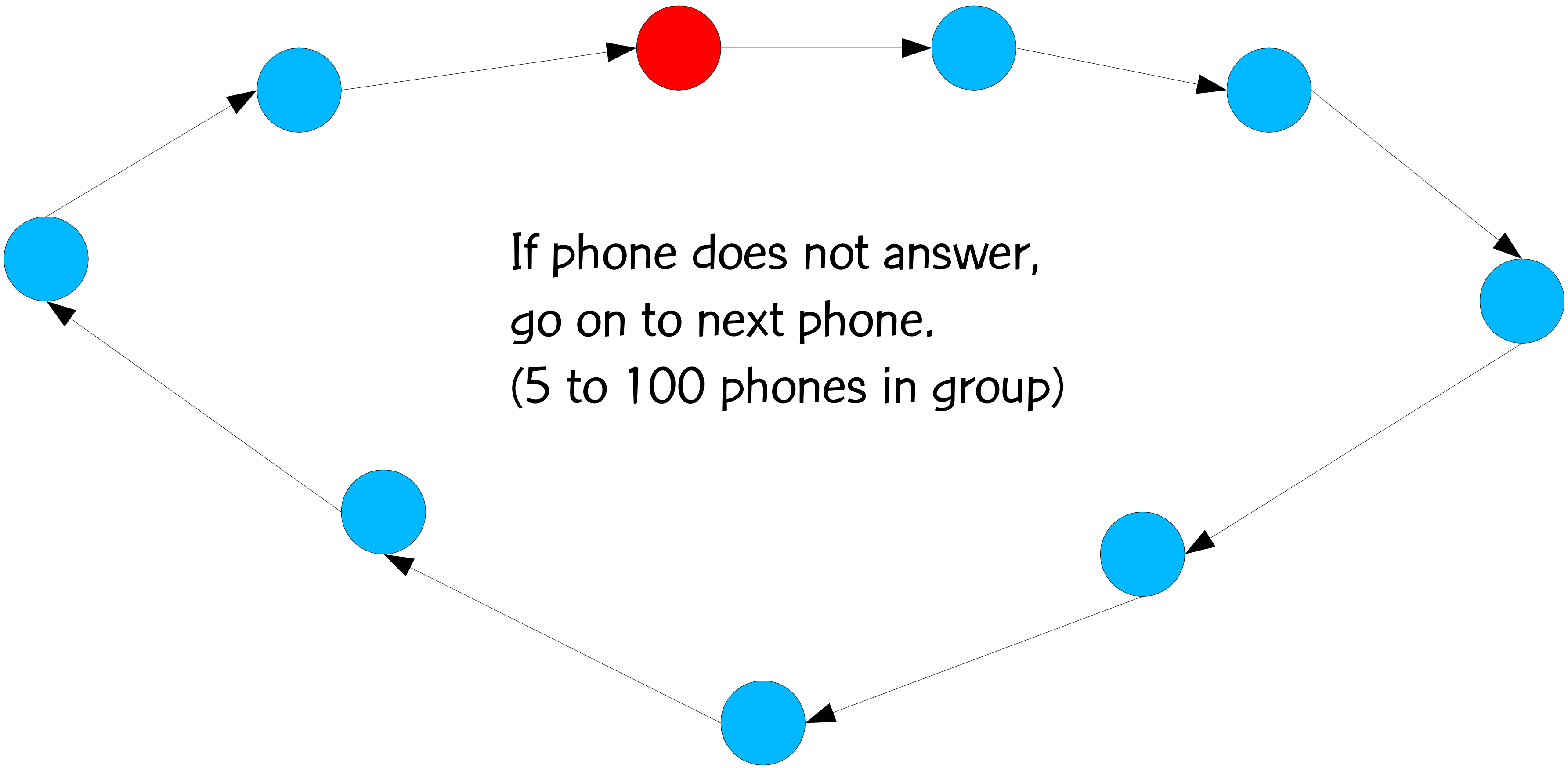

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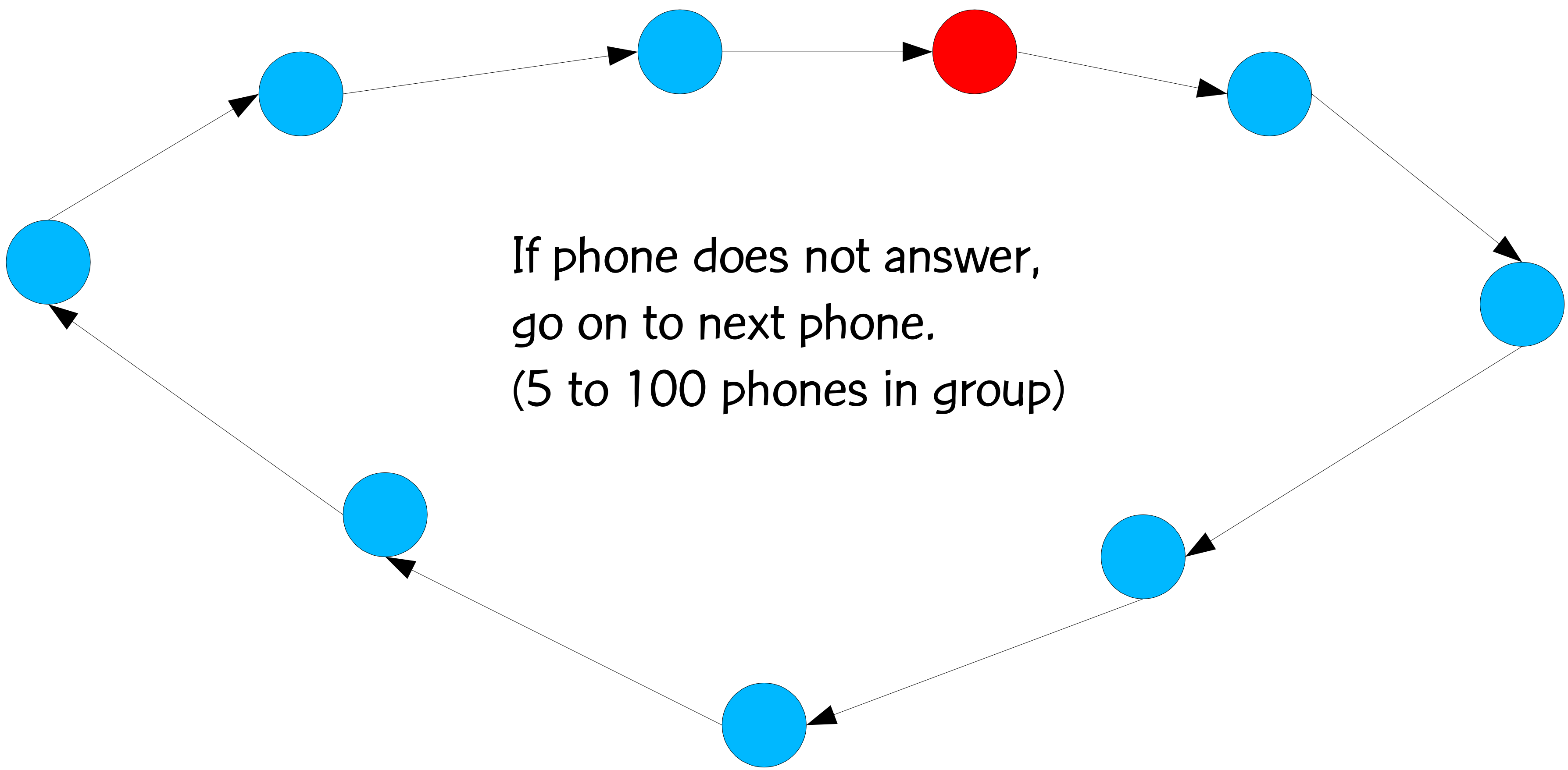

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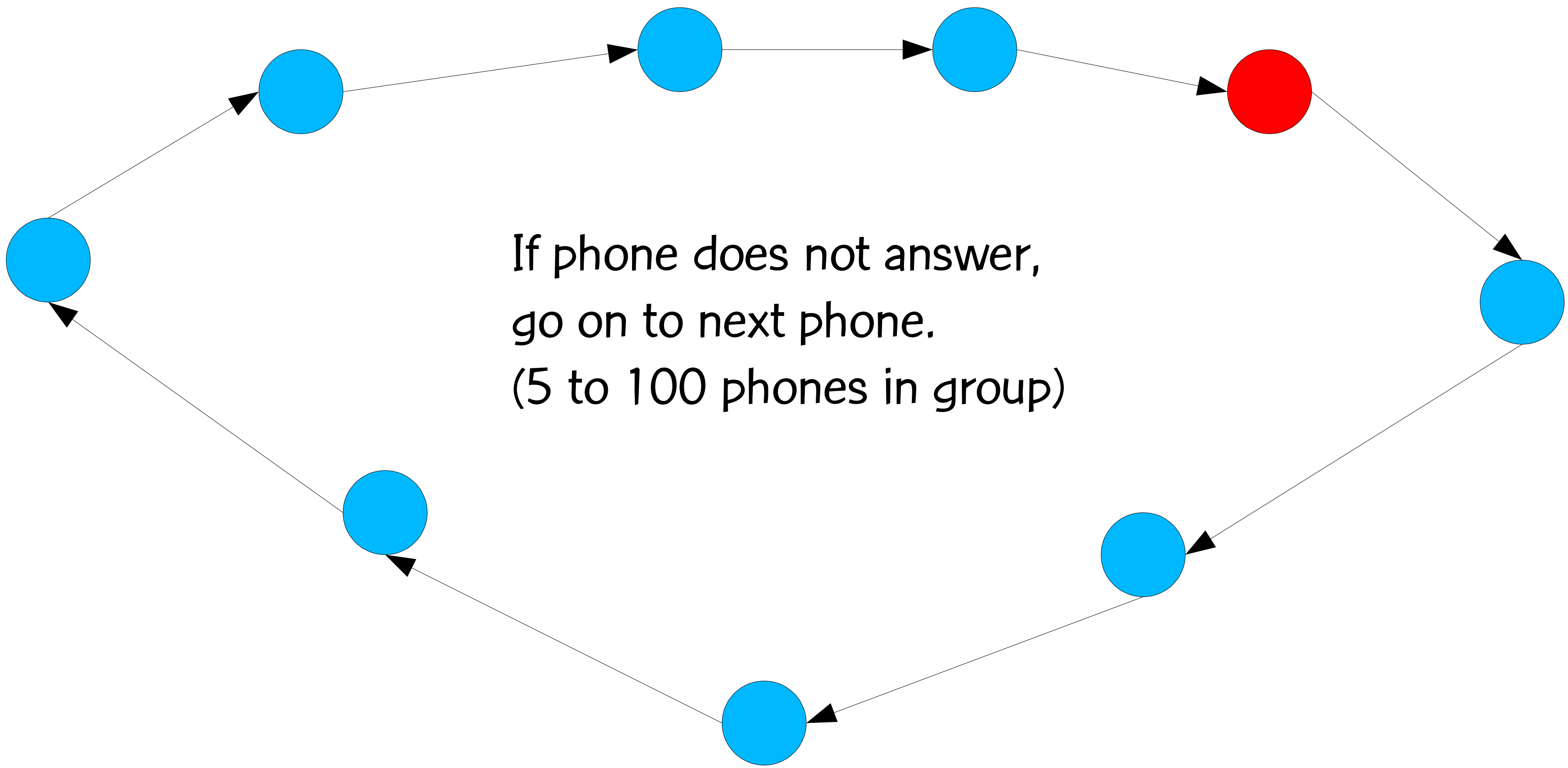

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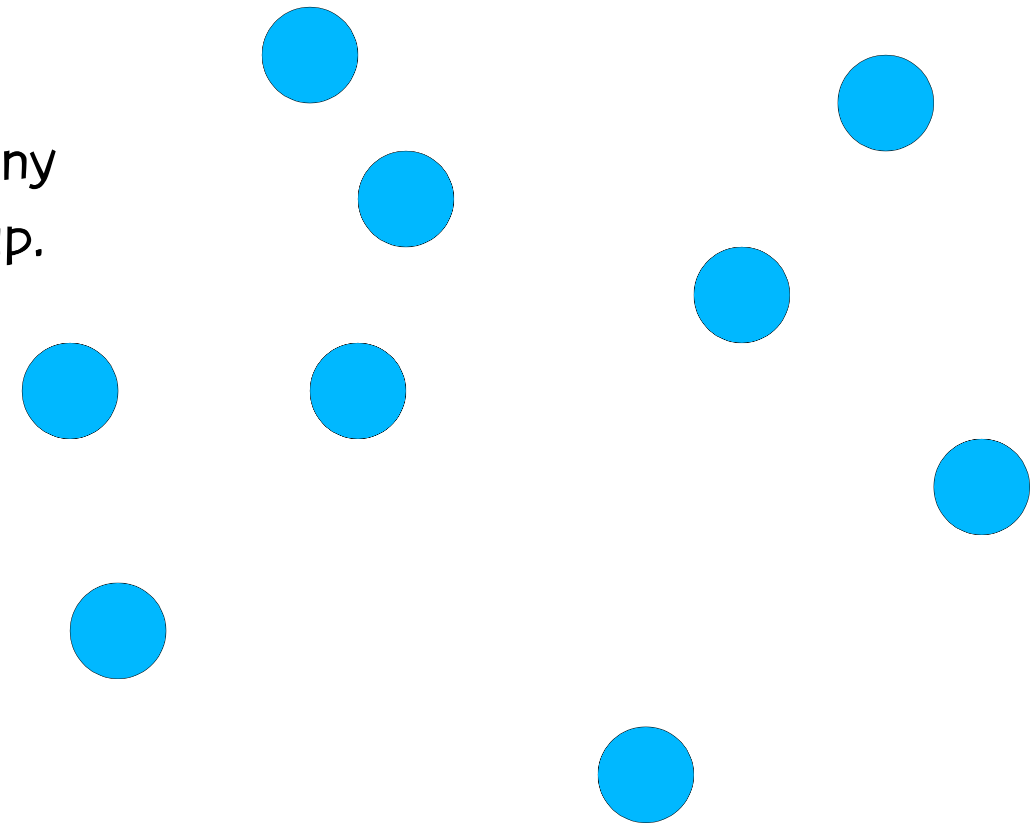
Call pickup (CPU)

Any phone in group can pickup call for any other phone in group.



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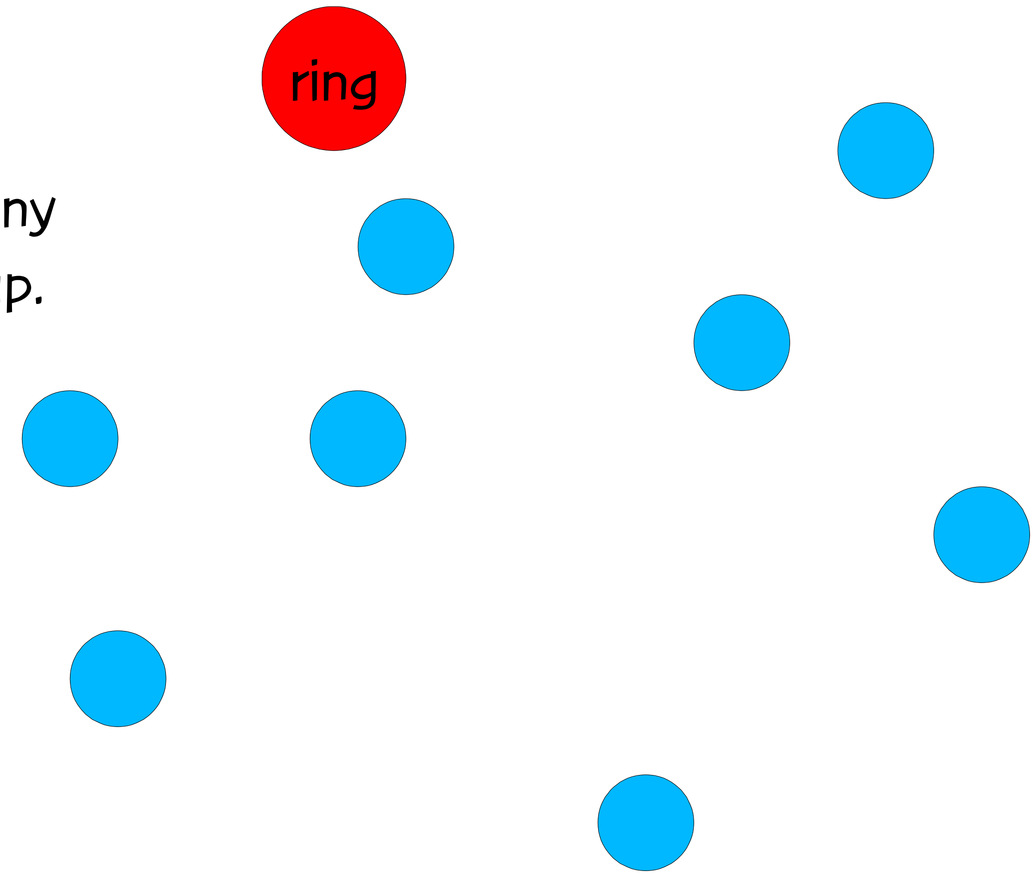

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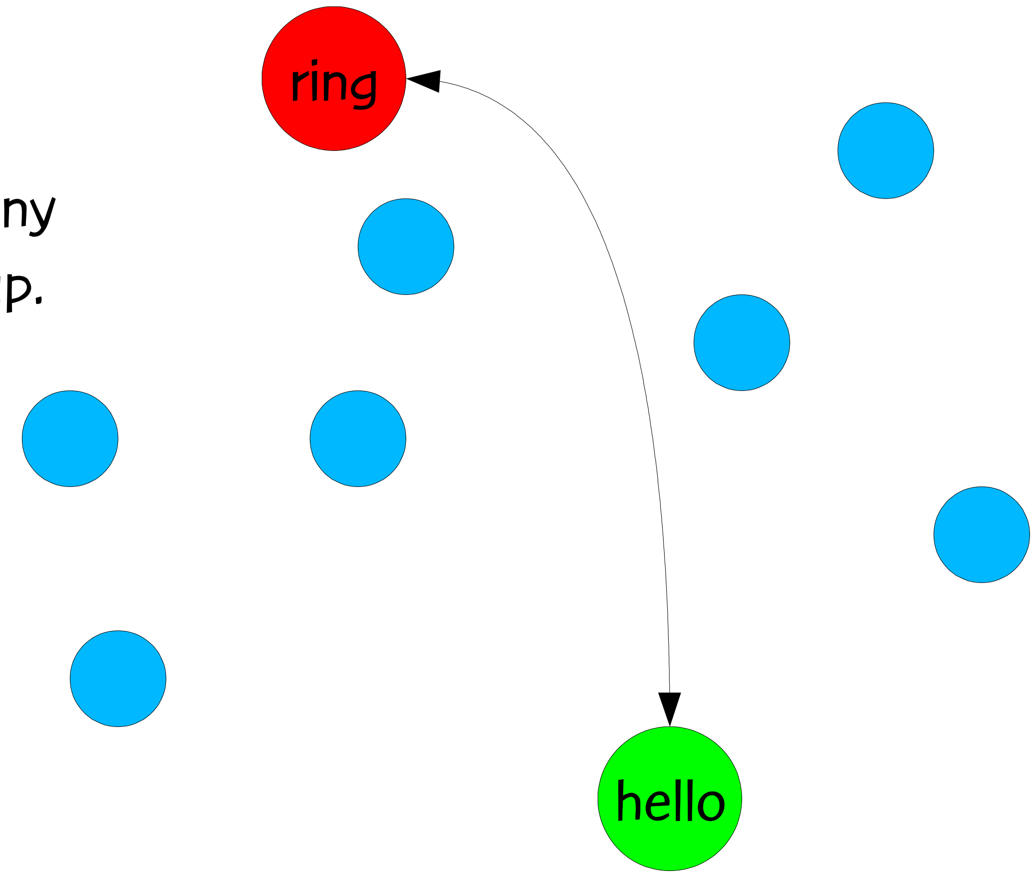

Intercomm (ICOM)

Allows speed dialing
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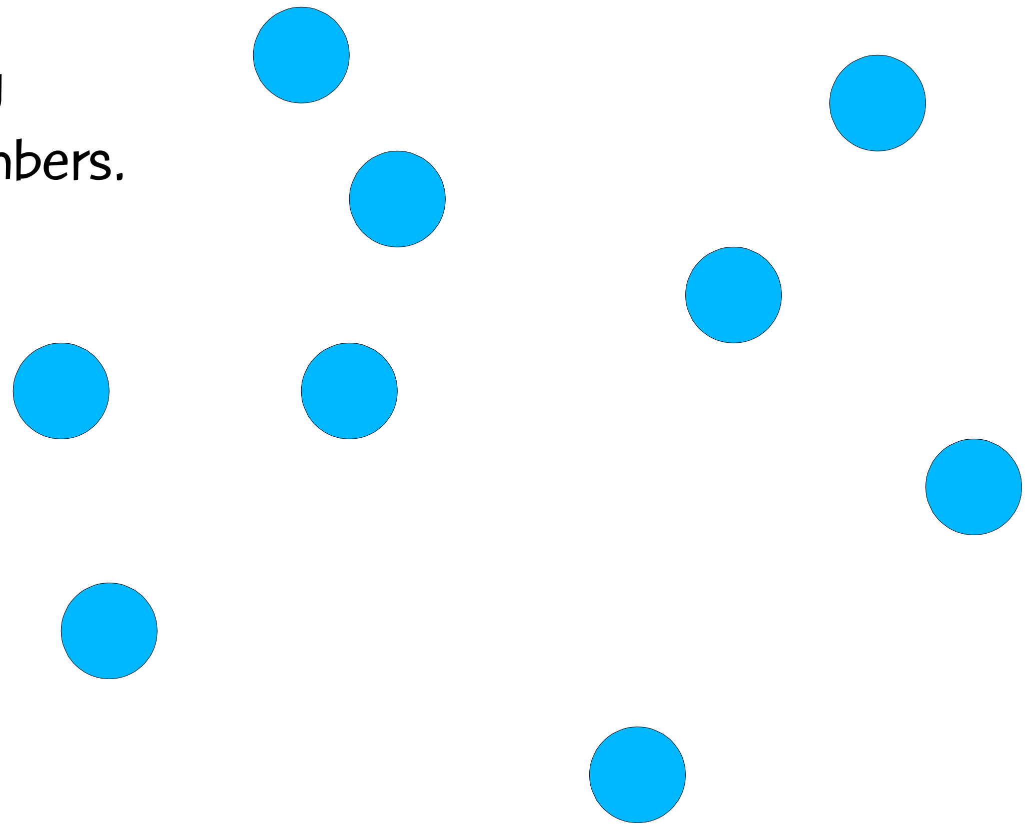

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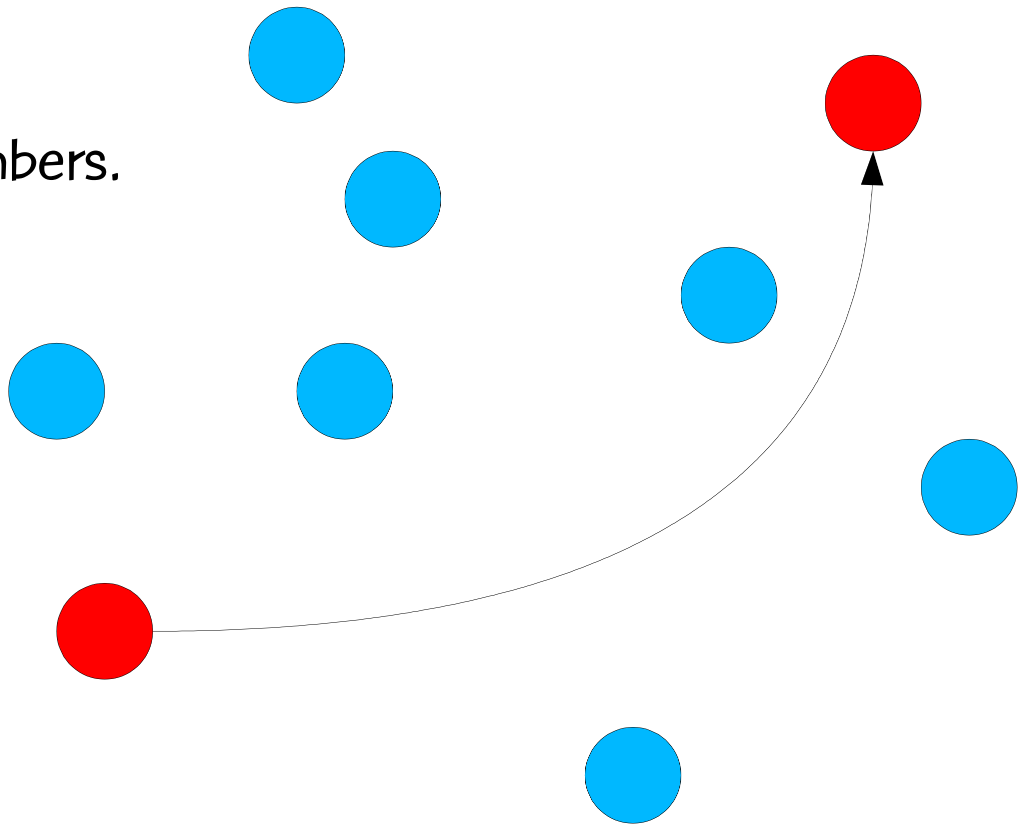

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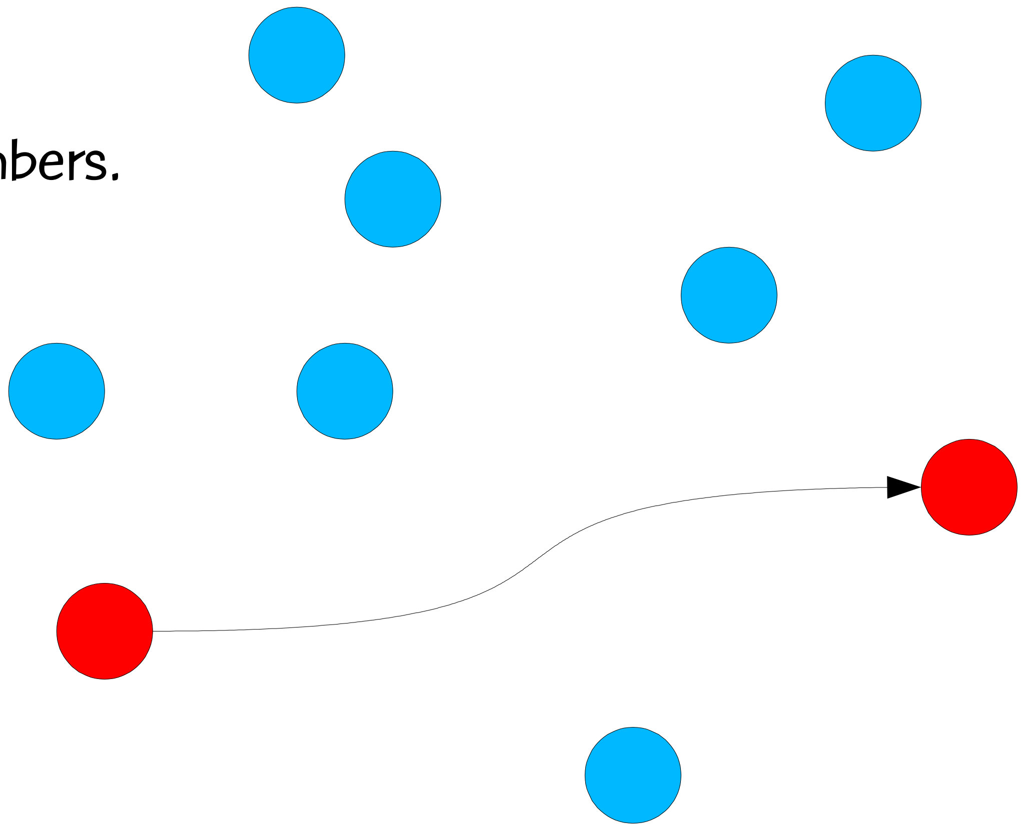

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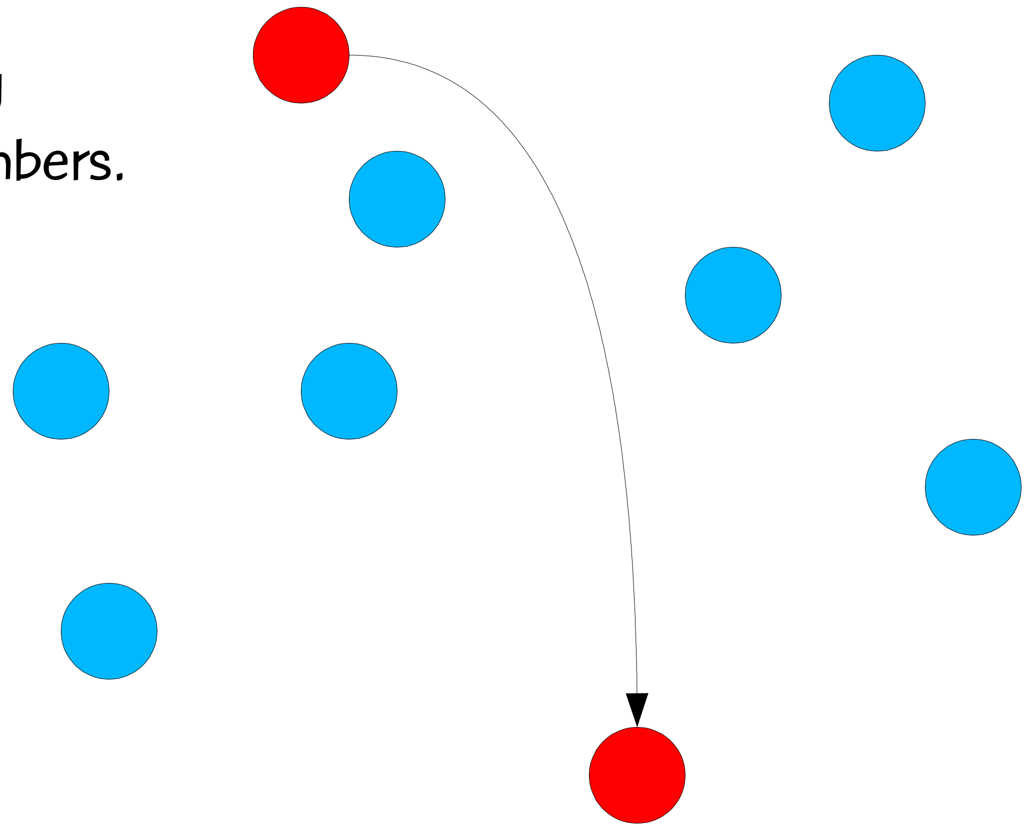
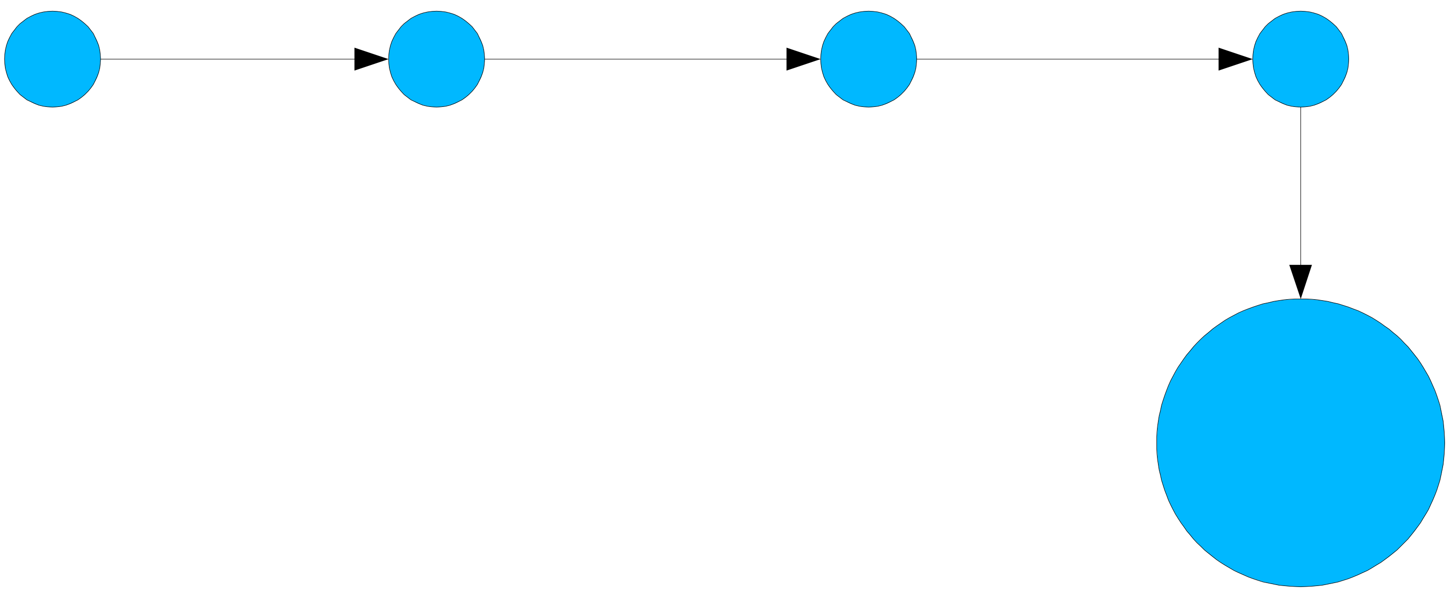


Series completion



Series completion

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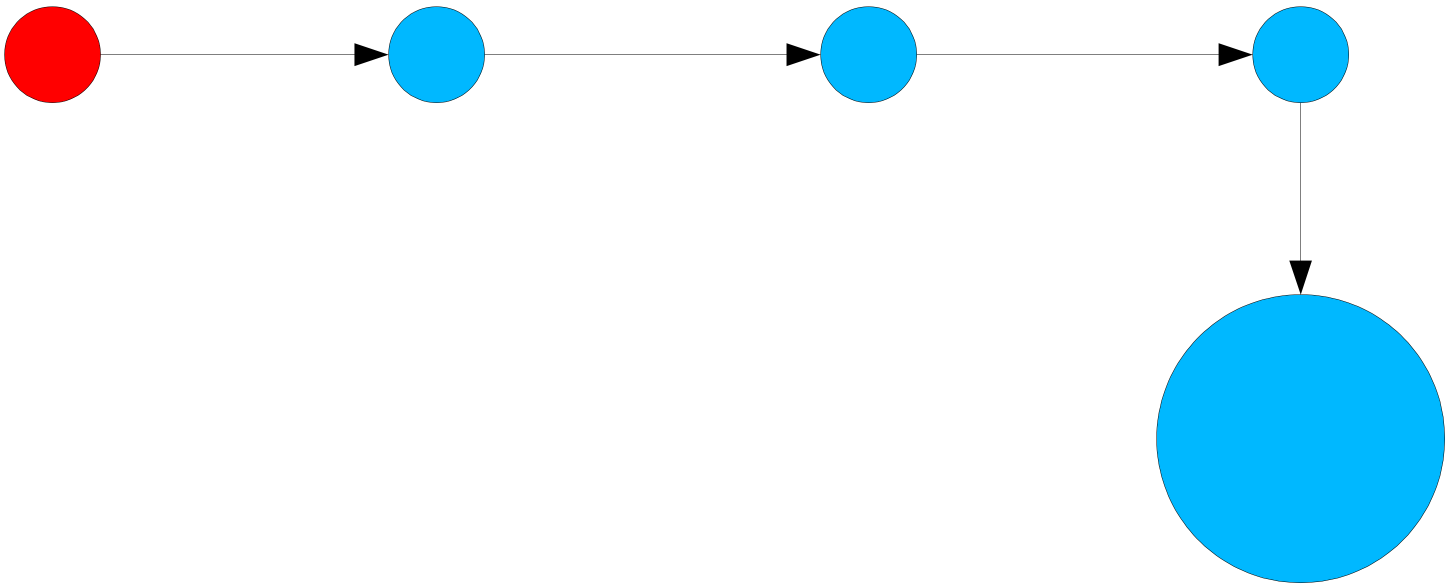

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If call not answered, it moves
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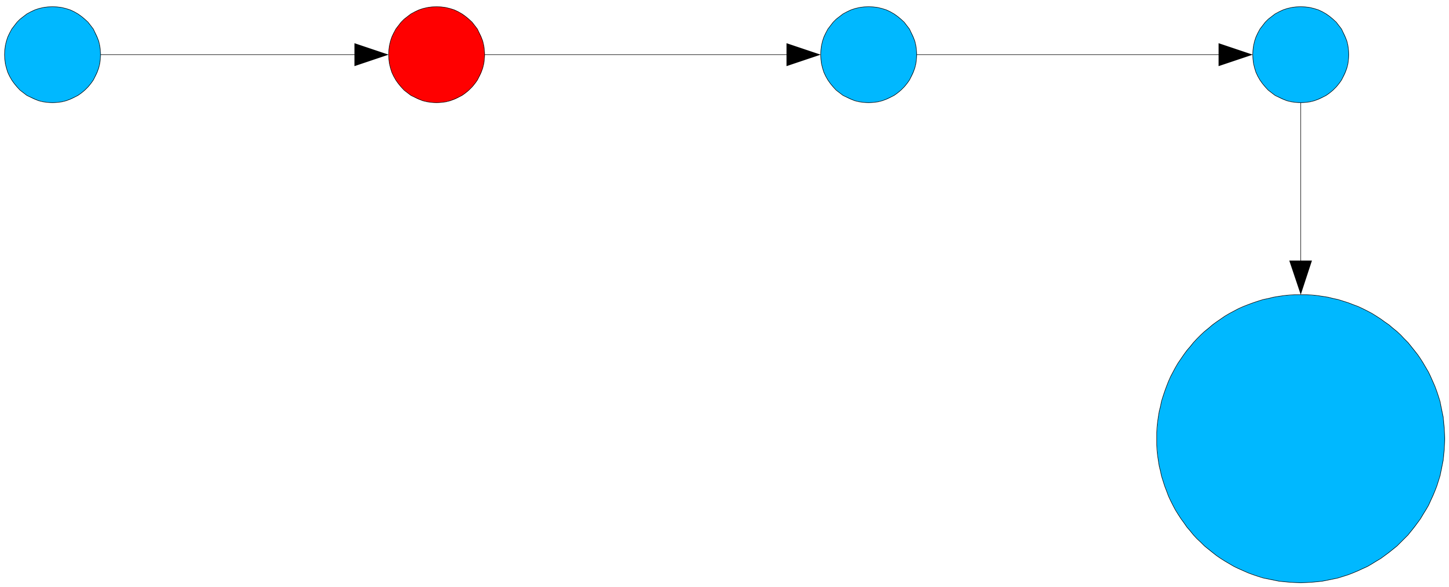

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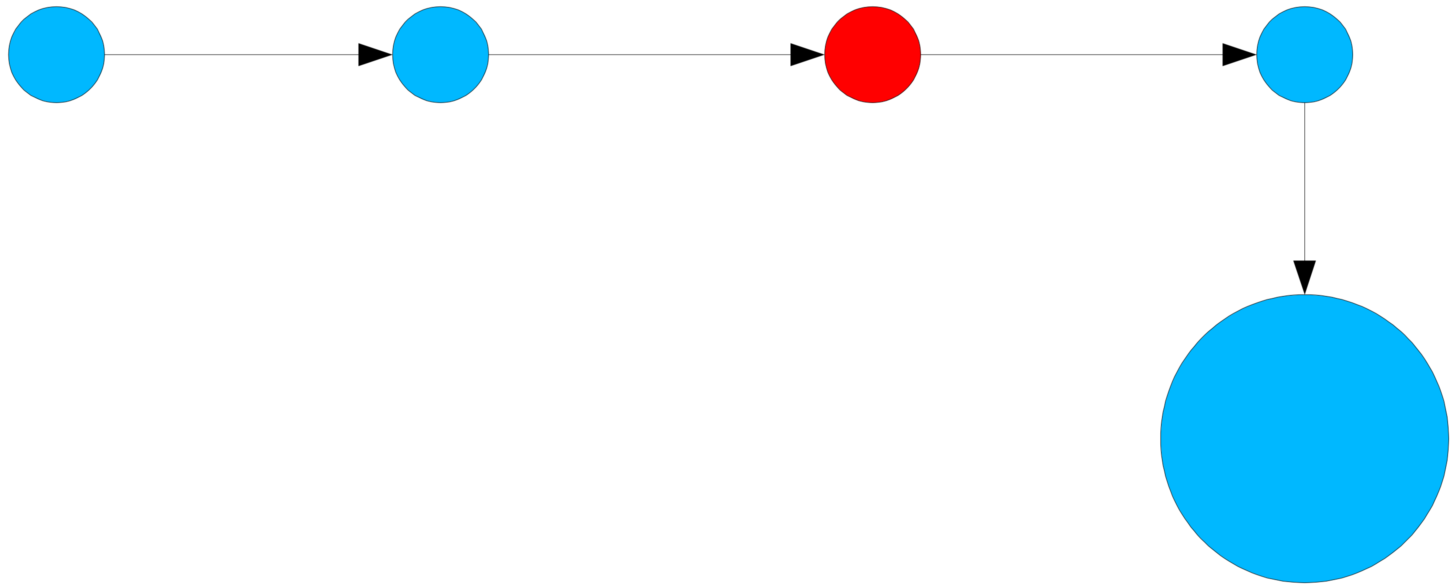

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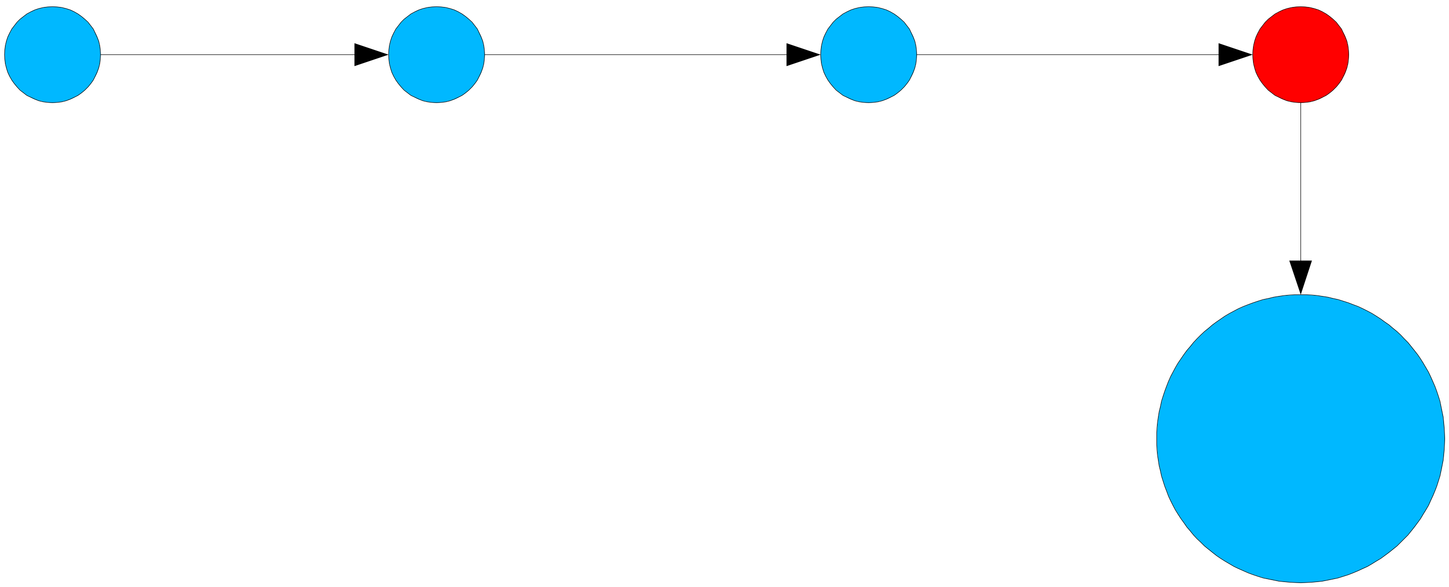

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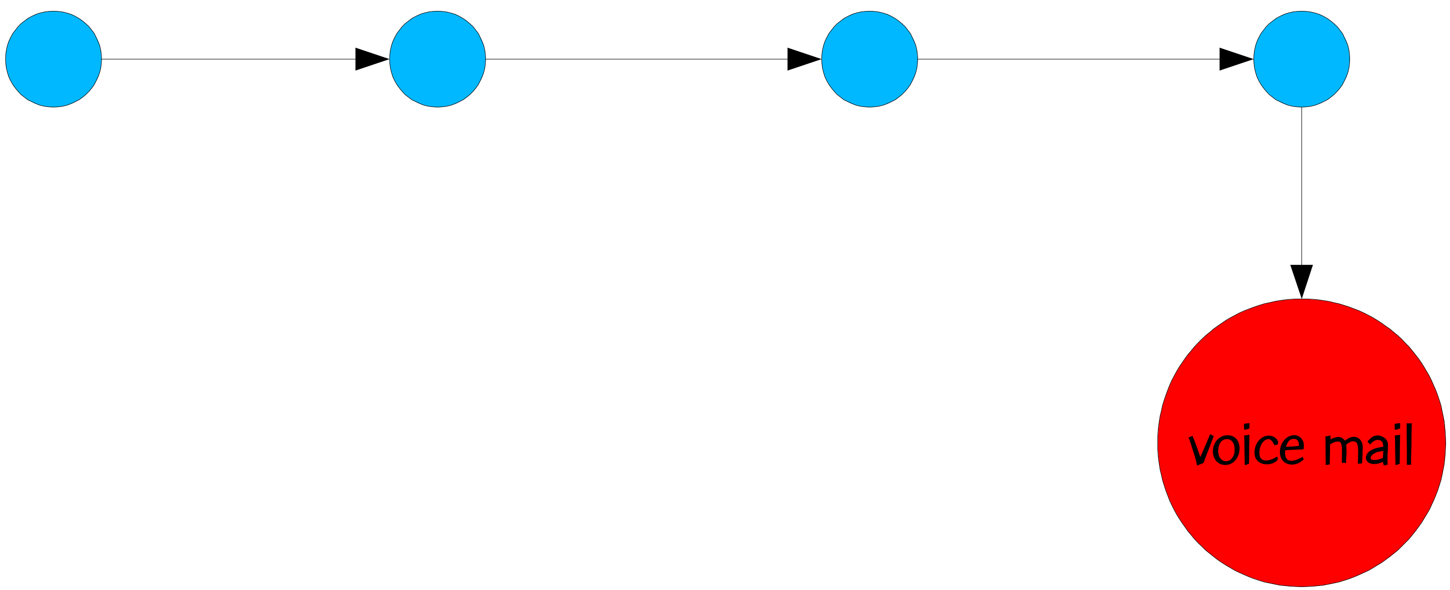

... until it is finally

answered by voice mail.

Shared TN

Assistant answers all
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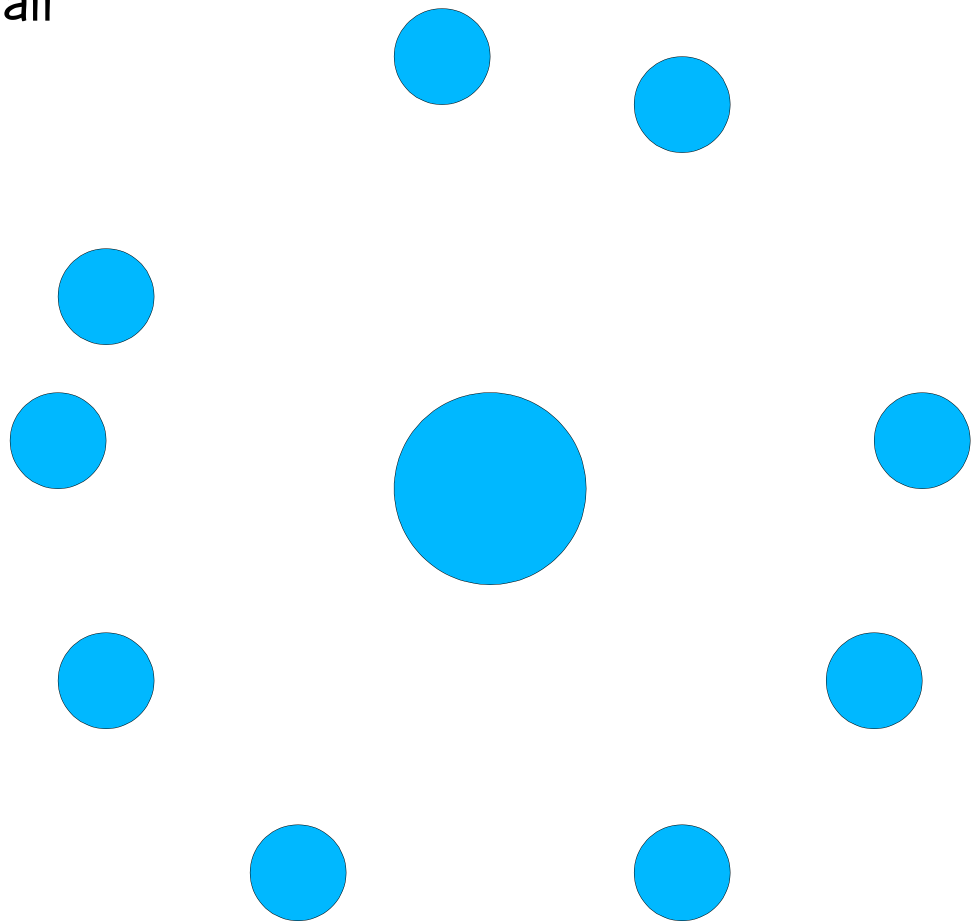

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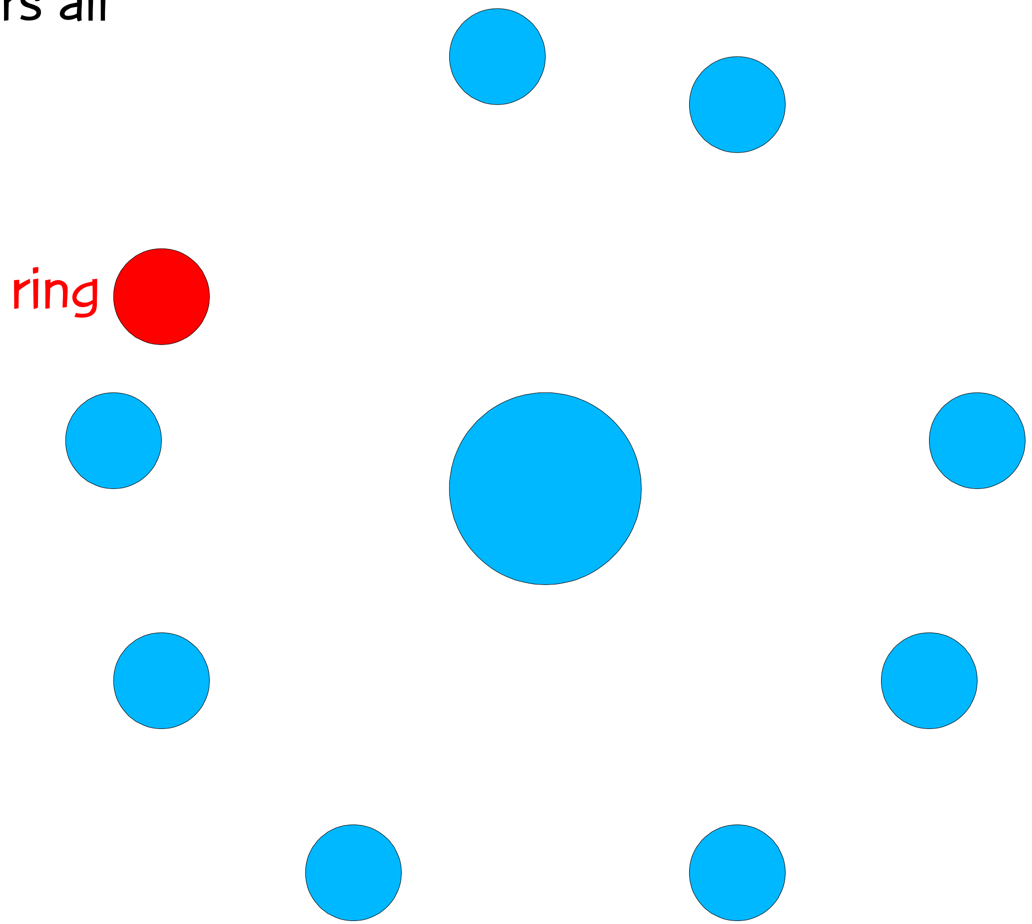

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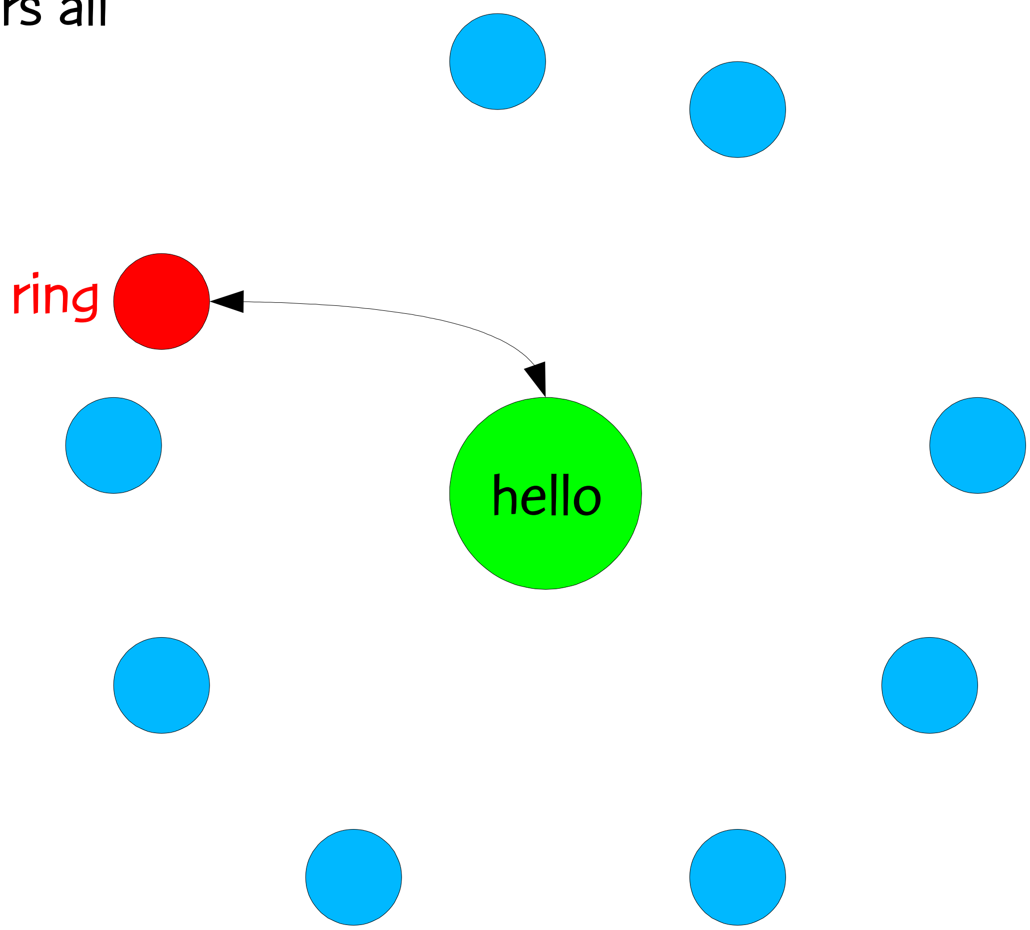

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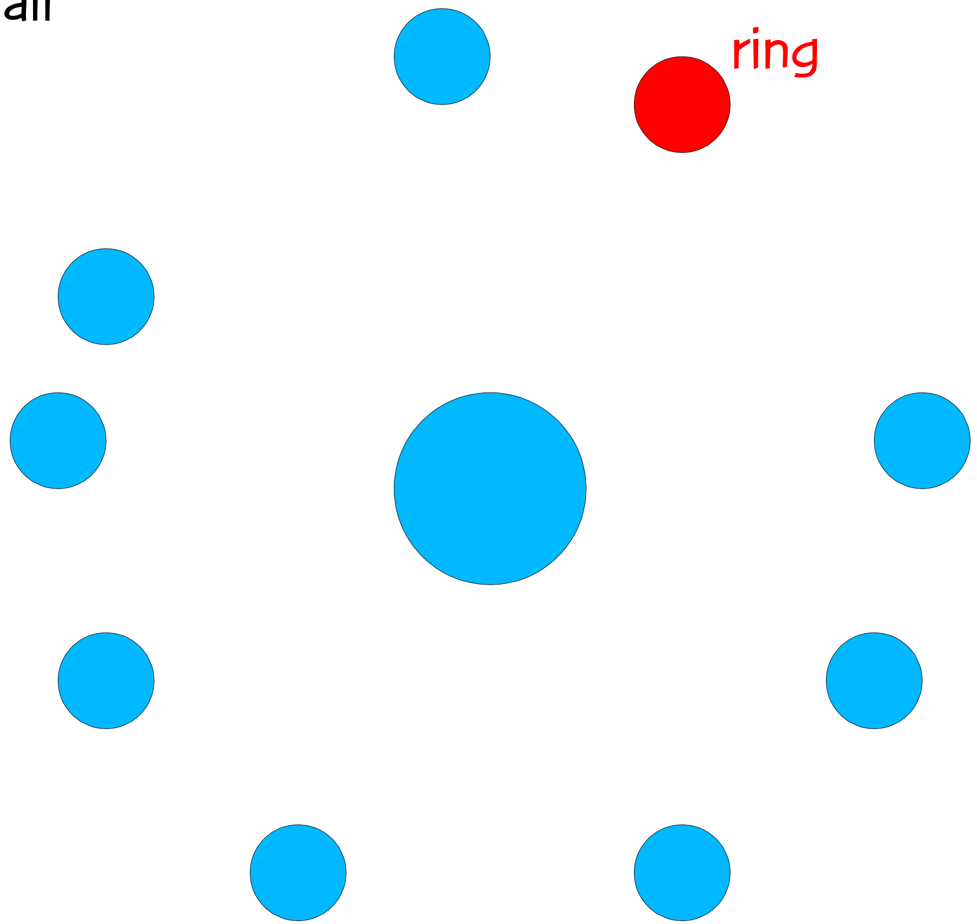

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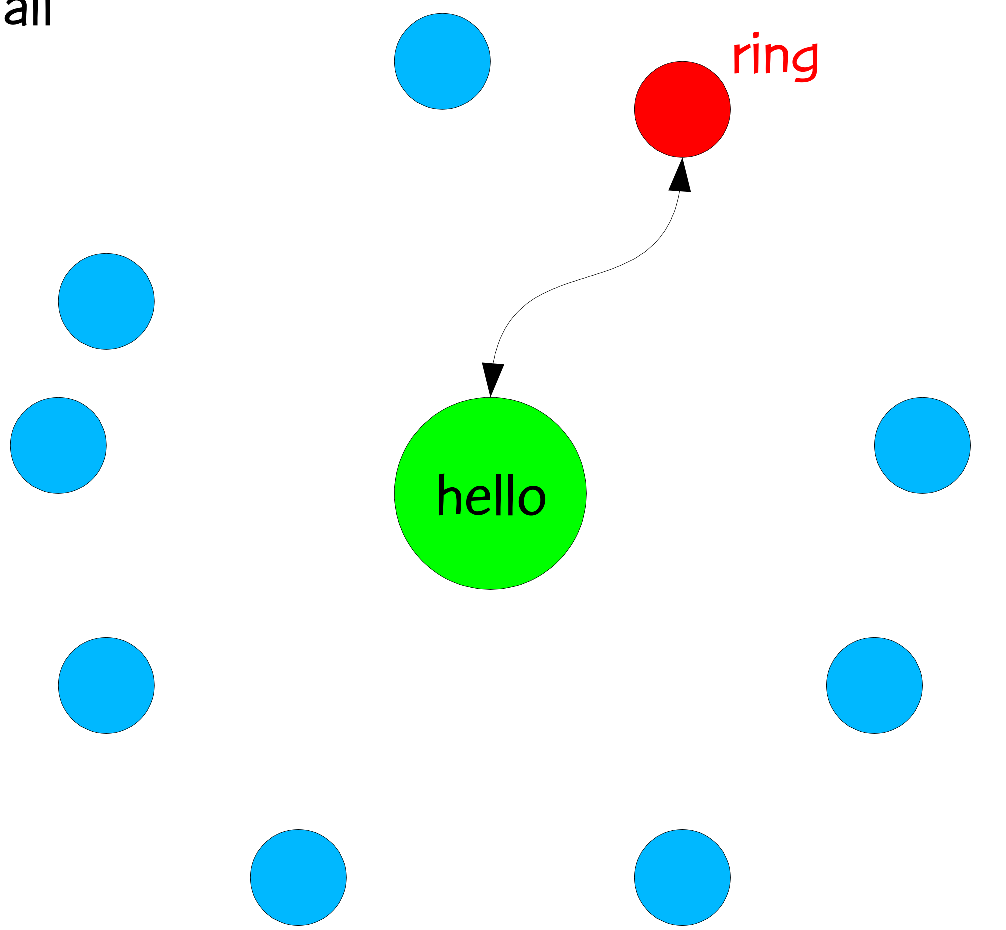

Real-world example

- 8 periods, 2855 phone numbers, 397 groups
- At most 375 phones can be moved in a period.
- Penalties:
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  - CPU : 4
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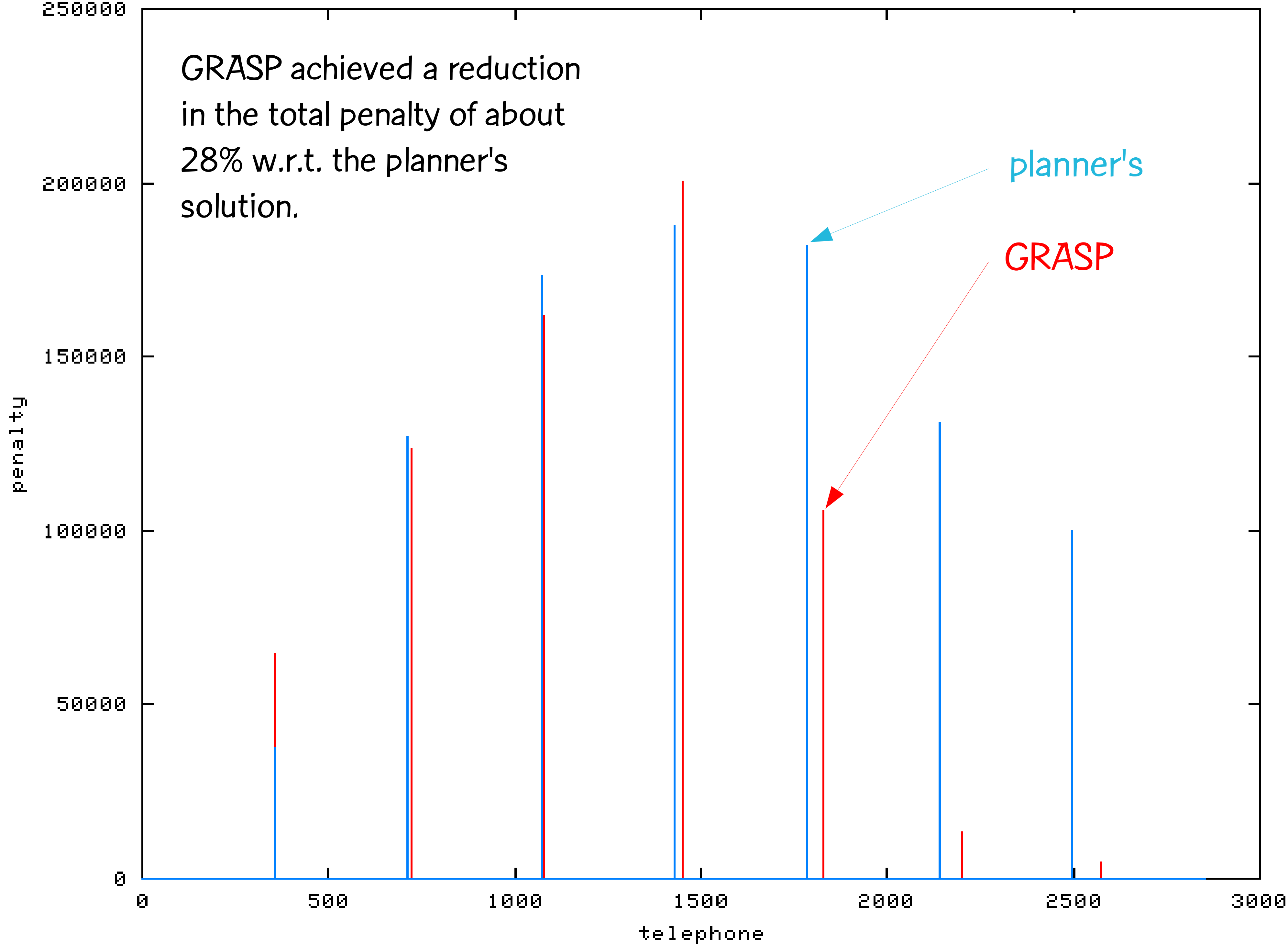
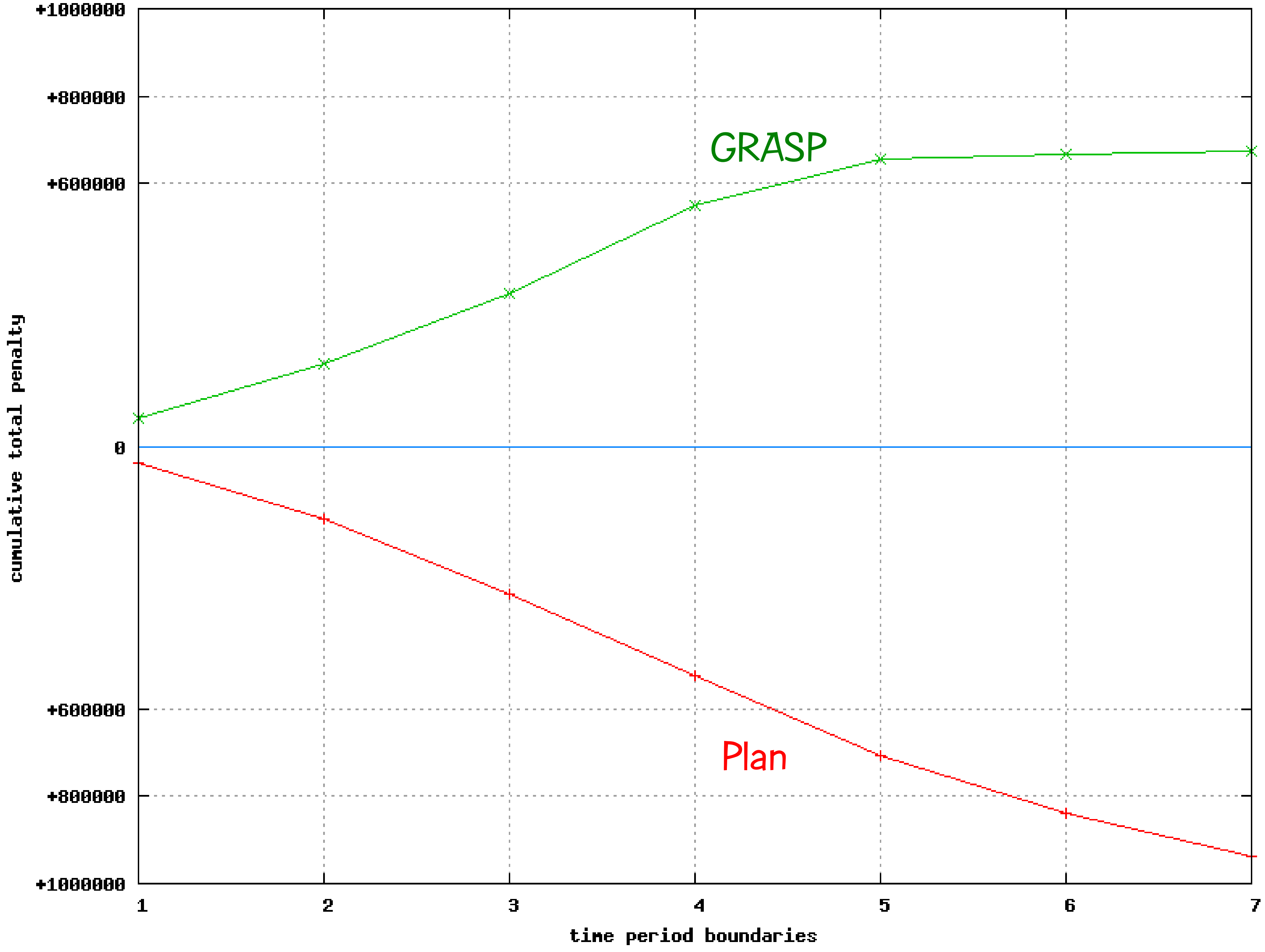


Your world. Delivered.



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Concluding remarks

- A special case of batch scheduling of multi-grouped units is a network migration problem that arises when traffic is migrated from an "old" (switched) network to a "new" (IP) network.
- One needs to determine the order in which switches are decommissioned so as to minimize the temporary capacity needed to carry traffic between the "old" network and the "new" network.
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My coauthor

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Rutgers University

The End