A Hybrid Multistart Heuristic for the Uncapacitated Facility Location Problem
Uncapacitated facility location problem

$c(f)$: setup cost of opening facility $f$

$d(u, f)$: distance between facility $d$ and user $u$.

$n (=11)$ potential facility locations

$m (=15)$ customers
Uncapacitated facility location problem

If 4 facilities are opened

n (=11) potential facility locations

m (=15) customers
Uncapacitated facility location problem

Customers home into nearest open facility.

If 4 facilities are opened
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Objective of optimization:

Minimize sum of the distances between customers and their nearest open facility plus the cost of opening the facilities.

Total cost = 61 + 75 = 136
Uncapacitated facility location problem

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Total cost = 58 + 75 = 133 < 136
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Total cost = 46 + 78 = 124 < 133
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Uncapacitated facility location problem

Total cost = 48 + 48 = 96 < 124
Facility location problem

- Set $F$ of potential facilities, each facility $f$ with a setup cost $c(f)$
- Set $U$ of users that must be served by a facility. The cost of serving user $u$ by facility $f$ is $d(u,f)$
- Facility location problem: Determine a set of facilities $S \subseteq F$ to open so as to minimize the total cost:
  \[
  \text{cost}(S) = \sum (c(f) : f \in S) + \sum \left[ \min \{d(u,f) : f \in S\} : u \in U \right]
  \]
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Uncapacitated facility location

- Customers home in to nearest open facility
- No limit on number of open facilities
- NP hard [Cournéjols, Nemhauser, & Wolsey, 1990]
- Perhaps the most common location problem, studied widely in literature both in theory & practice
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- Exact methods exist, e.g. [Conn and Cournéjols, 1990; Körkel, 1989]
- NP-hard nature makes heuristics a natural choice for larger instances
- Shmoys, Tardos, & Aardal (1997) present a 3.16-opt approximation algorithm
- Improvements, e.g. [Jain et al., 2002, 2003; Mahdian, Ye, & Zhang, 2002] have led to polynomial-time algorithms that find a solution within a factor of around 1.5 from the optimal.
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- Unfortunately, there is not much more room for improvement: Guha & Khuller (1999) established a lower bound of 1.463 for the approximation factor.
- In practice, approximation algorithms tend to be much closer for non-pathological instances: The 1.61-opt algorithm of Jain et al. (2003) was always within 2% of optimal in their experiments.
- Though interesting in theory, approximation algorithms are often outperformed in practice by more straightforward heuristics with no particular performance guarantees.
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- **Pioneering work on heuristics: Kuehn & Hamburger (1963)**
- Since then, more sophisticated heuristics have been applied:
  - Simulated annealing [Alves & Almeida, 1992]
  - Genetic algorithms [Kratica et al., 2001]
  - Tabu search [Ghosh, 2003; Michel & Van Hentenryck, 2003]
  - Complete local search with memory [Ghosh, 2003]
- **Dual-based methods have also shown promising results:**
  - Dual ascent [Erlenkotter, 1978]
  - Lagrangean dual ascent [Guignard, 1988]
  - Volume algorithm [Barahona & Chudak, 1999]
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- Hofer (2002) presented computational comparison of five methods:
  - JMS, an approximation algorithm of Jain et al. (2002)
  - MYZ, an approximation algorithm of Mahdian et al. (2002)
  - A swap-based local search
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Our algorithm

- In this talk, we provide an alternative that can be even better in practice.
- It is a hybrid multistart heuristic akin to the one we developed in Resende & Werneck (2004) for the p-median problem.
- A series of minor adaptations is enough to build a very robust algorithm, capable of obtaining near-optimal solutions for a wide variety of instances of the facility location problem.
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Our algorithm

- **Works in two phases:**
  - **Multistart routine with intensification:** Each iteration builds a randomized solution and applies local search to it. The resulting solution \( S \) is combined, in a process called path-relinking, with another solution from a set of elite solutions, resulting in \( S' \). The algorithm tries to insert \( S \) and \( S' \) into the elite set.
  - **Post-optimization:** Solutions from the elite set are combined with each other in a process that hopefully results in better solutions.

- The method is called HYBRID because it combines elements of several metaheuristics.
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HYBRID heuristic for location problems

```plaintext
function HYBRID (seed, maxit, elitesize)
1    randomize(seed);
2    init(elite, elitesize);
3    for i = 1 to maxit do
4        S ← randomizedBuild();
5        S ← localSearch(S);
6        S' ← select(elite, S);
7        if (S' ≠ NULL) then
8            S' ← pathRelinking(S, S');
9            add(elite, S');
10       endif
11    add(elite, S);
12   endfor
13    S ← postOptimize(elite);
14    return S;
end HYBRID
```
Reuse of p-median heuristic

- Although the HYBRID heuristic was originally proposed for the p-median problem, its framework can be applied to other problems: in this case, facility location.
- Recall that the p-median problem is very similar to facility location: the only difference is that instead of assigning costs to facilities, the p-median problem must specify p, the exact number of facilities to be opened.
- With minor adaptations, we can reuse several of the components used in Resende & Werneck (2004), such as the construction algorithms, local search, and path-relinking.
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Paper on HYBRID for p-median


• At iteration $i$, we determine the number $p_i$ of facilities to open.
  – For $i = 1$, $p_i = \lceil m/2 \rceil$;
  – For $i > 1$, we pick the average number of facilities opened in the first $i - 1$ iterations;

• We then execute procedure sample of the $p$-median variant of HYBRID:
  – At each step, choose $\left\lceil \log_2 \left( m/p_i \right) \right\rceil$ facilities uniformly at random and select the one that reduces the total cost the most.
Construction heuristic

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Local search

• Local search in p-median variant: given solution $S$, find two facilities $f_r \in S$, $f_i \notin S$ which, if swapped, leads to a better solution.
  – This keeps number of facilities constant.
  – We also allow pure insertions and pure deletions, as well as swaps.

• All possible insertions, deletions, and swaps are considered, and the best among those is performed.

• Local search stops (at local minimum) when no improving move exists.
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Local search

Largest $p$-median instance tested: 5934 users, Euclidean.
(preprocessing times not considered)
Local search paper


Path-relinking

[Glover (1996)]

- **Intensification**: takes two solutions $S_1$ and $S_2$
- Starts from $S_1$ and gradually transforms it into $S_2$
- Operations that change solution at each step are same as in local search: insertions, deletions, swaps
- However,
  - Only facilities in $S_2 \setminus S_1$ can be inserted
  - Only facilities in $S_1 \setminus S_2$ can be removed
- At each step, most profitable move is made
- Procedure returns best local optimal in path
- If no local optimal exists, one of the extremes is returned with equal probability
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Path-relinking

- Path is generated by selecting moves that introduce in the initial solution attributes of the guiding solution.
- At each step, all moves that incorporate attributes of the guiding solution are analyzed and best move is taken.

Diagram:
- Initial solution
- Guiding solution
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Output of PR usually is best solution in path.
Elite solutions

• To test whether a new solution should be inserted into the pool, we use a criterion based on symmetric difference between two solutions $S_a$ and $S_b$: $|S_a \setminus S_b| + |S_b \setminus S_a|$

• A new solution is inserted only if its symmetric difference to each cheaper solution already there is at least four.

• Moreover, if pool is full, the new solution must also cost less than the most expensive element in the pool. In this case, the new solution replaces the one (among those with equal or greater cost) it is most similar to.
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Intensification

• After each iteration, the solution $S$ obtained by the local search is combined with a solution $S'$ obtained from the pool.

• Solution $S'$ is chosen at random, with probability proportional to its symmetric difference to $S$:
  - This tends to lead to longer paths on which to search
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Post-optimization
Evolutionary path-relinking

a) **Start** with pool found at end of multistart phase: $P_0$; Set $k = 0$;

b) Combine with path-relinking all pairs of solutions in pool $P_k$;

c) Solutions obtained by combining solutions in $P_k$ are added to a new pool $P_{k+1}$ following same constraints for updates as before;

d) If best solution of $P_{k+1}$ is better than best solution of $P_k$, then set $k = k + 1$, and go to step (b);
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• Therefore, if we want to multiply the average running time of the algorithm by some factor $X$, we just multiply $N$ by $X$ and $E$ by $\sqrt{X}$, rounding off appropriately.
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Empirical results

Experimental setup

- Algorithm implemented in C++ and compiled with the SGI MIPSPro C++ compiler (v. 7.30) with flags –O3 – OPT:Olimit=6586
- Runs were done on an SGI Challenge with 28 196-MHz MIPS 10000 processors, but each execution was limited to a single processor
- All CPU times reported are measured by the getrusage function with a precision of 1/60 second
- Random number generator: Mersenne Twister (Matsumoto and Nishimura, 1998)
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Test problems

- Algorithm was tested on all classes from UflLib (Hoefer, 2003) and on class GHOSH, described in Ghosh (2003).
- In every case, the number of users and potential facilities is the same (locations are the same).

http://www.mpi-sb.mpg.de/units/agl/projects/benchmarks/UflLib
Empirical results

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</table>
| BK             | Bilde & Krarup (1977) | 200 instances, 30 to 100 users      | d ~ [0,1000]  
|                |                    |                                     | c ≥ 1000                                   |
| FPP            | Kochetov (2003)    | 80 instances, 133 & 307 users        | Meant to be challenging for algorithms based on local search. |
| GAP            | Kochetov (2003)    | 120 instances, 100 users            | Large duality gaps. Hard for dual-based methods. |
| GHOSH          | Ghosh (2003)       | 90 instances, 250, 500, & 750 users | d ~ [1000,2000]  
|                |                    |                                     | A: c ~ [100,200]  
|                |                    |                                     | B: c ~ [1000,2000]  
|                |                    |                                     | C: c ~ [10K,20K]    |

**Test problems**

BK used in Hoefer's comparative analysis.
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<td>Galvão &amp; Raggi (1989)</td>
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<td>M*</td>
<td>Kratica et al. (2001)</td>
<td>22 instances, 100 to 2000 users</td>
<td>Meant to be close to real-life applications: many near-optimal solutions.</td>
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<td>MED</td>
<td>Ahn et al. (1998); Barahona &amp; Chudak (1999)</td>
<td>18 instances, 500 to 3000 users</td>
<td>Random points in unit square, Euclidean distances with 4 signif. digits.</td>
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<td>ORLIB</td>
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Test problems

GR, M*, MED, and ORLIB used in Hoefer's comparative analysis.
Empirical results

Quality assessment

- Standard version of algorithm
- Run ten times on each instance with ten random number seeds (1,…,10)
- Compare to optima for FPP, GAP, BK, GR, and ORLIB and best upper bounds for MED and M*
- Geometric means given for times.

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Empirical results

Comparative analysis

- We have seen that our algorithm produces very good quality solutions on most of the classes of instances tested.
- On their own, however, these results don't mean much.
- Any reasonably scalable algorithm, given enough time, should be able to find good solutions.
- With this in mind: we compare our algorithm with the best algorithm from Hoefer's analysis: the tabu search of Michel and Van Hentenryck (2003)
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• We downloaded TABU from UflLib and ran it on our computer with 500 iterations (as in Hoefer’s experiments).
• Since TABU was faster than our standard version, we compare with a faster HYBRID with $N = 8$ and $E = 5$.
• Both algorithms were run 10 times on each instance

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Longer runs

- Both HYBRID and TABU should benefit if given more time to solve instances in GAP and FPP.
- We ran TABU with 1000, 2000, 4000, ..., 64000 iterations and HYBRID with N:E pairs 4:3, 8:5, 16:7, 32:10 (standard HYBRID), 64:14, 128:20, 256:28, and 512:40.
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<table>
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<th>elite</th>
<th>% error</th>
<th>time</th>
<th>iterations</th>
<th>% error</th>
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Results are means over ten runs.

**GAP class**

**Time in seconds (196MHz R10000)**
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<th>time</th>
<th>iterations</th>
<th>% error</th>
<th>time</th>
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</table>

Means over ten runs.

FPP class

Time in seconds (196MHz R10000)
FPP class

Average error (percentage)

Mean time (seconds)

- tabu
- tabu-ms
- multistart+pr
- multistart
- hybrid

Uncapacitated facility location

Software availability

Our software (local search, and hybrid heuristics for p-median and facility location) as well as all test instances used in our studies are available for download at:

http://www.research.att.com/~mgcr/popstar