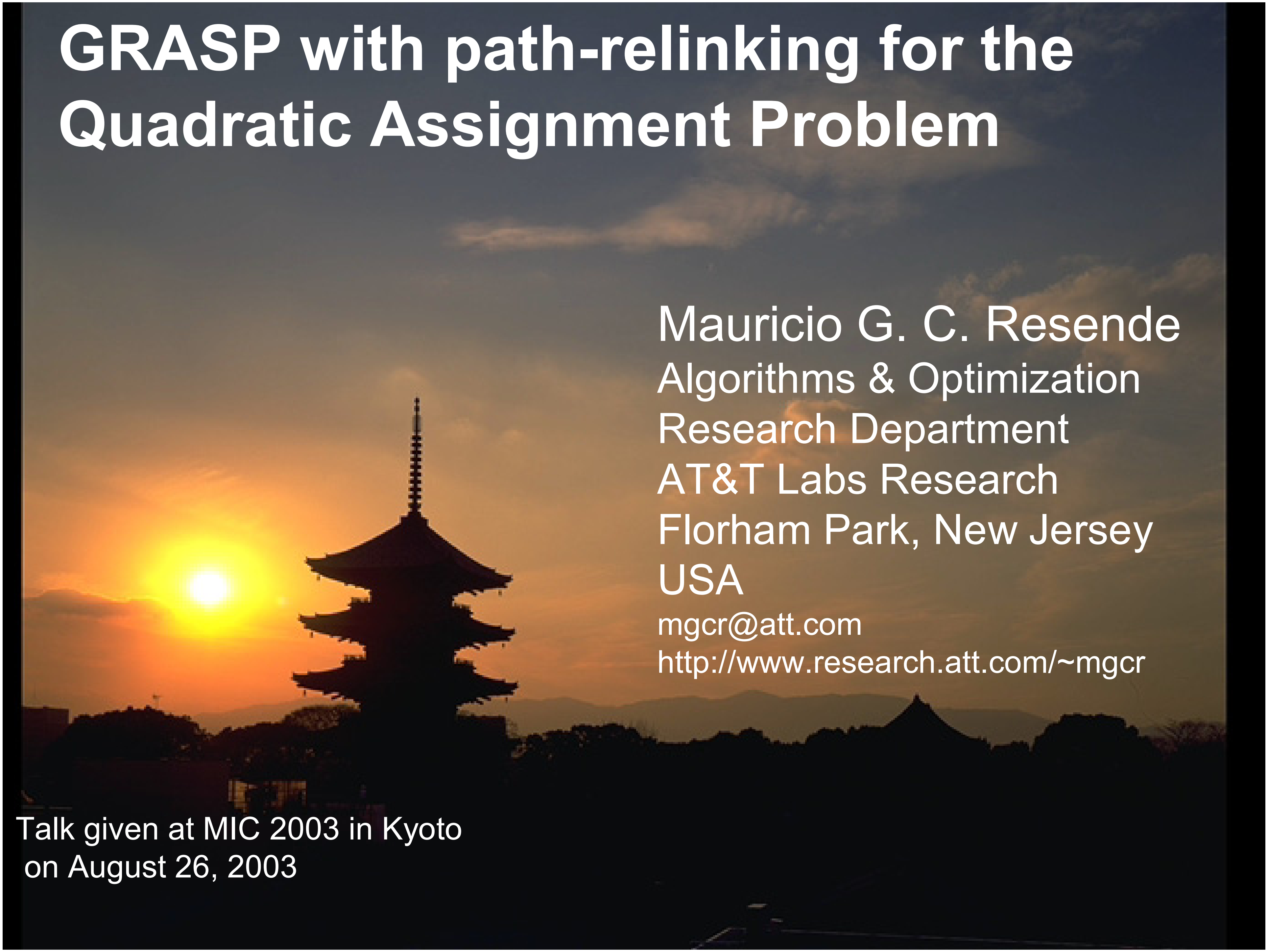


GRASP with path-relinking for the Quadratic Assignment Problem

Mauricio G. C. Resende
Algorithms & Optimization
Research Department
AT&T Labs Research
Florham Park, New Jersey
USA
mgcr@att.com
http://www.research.att.com/~mgcr

Talk given at MIC 2003 in Kyoto
on August 26, 2003

Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

Joint work with Carlos Oliveira and Panos Pardalos (U. of Florida)

AT&T

Quadratic assignment problem (QAP)

- Given N facilities $f_1, f_2, \ldots, f_N$ and N locations $l_1, l_2, \ldots, l_N$
- Let $A^{N \times N} = (a_{i,j})$ be a positive real matrix where $a_{i,j}$ is the flow between facilities f_i and f_j
- Let $B^{N \times N} = (b_{i,j})$ be a positive real matrix where $b_{i,j}$ is the distance between locations l_i and l_j

AT&T

Quadratic assignment problem (QAP)

- Let p: {1,2,...,N} →{1,2,...,N} be an assignment of the N facilities to the N locations
- Define the cost of assignment p to be

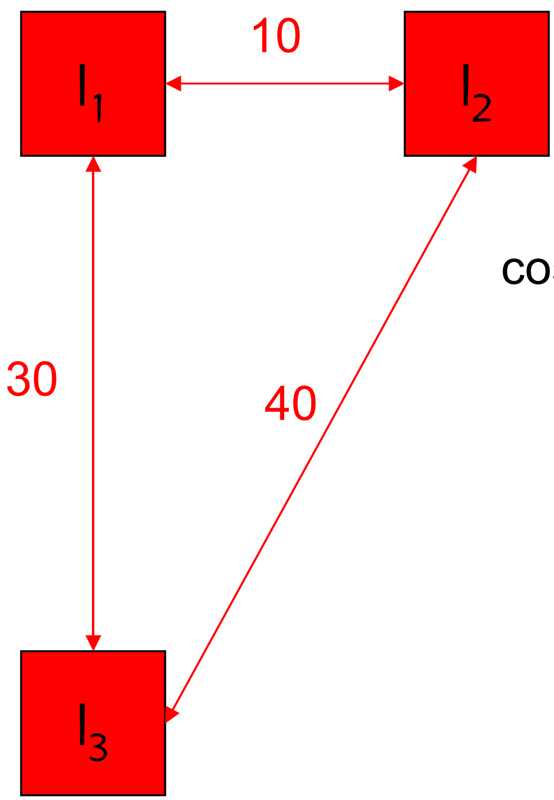
$$c(p) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{i,j} b_{p(i),p(j)}$$

- QAP: Find a permutation vector $p \in \prod_N$ that minimizes the assignment cost:

$$\min c(p): \text{ subject to } p \in \prod_N$$

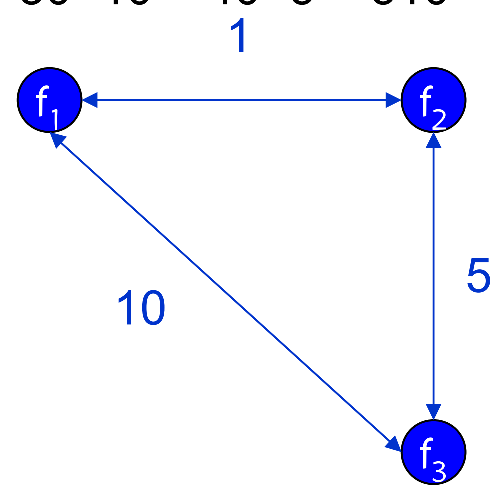
AT&T

Quadratic assignment problem (QAP)



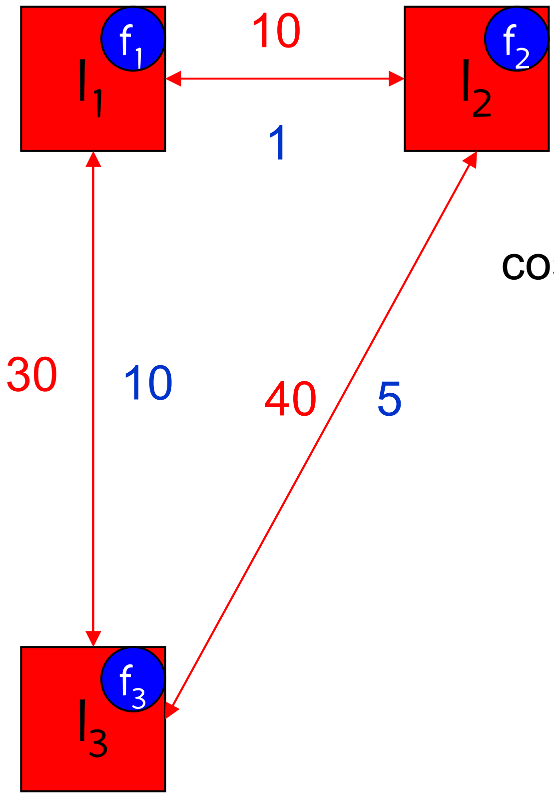
locations and distances

cost of assignment: 10×1+ 30×10 + 40×5 = 510



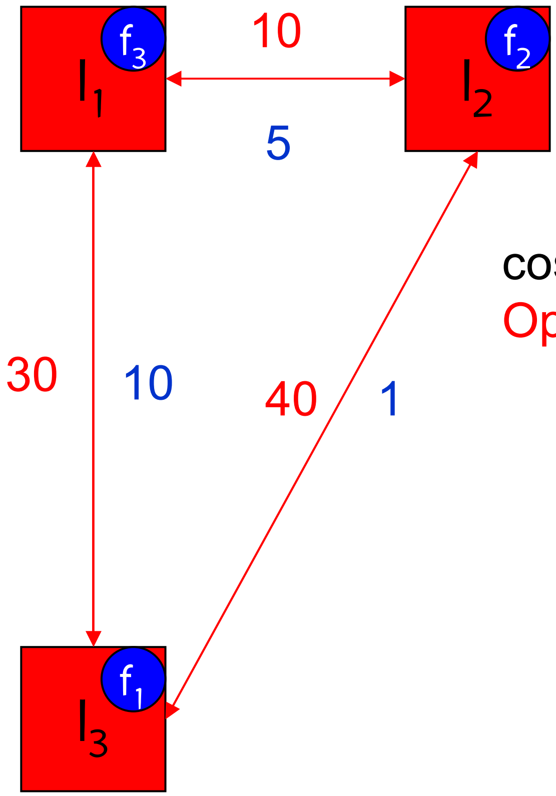
facilities and flows

Quadratic assignment problem (QAP)



cost of assignment: 10×5+ 30×10 + 40×1 = 390

Quadratic assignment problem (QAP)



cost of assignment: 10×10+ 30×5 + 40×1 = 290

Optimal!

GRASP for QAP

- GRASP ✷ multi-start metaheuristic: greedy randomized construction, followed by local search (Feo & Resende, 1989, 1995; Festa & Resende, 2002; Resende & Ribeiro, 2003)
- GRASP for QAP
  - Li, Pardalos, & Resende (1994): GRASP for QAP
  - Resende, Pardalos, & Li (1996): Fortran suboroutines for dense QAPs
  - Pardalos, Pitsoulis, & Resende (1997): Fortran subroutines for sparse QAPs
  - Fleurent & Glover (1999): memory mechanism in construction

AT&T

GRASP for QAP

```
repeat {
    x = GreedyRandomizedConstruction(▪);
    x = LocalSearch(x);
    save x if best so far;
}
return x;
```

Construction

- Stage 1: make two assignments $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$
- Stage 2: make remaining N−2 assignments of facilities to locations, one facility/location pair at a time

Stage 1 construction

- sort distances $b_{i,j}$ in increasing order:
  $b_{i(1),j(1)} \leq b_{i(2),j(2)} \leq \cdots \leq b_{i(N),j(N)}$.
- sort flows $a_{k,l}$ in decreasing order:
  $a_{k(1),l(1)} \geq a_{k(2),l(2)} \geq \cdots \geq a_{k(N),l(N)}$.
- sort products:
  $a_{k(1),l(1)} \cdot b_{i(1),j(1)}, a_{k(2),l(2)} \cdot b_{i(2),j(2)}, \ldots, a_{k(N),l(N)} \cdot b_{i(N),j(N)}$
- among smallest products, select $a_{k(q),l(q)} \cdot b_{i(q),j(q)}$ at random:
  corresponding to assignments $\{f_{k(q)} \rightarrow l_{i(q)} ; f_{l(q)} \rightarrow l_{j(q)}\}$

AT&T

Stage 2 construction

- If $\Omega = \{(i_1,k_1),(i_2,k_2), \ldots, (i_q,k_q)\}$ are the q assignments made so far, then
- Cost of assigning $f_j \rightarrow l_l$ is $$c_{j,l} = \sum_{i,k \in \Gamma} a_{i,j} b_{k,l}$$
- Of all possible assignments, one is selected at random from the assignments of smallest costs and added to Ω

Sped up in Pardalos, Pitsoulis, & Resende (1997) for QAPs with sparse A or B matrices.

AT&T

Swap based local search

a) For all pairs of assignments $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$, test if swapped assignment $\{f_i \rightarrow l_l ; f_j \rightarrow l_k\}$ improves solution.

b) If so, make swap and return to step (a)

repeat (a)-(b) until no swap improves current solution

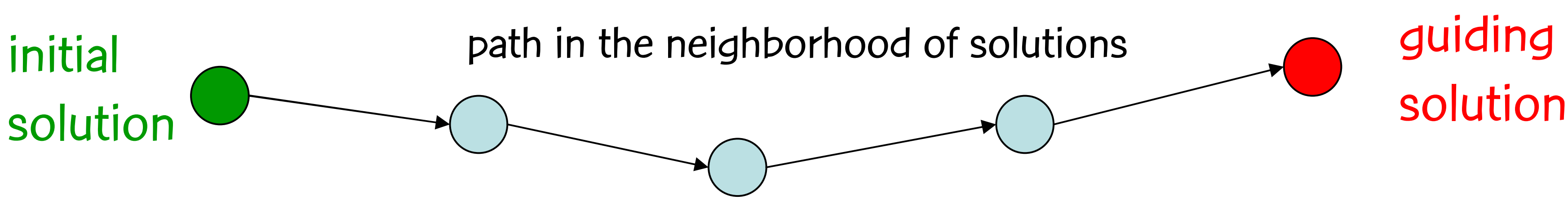
AT&T

Path-relinking

- Path-relinking:
  - Intensification strategy exploring trajectories connecting elite solutions: Glover (1996)
  - Originally proposed in the context of tabu search and scatter search.
  - Paths in the solution space leading to other elite solutions are explored in the search for better solutions:
    - selection of moves that introduce attributes of the guiding solution into the current solution

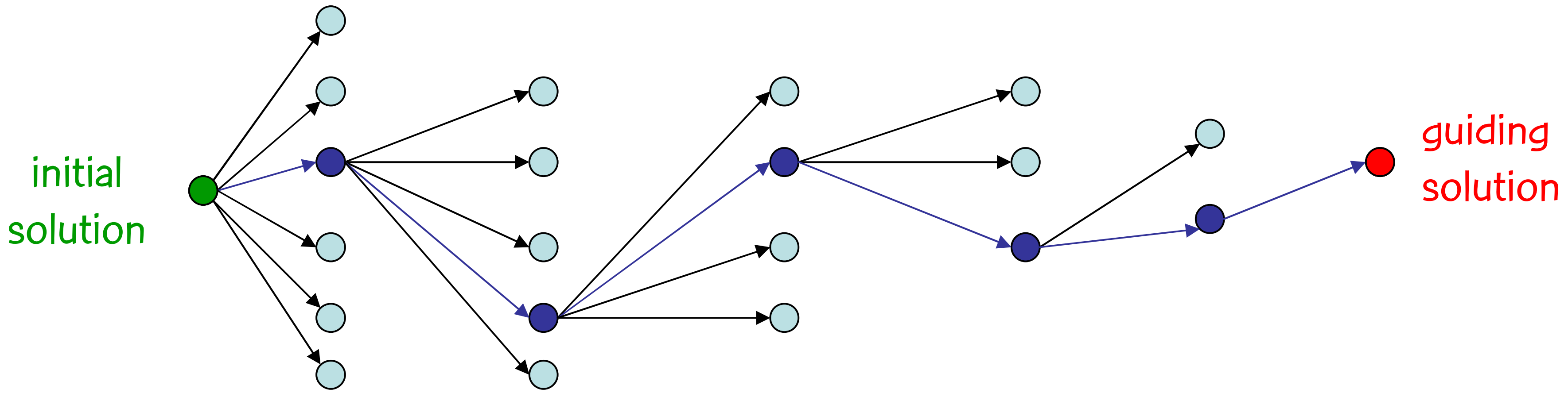
Path-relinking

- Exploration of trajectories that connect high quality (elite) solutions:



Path-relinking

- Path is generated by selecting moves that introduce in the initial solution attributes of the guiding solution.
- At each step, all moves that incorporate attributes of the guiding solution are evaluated and the best move is selected:



Path-relinking

Combine solutions x and y

Δ(x,y): symmetric difference between x and y

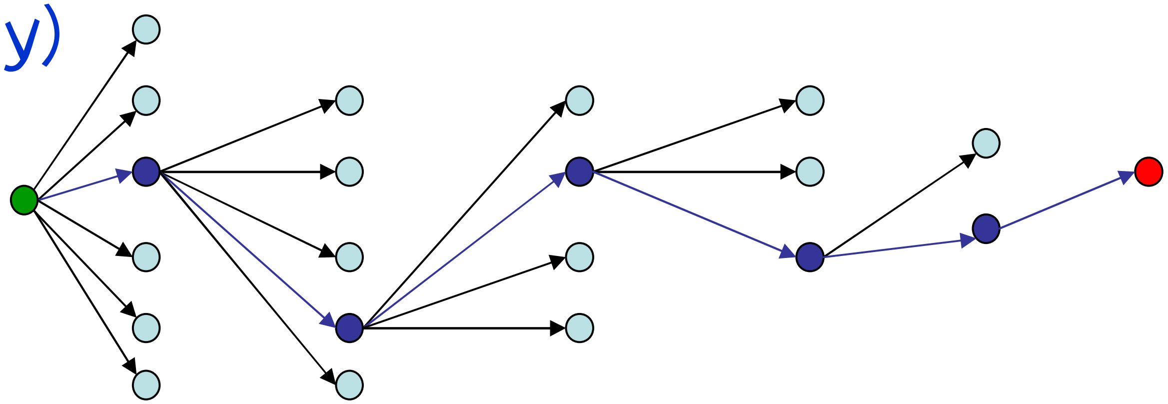
while (|Δ(x,y)| > 0) {

evaluate moves corresponding in Δ(x,y)

make best move

update Δ(x,y)

}



GRASP with path-relinking

- Originally used by Laguna and Martí (1999).
- Maintains a set of elite solutions found during GRASP iterations.
- After each GRASP iteration (construction and local search):
  - Use GRASP solution as initial solution.
  - Select an elite solution uniformly at random: guiding solution.
  - Perform path-relinking between these two solutions.

GRASP with path-relinking

Repeat for Max_Iterations:

Construct a greedy randomized solution.

Use local search to improve the constructed solution.

Apply path-relinking to further improve the solution.

Update the pool of elite solutions.

Update the best solution found.

AT&T

PR for QAP (permutation vectors)

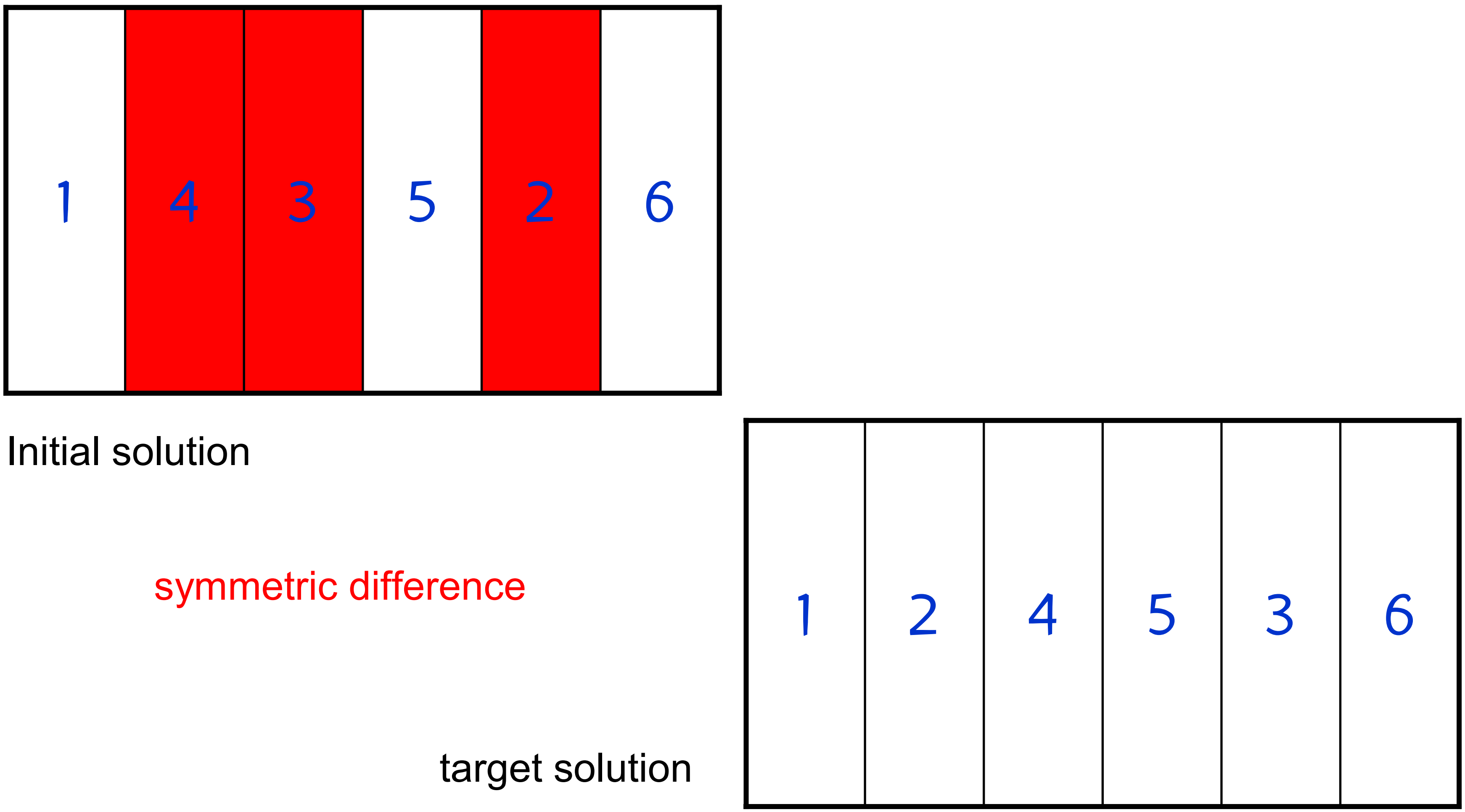
1	4	3	5	2	6

Initial solution

1	2	4	5	3	6

target solution

PR for QAP (permutation vectors)



PR for QAP (permutation vectors)

1	4	3	5	2	6

Initial solution

moves: swap 4 & 2 then 3 & 4*
swap 4 & 3 then 2 & 3

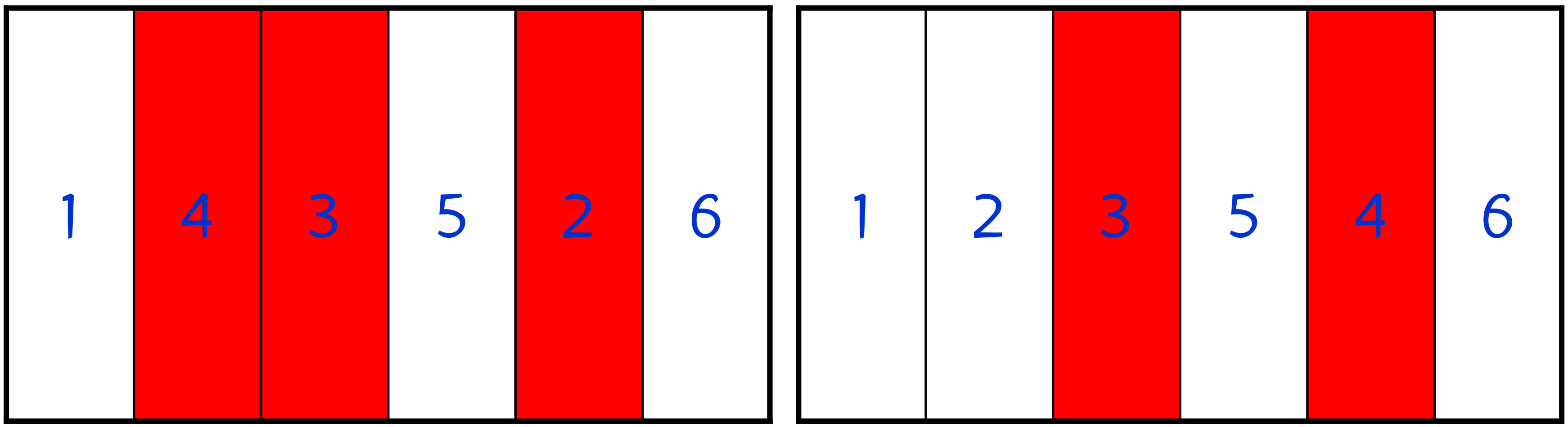
* best improvement

target solution

1	2	4	5	3	6

AT&T

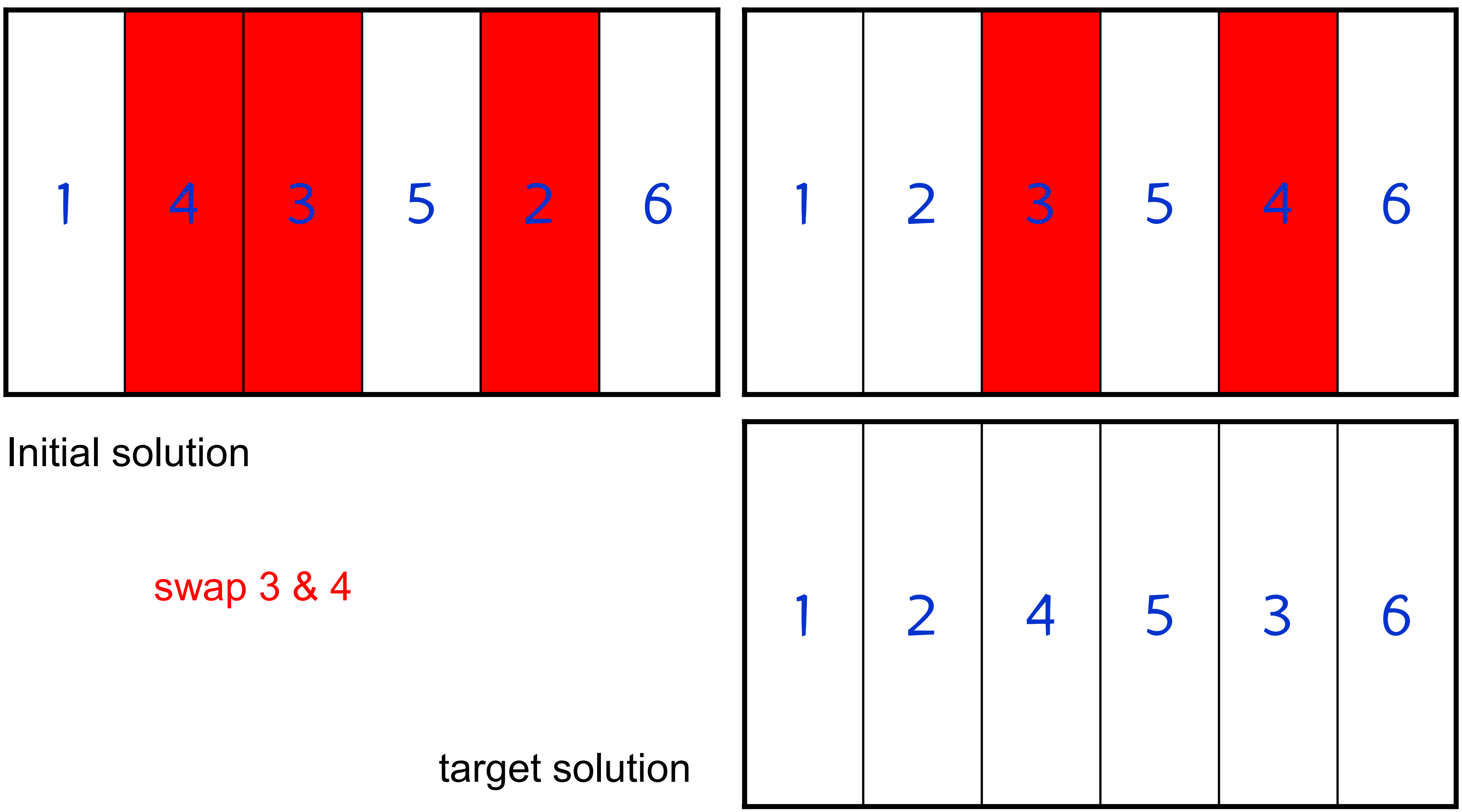
PR for QAP (permutation vectors)



Initial solution

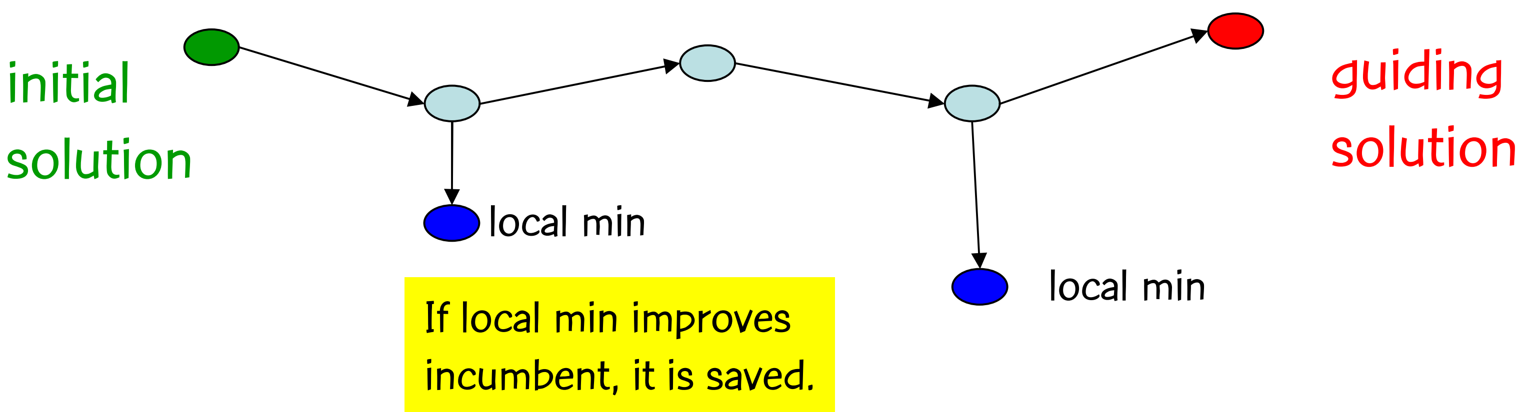
swap 4 & 2

PR for QAP (permutation vectors)



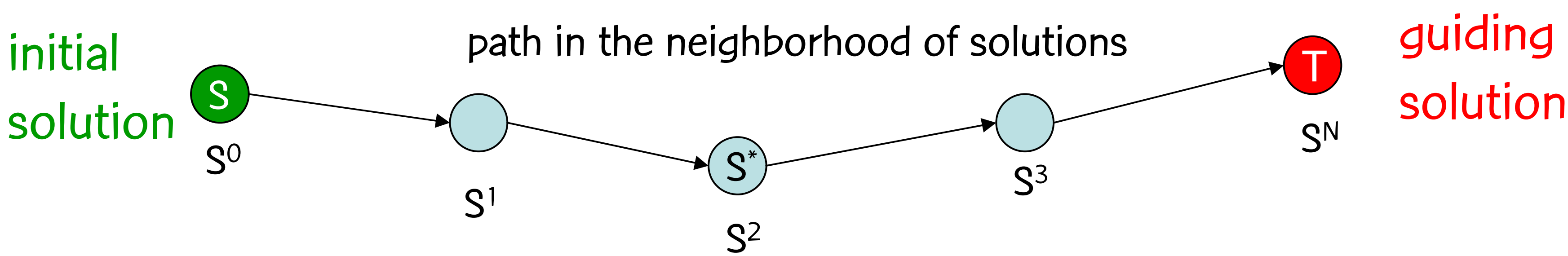
Path-relinking for QAP

If swap improves solution: local search is applied



Path-relinking for QAP

Results of path relinking: S^*

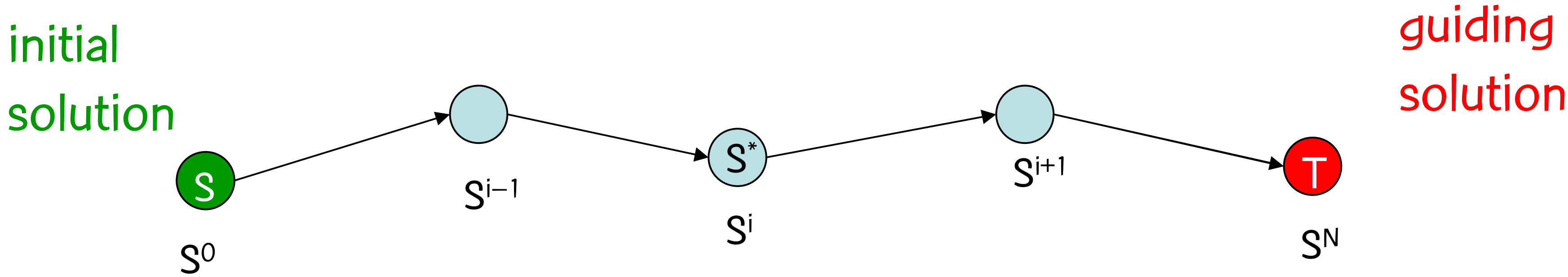


If $c(S^*) < \min\{c(S), c(T)\}$, and $c(S^*) \leq c(S^i)$, for $i=1,\dots,N$, i.e. S^* is best solution in path, then S^* is returned.

Path-relinking for QAP

S^i is a local minimum w.r.t. PR:

$c(S^i) < c(S^{i-1})$ and $c(S^i) < c(S^{i+1})$, for all $i=1,\ldots,N$.



If path-relinking does not improve (S,T), then if S^i is a best local min w.r.t. PR: return $S^* = S^i$

If no local min exists, return S*=argmin{S,T}

AT&T

PR pool management

- S^* is candidate for inclusion in pool of elite solutions (P)
- If $c(S^*) < c(S^e)$, for all $S^e \in P$, then S^* is put in P
- Else, if $c(S^*) < \max\{c(S^e), S^e \in P\}$ and $|\Delta(S^*,S^e)| \geq 3$, for all $S^e \in P$, then S^* is put in P
- If pool is full, remove $\text{argmin}\,\{|\Delta(S^*,S^e)|, \forall\, S^e \in P \text{ s.t. } c(S^e) \geq c(S^*)\}$

PR pool management

S is initial solution for path-relinking: favor choice of target solution T with large symmetric difference with S.

This leads to longer paths in path-relinking.

Probability of choosing $S^e \in P$:

$$p(S^e) = \frac{|\Delta(S, S^e)|}{\sum_{R \in P} |\Delta(S, R)|}$$

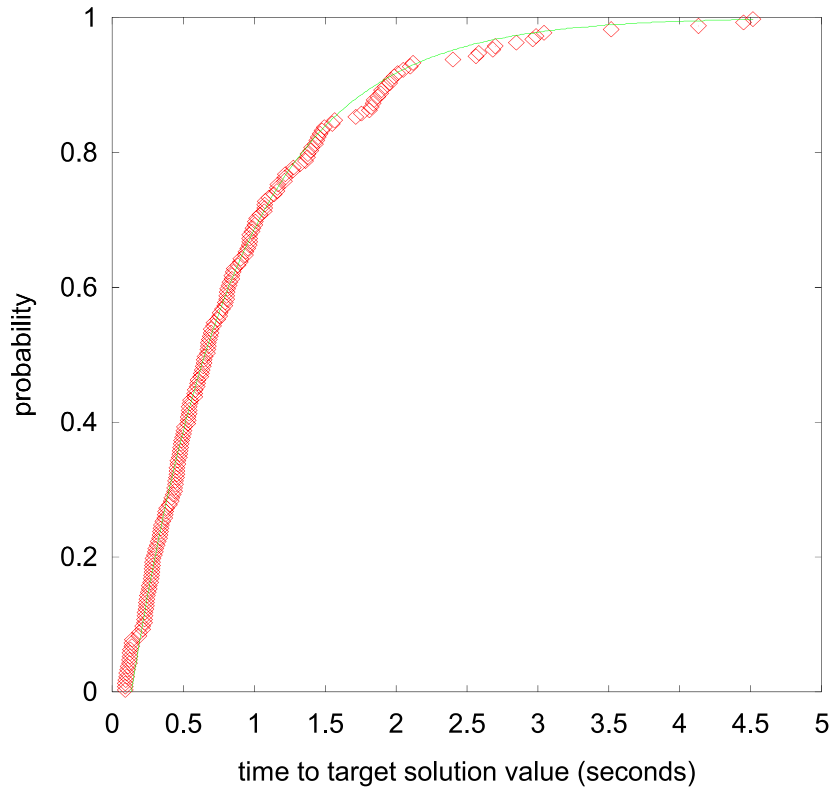
Experimental results

- Compare GRASP with and without path-relinking.
- New GRASP code in C outperforms old Fortran codes: we use same code to compare algorithms
- All QAPLIB (Burkhard, Karisch, & Rendl, 1991) instances of size $N \leq 40$
- 100 independent runs of each algorithm, recording CPU time to find the best known solution for instance

Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter α chosen at random in interval [0,1] at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

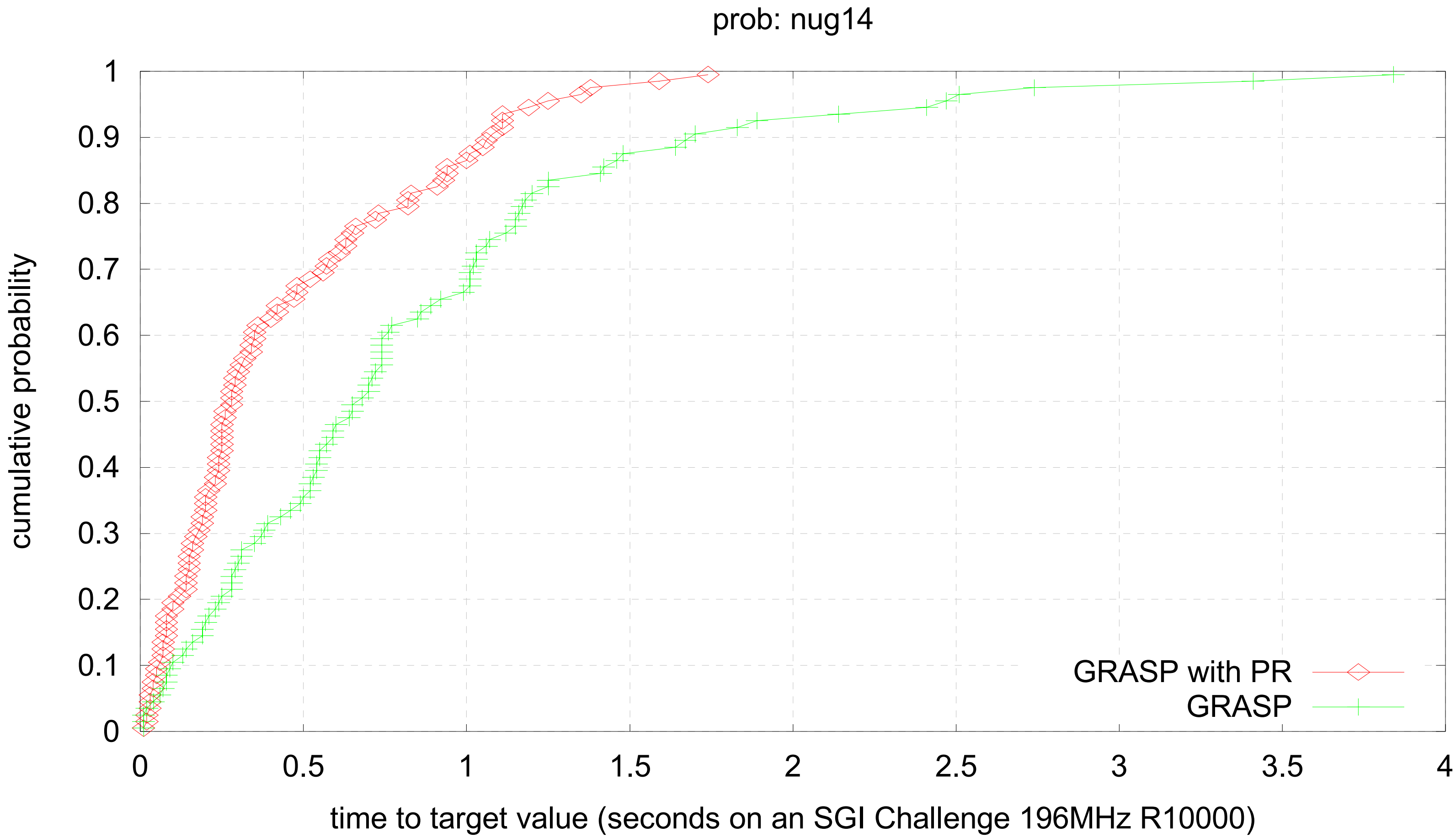
Time-to-target-value plots



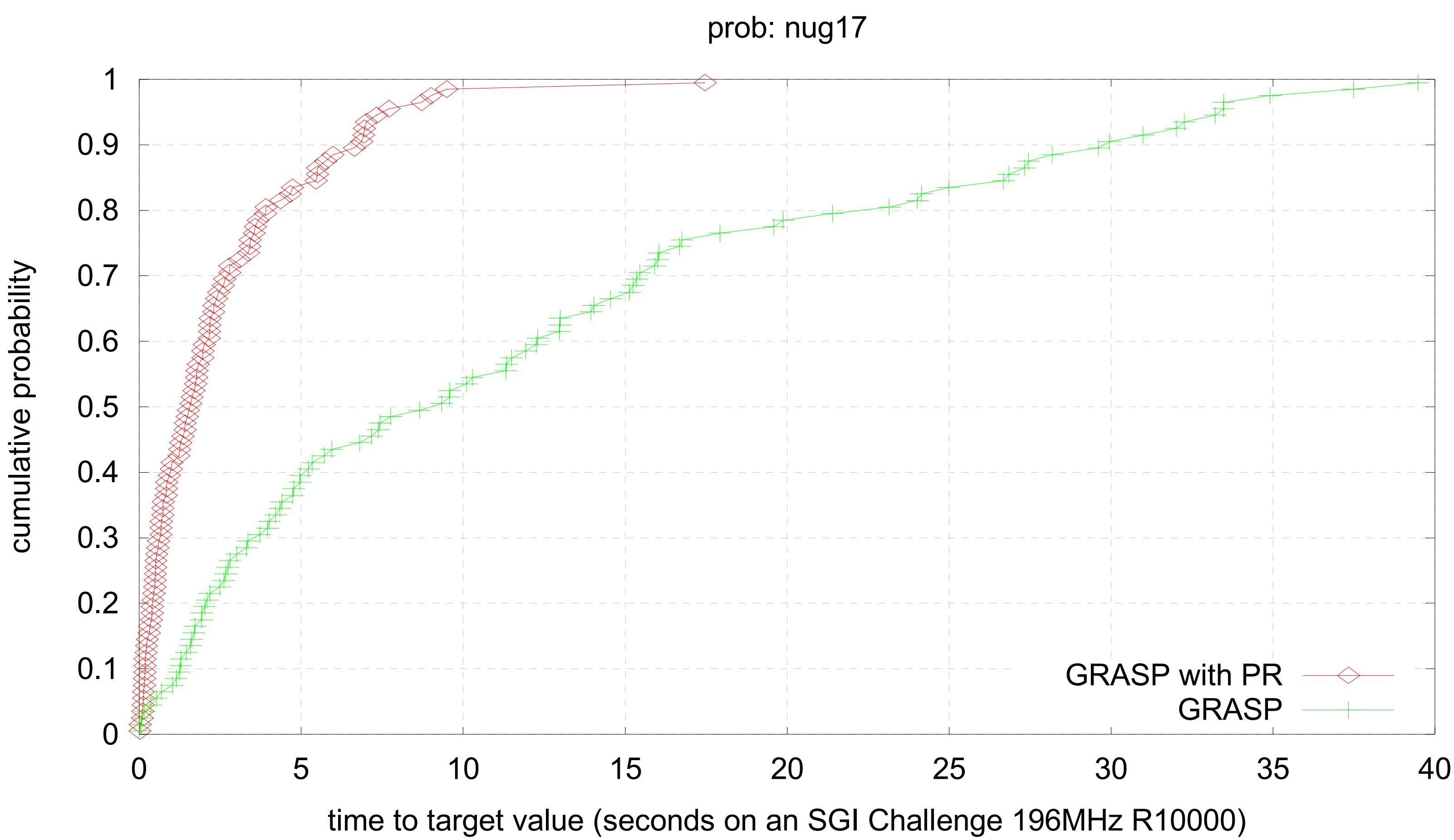
Sort times such that $t_1 \leq t_2 \leq \cdots \leq t_{100}$ and plot $\{t_i, p_i\}$, for i=1,...,N, where $p_i = (i-.5)/100$

Random variable time-to-target-solution value fits a two-parameter exponential distribution (Aiex, Resende, & Ribeiro, 2002).

nug14 (N=14; look4=1014)

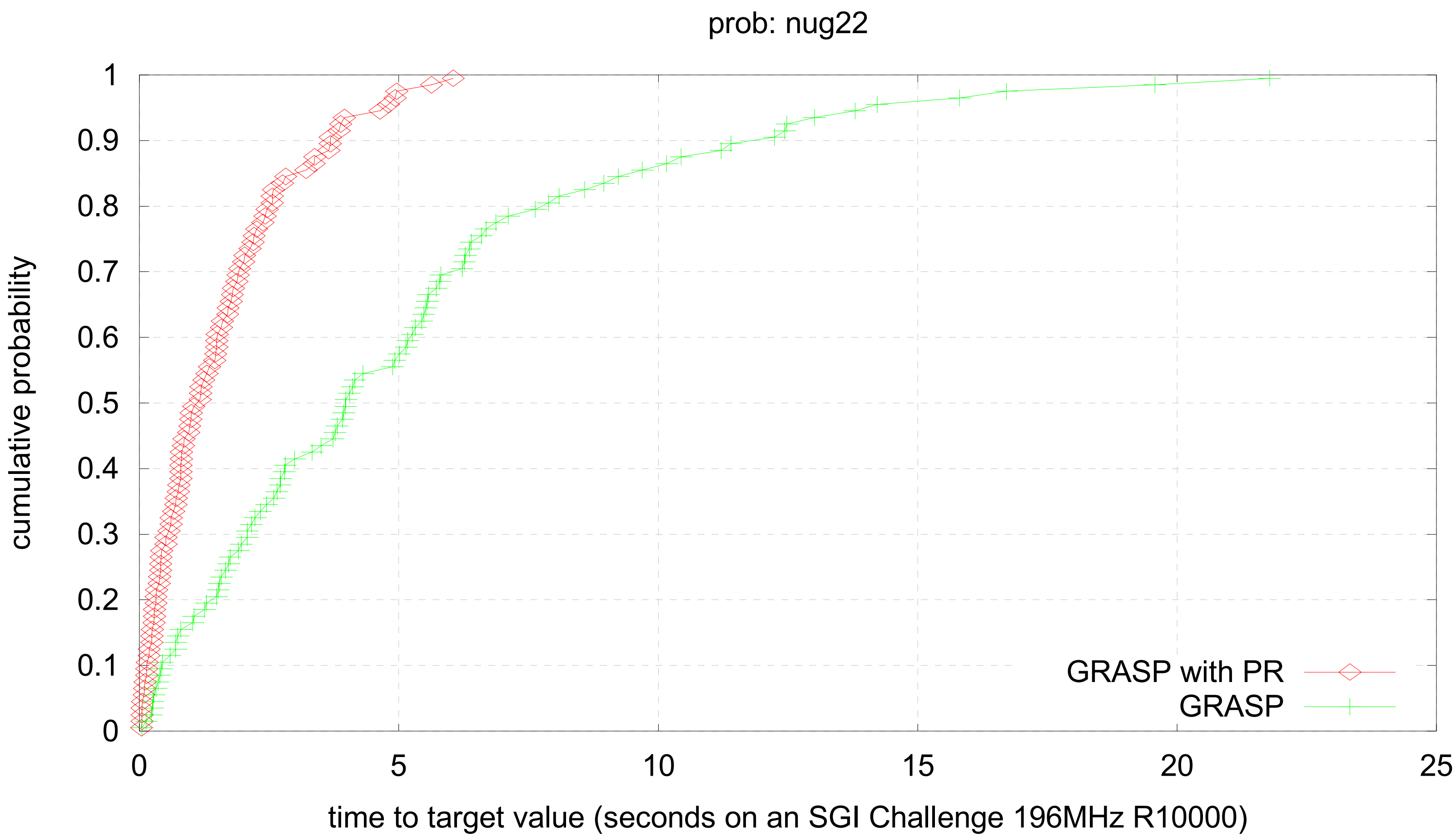


nug17 (N=17; look4=1732)

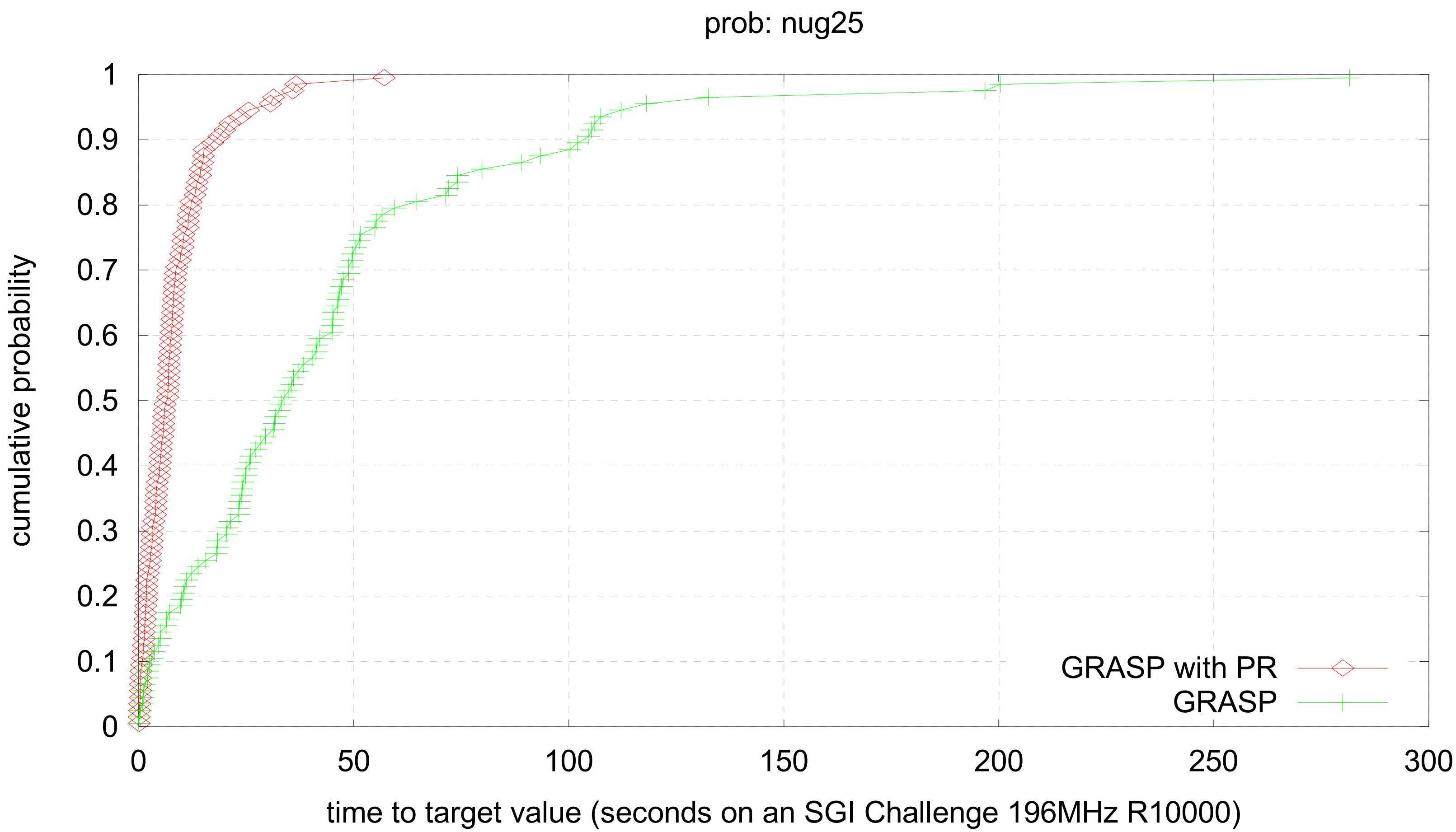


AT&T

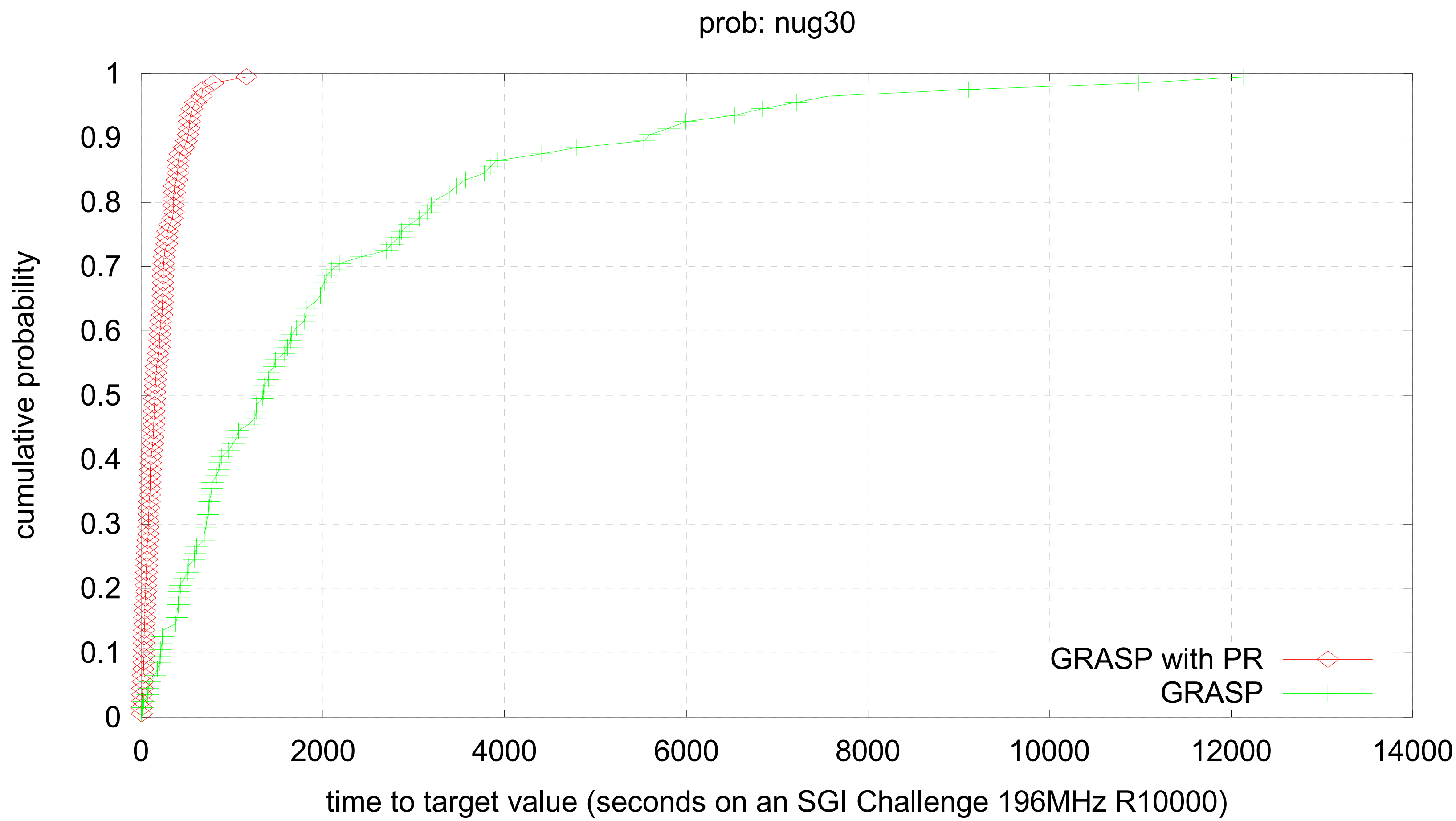
nug22 (N=22; look4=3596)



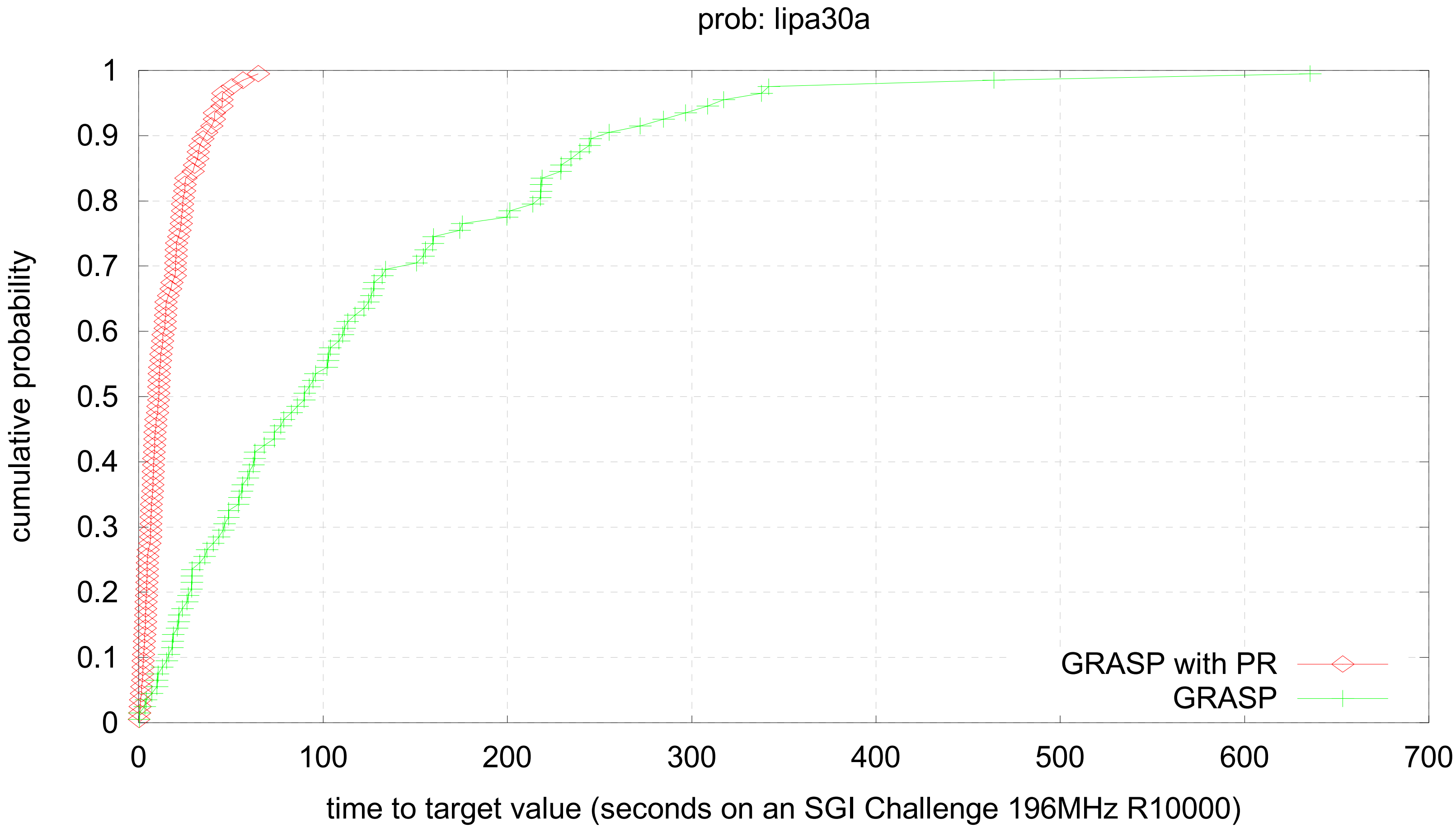
nug25 (N=25; look4=3744)



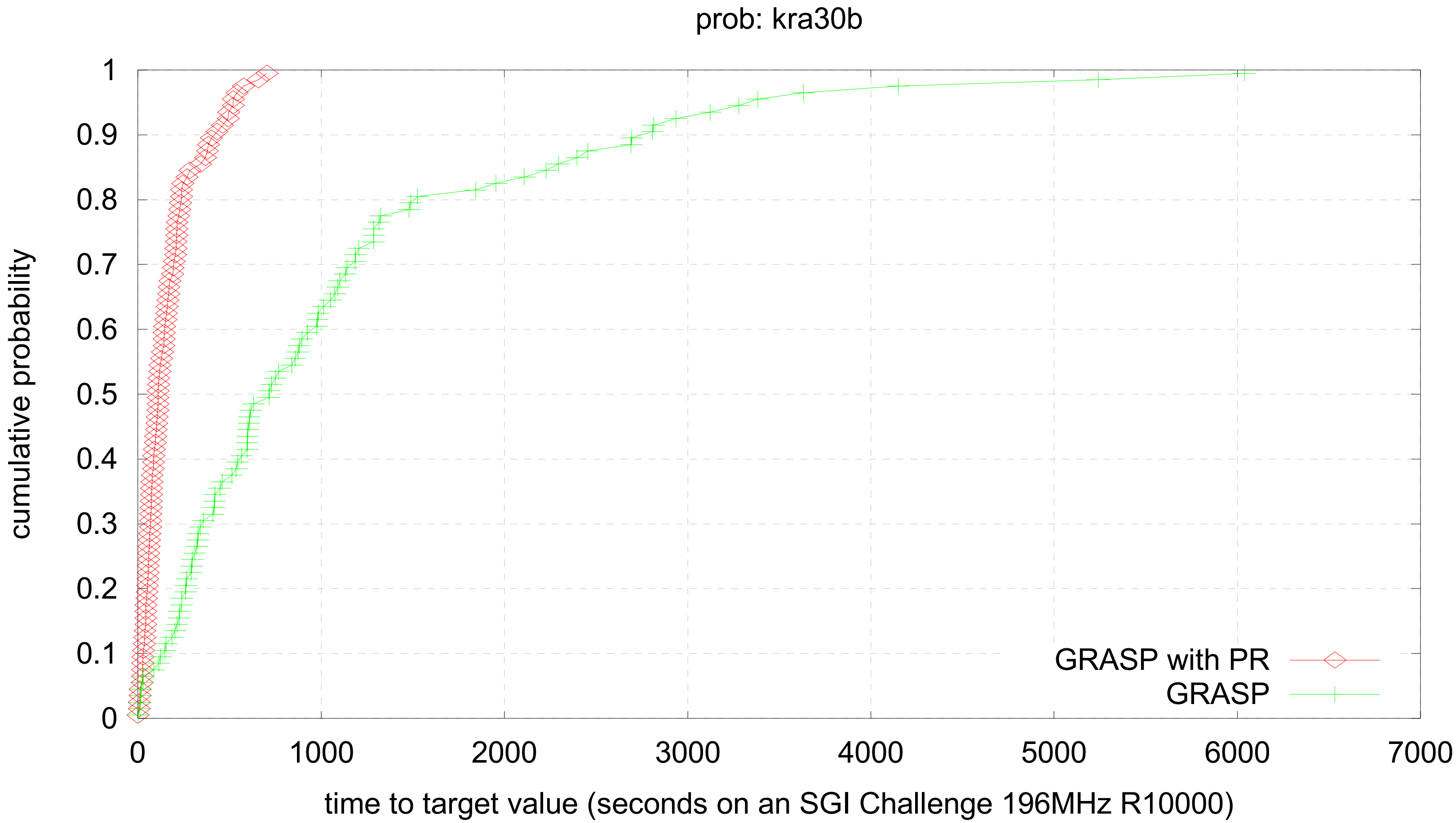
nug30 (N=30; look4=6124)



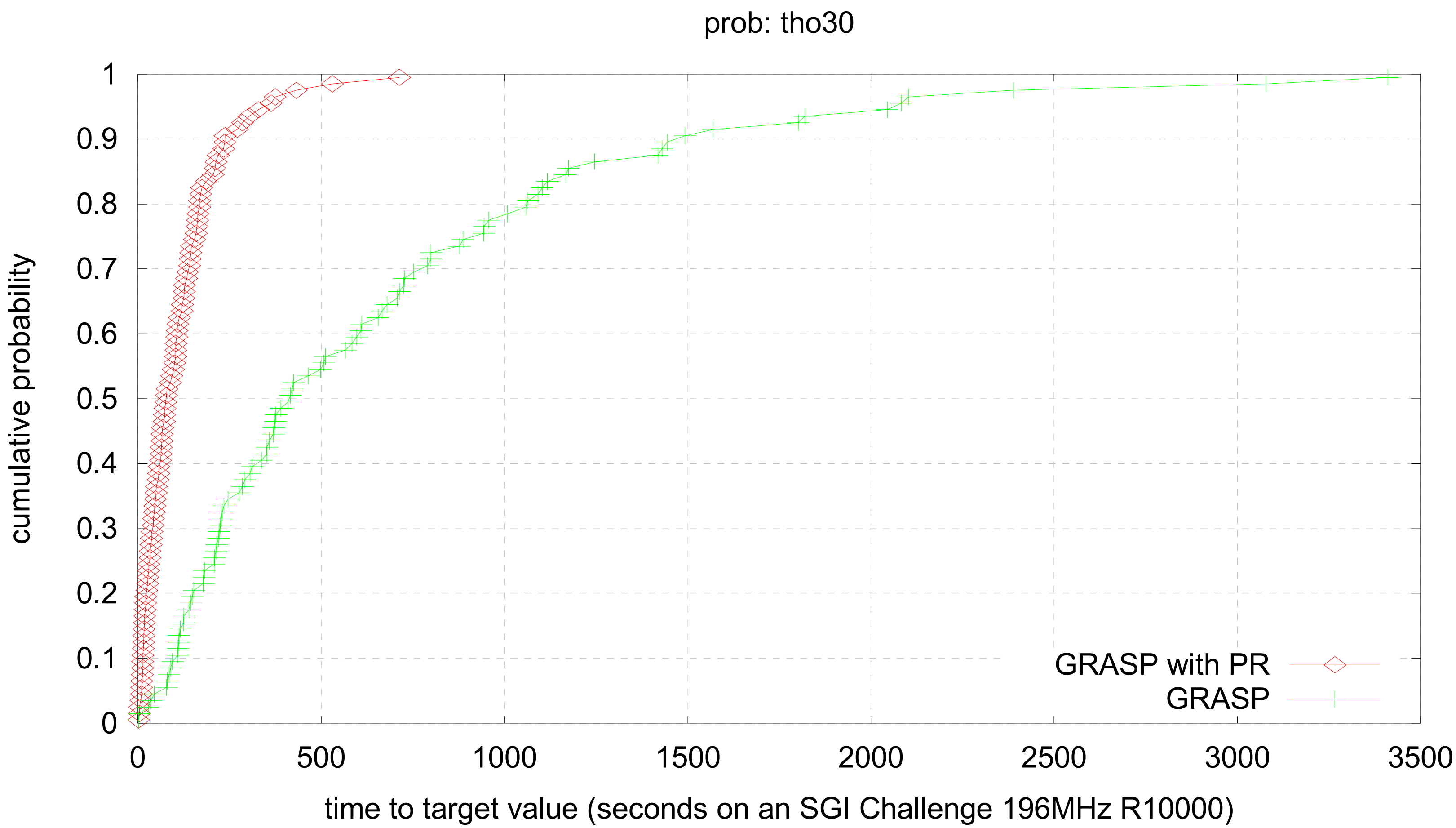
lipa30a (N=30; look4=13178)



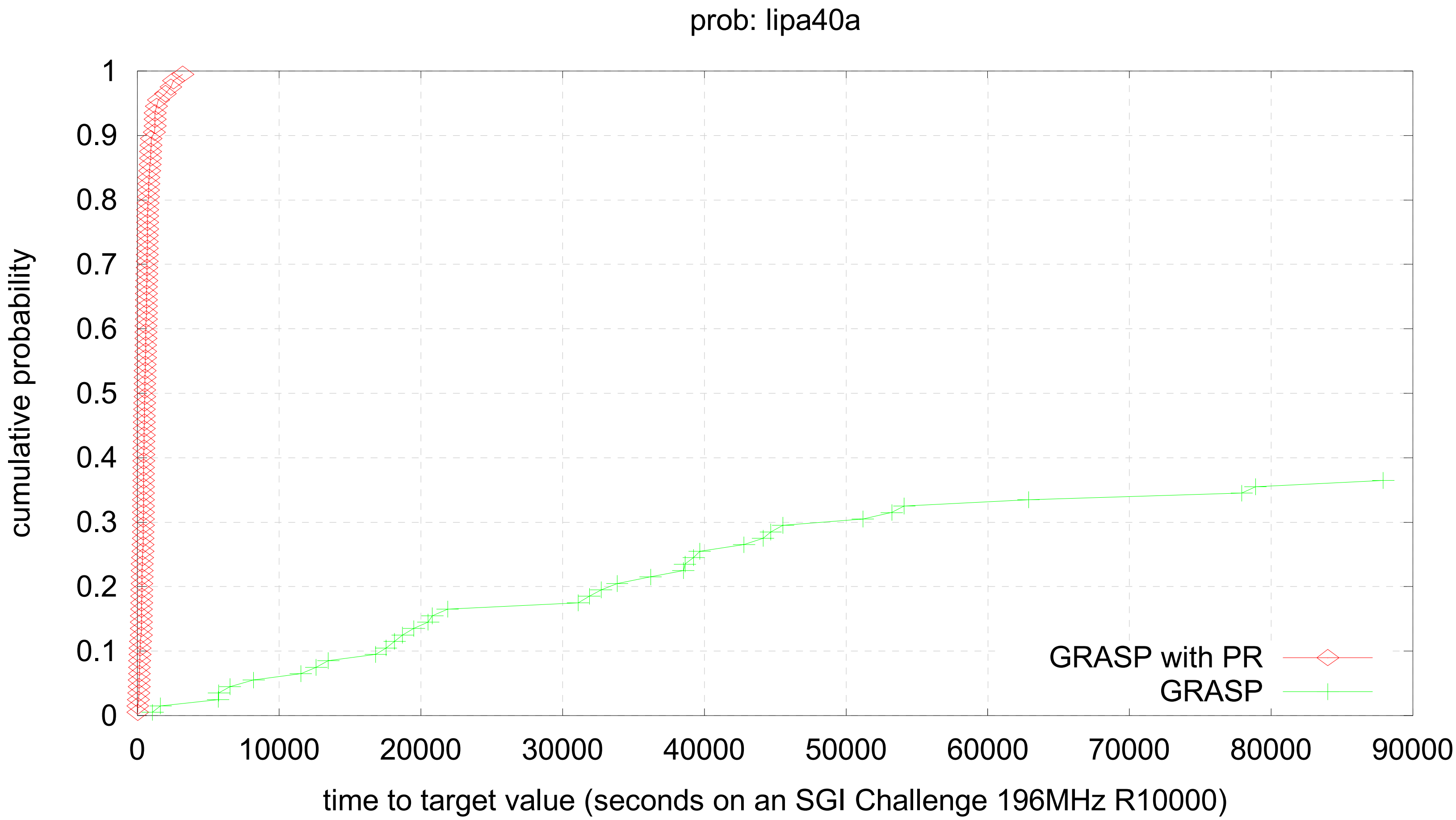
kra30b (N=30; look4=91420)



tho30 (N=30; look4=149936)



lipa40a (N=50; look4=31538)



Concluding remarks

- New heuristic for the QAP is described.
- Path-relinking shown to improve performance of GRASP on all instances.
- Final paper will compare GRASP+PR with other heuristics for QAP on larger instances from QAPLIB.
- We intend to make the code available on the web site http://www.research.att.com/~mgcr