A memetic algorithm for routing optimization in networks using exponential flow splitting

Talk given at the Ninth INFORMS Telecommunications Conference, University of Maryland, College Park, MD
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Joint work with Luciana Buriol,
Marcus Ritt, and Roger Reis ~ UFRGS, Brazil.

Summary of talk

- Genetic algorithm with random keys
- OSPF routing
- Memetic algorithm for OSPF routing
- DEFT routing
- Implementation issues
- Numerical results for OSPF and DEFT
- Concluding remarks

Genetic algorithms with random keys

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.

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- Individuals are strings of real-valued numbers (random keys) in the interval [0,1].

S = (0.25, 0.19, 0.67, 0.05, 0.89)
s(1) s(2) s(3) s(4) s(5)

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Individuals are strings of real-valued numbers (random keys) in the interval [0,1].
- Sorting random keys results in a sequencing order.

S = (0.25, 0.19, 0.67, 0.05, 0.89)
s(1) s(2) s(3) s(4) s(5)

S' = (0.05, 0.19, 0.25, 0.67, 0.89)
s(4) s(2) s(1) s(3) s(5)

Sequence: 4 – 2 – 1 – 3 – 5

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GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)

a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)

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- For each gene, flip a biased coin to choose which parent passed the allele to the child.

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Every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

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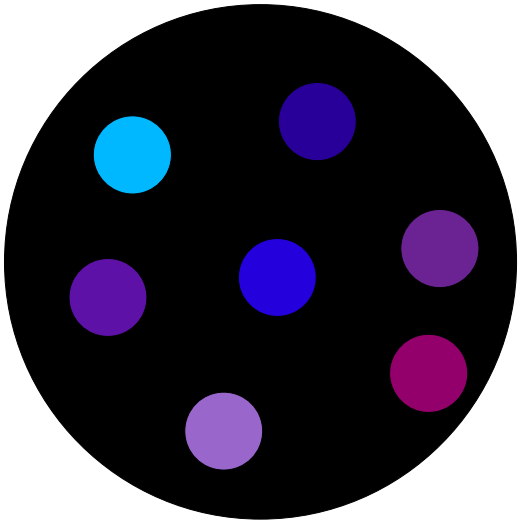
GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Initial population is made up of P chromosomes, each with N genes, each having a value (allele) generated uniformly at random in the interval [0,1].



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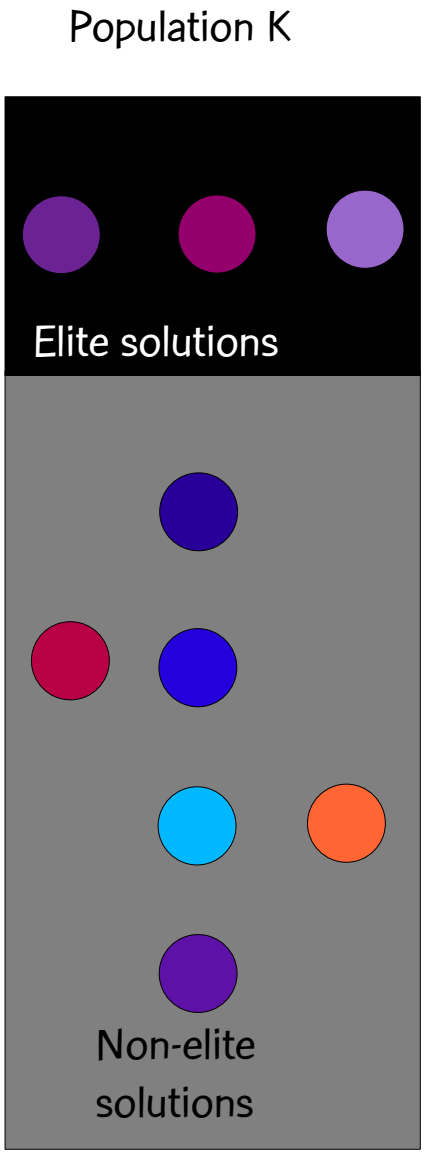
GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions, non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.



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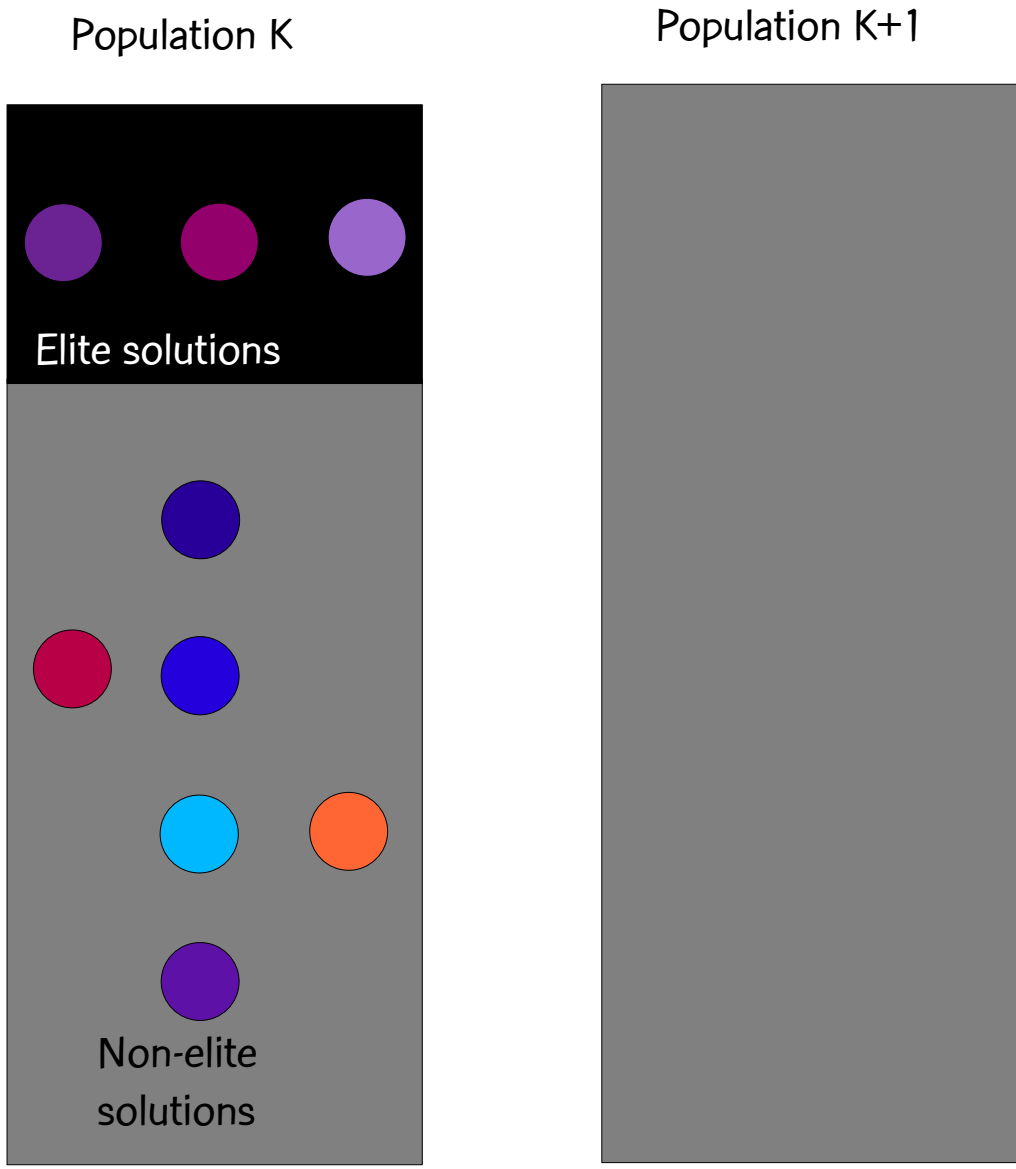
GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Evolutionary dynamics



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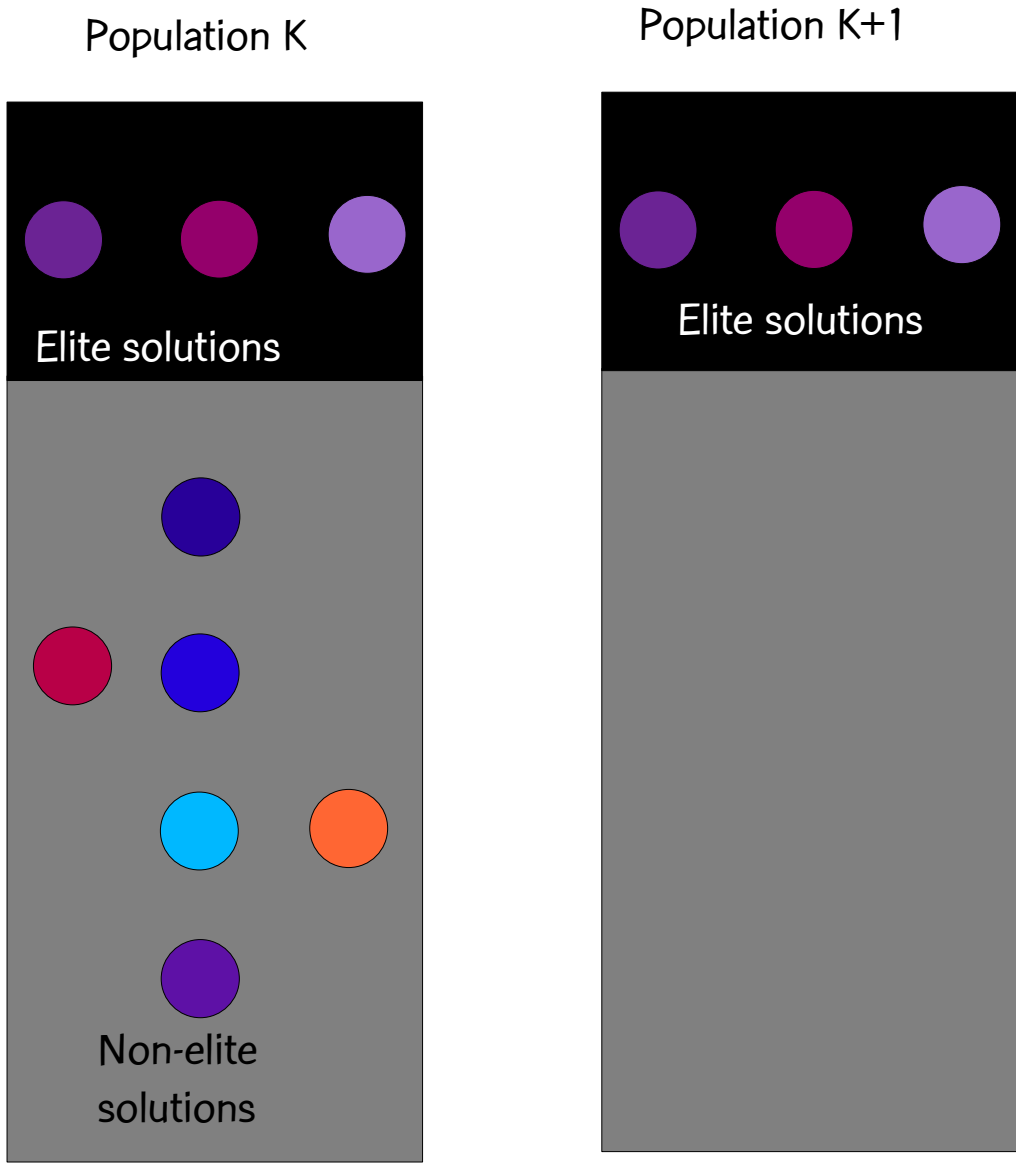
GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Evolutionary dynamics
  - Copy elite solutions from population K to population K+1



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  - Copy elite solutions from population K to population K+1
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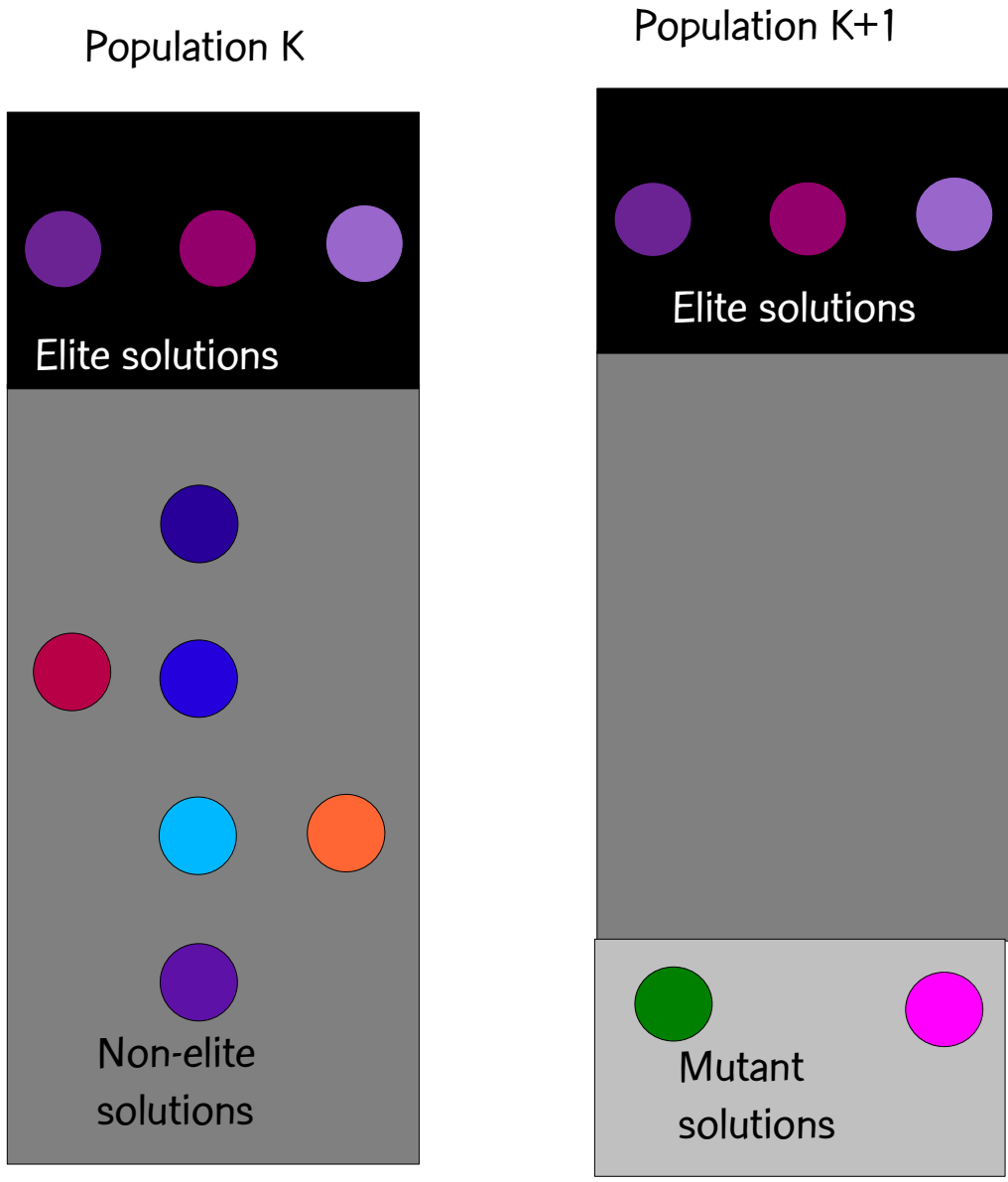

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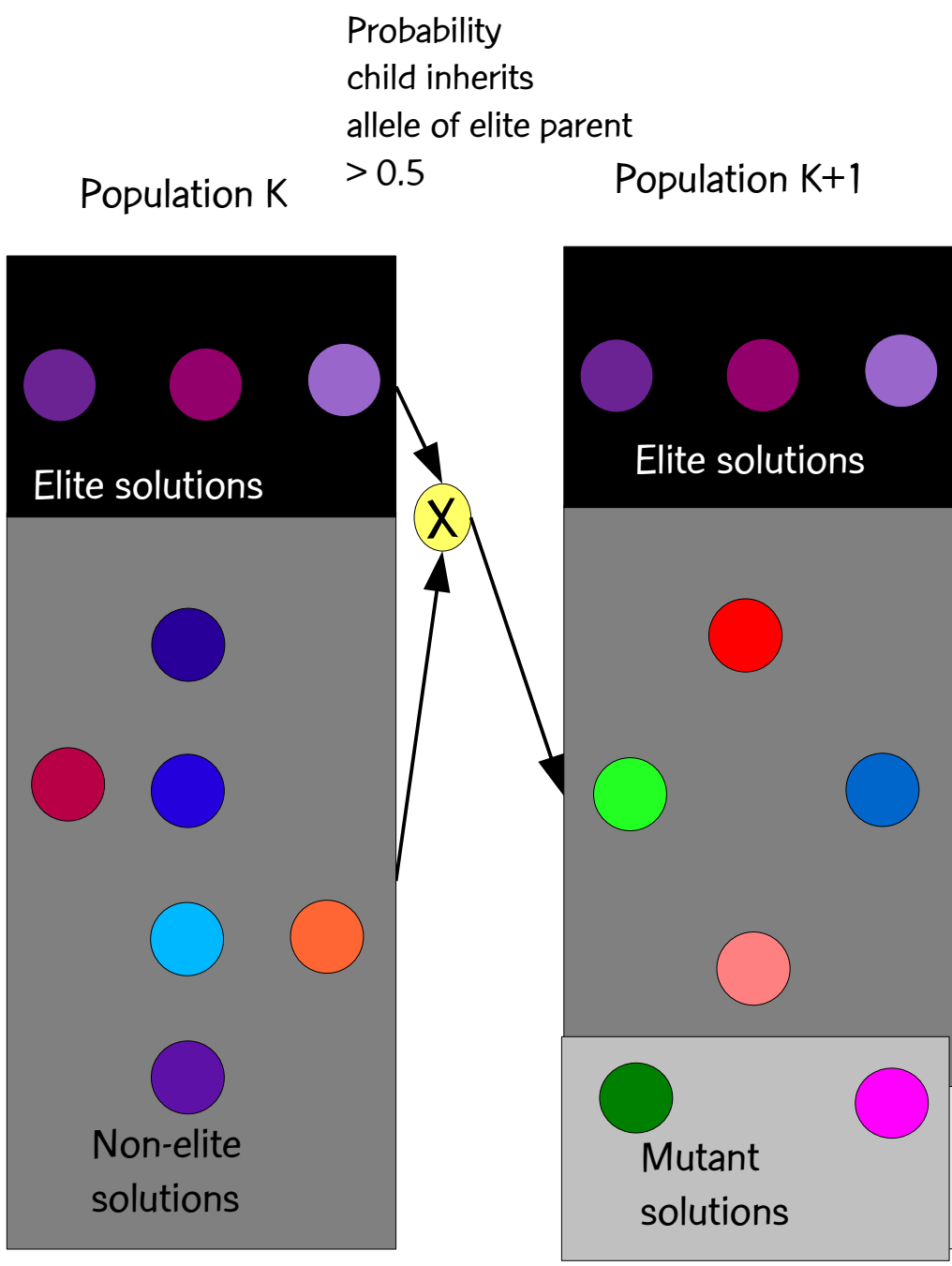
GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Evolutionary dynamics
  - Copy elite solutions from population K to population K+1
  - Add R random solutions (mutants) to population K+1
  - While K+1-th population < P
    - Mate elite solution with non elite to produce child in population K+1. Mates are chosen at random.



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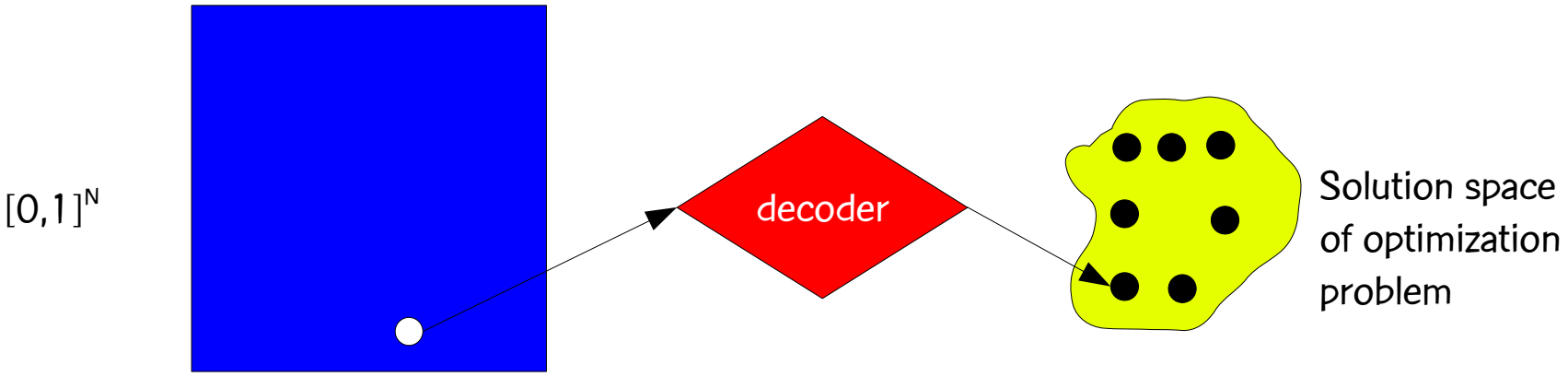
Decoders

- A decoder is a deterministic algorithm that takes as input a random-key vector and returns a feasible solution of the optimization problem and its cost.
- Bean (1994) proposed decoders based on sorting the random-key vector to produce a sequence.
- A random-key GA searches the solution space indirectly by searching the space of random keys and using the decoder to evaluate fitness of the random key.



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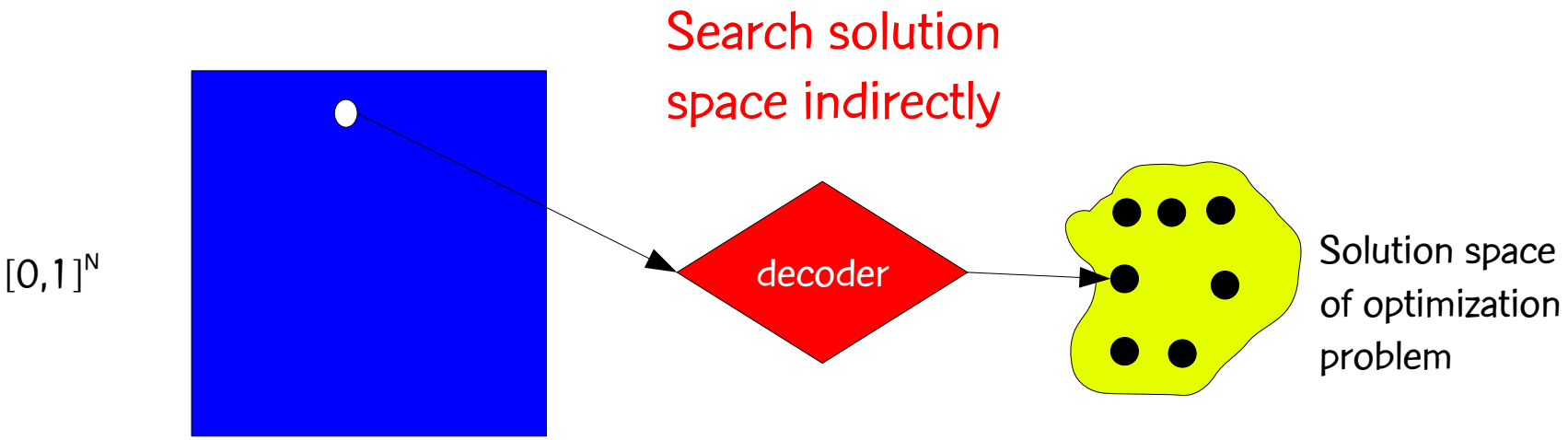

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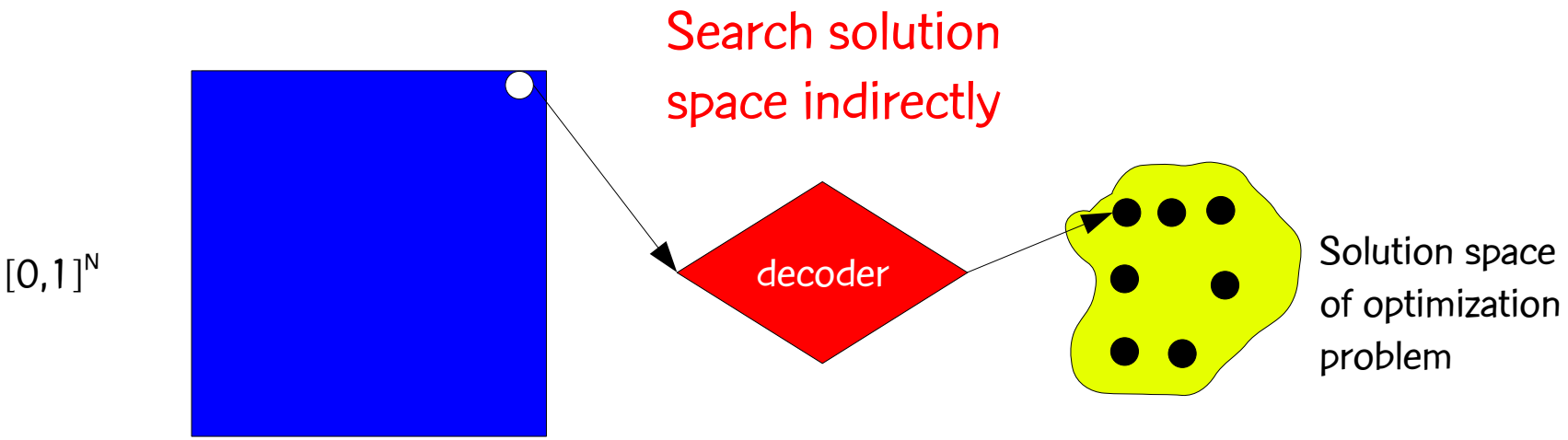

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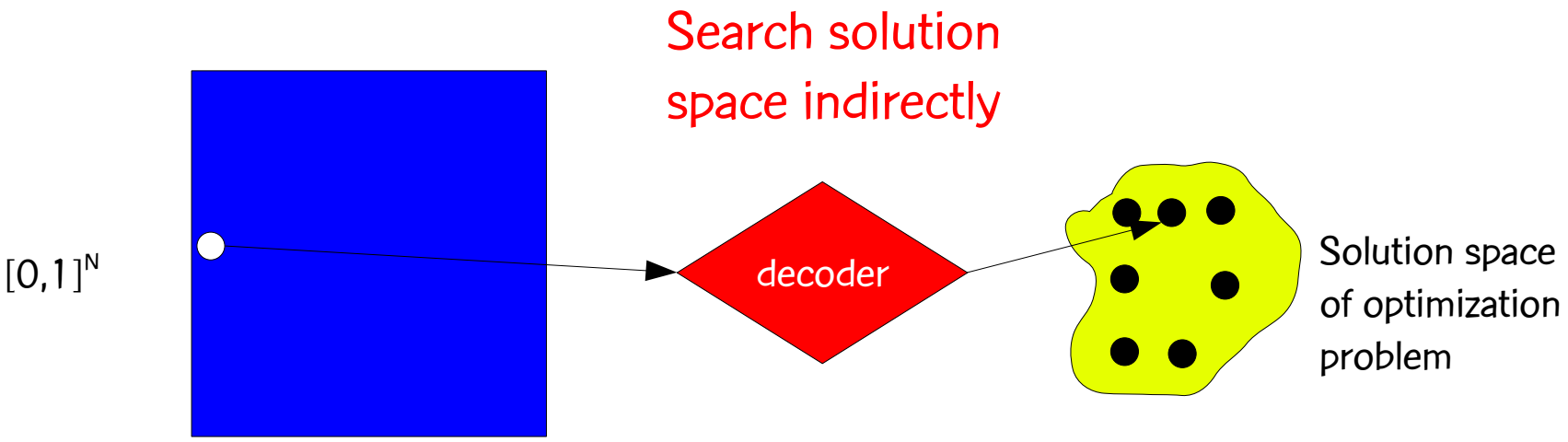

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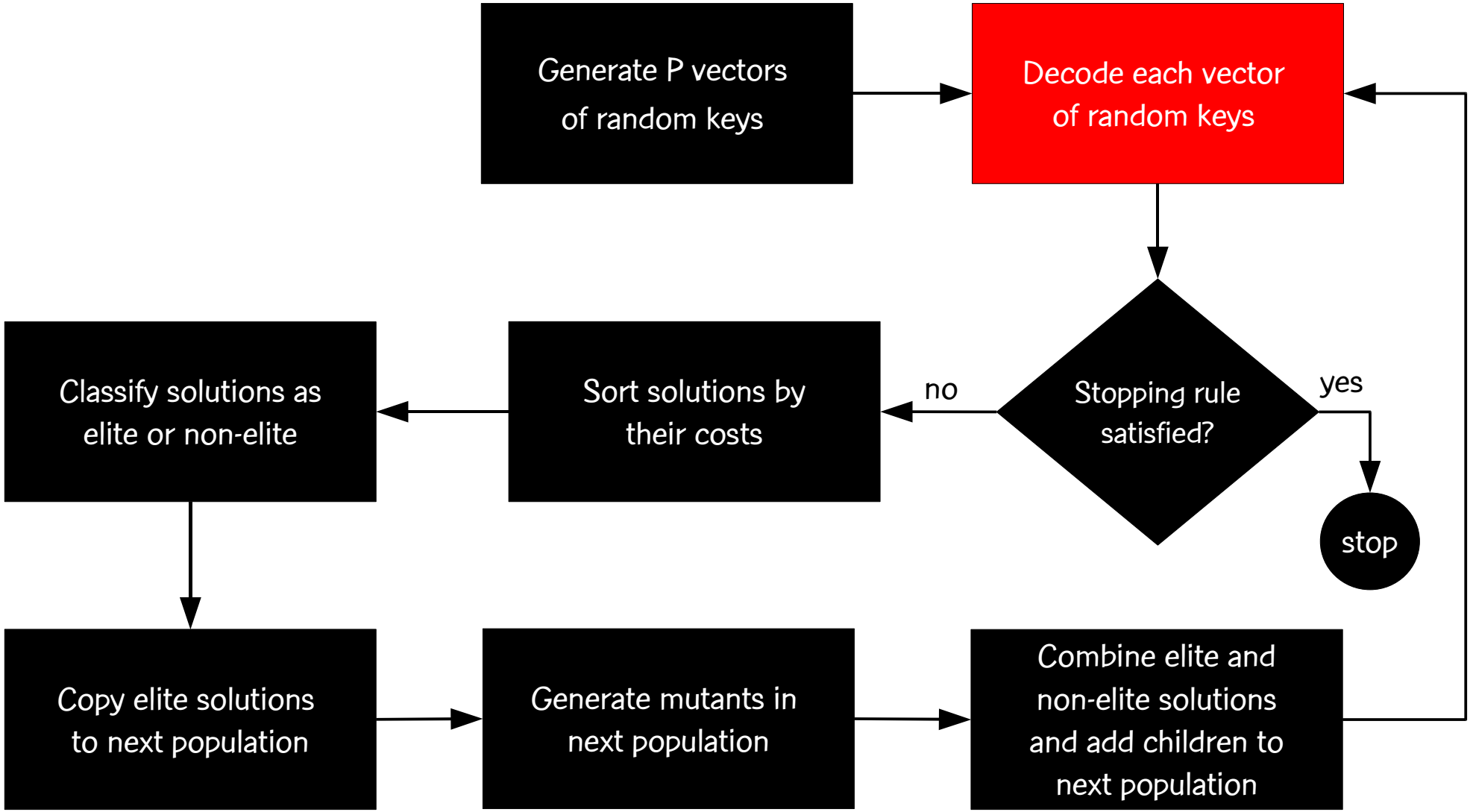
Framework for random-key genetic algorithms



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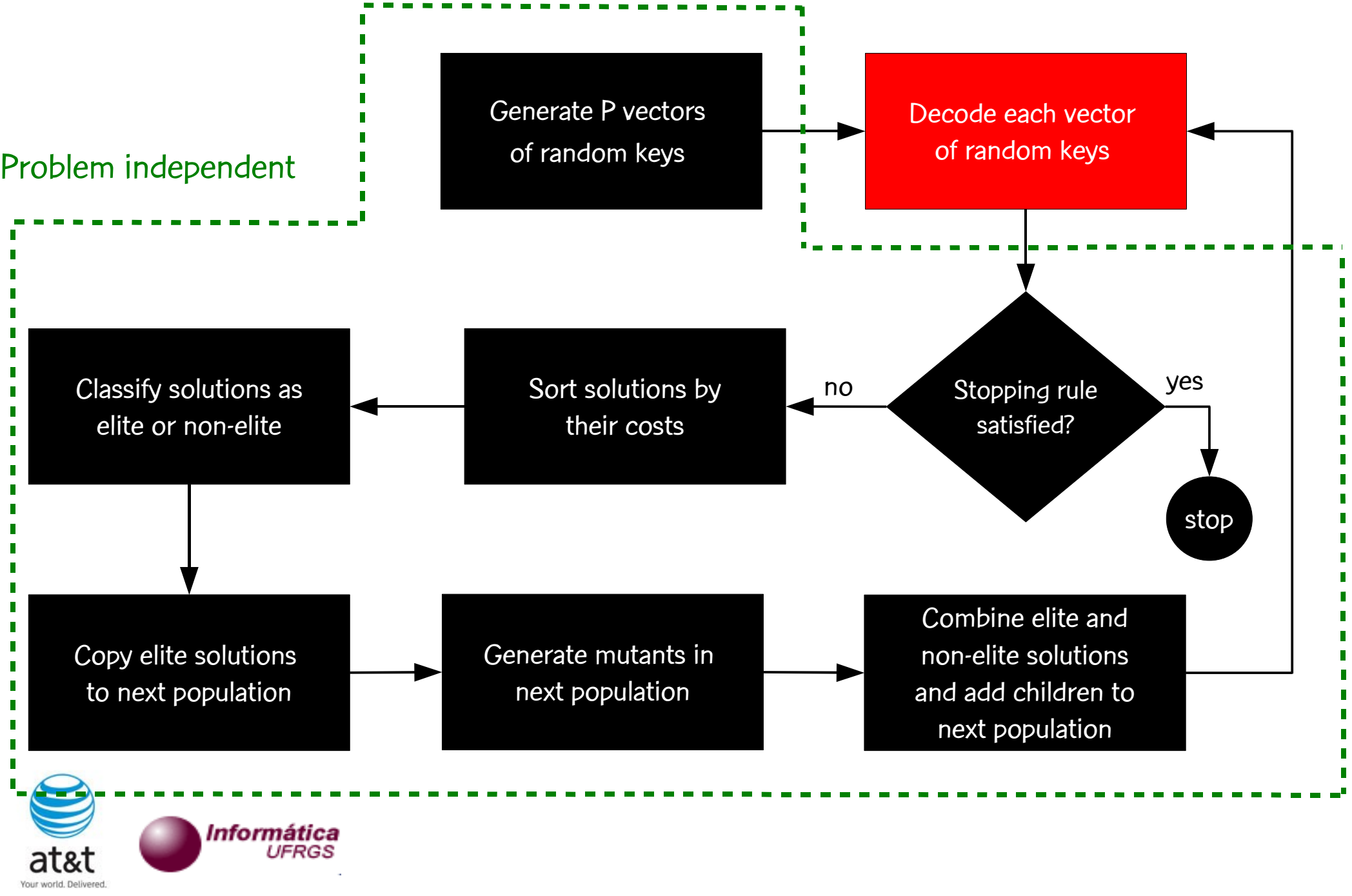

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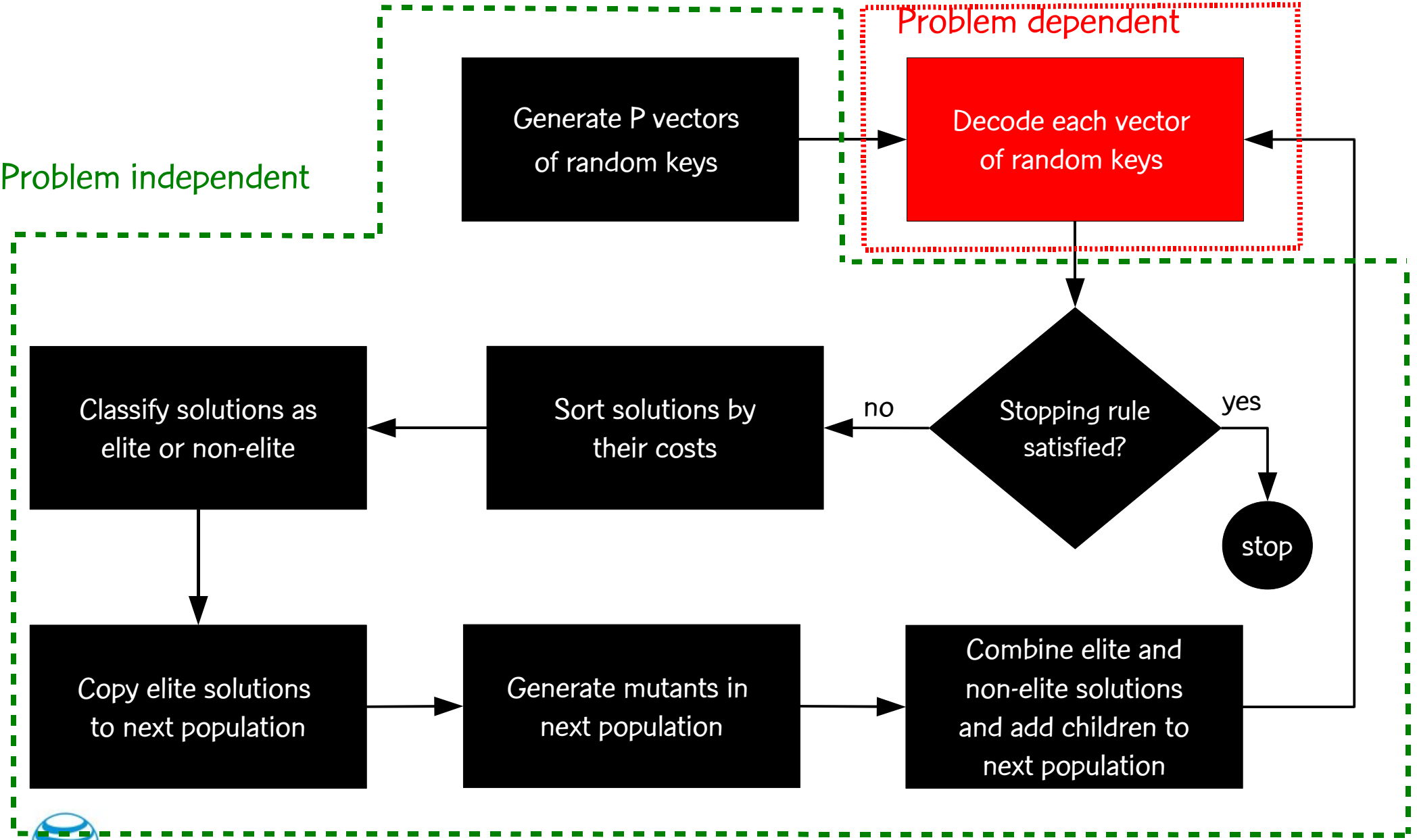

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OSPF routing in IP networks

Routing in IP networks

- Protocol: In OSPF, traffic is routed on shortest weight paths from origination router to destination router.
- Splitting: If more than one link out of a router is on a shortest weight path, traffic is evenly distributed on those links.
- Weight setting problem: Determine OSPF weights such that if traffic is routed according to OSPF protocol, network congestion is minimized.

Minimization of congestion

- Consider the directed capacitated network $G = (N, A, c)$, where N are routers, A are links, and c_a is the capacity of link $a \in A$.
- We use the measure of Fortz & Thorup (2000) to compute congestion:

$$\Phi = \Phi_1(l_1) + \Phi_2(l_2) + \ldots + \Phi_{|A|}(l_{|A|})$$

where l_a is the load on link $a \in A$,

$\Phi_a(l_a)$ is piecewise linear and convex,

$\Phi_a(0) = 0$, for all $a \in A$.

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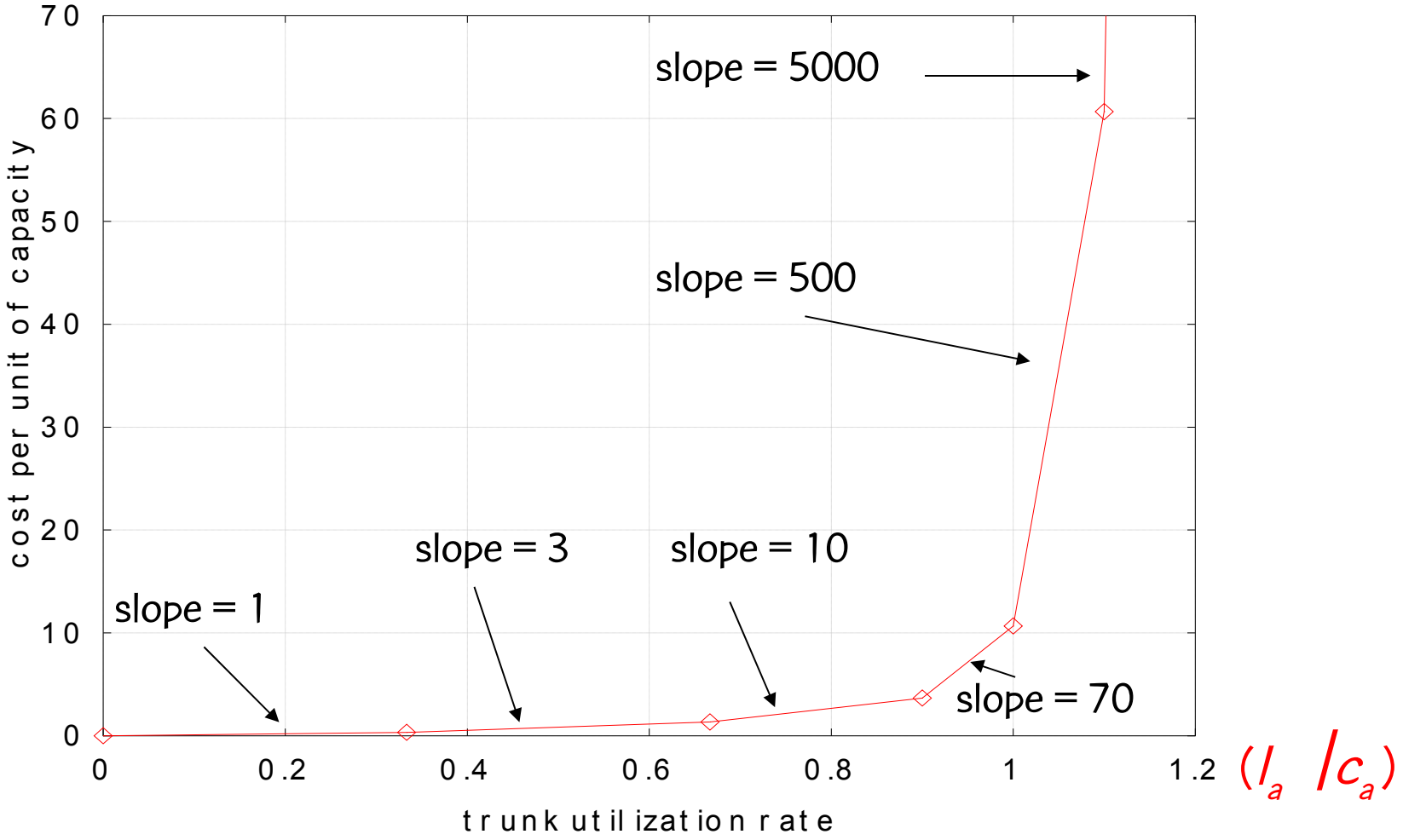
Piecewise linear and convex $\Phi_a(I_a)$ link congestion measure



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Genetic algorithm for OSPF routing in IP networks

Ericsson, R., & Pardalos (J. Comb. Opt., 2002)

- Chromosome:
  - A vector X of N random keys, where N is the number of links. The i-th random key corresponds to the i-th link weight.
- Decoder:
  - For i = 1,N: set w(i) = ceil (X(i) $\times$ w_{max})
  - Compute shortest paths and route traffic according to OSPF.
  - Compute load on each link, compute link congestion, add up all link congestions to compute network congestion.

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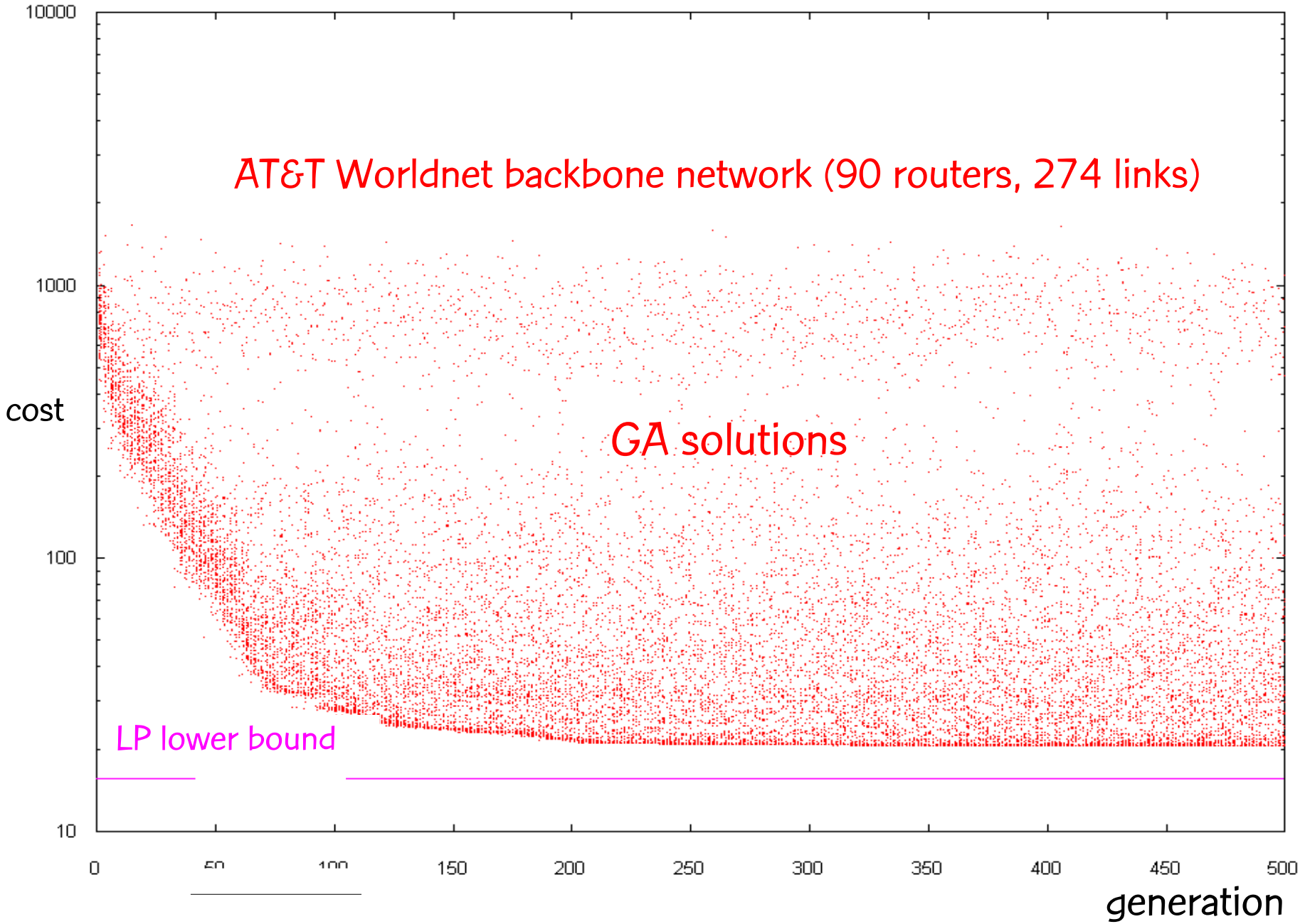

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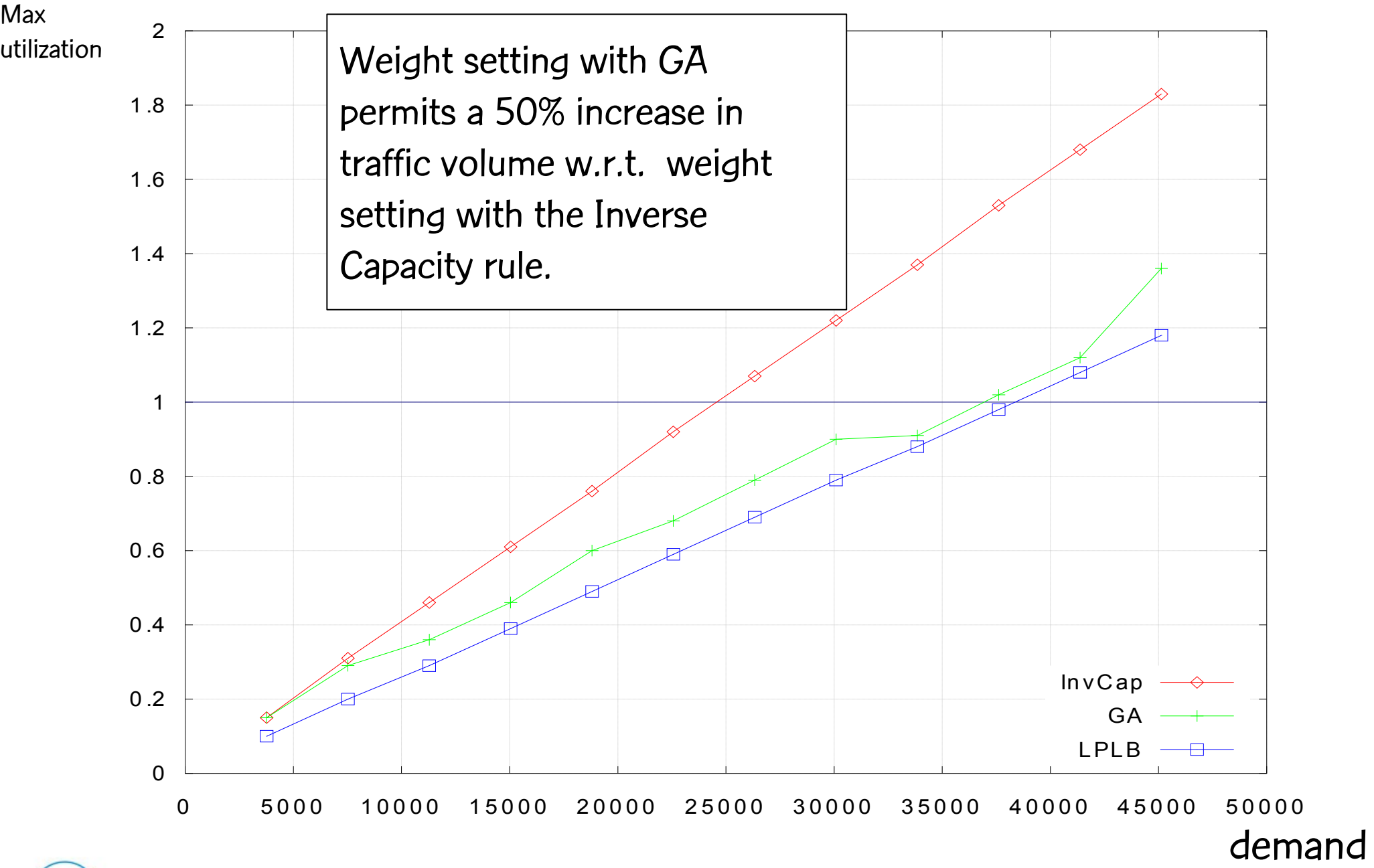
AT&T Worldnet backbone network (90 routers, 274 links)



Weight setting with GA permits a 50% increase in traffic volume w.r.t. weight setting with the Inverse Capacity rule.

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Memetic algorithms for OSPF routing in IP networks

Buriol, R., Ribeiro, and Thorup (Networks, 2005)

- Chromosome:
  - A vector X of N random keys, where N is the number of links. The i-th random key corresponds to the i-th link weight.
- Decoder:
  - For i = 1,N: set $w(i) = ceil\ (\ X(i) \times w_{max}\ )$
  - Compute shortest paths and route traffic according to OSPF.
  - Compute load on each link, compute link congestion, add up all link congestions to compute network congestion.
  - Apply fast local search to improve weights.

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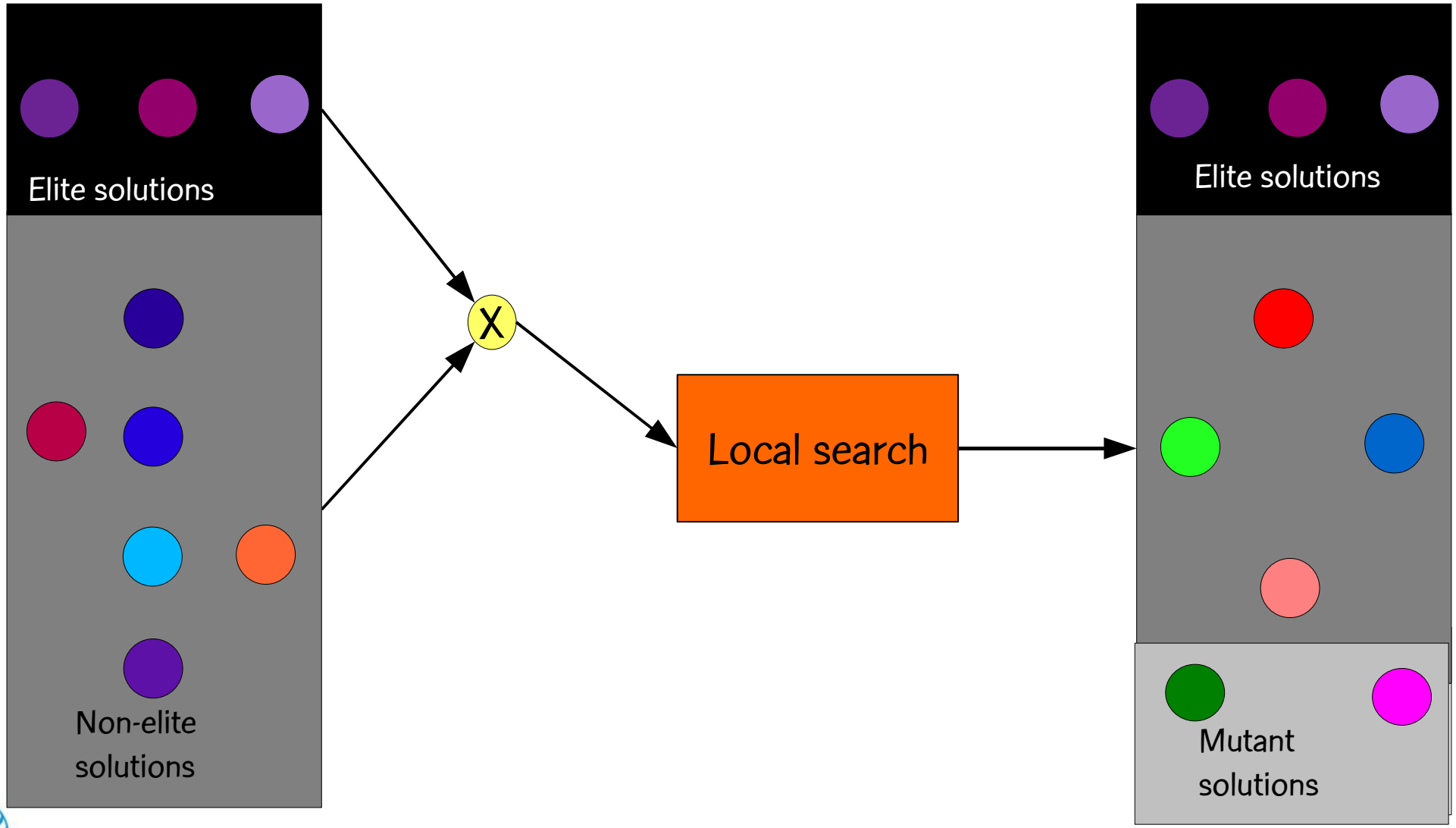
Memetic algorithm: Optimized crossover = crossover + local search



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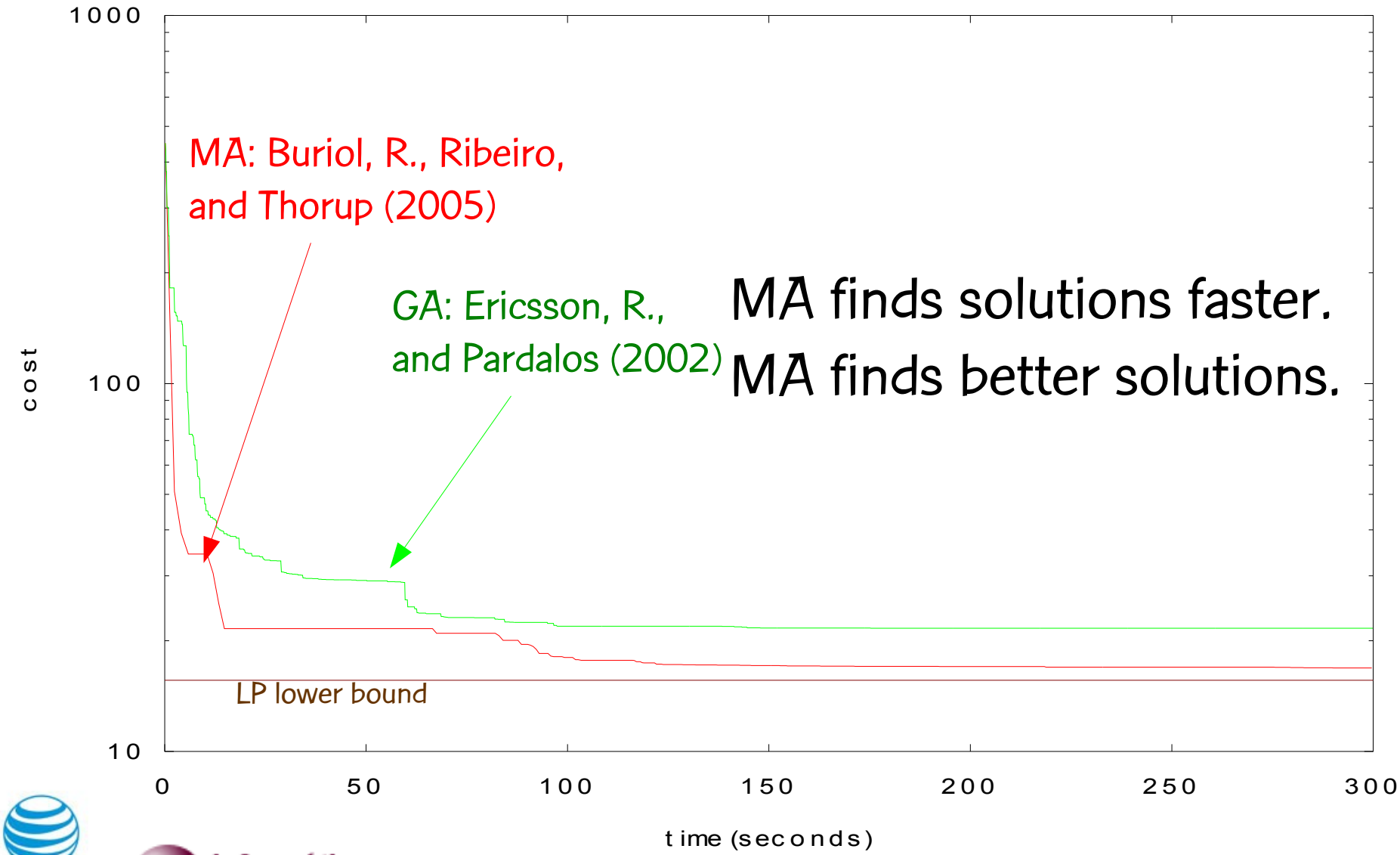
Fast local search

- Let A^* be the set of five arcs $a \in A$ having largest Φ_a values.
- Scan arcs $a \in A^*$ from largest to smallest Φ_a:
  - Increase arc weight, one unit at a time, in the range $[w_a , w_a + \lceil (w_{max} - w_a)/4 \rceil]$
  - If total cost Φ is reduced, restart local search.

Effect of decoder with fast local search



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DEFT routing in IP networks

DEFT routing

- Proposed by Dahai Xu, Mung Chiang, and Jennifer Rexford, DEFT: Distributed Exponentially-weighted Flow spliTting, INFOCOM 2007

- Flow is routed on all links that lead to the destination. An *exponential penalty* is used to assign less flow to links that are on longer paths.

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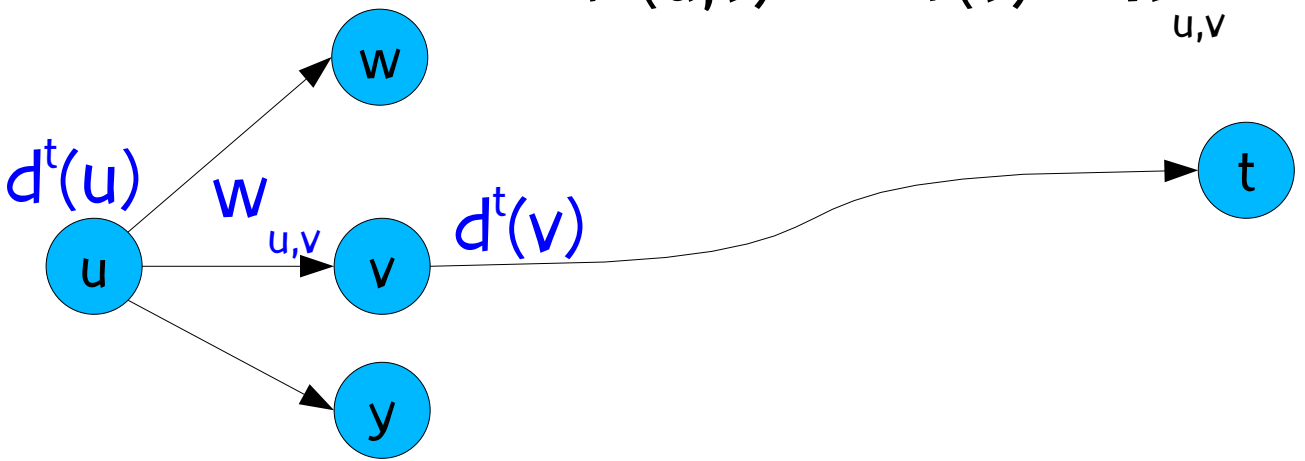
DEFT routing

- Consider each forward link (u,v) outgoing a given node u.
- Denote by $w_{u,v}$ the real-valued weight of link (u,v) and $d^t(u)$ as the distance of node u from target t.
- The gap $h^t(u,v)$ between u and v is calculated as:

$$h^t(u,v) = d^t(v) + w_{u,v} - d^t(u)$$



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DEFT routing

- Exponential function:

if $d^t(u) > d^t(v)$ then $\Gamma[h^t(u,v)] = \exp[-h^t(u,v)]$,

otherwise $\Gamma[h^t(u,v)] = 0$

- The total flow $f^t(u)$ out of node u and destined to node t is split according to:

$$f^t(u,v) = f^t(u)\ \Gamma[h^t(u,v)] / \Sigma_{(u,j)\in E} \Gamma[h^t(u,j)]$$

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OSPF vs. DEFT

- Xu, Chiang, and Rexford (2007) proposed a two-stage iterative method, where each stage solves a non-linear programming problem making use of a primal-dual interior point algorithm. There method produces real-valued weights.
- In experiments described in their paper, their approach finds near-optimal solutions (i.e. near the linear programming lower bounds).

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Memetic algorithm for DEFT weight setting

- Similar to memetic algorithm for OSPF weight setting
  - GA with random keys
  - fast local search
- Decoder is the only difference
  - weights are set as in MA for OSPF
  - shortest paths and gaps are determined, penalties defined, and flows computed
  - fast local search is adapted for DEFT

Implementation issues for OSPF and DEFT: dynamic updates

- The local search makes lots of unit link weight changes
- Each changed network has to be re-evaluated
  - calculate new shortest paths
  - calculate new network flow
- In both algorithms distances and flows should be updated and not recomputed from scratch

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Implementation issues for OSPF and DEFT: dynamic distances

- Our MA for OSPF uses an improved version of the algorithm of Ramalingan and Reps
  - Buriol, Resende, Thorup: Speeding up dynamic shortest path algorithms, INFORMS J. Comput. 2008
- For DEFT, the algorithm is slightly faster because it only needs to update the distances and not the shortest path graphs.

Implementation issues for OSPF and DEFT: dynamic flows

- Given a change in a link flow, the flow in the graph can be updated in both protocols.
- Updates affect a larger portion of the graph in DEFT than in OSPF.
- For example: a link weight increase on a non-shortest path does not require updating in OSPF but does so in DEFT

Preliminary experimental results

- We tested 6 instances with 7 different demand matrices
- Results are averages over 3 random seeds
- Stopping criterion: 2000 generations or 500 generations without improvement
- Each run takes about 1 hour on a SGI Altrix (1.6Ghz Itanium 2 processor)

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Preliminary experimental results

- DEFT reduces network congestion when compared to OSPF
  - NLP as well as MA
  - Running times are about the same for both OSPF & DEFT MA implementations
- Optimality gap is small for both (MA, NLP)
- Some instances permit large improvements (hier50b, hier100a), others almost none (waxman)

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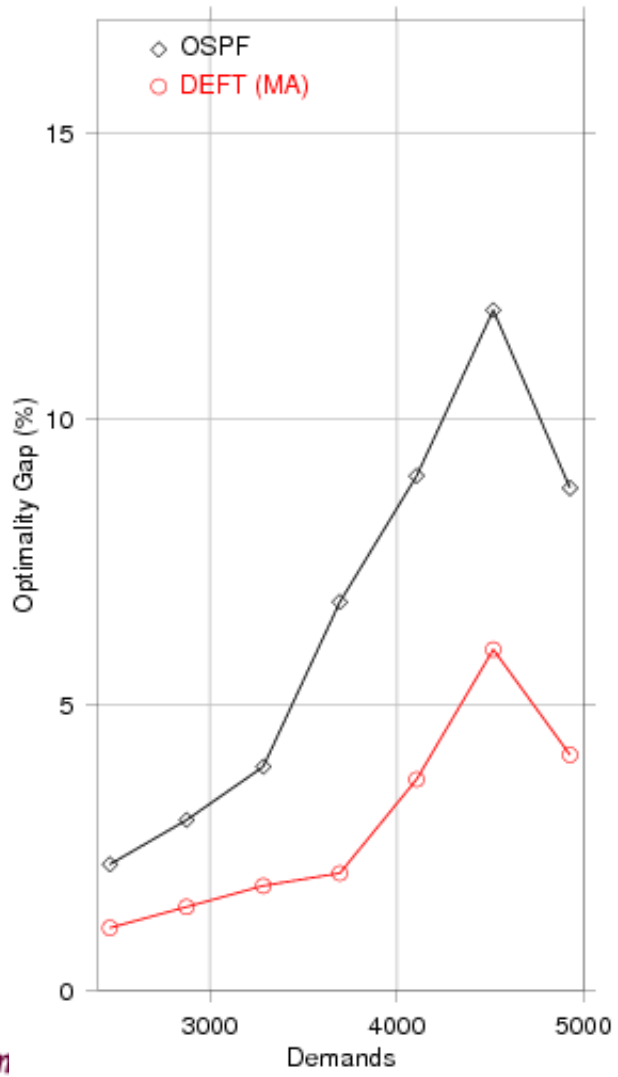
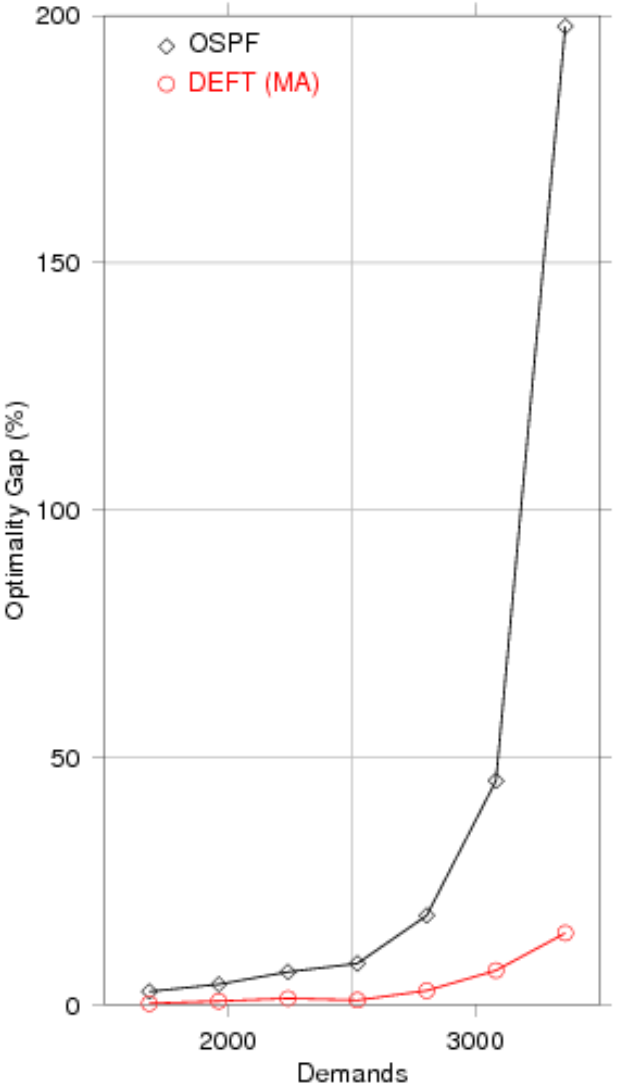
OSPF vs. DEFT

Two level hierarchy with 50 nodes

hier50a
148 links



Hier50b
212 links



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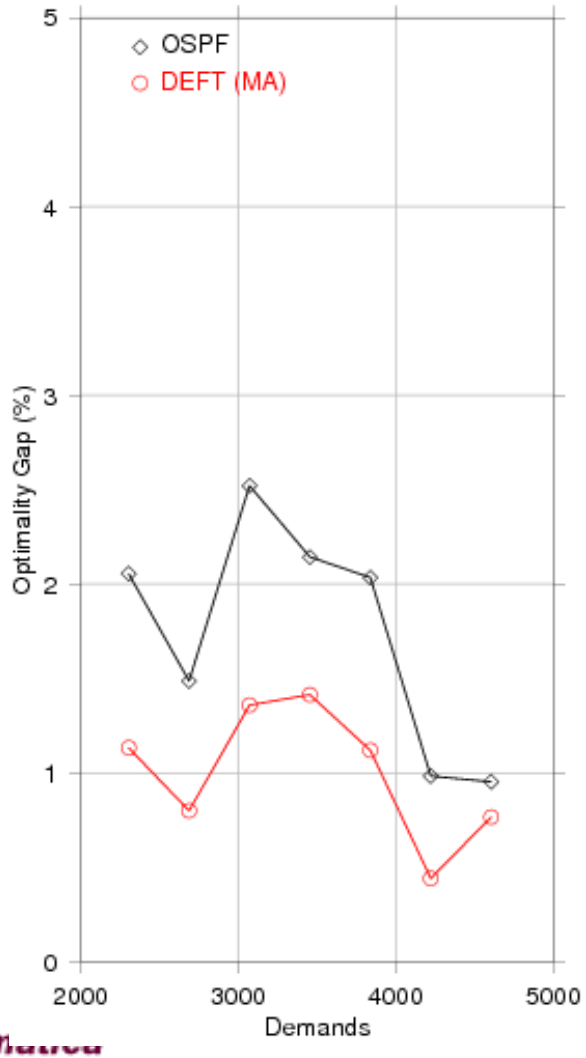
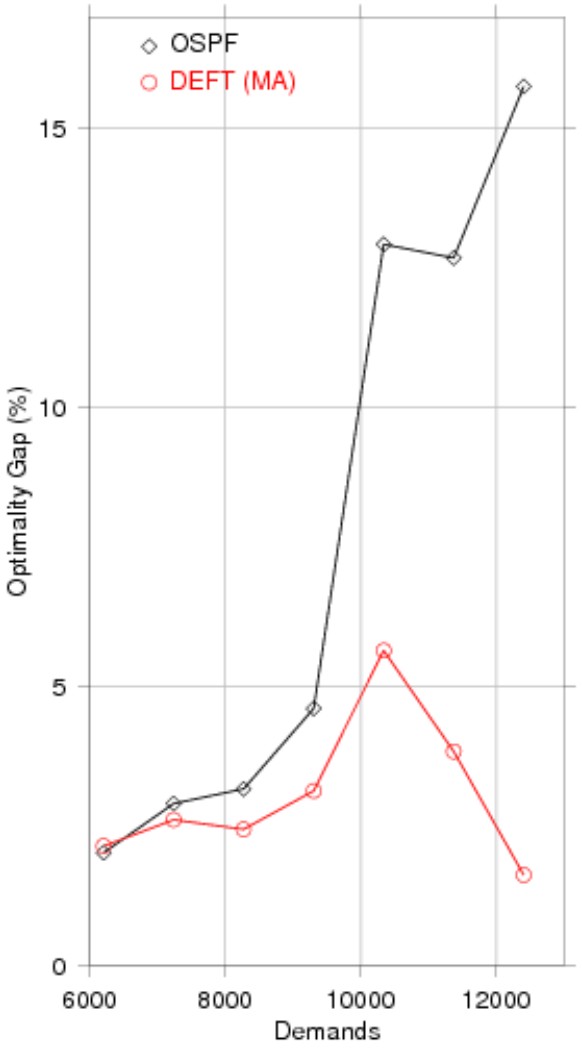
OSPF vs. DEFT

Two level hierarchy with 100 nodes

Hier100
280 links



Hier100a
360 links



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OSPF vs. DEFT

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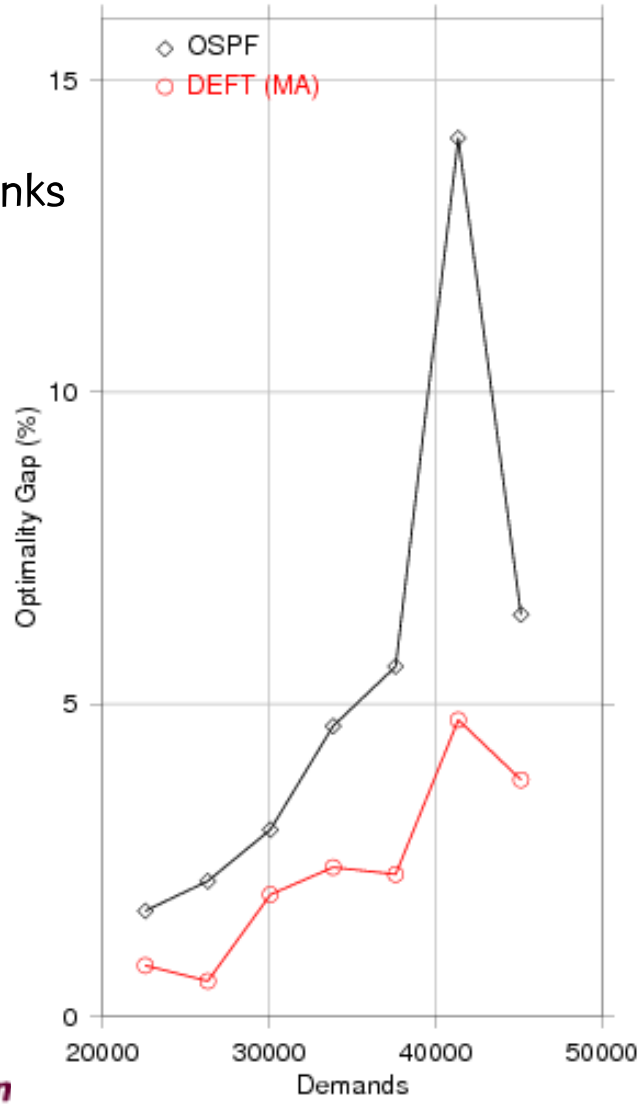
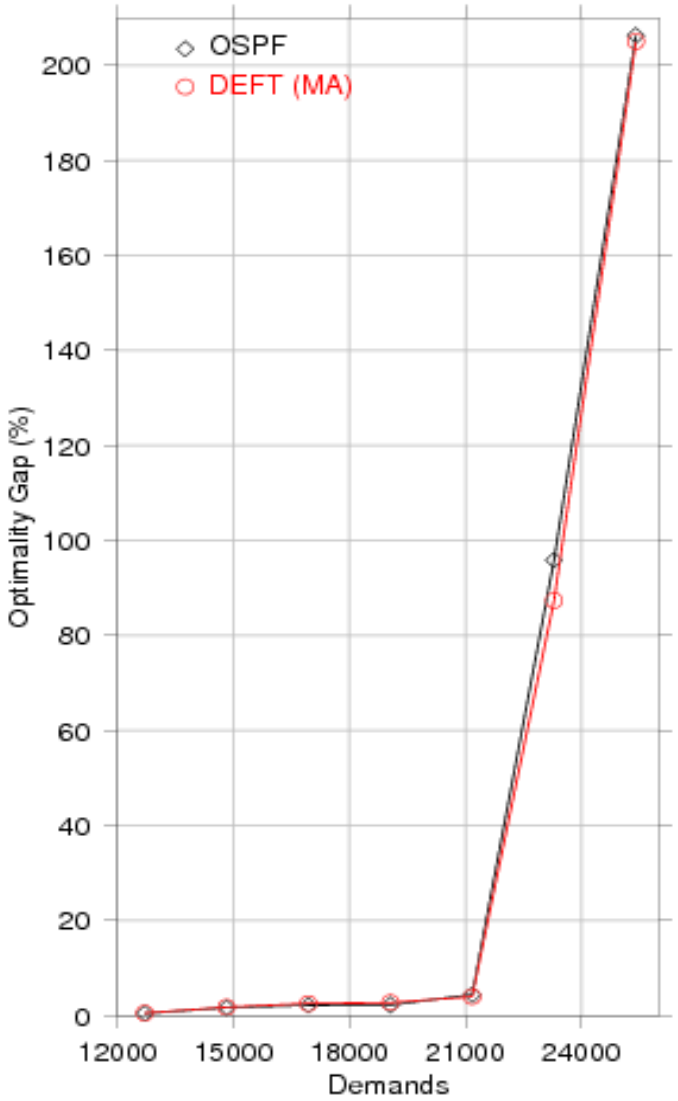
90 nodes, 274 links



Waxman
50 nodes,
169 links



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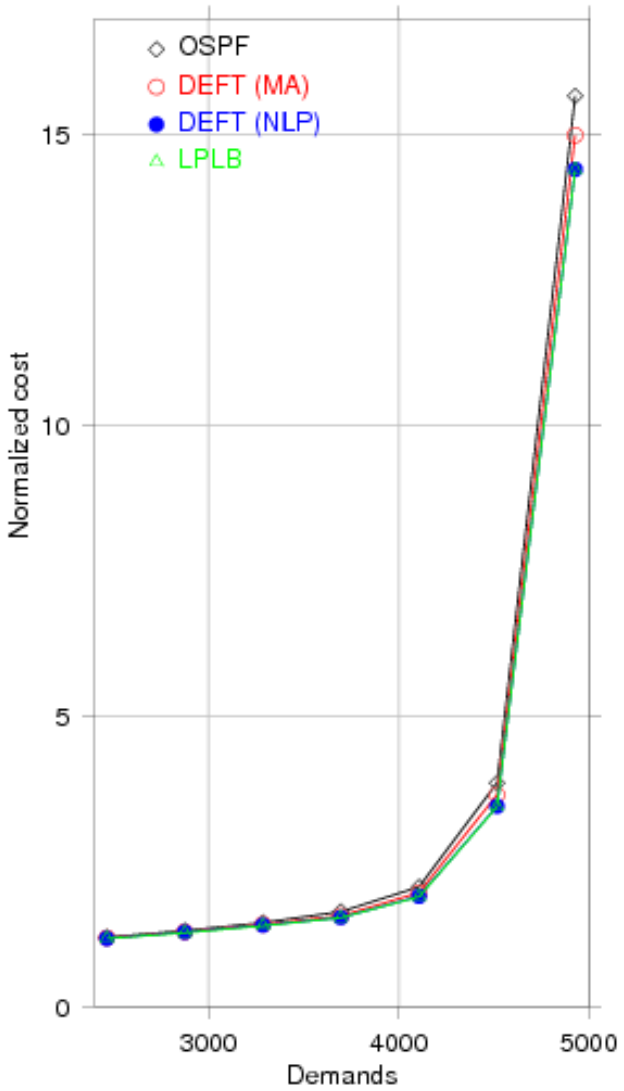
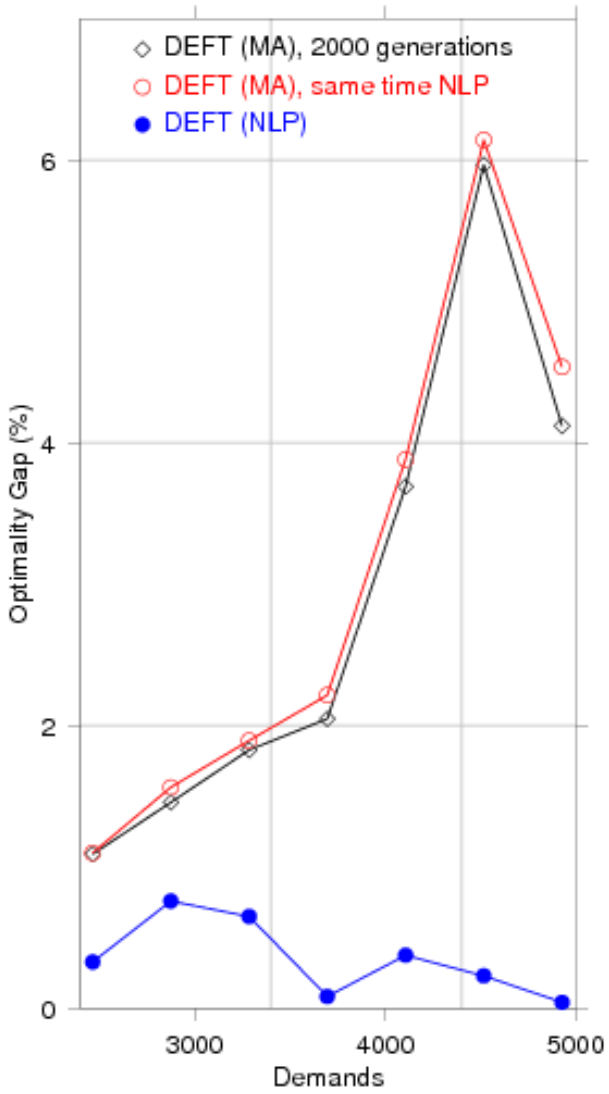
DEFT
Comparison of NLP and MA approaches

Hier50a
50 nodes, 148 links





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Conclusion

- DEFT routing archives better results than OSPF routing
- Further experiments are necessary to conclude if it is worthwhile to switch to DEFT
- Memetic algorithm is a flexible tool
  - permits survivable network design (e.g. multiplicities)
  - permits parallelization (e.g. island methods)

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Future work

- Compare the network delay of DEFT and OSPF
- Explore DEFT with real weights
  - Better local search using line search algorithms
  - Continuous optimization methods, e.g. continuous GRASP
- Parallelize the memetic algorithm
- Implement survivable network design using DEFT

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Thanks to...

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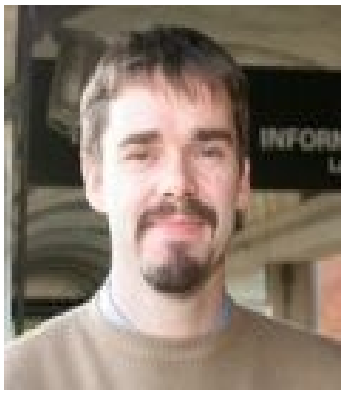

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The End

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