A genetic algorithm with optimized crossover for the weight setting problem in OSPF routing

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Joint work with:

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C. C. Ribeiro
M. Thorup

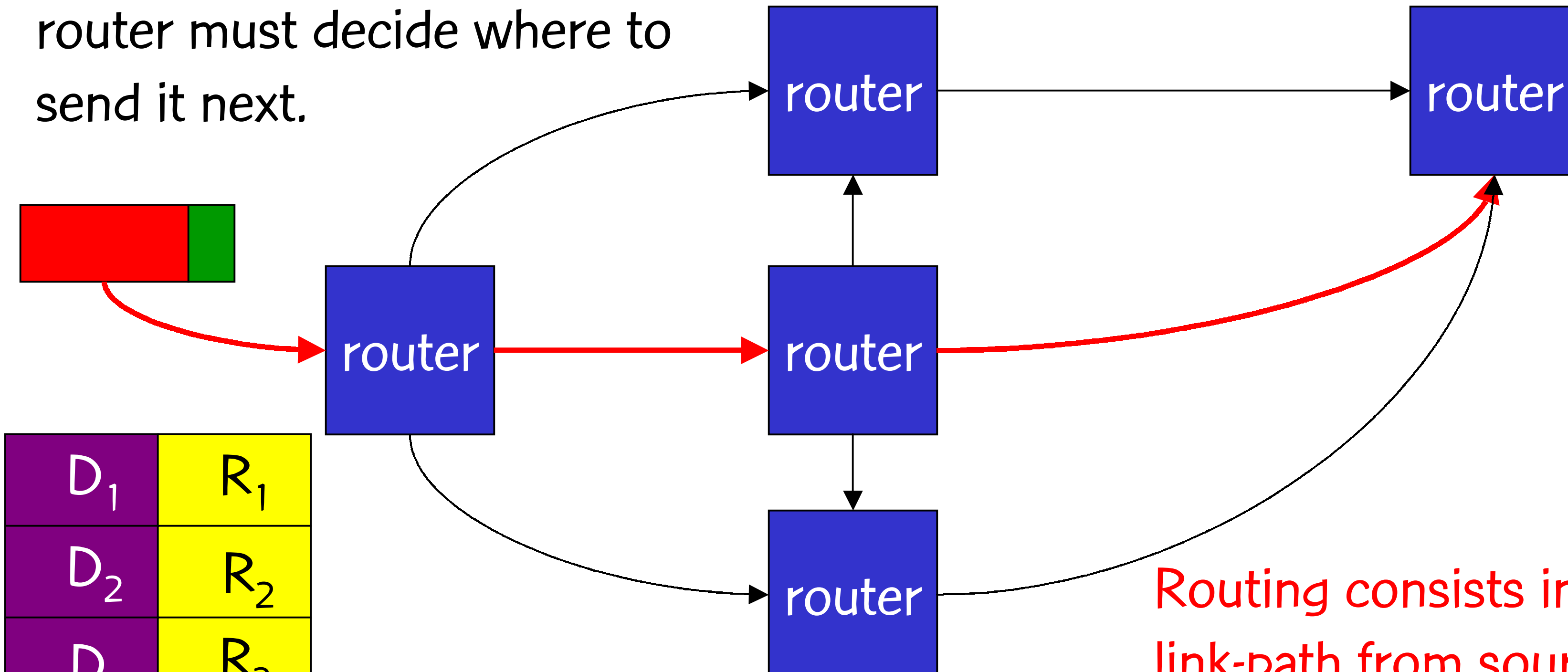
Internet traffic engineering

- Objective: make more efficient use of existing network resources.
- Routing of traffic can have a major impact on efficiency of network resource utilization.

Packet routing

When packet arrives at router, router must decide where to send it next.

Packet's final destination.



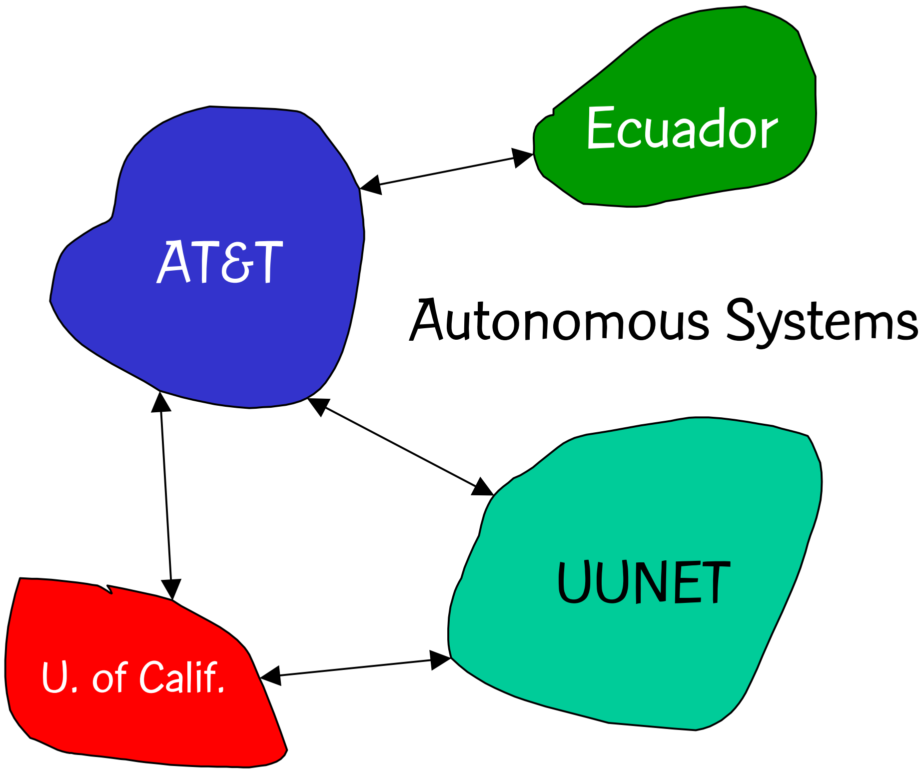
D_1	R_1
D_2	R_2
D_3	R_3
D_4	R_4

Routing table

Routing consists in finding a link-path from source to destination.

OSPF (Open Shortest Path First)

- OSPF is a commonly used intra-domain routing protocol (IGP).
- Routers exchange routing information with all other routers in the autonomous system (AS).
  - Complete network topology knowledge is available to all routers, i.e. state of all routers and links in the AS.



OSPF routing

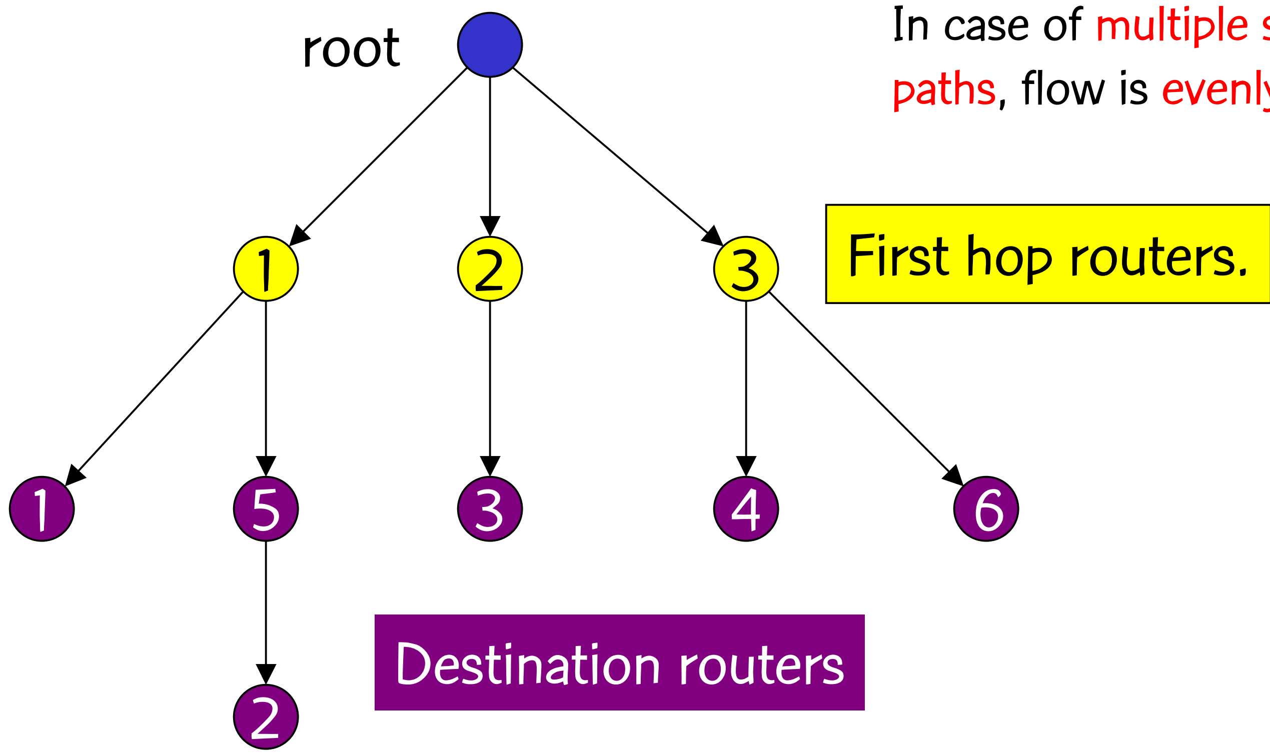
- Assign an integer weight $\in [1, w_{max}]$ to each link in AS. In general, $w_{max} = 65535 = 2^{16} - 1$.
- Each router computes tree of shortest weight paths to all other routers in the AS, with itself as the root, using Dijkstra's algorithm.

OSPF routing

Routing table

D_1	R_1
D_2	R_1
D_3	R_2
D_4	R_3
D_5	R_1
D_6	R_3

Routing table is filled
with first hop routers
for each possible destination.
In case of multiple shortest paths, flow is evenly split.



OSPF routing

Routing table

D_1	R_1
D_2	R_1
D_3	R_2
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D_5	R_1
D_6	R_3

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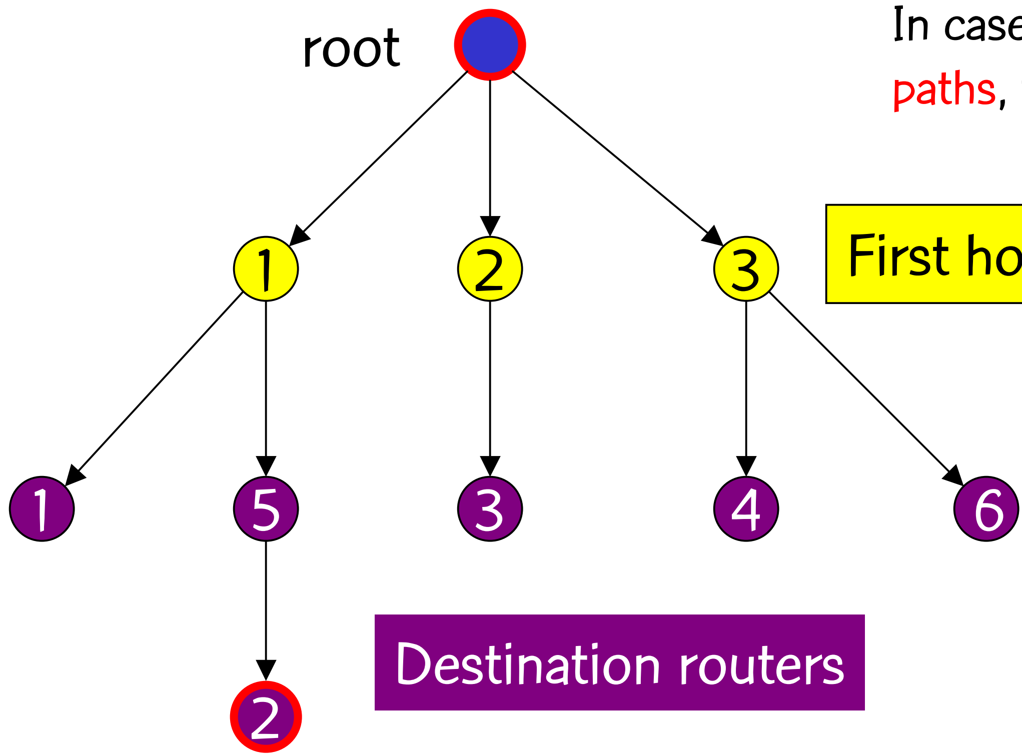

First hop routers.

OSPF routing

Routing table

D_1	R_1
D_2	R_1
D_3	R_2
D_4	R_3
D_5	R_1
D_6	R_3

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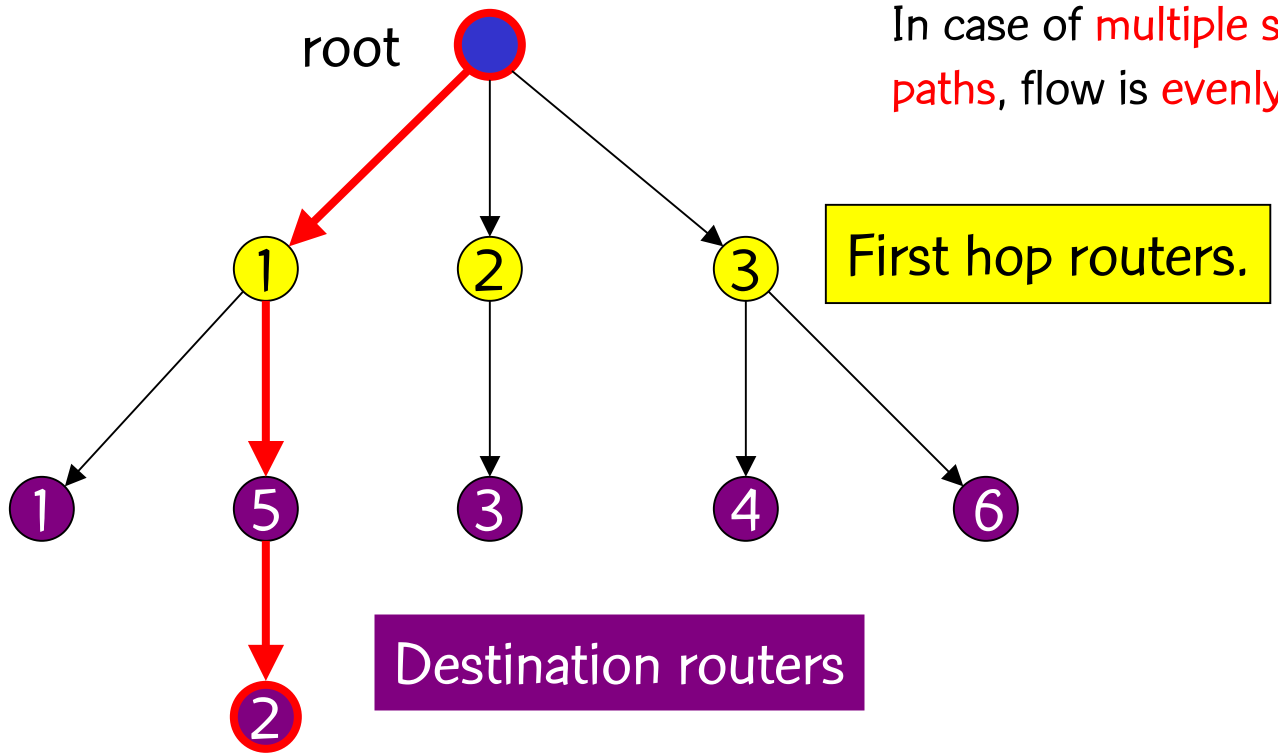

OSPF routing

Routing table

D_1	R_1
D_2	R_1
D_3	R_2
D_4	R_3
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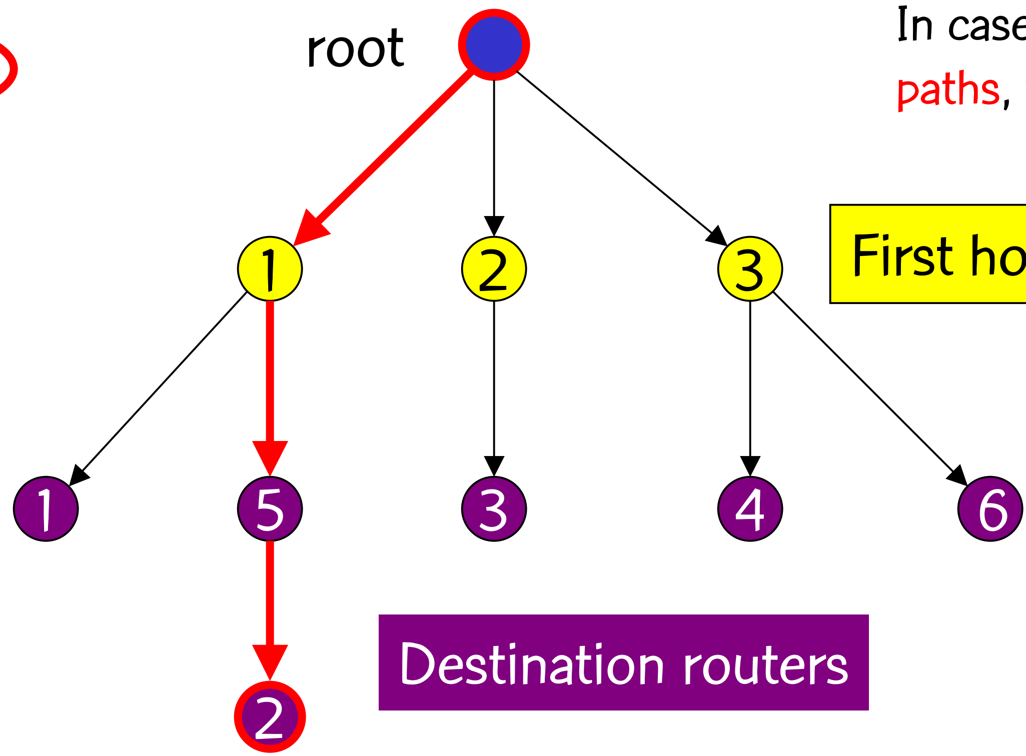


First hop routers.

OSPF weight setting

- OSPF weights are assigned by network operator.
  - CISCO assigns, by default, a weight proportional to the inverse of the link bandwidth (Inv Cap).
  - If all weights are unit, the weight of a path is the number of hops in the path.
- We propose a hybrid genetic algorithm to find good OSPF weights.
  - Memetic algorithm
  - Genetic algorithm with optimized crossover

Minimization of congestion

- Consider the directed capacitated network $G = (N,A,c)$, where N are routers, A are links, and c_a is the capacity of link $a \in A$.
- We use the measure of Fortz & Thorup (2000) to compute congestion:

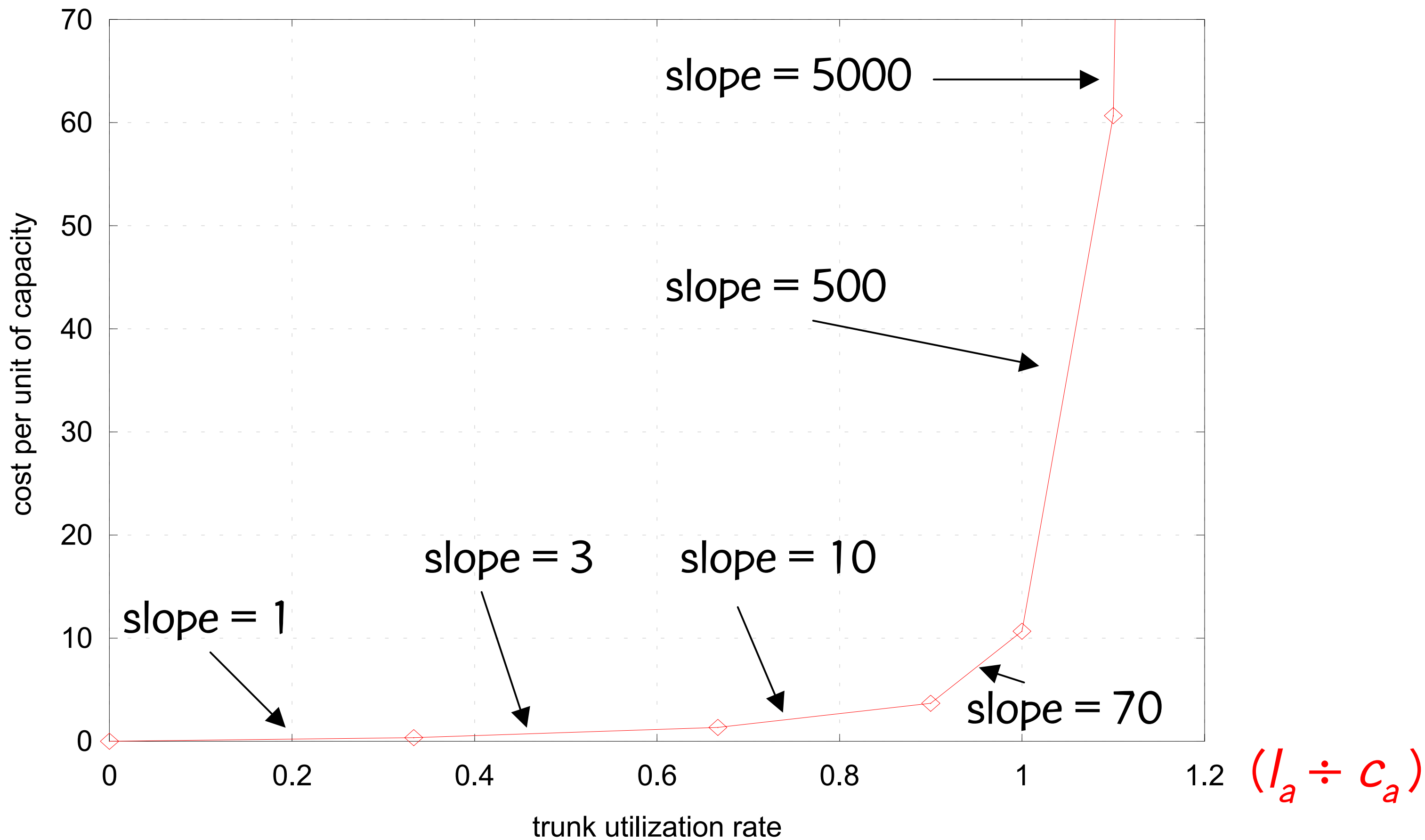
$$\Phi = \Phi_1(l_1) + \Phi_2(l_2) + \ldots + \Phi_{|A|}(l_{|A|})$$

where l_a is the load on link $a \in A$,

$\Phi_a(l_a)$ is piecewise linear and convex,

$\Phi_a(0) = 0$, for all $a \in A$.

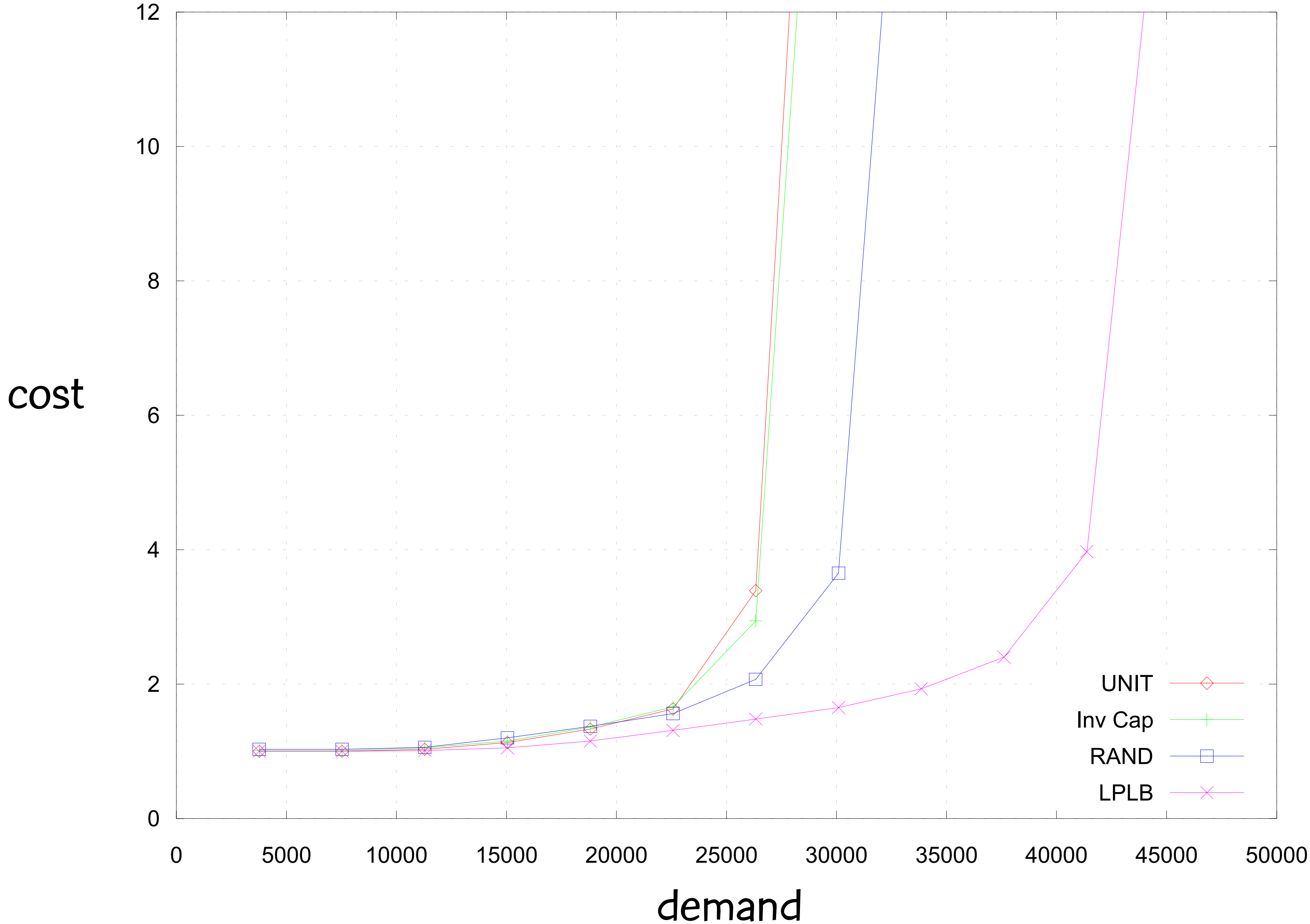
Piecewise linear and convex $\Phi_a(l_a)$ link congestion measure



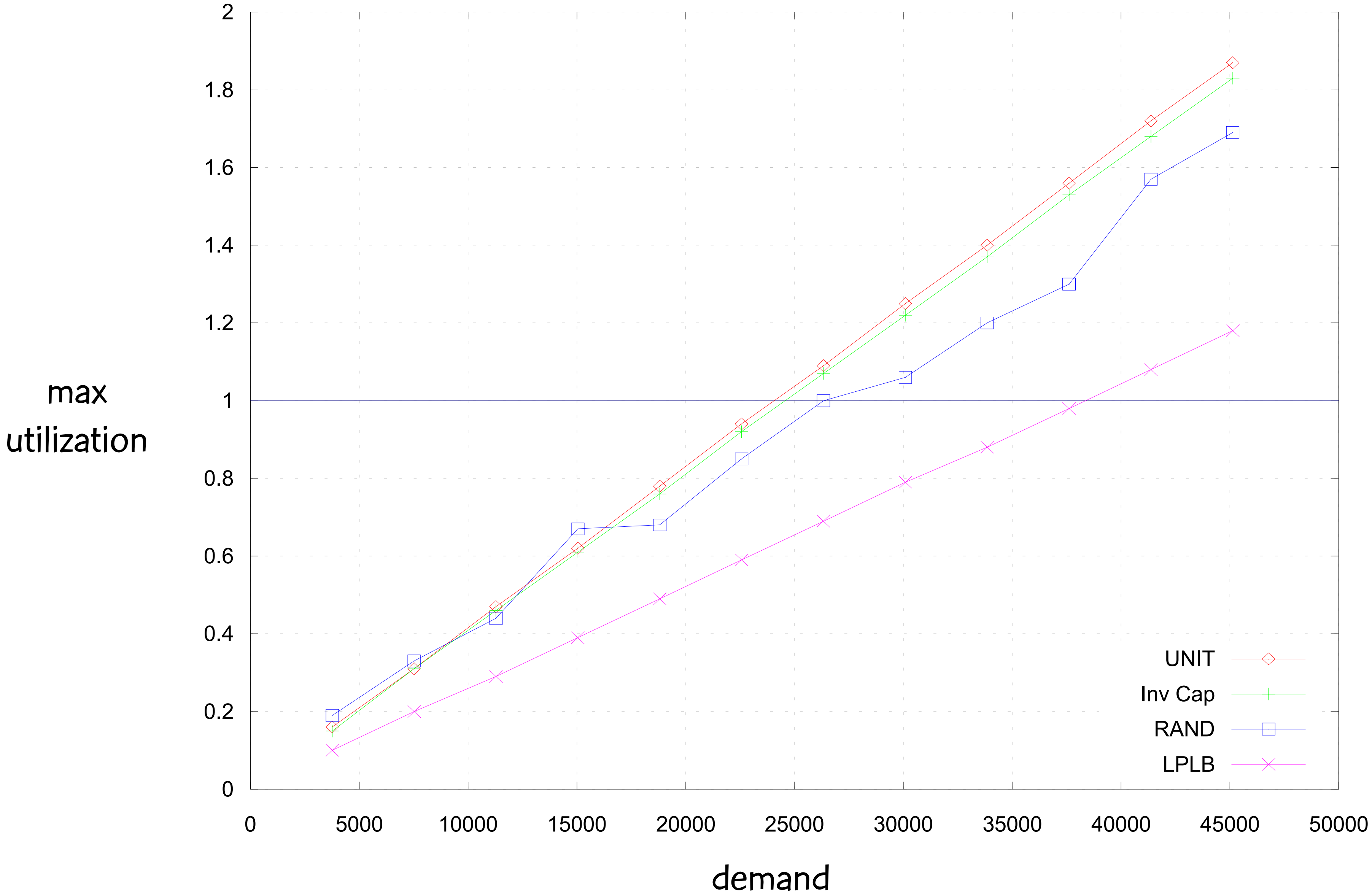
OSPF weight setting problem

- Given a directed network $G = (N, A)$ with link capacities $c_a \in A$ and demand matrix $D = (d_{s,t})$ specifying a demand to be sent from node s to node t:
  - Assign weights $w_a \in [1, w_{max}]$ to each link $a \in A$, such that the objective function Φ is minimized when demand is routed according to the OSPF protocol.

AT&T Worldnet backbone network (90 routers, 274 links)



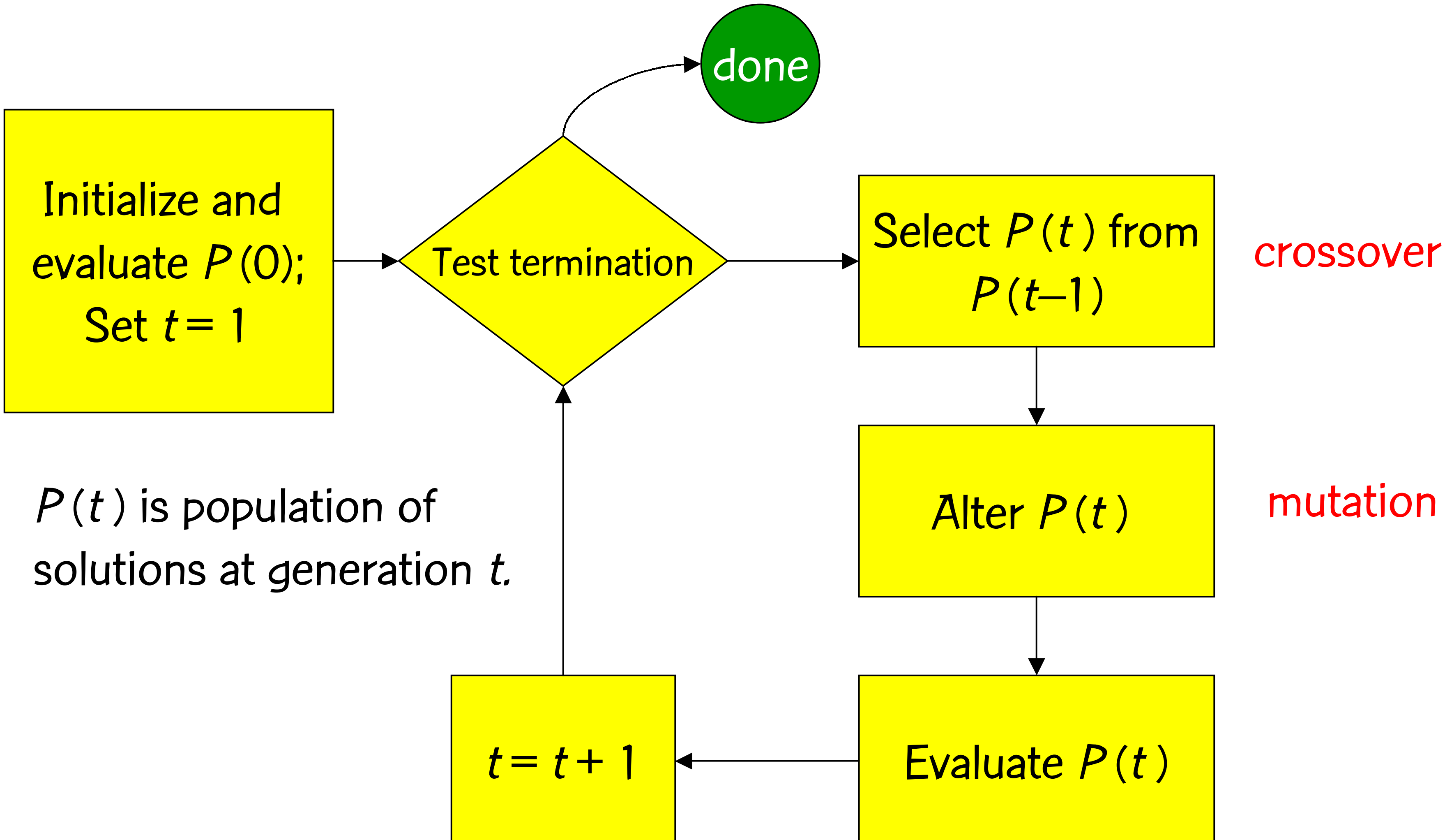
AT&T Worldnet backbone network (90 routers, 274 links)



Previous work

- Avoid multiple shortest paths (small networks):
  - Lin & Wang (1993): use Lagrangian relaxation and consider networks with up to 26 nodes.
  - Rodrigues & Ramakrishnan (1994): use local search with single descent and work on small networks with at most 16 nodes and 18 links.
  - Bley et at. (1998): use local search with single descent and consider small networks with up to 13 links.
- Allow multiple shortest paths (realistically sized networks):
  - Fortz & Thorup (2000): use tabu search type local search on large networks with up to 100 nodes and 503 links.
  - Ericcson, R., & Pardalos (2002): genetic algorithm

Genetic algorithms



$P(t)$ is population of solutions at generation t.

Solution encoding

- A population consists of $nPop = 50$ integer weight arrays: $w = (w_1, w_2, ..., w_{|A|})$,

  where $w_a \in [1, w_{max} = 20]$
- All possible weight arrays correspond to feasible solutions.

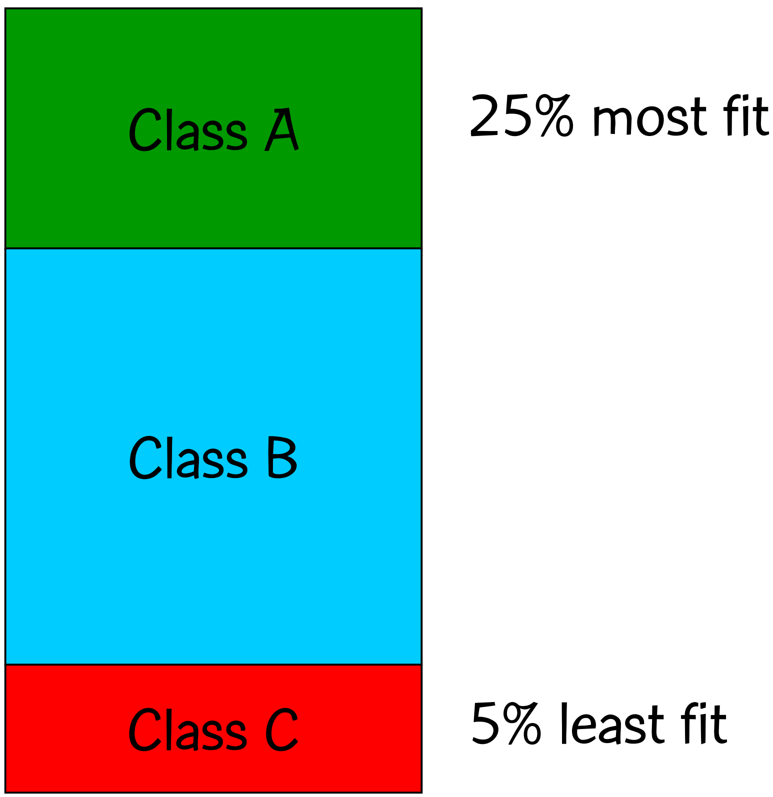
Initial population

- *nPop* solutions, with each weight randomly generated, uniformly in the interval $[1, w_{max}/3]$.

Solution evaluation

- For each demand pair (s,t), route using OSPF, computing demand pair loads $l_a^{s,t}$ on each link $a \in A$.
- Add up demand pair loads on each link $a \in A$, yielding total load l_a on link.
- Compute link congestion cost $\Phi_a(l_a)$ for each link $a \in A$.
- Add up costs: $\Phi = \Phi_1(l_1) + \Phi_2(l_2) + \ldots + \Phi_{|A|}(l_{|A|})$

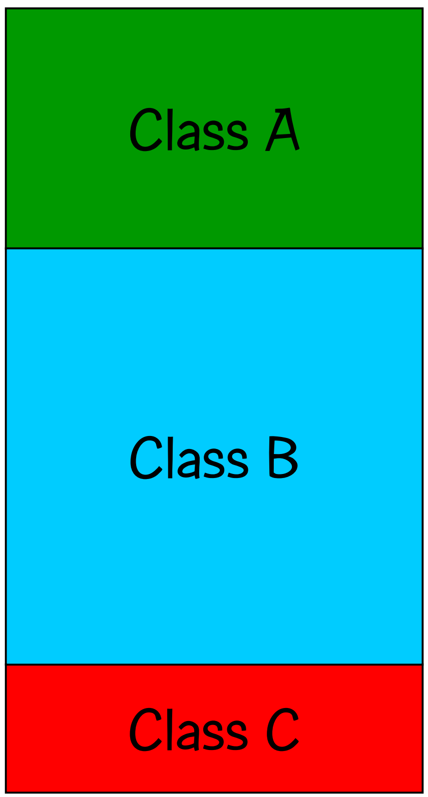
Population partitioning



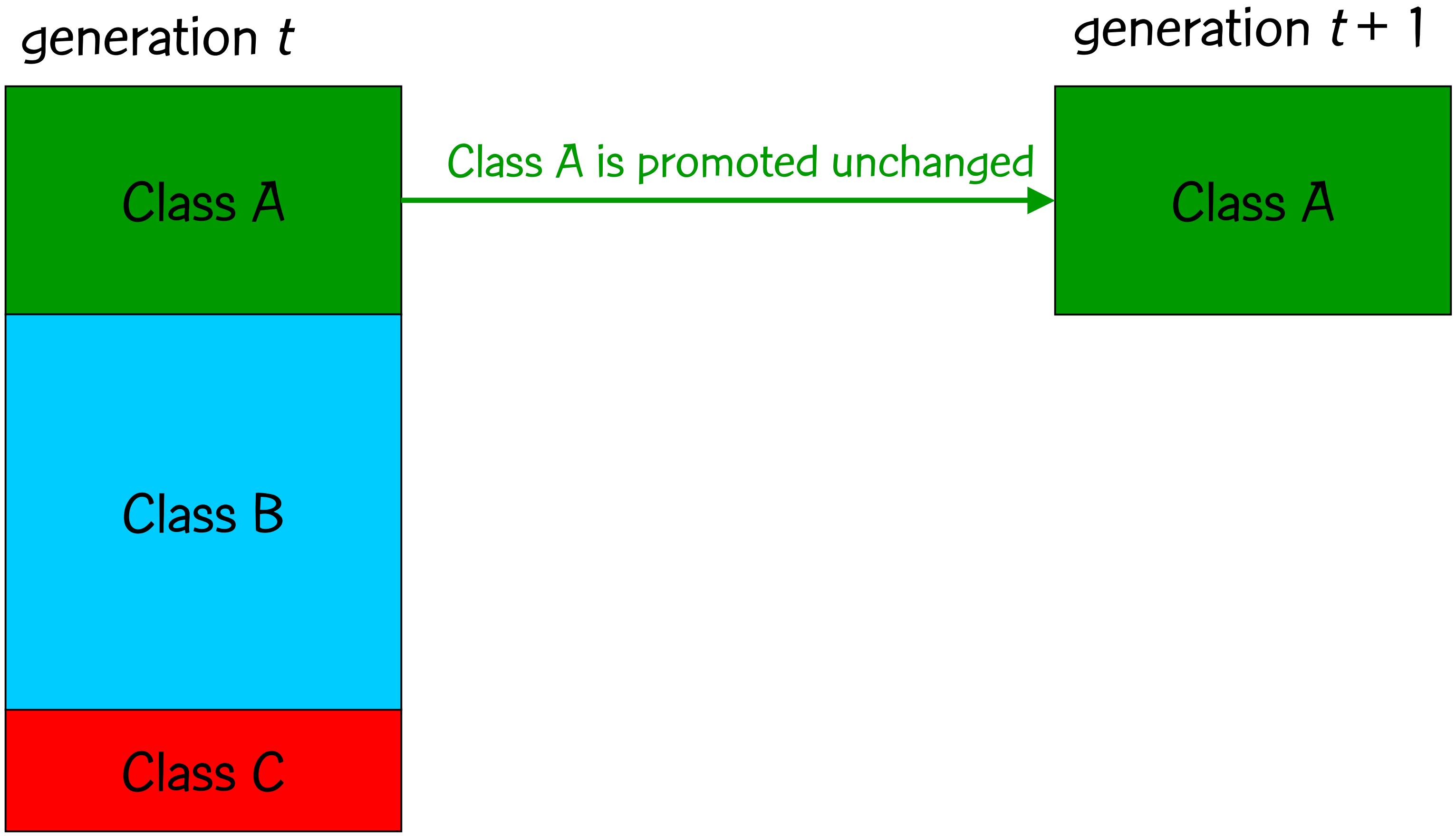
Population is sorted according to solution value Φ and solutions are classified into three categories.

Population dynamics

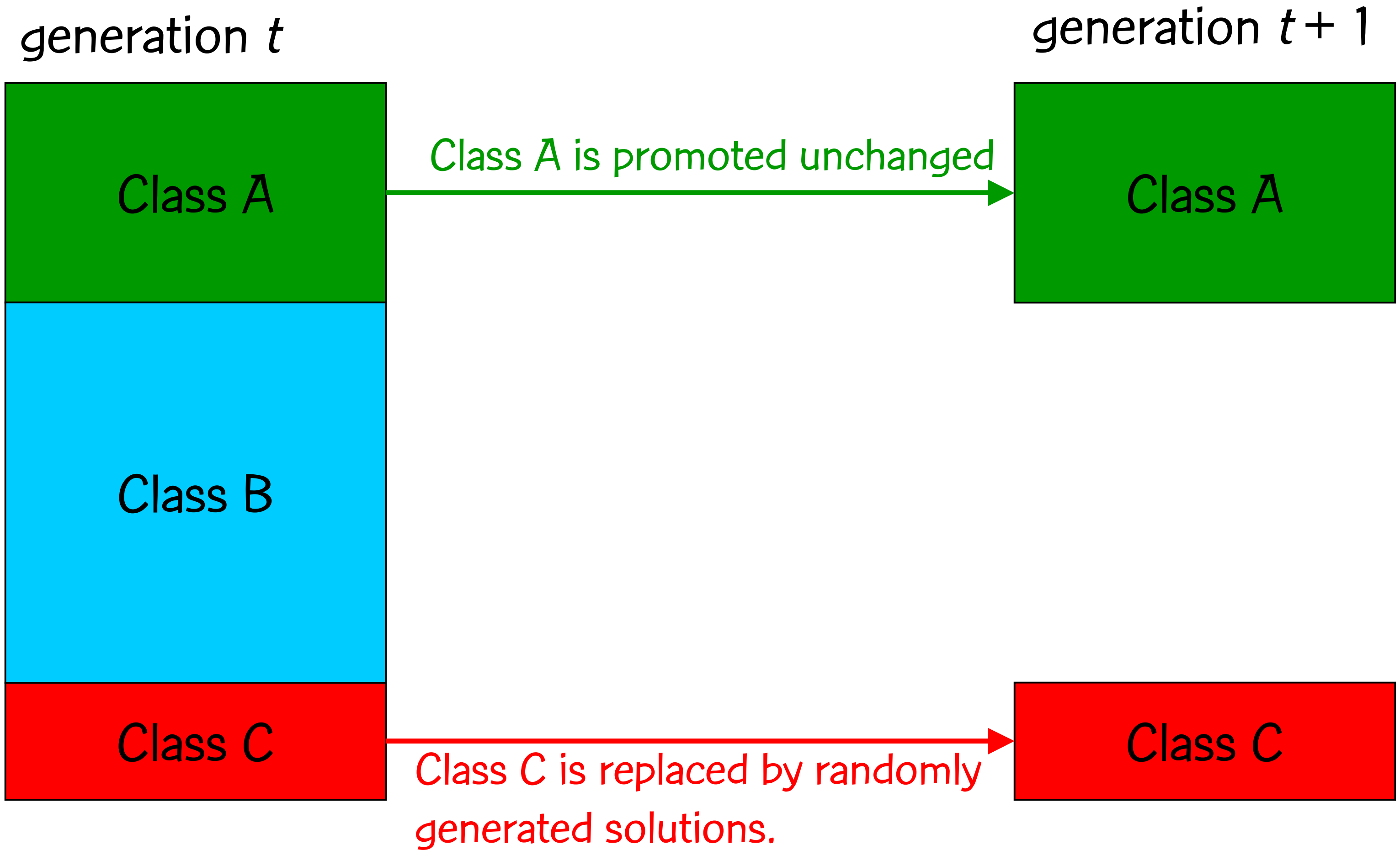
generation t



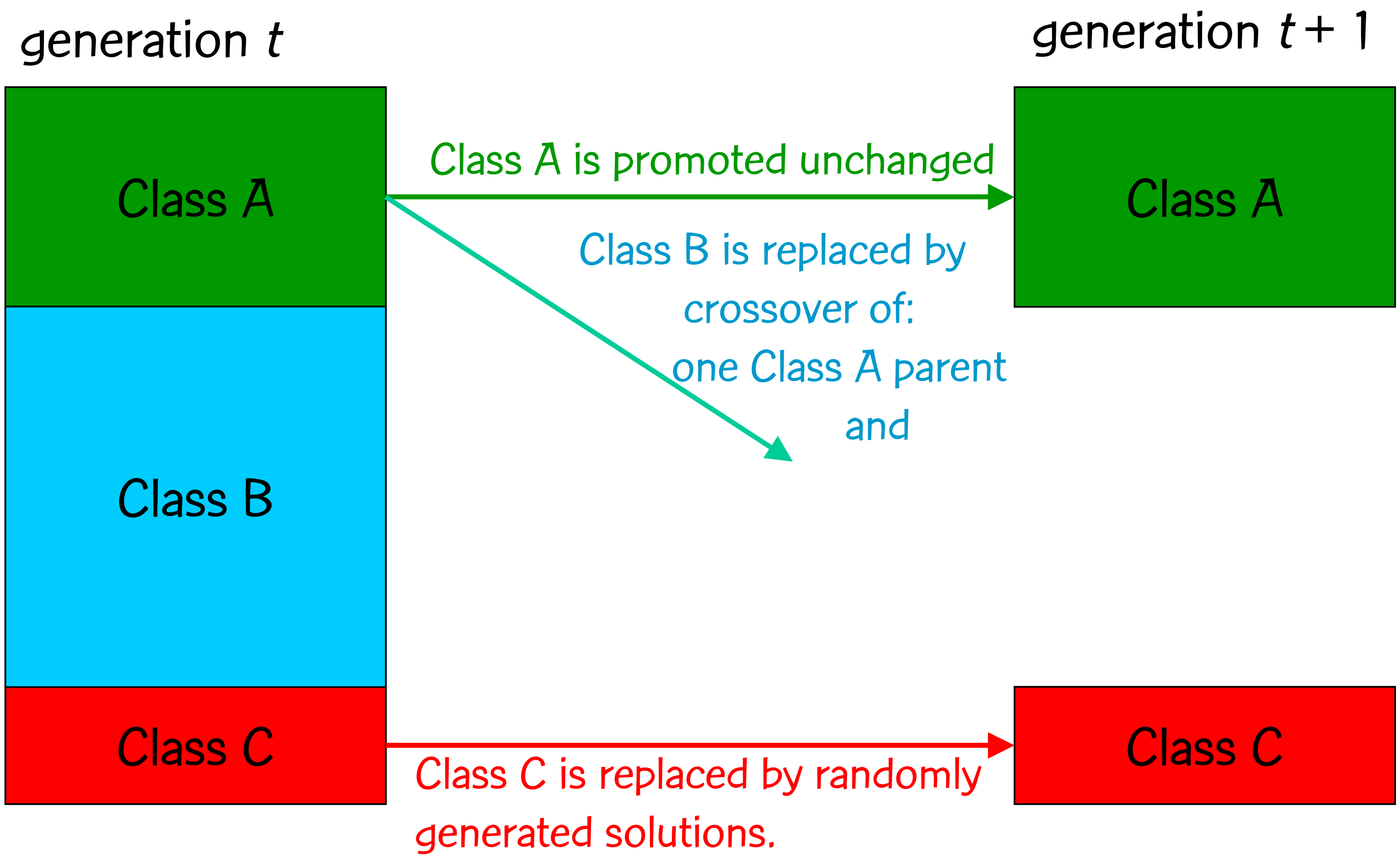
Population dynamics



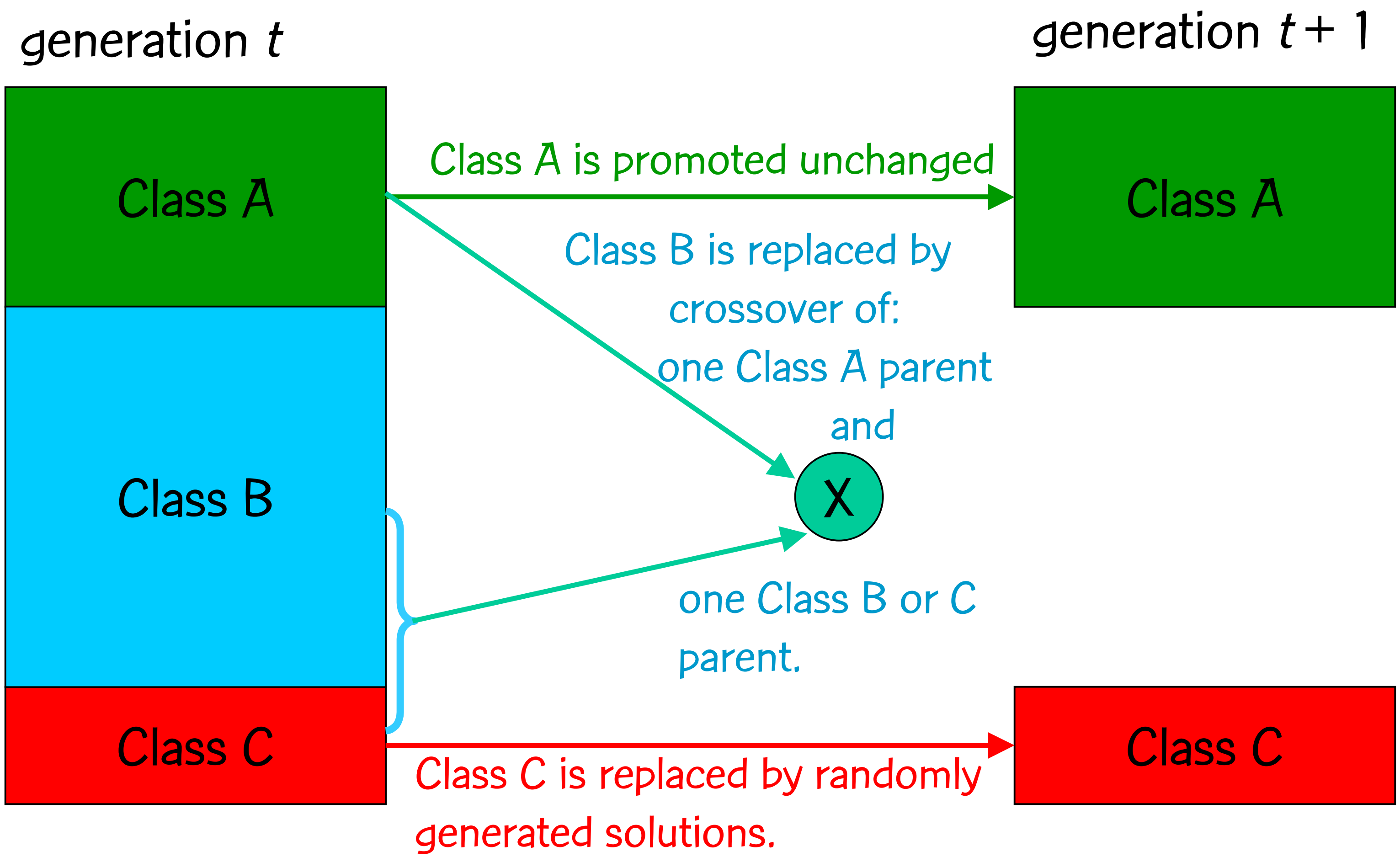
Population dynamics



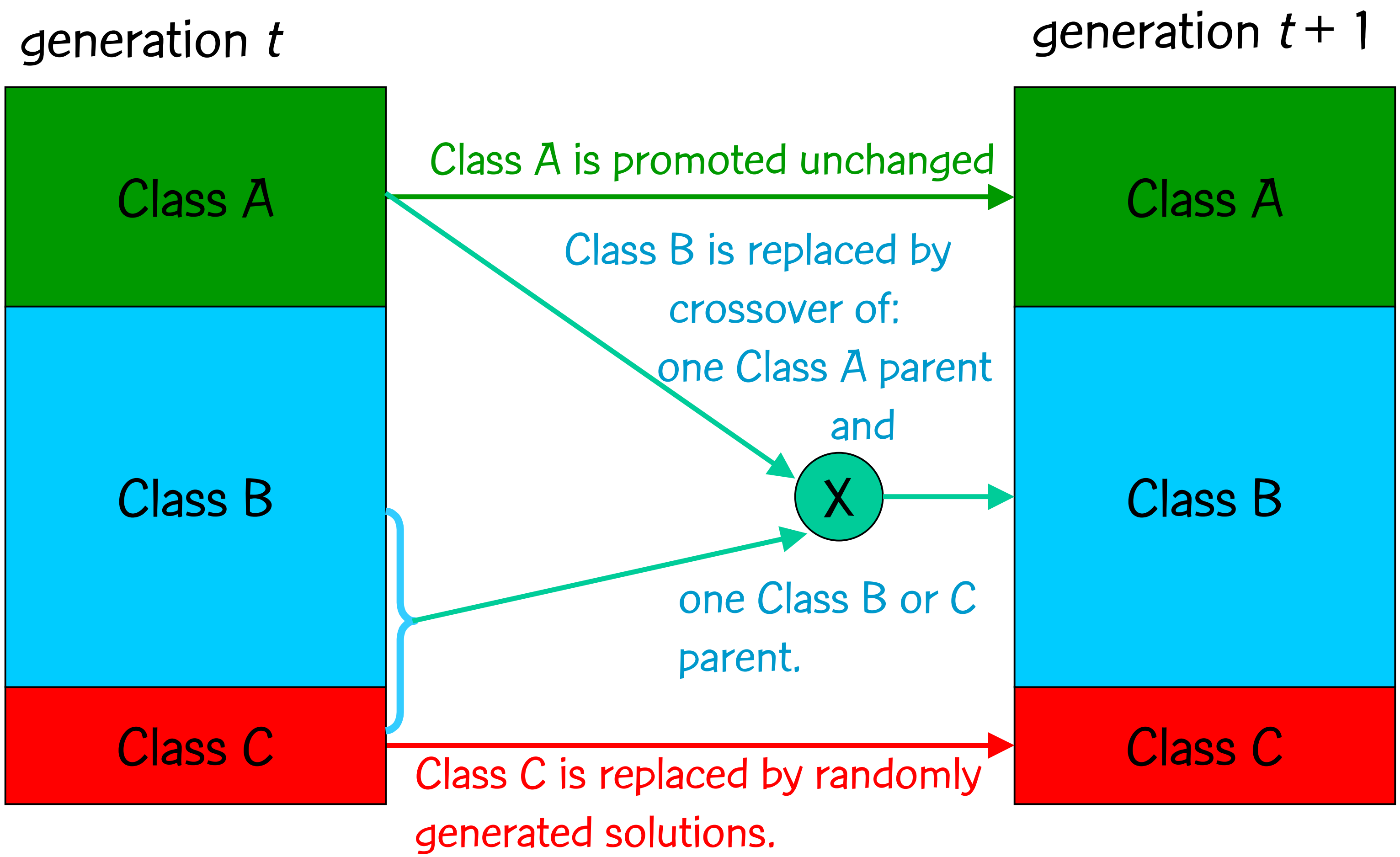
Population dynamics



Population dynamics



Population dynamics



Parent selection

- Parents are chosen at random:
  - one parent from Class A (elite).
  - one parent from Class B or C (non-elite).
- Reselection is allowed, i.e. parents can breed more than once per generation.
- Better individuals are more likely to reproduce.

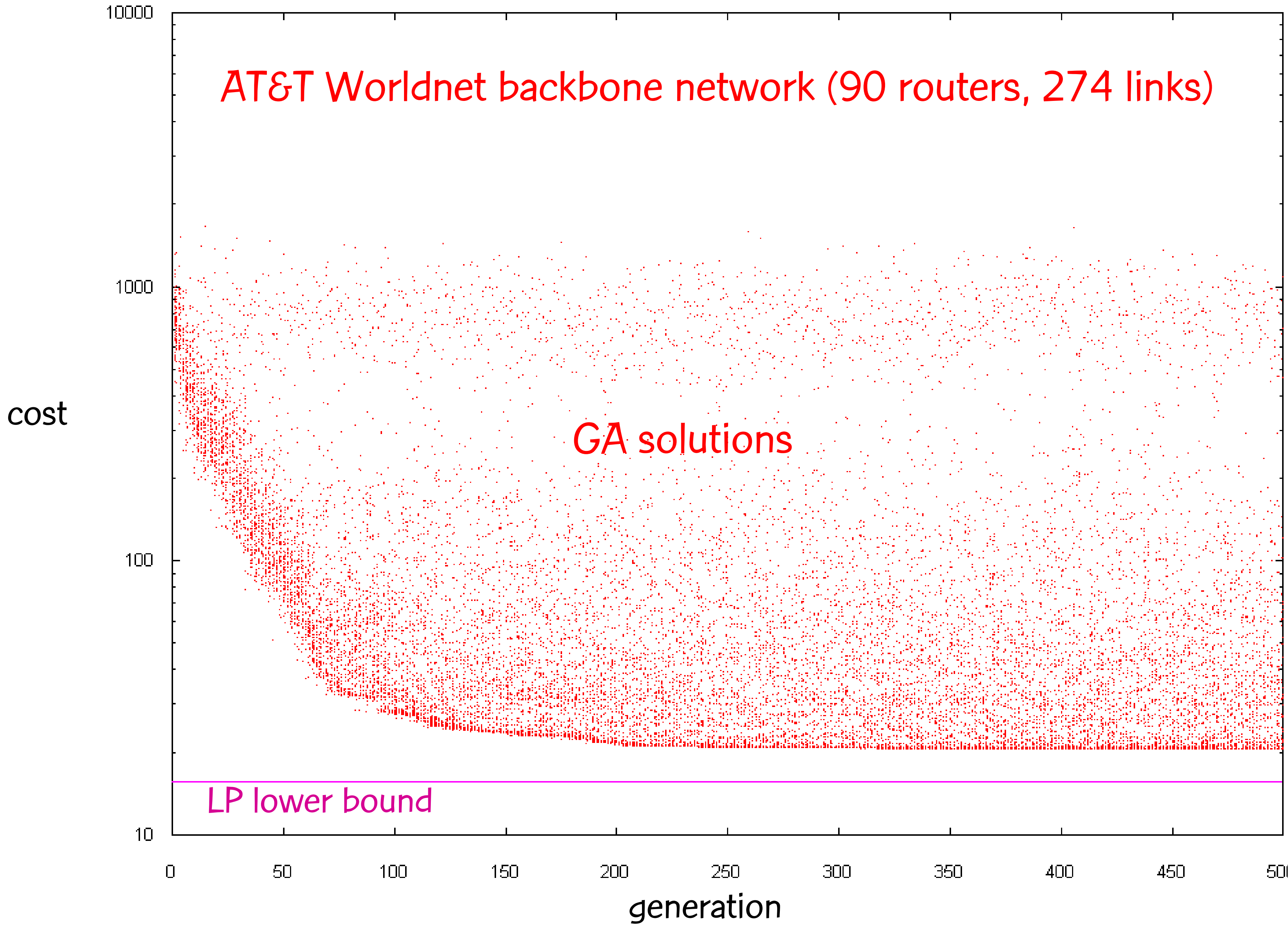
Crossover with random keys (Bean, 1994)

Crossover combines elite parent p_1 with non-elite parent p_2 to produce child c:

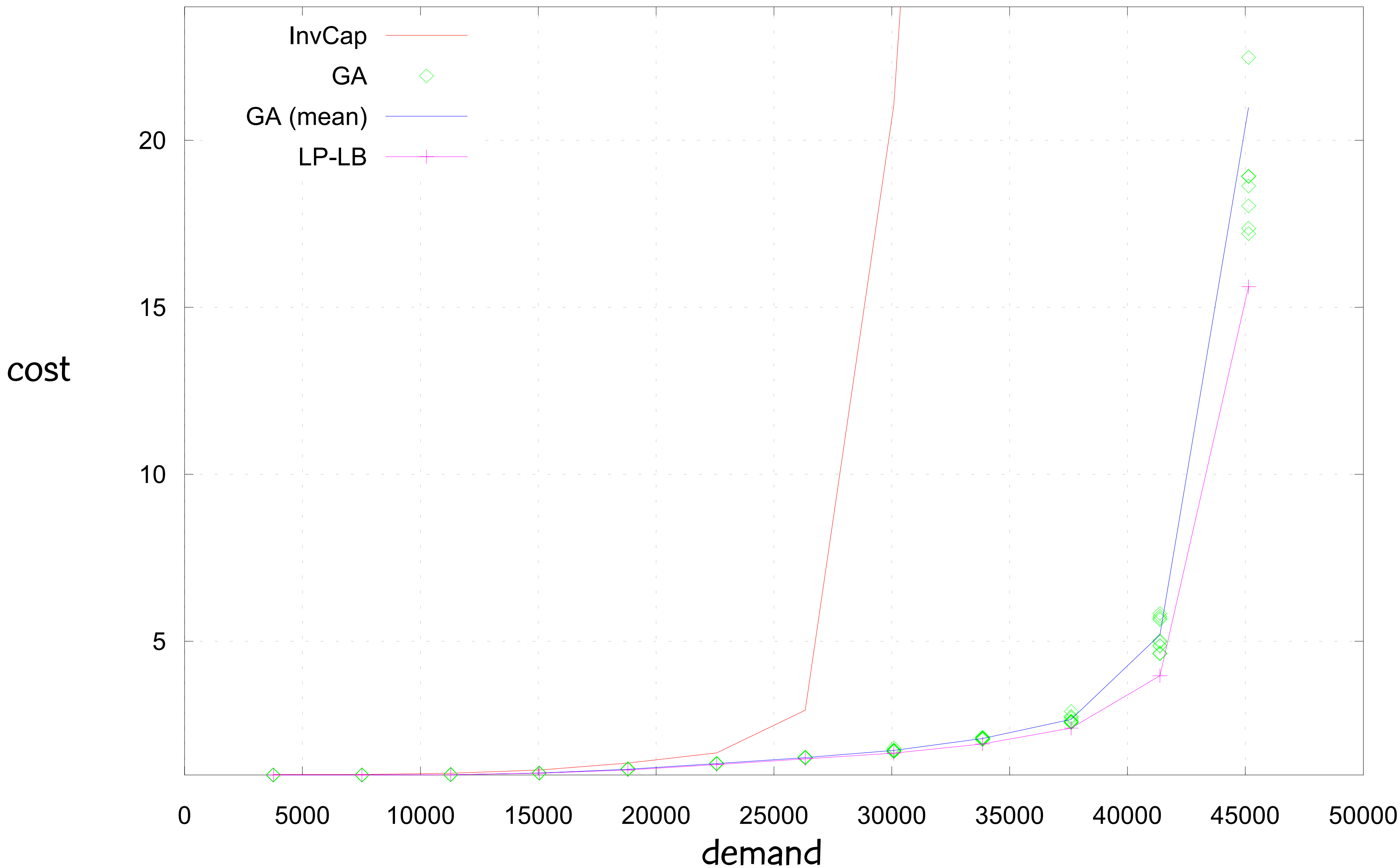
With small probability child has single gene mutation.

Child is more likely to inherit gene of elite parent.

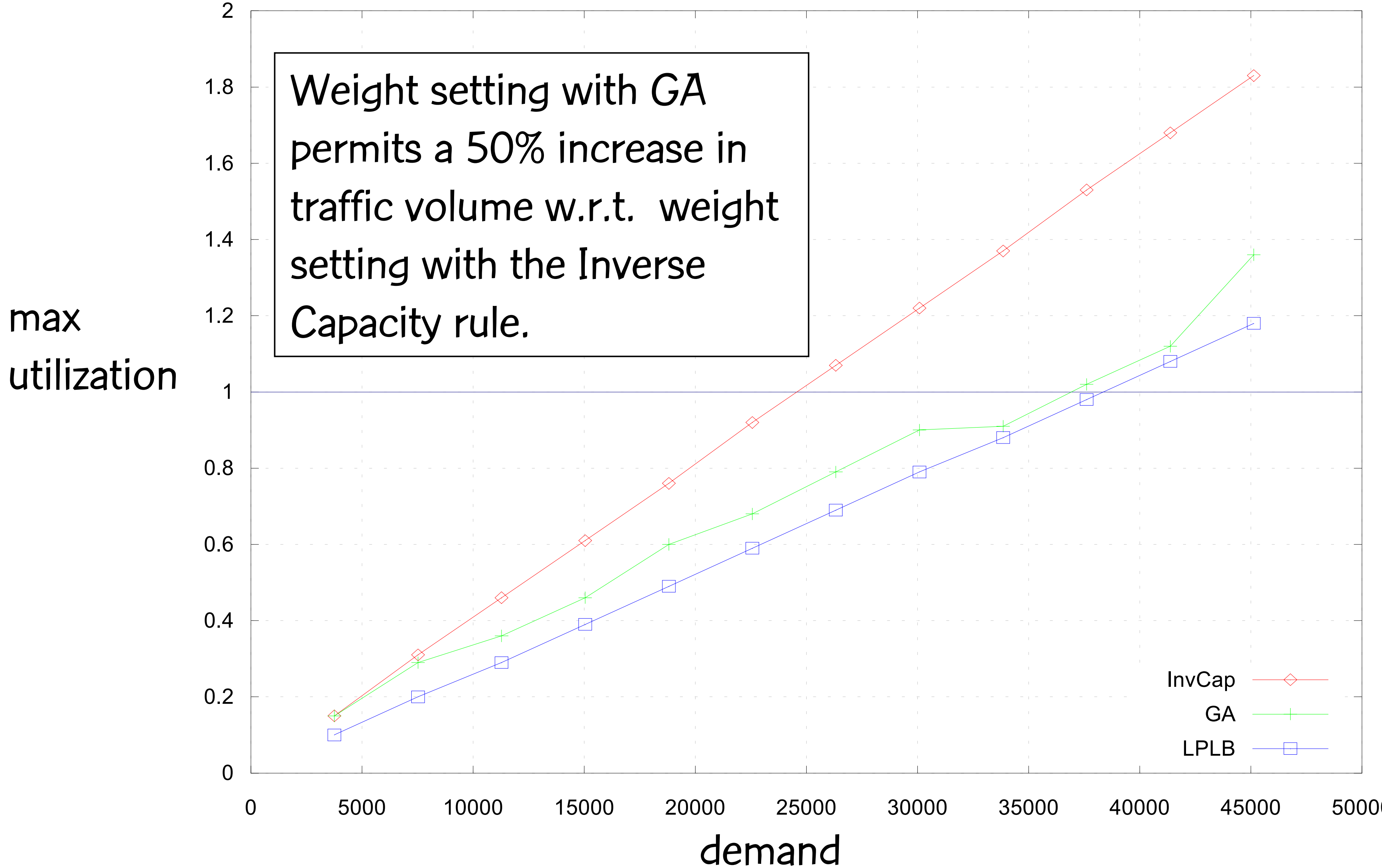
for all genes $i = 1,2,\ldots,|A|$ do
  if $\text{rrandom}[0,1] < 0.01$ then
    $c[i] = \text{irandom}[1, w_{max}]$
  else if $\text{rrandom}[0,1] < 0.7$ then
    $c[i] = p_1[i]$
  else $c[i] = p_2[i]$
end


AT&T Worldnet backbone network (90 routers, 274 links)
GA solutions
LP lower bound
cost
generation
10000
1000
100
10
0
50
100
150
200
250
300
350
400
450
500

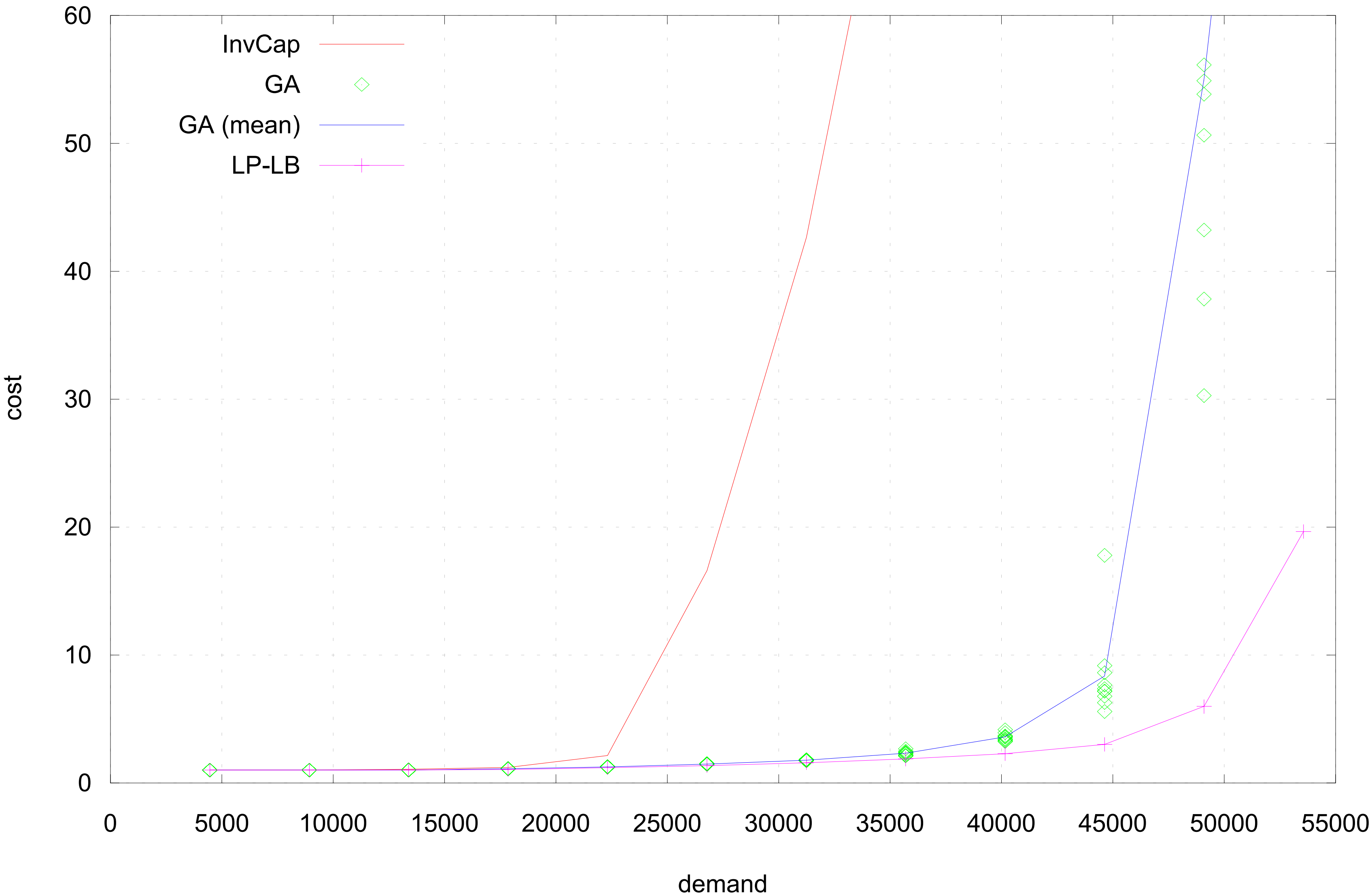
AT&T Worldnet backbone network (90 routers, 274 links)



AT&T Worldnet backbone network (90 routers, 274 links)

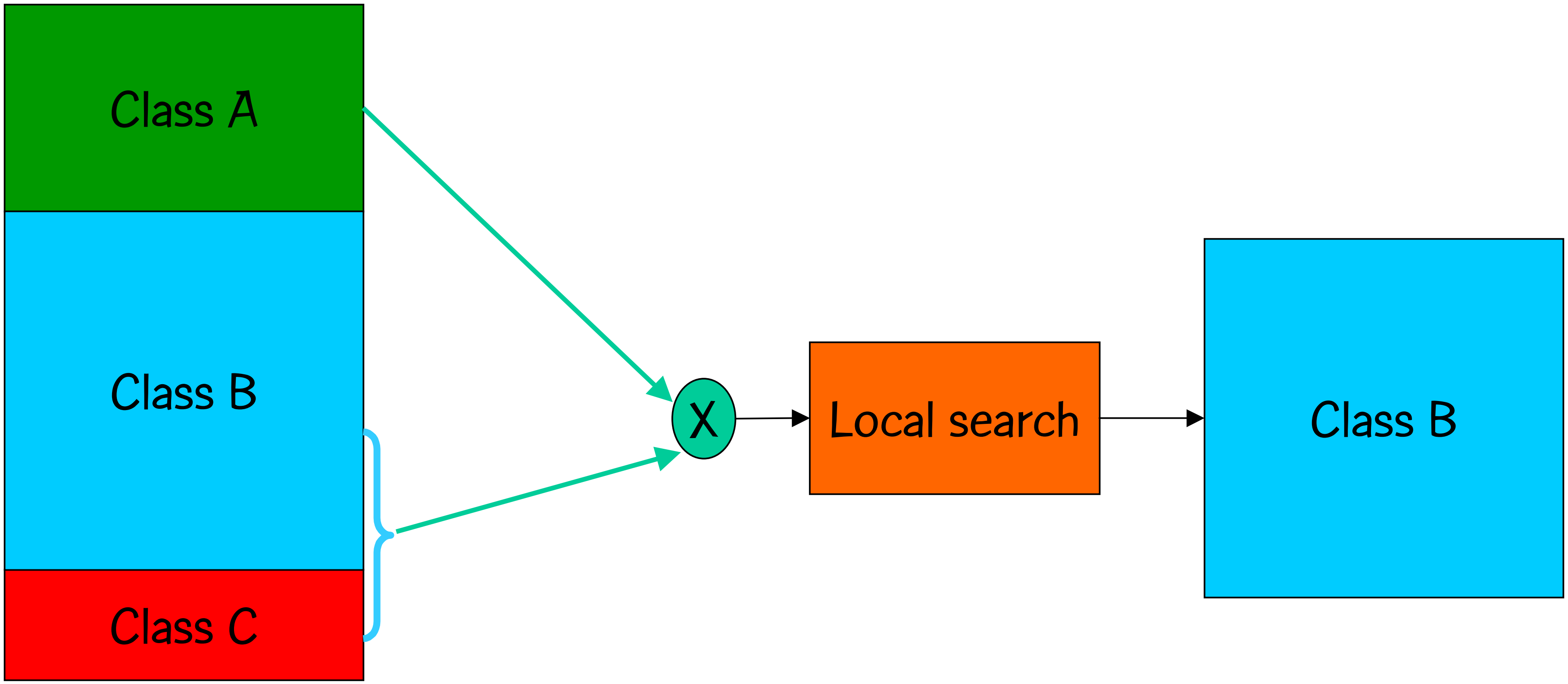


Rand50a: random graph with 50 nodes and 245 arcs.



1 hour run

Optimized crossover = crossover + local search



Fast local search

- Let A^* be the set of five arcs $a \in A$ having largest Φ_a values.
- Scan arcs $a \in A^*$ from largest to smallest Φ_a:
  - Increase arc weight, one unit at a time, in the range $\left[w_a, w_a + \lceil (w_{max} - w_a)/4 \rceil\right]$
  - If total cost Φ is reduced, restart local search.

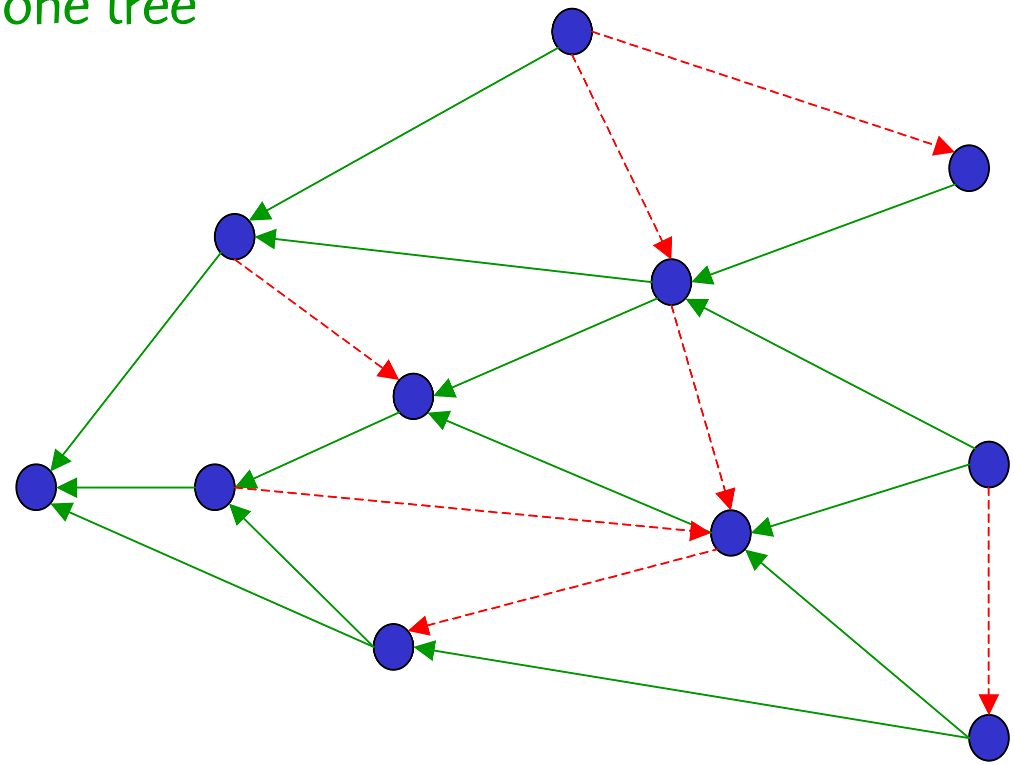
Dynamic shortest path

- In local search, when arc weight increases, shortest path trees:
  - may change completely (rarely does)
  - may remain unchanged (e.g. arc not in a tree)
  - may change partially
    - Few trees change
    - Small portion of tree changes

Does not make sense to recompute trees from scratch.

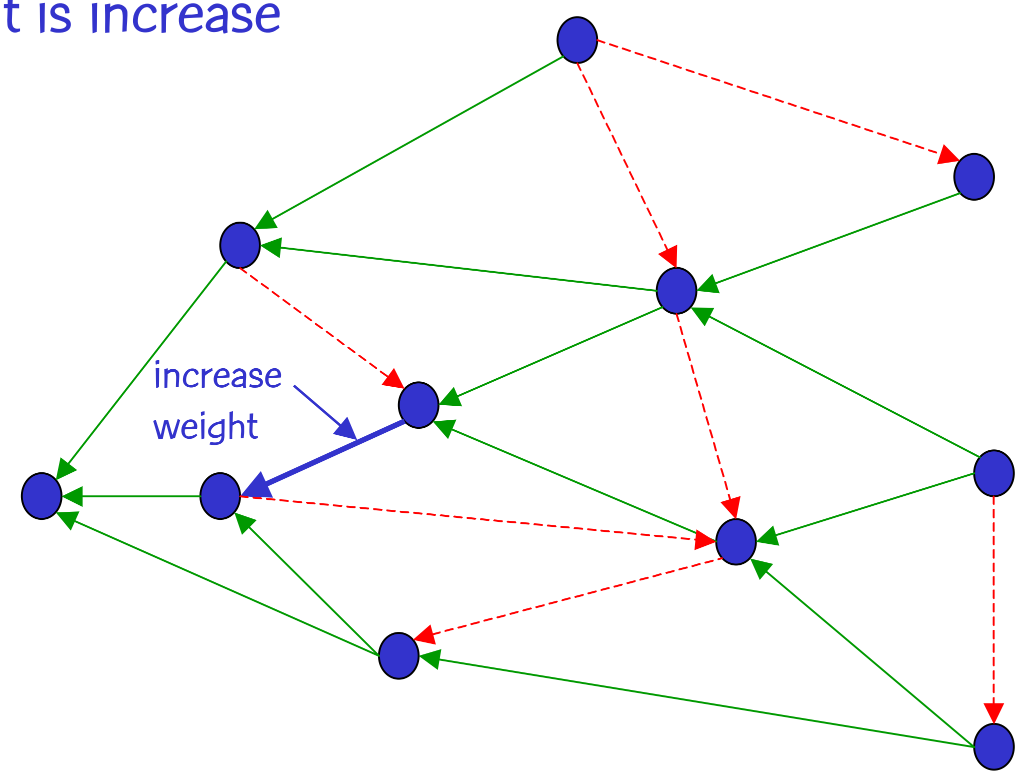
Dynamic shortest path

Consider one tree at a time.



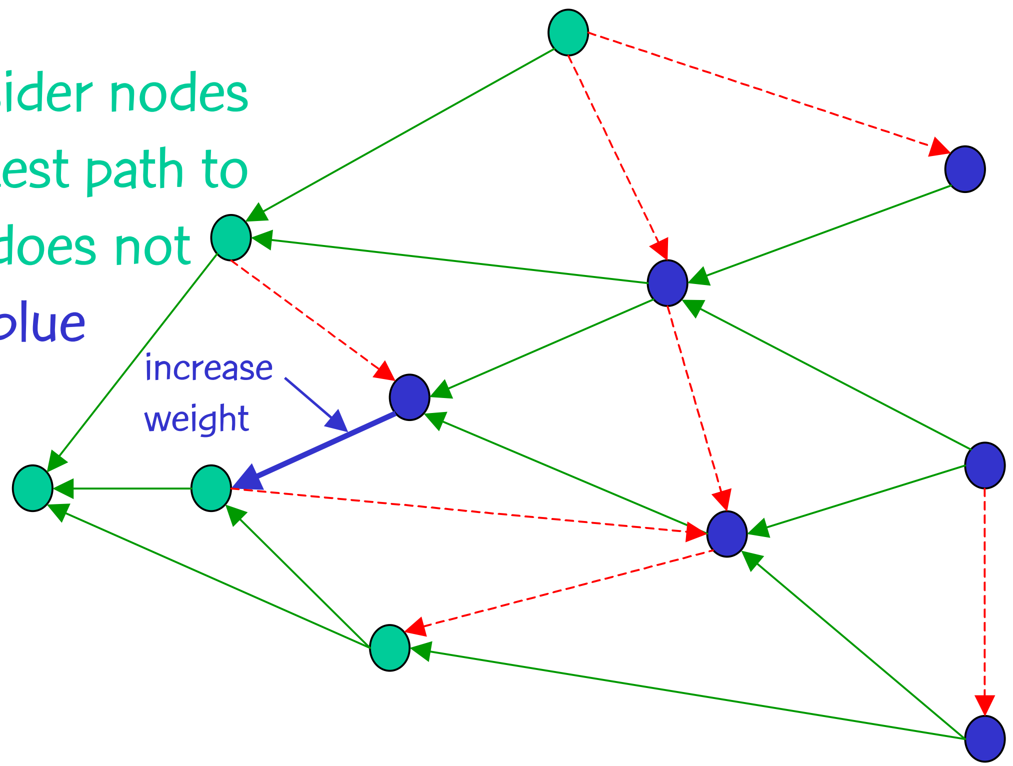
Dynamic shortest path

Arc weight is increase by 1.

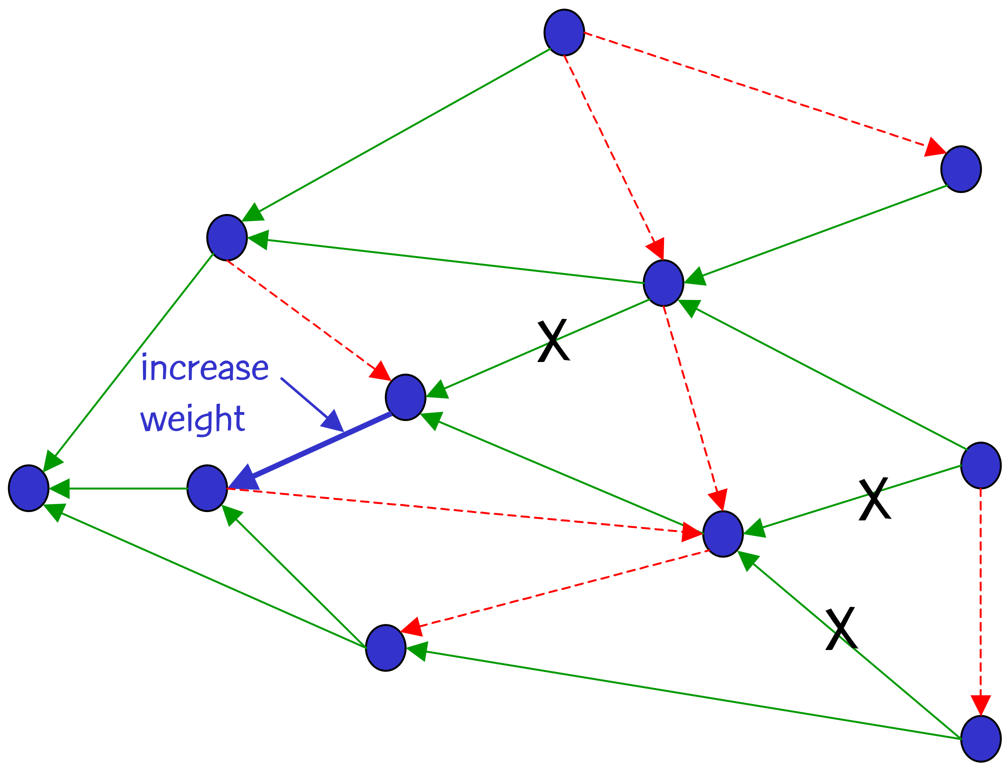


Dynamic shortest path

Do not consider nodes whose shortest path to destination does not go through blue arc.

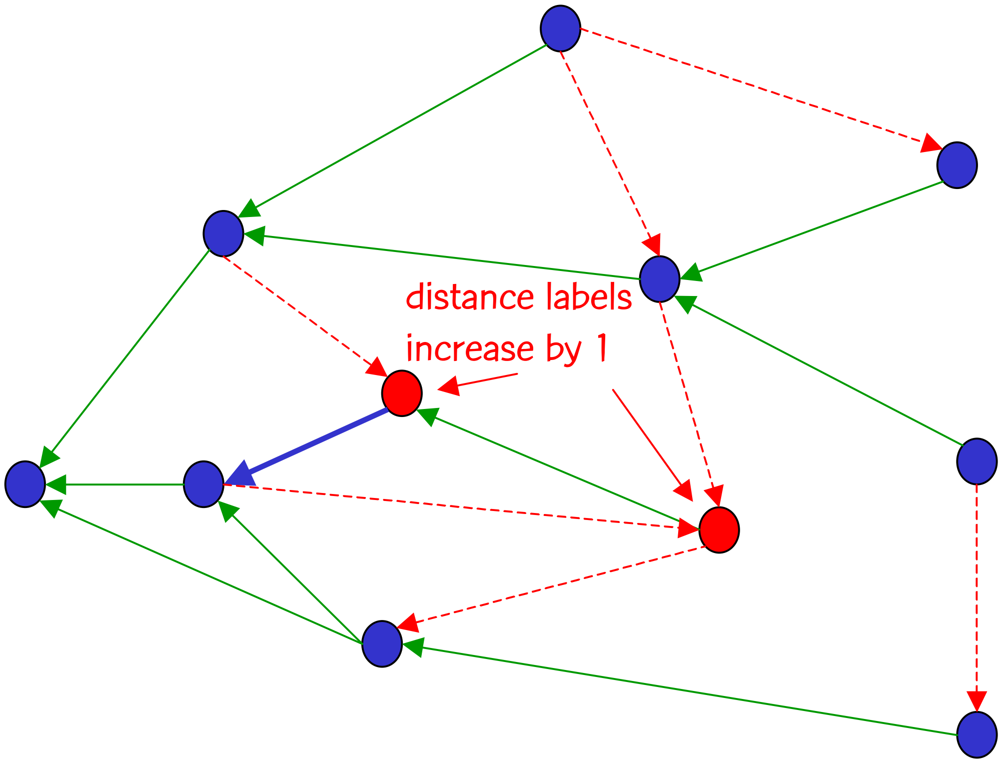


Dynamic shortest path



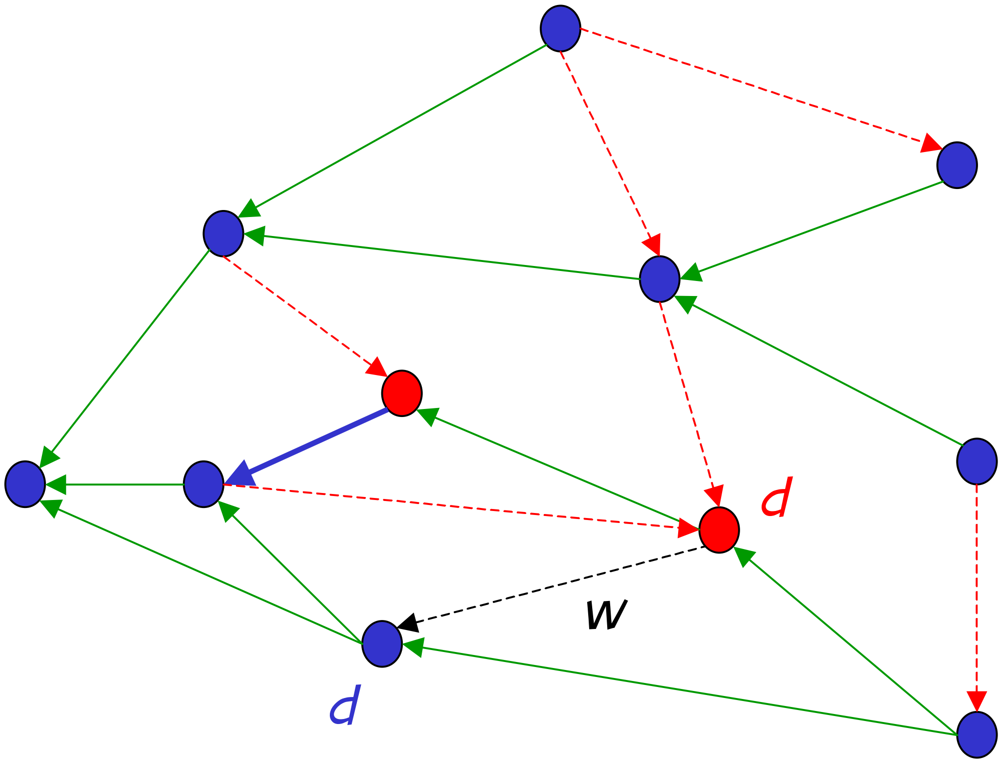
Arc (u,v) is removed from tree since alternative paths from node u to the destination node exist.

Dynamic shortest path

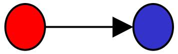


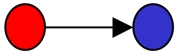
Shortest paths from red nodes must traverse blue arc.

Dynamic shortest path

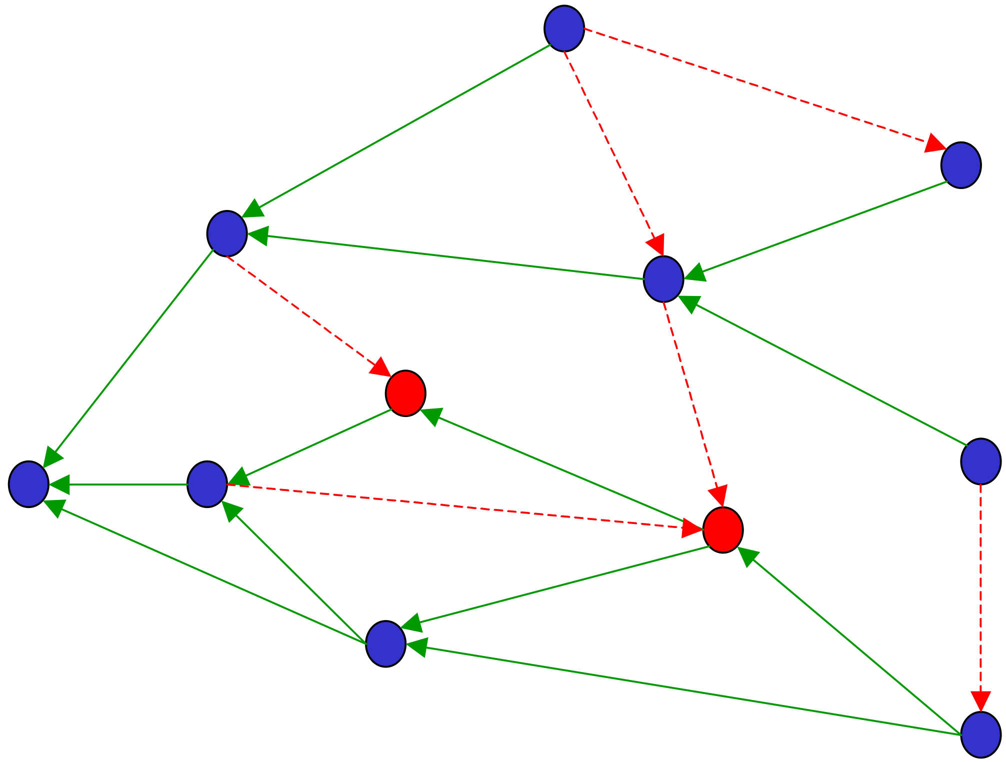


Test all arcs of type



If $d - d = w$, then  enters tree.

Dynamic shortest path

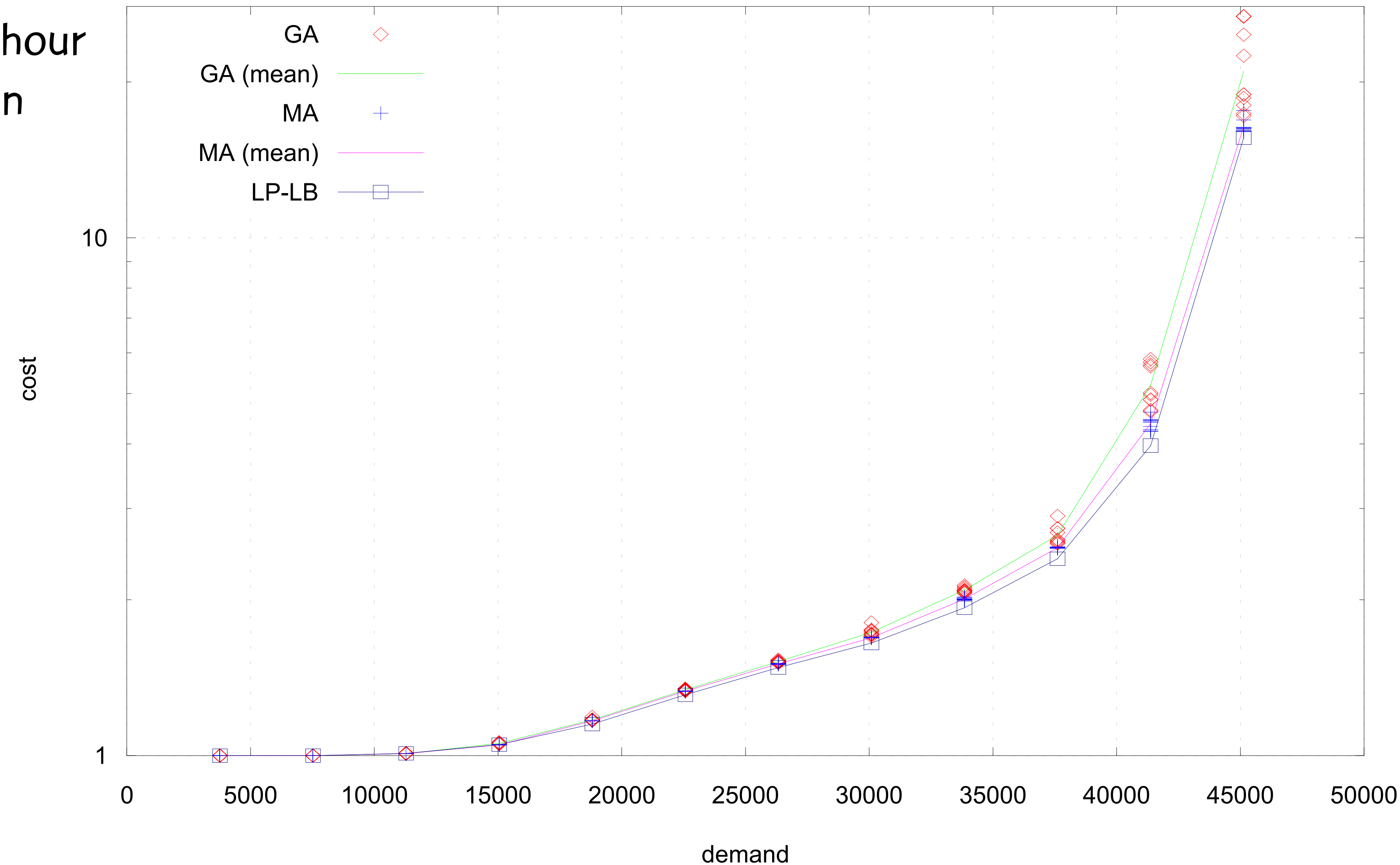


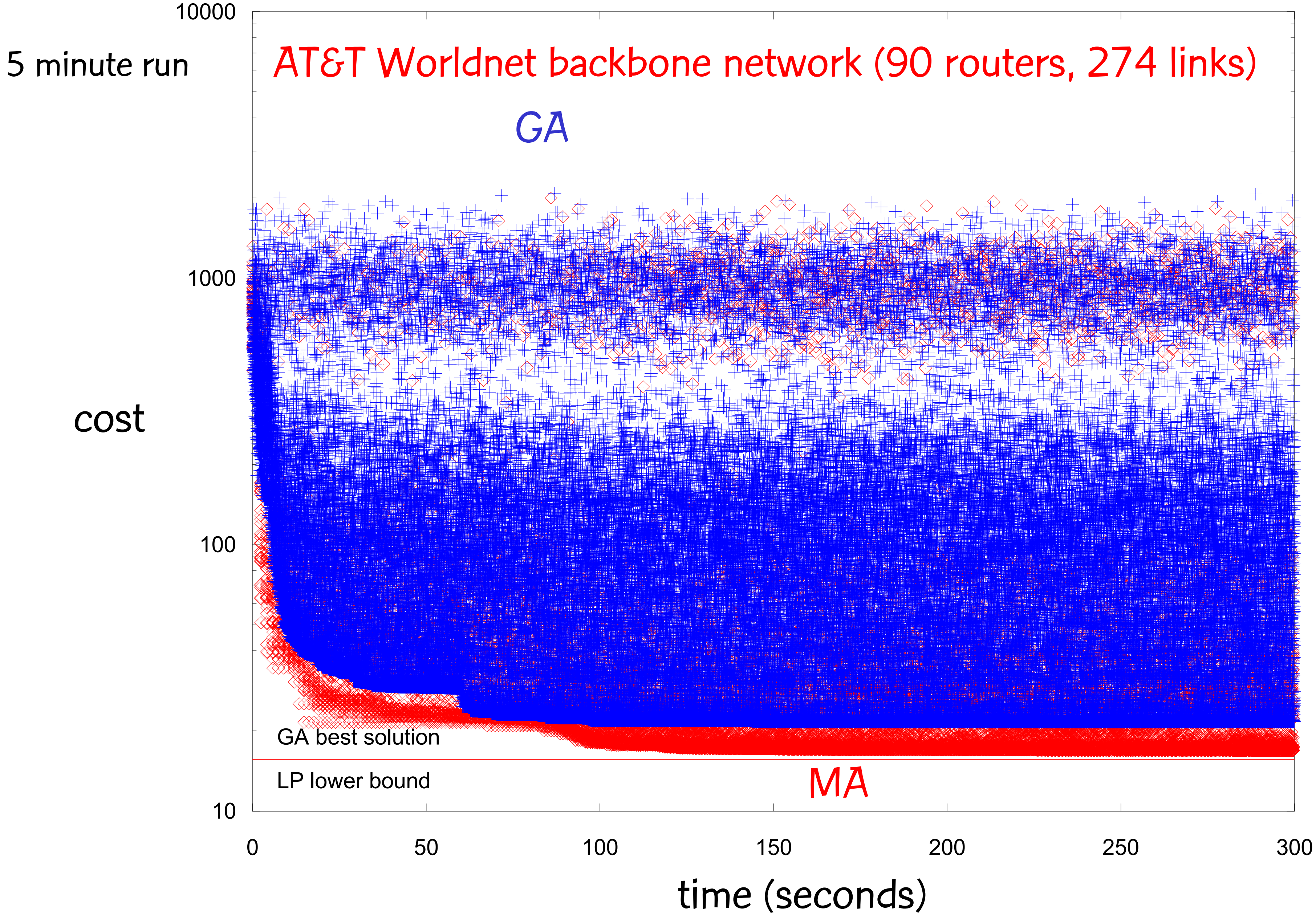
Dynamic shortest path

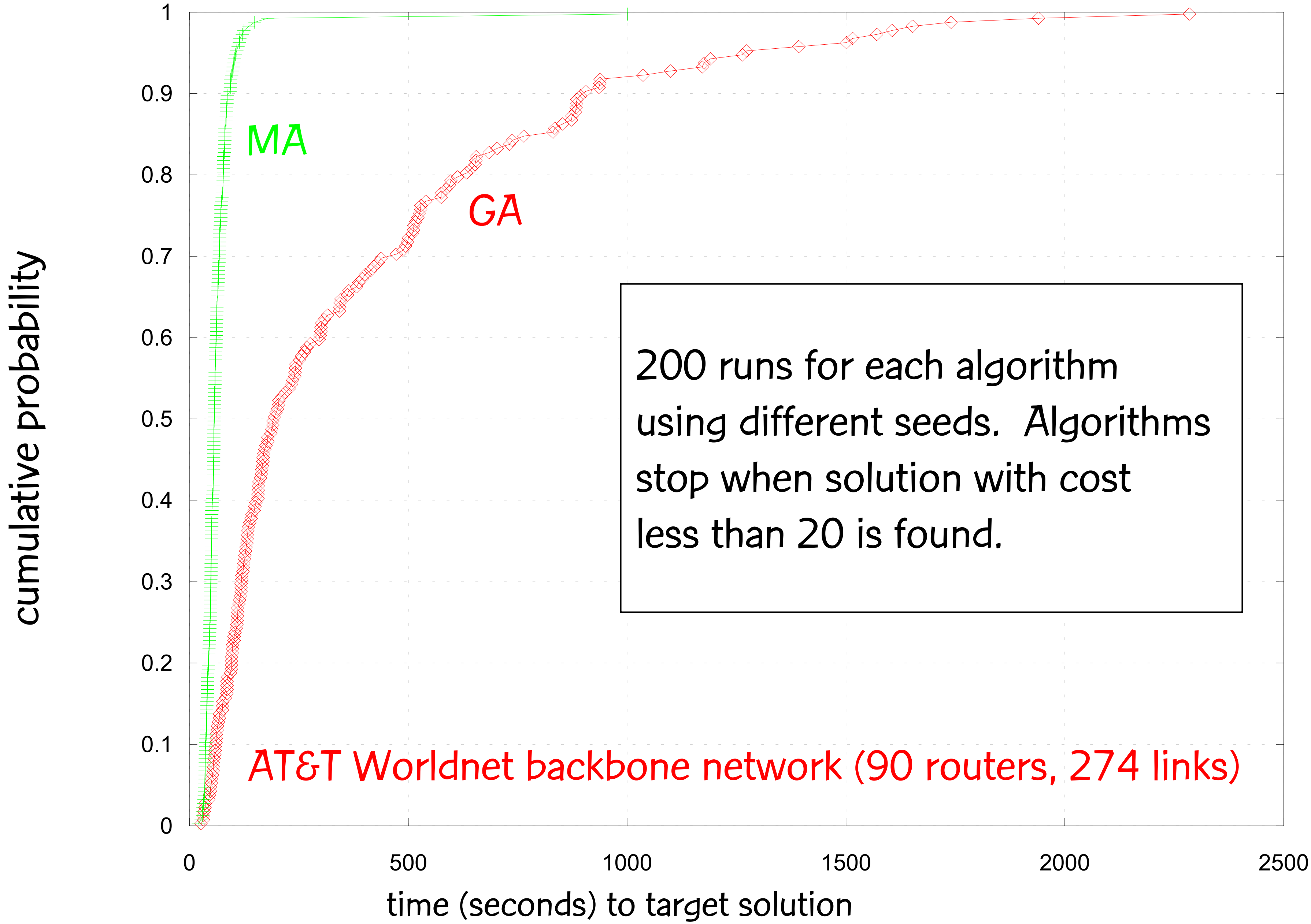
- Ramalingam & Reps (1996) allow arbitrary arc weight change.
- We specialized the Ramalingam & Reps algorithm for unit arc weight change.
  - Avoid use of heaps
  - Achieve a factor of 4 speedup w.r.t. Ramalingam & Reps on these test problems

AT&T Worldnet backbone network (90 routers, 274 links)

1 hour run



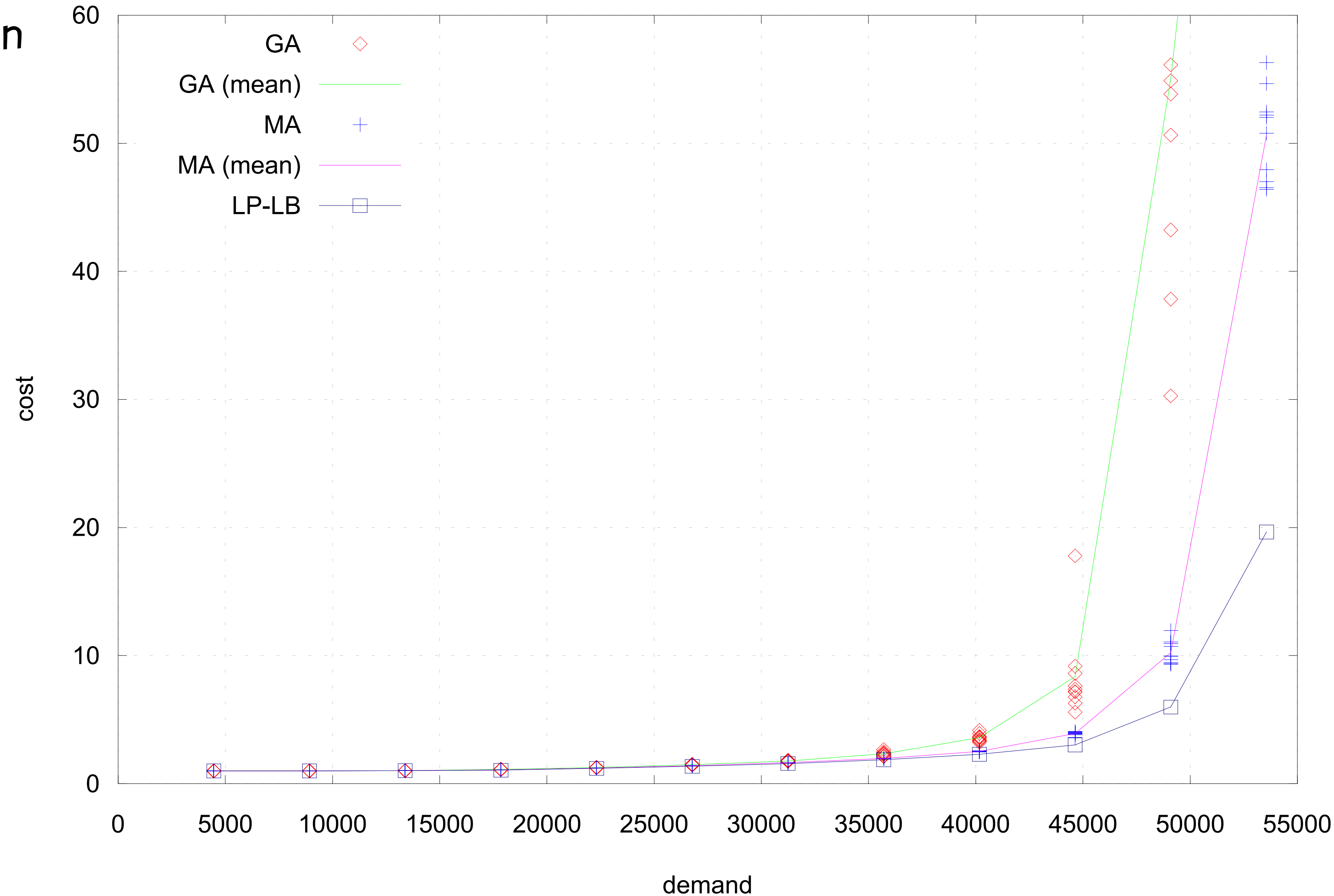

10000
5 minute run
AT&T Worldnet backbone network (90 routers, 274 links)
GA
1000
cost
100
GA best solution
LP lower bound
MA
10
0
50
100
150
200
250
300
time (seconds)



cumulative probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MA
GA
200 runs for each algorithm using different seeds. Algorithms stop when solution with cost less than 20 is found.
AT&T Worldnet backbone network (90 routers, 274 links)
0
500
1000
1500
2000
2500
time (seconds) to target solution

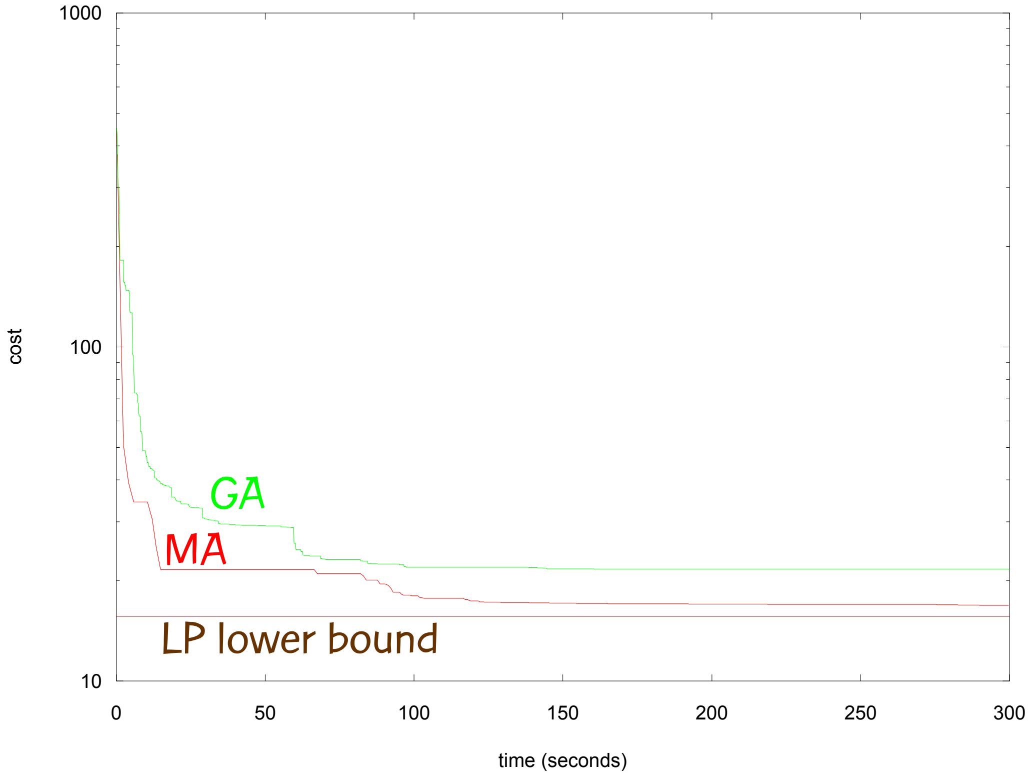
Rand50a: random graph with 50 nodes and 245 arcs.

1 hour run



Remark

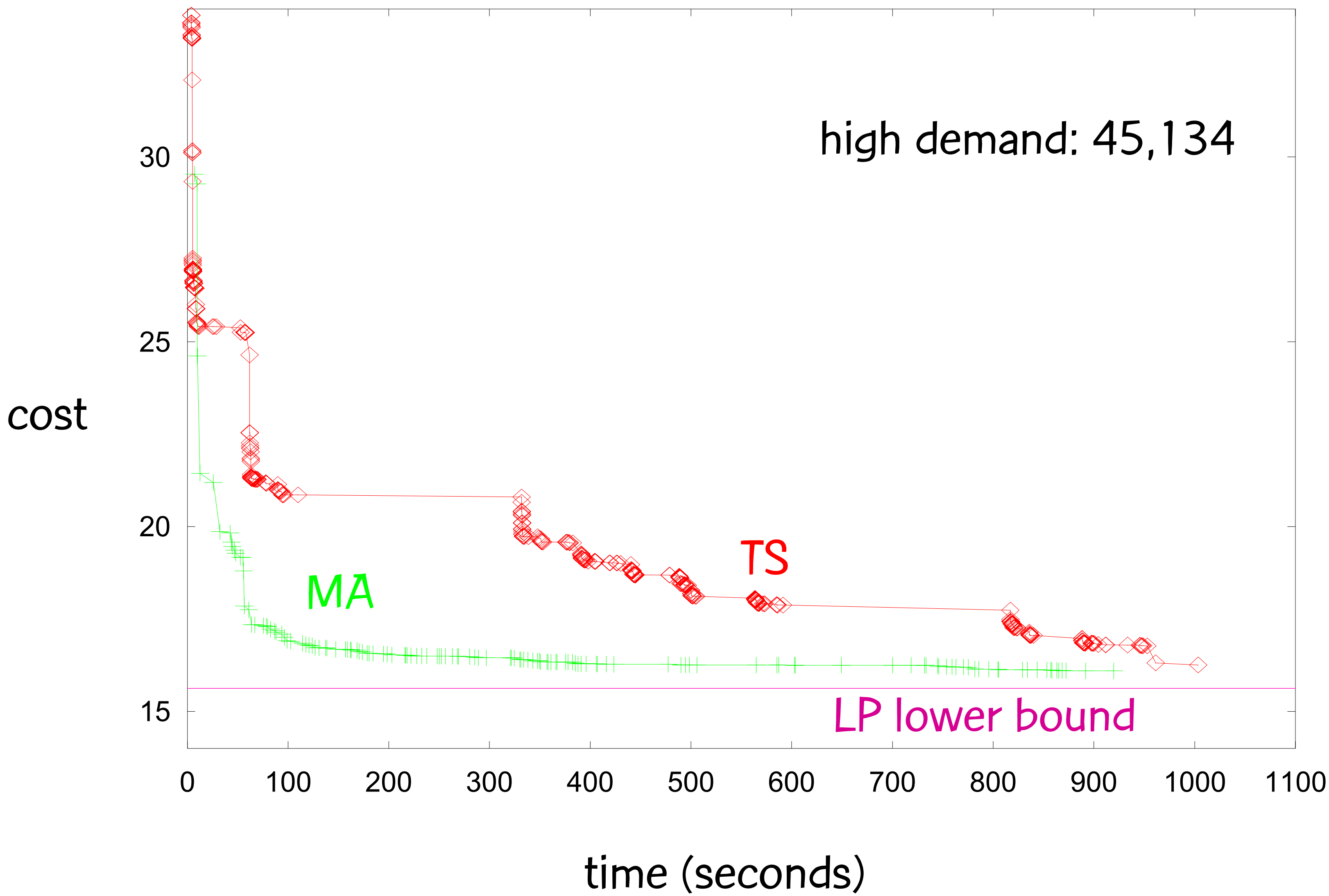
- Memetic algorithm (MA) improves over pure genetic algorithm (GA) in two ways:
  - Finds solutions faster
  - Finds better solutions



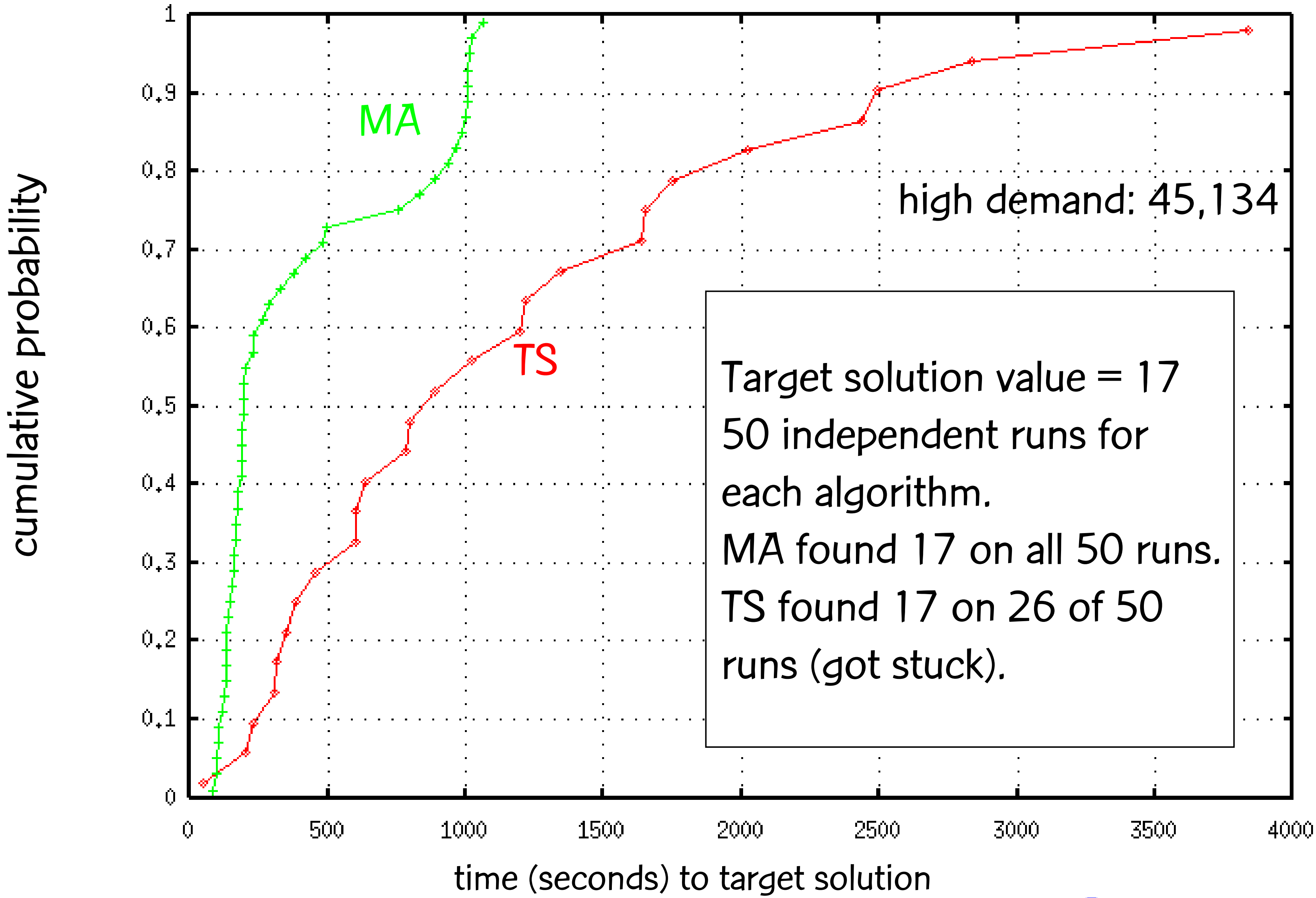
Tabu search (TS) of Fortz & Thorup (2000)

- Starts with randomly generated solution
- At each step:
  – Explores neighborhood and moves to best unvisited solution in neighborhood (can move to a worse solution) using Ramalingam & Reps dynamic shortest path algorithm.
  – If 300 consecutive non-improving steps occur, 10% of the weights change randomly by up to 2 units, and the method restarts.

AT&T Worldnet backbone network (90 routers, 274 links)

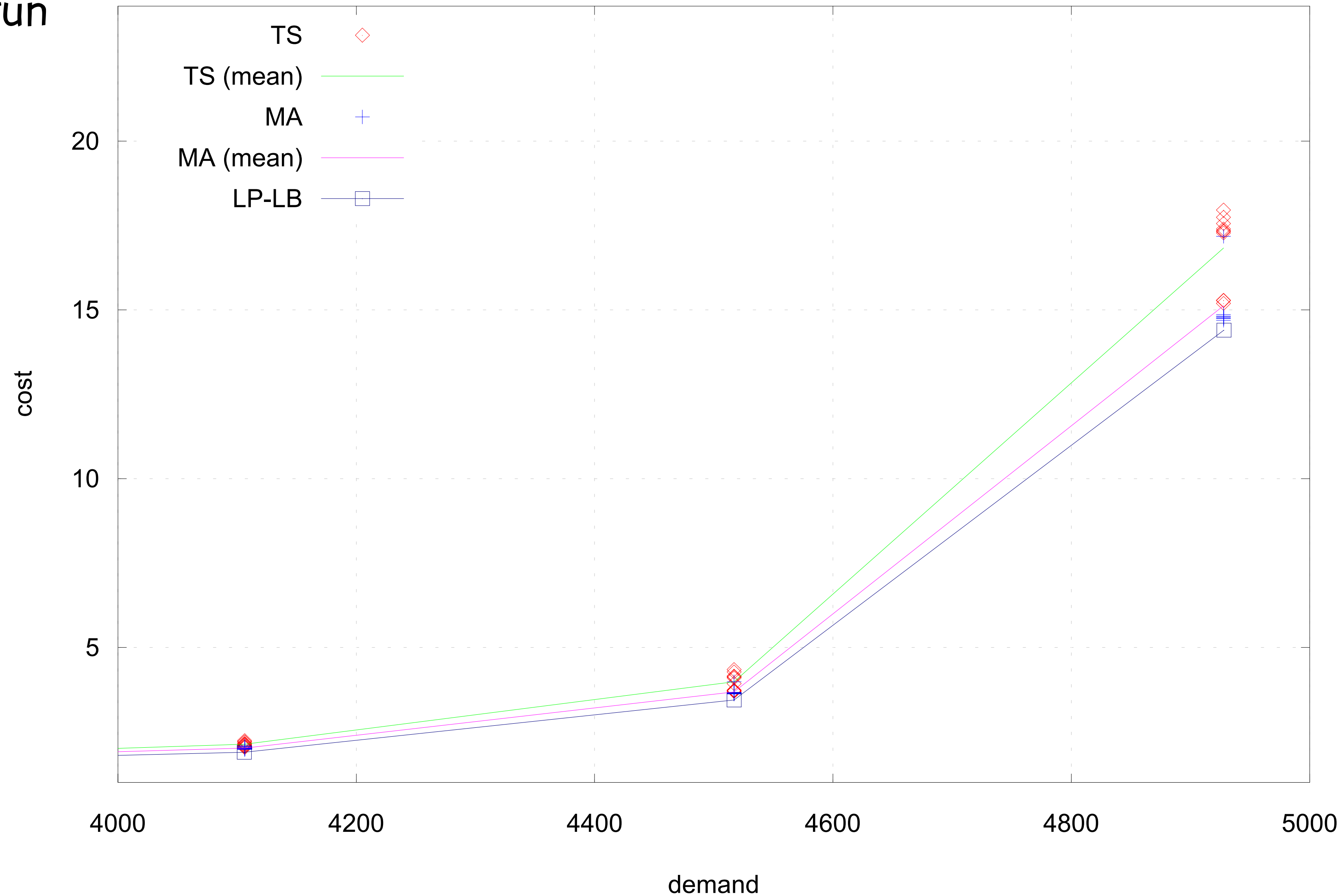


AT&T Worldnet backbone network (90 routers, 274 links)



2-level hierarchical graph (50 nodes, 148 arcs)

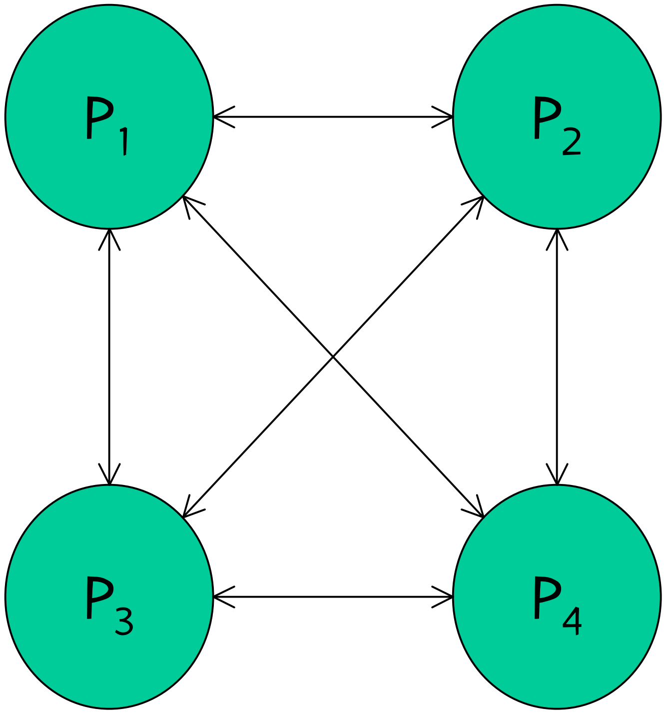
1 hour run



Remarks

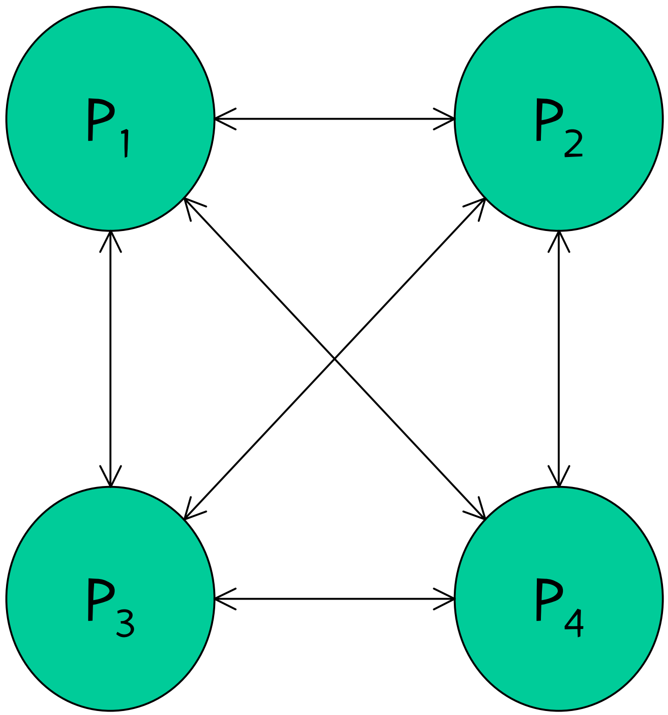
- MA is more robust than TS (does not get stuck as often)
- On some test problems, MA finds better solutions than does found by TS (e.g. AT&T and 2-level hierarchical graphs)
- On other test problems, TS finds better solutions solutions than those found by MA (e.g. random graphs and Waxman graphs)
- MA can be easily implemented in parallel (island model)

Collaborative parallel implementation



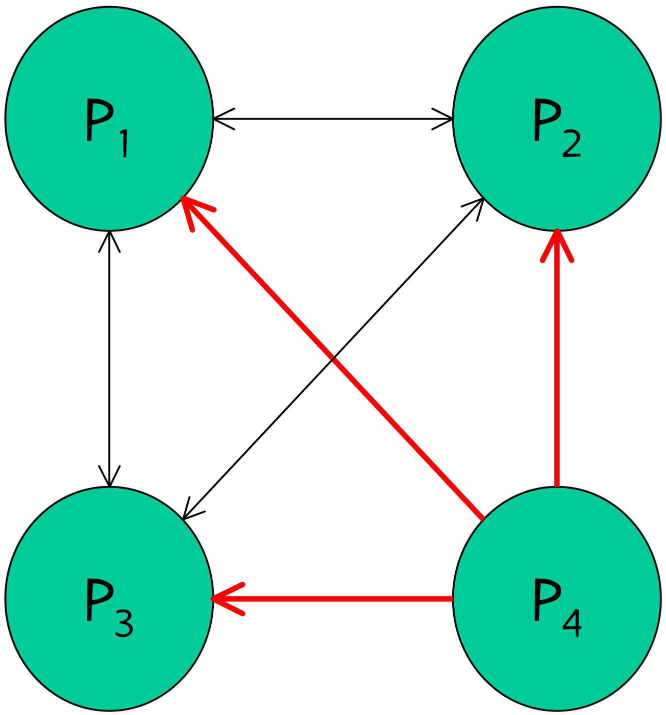
MPI: Message Passing Interface

Collaborative parallel implementation



If P_4 finds a new incumbent solution.

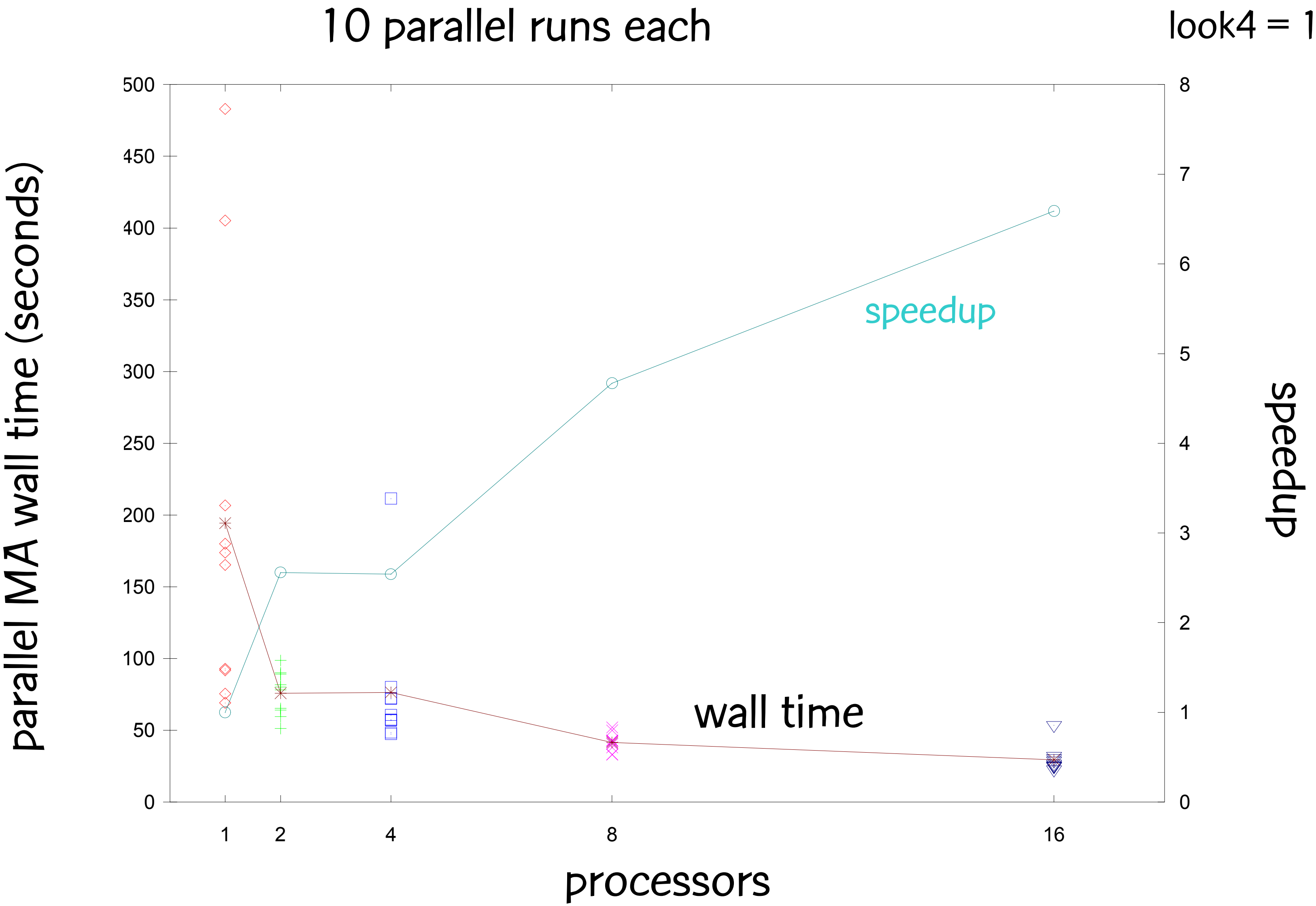
Collaborative parallel implementation



If P_4 finds a new incumbent solution.
Incumbent solution is broadcast to P_1, P_2, P_3.

AT&T Worldnet backbone network (90 routers, 274 links)

demand = 45134
look4 = 18



Concluding remarks

- Slides of this talk can be downloaded from: http://www.research.att.com/~mgcr/talks/ma-ospf.pdf
- Paper on GA: M. Ericsson, M.G.C. Resende, and P.M. Pardalos, "A genetic algorithm for the weight setting problem in OSPF routing," J. Comb. Opt., vol. 6, pp. 299-333, 2002.
- Paper on GA with opt. crossover (L.S. Buriol, M.G.C. Resende, C.C. Ribeiro, and M. Thorup), soon at :

  http://www.research.att.com/~mgcr/doc/ma-ospf.pdf

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Photos from their homepages.

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