## Biased random-key

## genetic algorithms

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## Summary

- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
- Encoding / Decoding
- Initial population
- Evolutionary mechanisms
- Problem independent / problem dependent components
- Multi-start strategy
- Specifying a BRKGA
- Application programming interface (API) for BRKGA
- Applications
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing
- BRKGA for unequal area facility layout
- Concluding remarks


## Reference



## J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:
http://mauricio.resende.info/doc/srkga.pdf

## Encoding solutions with random keys

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- A random key is a real random number in the continuous interval $[0,1$ ).


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- A random key is a real random number in the continuous interval $[0,1$ ).
- A vector $X$ of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.


## Encoding with random keys: Sequencing

Encoding

$$
\begin{array}{rrrrrr}
{[ } & 1, & 2, & 3, & 4, & 5] \\
X & {[0.099,} & 0.216, & 0.802, & 0.368,0.658]
\end{array}
$$

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Decode by sorting vector of random keys

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## Encoding with random keys: Sequencing

Therefore, the vector of random keys:
$X=[0.099,0.216,0.802,0.368,0.658]$ encodes the sequence: 1-2-4-5-3

## Encoding with random keys: Subset selection (select 3 of 5 elements)

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## Encoding with random keys: Assigning integer

 weights $\in[0,10]$ to a subset of 3 of 5 elementsEncoding

$$
\begin{aligned}
& \text { [ 1, 2, 3, 4, } \left.5 \left\lvert\, \begin{array}{lllllll}
5
\end{array}\right.\right] \\
& X=[0.099,0.216,0.802,0.368,0.658 \mid 0.4634,0.5611,0.2752,0.4874,0.0348]
\end{aligned}
$$

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$$
\begin{gathered}
{\left[\begin{array}{cccccccccr}
{[ } & 1, & 2, & 3, & 4, & 5 \mid & 1, & 2, & 3, & 4, \\
\mathrm{x}=\left[\begin{array}{ll}
0.099
\end{array}\right] \\
0.216,0.802, & 0.368, & 0.658 & 0.4634, & 0.5611,0.2752, & 0.4874,0.0348
\end{array}\right]}
\end{gathered}
$$

Decode by sorting the first 5 keys and assign as the weight the value $W_{i}=$ floor $\left[10 X_{5+i}\right]+1$ to the 3 elements with smallest keys $X_{i}$ for $i=1, \ldots, 5$.

## Encoding with random keys: Assigning integer

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$X=[0.099,0.216,0.802,0.368,0.658 \mid 0.4634,0.5611,0.2752,0.4874,0.0348]$ encodes the weight vector $W=(5,6,-, 5,-)$

## Genetic algorithms and random keys

## GAs and random keys

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- Individuals are strings of
real-valued numbers
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$$
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& s=\left(\begin{array}{lllll}
0.25, & 0.19, & 0.67, & 0.05, & 0.89
\end{array}\right) \\
& s(1) \\
& s(2) s(3) \\
& \hline
\end{aligned}(4) \quad s(5)
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\end{aligned}
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- Sorting random keys results

$$
\begin{aligned}
\mathrm{S}^{\prime}=\left(\begin{array}{rlll}
0.05, & 0.19, & 0.25, & 0.67, \\
\mathrm{~s}(4) & \mathrm{s}(2) & \mathrm{s}(1) & \mathrm{s}(3) \\
& \mathrm{s}(5)
\end{array}\right.
\end{aligned}
$$

Sequence: 4-2-1-3-5

## GAs and random keys

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parametrized uniform
CrOSSOVEY (Spears \& DeJong, 1990)

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& a=(0.25,0.19,0.67,0.05,0.89) \\
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- Mating is done using
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$$ coin to choose which parent passes the allele (key, or value of gene) to the child.

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## GAs and random keys

Initial population is made up of $P$ random-key vectors, each with N keys, each having a value generated uniformly at random in the interval $[0,1)$.


## GAs and random keys

Population K
At the K-th generation, compute the cost of each solution ...


## GAs and random keys

 At the K-th generation, compute the cost of each solution and partition the solutions into two sets:

## GAs and random keys

 At the K-th generation, compute the cost of each solution and partition thePopulation K


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Population K


## GAs and random keys

Population K
Population K+1

## Evolutionary dynamics



## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$

Population K


BRKGA

Population K+1

amazon

## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$

Population K


Population K+1


## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$
- While $\mathrm{K}+1$-th population $<\mathrm{P}$
- RANDOM-KEY GA: Use any two solutions in population K to produce child in population $\mathrm{K}+1$. Mates are chosen at random.

Population K


## Biased random key genetic algorithm

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- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.


## How RKGA \& BRKGA differ

## RKGA

BRKGA
both parents chosen at random from entire population

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RKGA
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## BRKGA

both parents chosen at random but one parent chosen from population of elite solutions
either parent can be parent $A$ in parametrized uniform crossover
best fit parent is parent $A$ in parametrized uniform crossover

## Biased random key GA

BRKGA: Probability
child inherits key of elite
Population K parent>0.5 Population K+1

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$
- While $\mathrm{K}+1$-th population $<\mathrm{P}$
- RANDOM-KEY GA: Use any two solutions in population K to produce child in population $\mathrm{K}+1$. Mates are chosen at random.
- BIASED RANDOM-KEY GA: Mate elite solution with other solution of population K to produce child in population $\mathrm{K}+1$. Mates are chosen at random.



## Paper comparing BRKGA and Bean's

## Method



Gonçalves, R., and Toso, "An experimental comparison of biased and unbiased random-key genetic algorithms",
Pesquisa Operacional, vol. 34, pp. 143-164, 2014.







## Observations

- Random method: keys are randomly generated so solutions are always vectors of random keys


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- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent $>0.5$ Not so in the RKGA of Bean.
- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs ... help escape local optima)


## Framework for biased random-key genetic algorithms



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Generate $P$ vectors of random keys

Decode each vector of random keys

Classify solutions as elite or non-elite






## Framework for biased random-key genetic algorithms

Classify solutions as elite or non-elite


## Decoding of random key vectors can be done in parallel



## Is a BRKGA any different from applying

 the decoder to random keys?- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to $\mathrm{P}-1$
- Each iteration, best solution is maintained in elite set and $\mathrm{P}-1$ random key vectors are generated as mutants ... no mating is done since population already has P individuals

Network monitor location problem (opt $=23$ ) solution
n18日-i2-n180-b18日: GA and randon multi-start iterates


## BRKGA in multi-start strategy





In most of the independent runs, the algorithm finds the target solution in relatively few iterations:


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In most of the independent runs, the algorithm finds the target solution in relatively few iterations: $50 \%$ of the runs take fewer than 192 iterations


In most of the independent runs, the algorithm finds the target solution in relatively few iterations: $75 \%$ of the runs take fewer than 345 iterations


However, some runs take much longer: $10 \%$ of the runs take over 1000 iterations


However, some runs take much longer: 5\% of the runs take over 2000 iterations


However, some runs take much longer: $2 \%$ of the runs take over 9715 iterations


However, some runs take much longer: the longest run took 11607 iterations


Probability that algorithm will take over 345 iterations: $25 \%=1 / 4$


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By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: $25 \%=$ 1/4

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 X probability of taking over 690 iterations given it took over $345=$ $1 / 4 \times 1 / 4=1 / 4^{2}$




Probability that algorithm will still be running after K periods of 345 iterations: $1 / 4^{\mathrm{K}}$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1 / 4^{5} \cong$ 0.0977\%

This is much less than the $5 \%$ probability that the algorithm without restart will take over 2000 iterations.

## Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S=\left\{\tau_{1}, T_{2}, T_{3}, \ldots\right\}$ which define epochs $\tau_{1}, \quad \tau_{1}+\tau_{2}, \tau_{1}+\tau_{2}+\tau_{3}, \ldots$ when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $\tau_{1}=\tau_{2}=\tau_{3}=\cdots=\tau^{*}$, where $\tau^{*}$ is a constant.


## Restart strategy for BRKGA

- Recall the restart strategy of Luby et al. where equal time intervals $T_{1}=T_{2}=T_{3}=\cdots=T^{*}$ pass between restarts.
- Strategy requires $\mathrm{T}^{*}$ as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
- choosing $\mathrm{T}^{*}$ too small: restart variant may take long to converge
- choosing т $^{*}$ too big: restart variant may become like norestart variant


## Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy restart(K)


## Example of restart strategy for BRKGA: Telecom application



## Specifying a BRKGA

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- Parameters


## Specifying a biased random-key GA

## Parameters:

- Size of population
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion


## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of $N$, say N or 2 N
- Size of elite partition
- Size of mutant set
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## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set
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## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
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## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set: $5-15 \%$ of population
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- Restart strategy parameter
- Stopping criterion


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- Size of population: a function of N , say N or 2 N
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- Restart strategy parameter: a function of N , say 2 N or 10 N
- Stopping criterion


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- Child inheritance probability: $>0.5$, say 0.7
- Restart strategy parameter: a function of N, say 2 N or 10 N
- Stopping criterion: e.g. time, \# generations, solution quality, \# generations without improvement


## brkgaAPI: A C++ API for BRKGA

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.


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- population management
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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
- population management
- evolutionary dynamics
- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.


## brkgaAPI: A C++ API for BRKGA



Paper: Rodrigo F. Toso and M.G.C.R., "A C++ Application Programming Interface for Biased Random-Key Genetic Algorithms,"

Optimization Methods \& Software, vol. 30, pp. 81-93, 2015.

Software: http://mauricio.resende.info/src/brkgaAPI

## An example BRKGA:

 Packing weighted rectangles
## Reference



# J.F. Gonçalves and R., "A parallel multipopulation genetic algorithm for a constrained two-dimensional orthogonal <br> packing problem," Journal of Combinatorial Optimization, vol. 22, pp. 180-201, 2011. 

Tech report:
http://mauricio.resende.info/doc/pack2d.pdf

## Constrained orthogonal packing

- Given a large planar stock rectangle (W, H) of width $W$ and height $H$;


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## Constrained orthogonal packing

- r[i] rectangles of type $\mathrm{i}=1, \ldots, \mathrm{~N}$ are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;



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0 \leq P[i] \leq r[i] \leq Q[i]
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\text { Suppose } 5 \leq r[1] \leq 12
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## Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

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v[1] r[1]+v[2] r[2]+\cdots+v[N] r[N]
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$$



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Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

$$
v[1] r[1]+v[2] r[2]+\cdots+v[N] r[N]
$$



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$$



## Applications

Problem arises in several production processes, e.g.

- Textile
- Glass
- Wood
- Paper
where rectangular figures are cut from large rectangular sheets of materials.

2D-HopperTP12-1-49-3576.txt: 3576


Hopper \& Turton, 2001 Instance 4-1 $60 \times 60$ Value: 3576

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

2D-HopperTP12-1-49-3585.txt: 3585


BRKGA

2D-HopperTP12-1-49-3586.txt: 3586


Hopper \& Turton, 2001 Instance 4-2 $60 \times 60$ Value: 3586

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

2D-HopperTP12-1-49-3591.txt: 3591


Hopper \& Turton, 2001 Instance 4-2 $60 \times 60$ Value: 3591

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

2D-HopperTP12-1-49-3591.txt: 3591


Hopper \& Turton, 2001 Instance 4-2 $60 \times 60$ Value: 3591
New best known solution! Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

## BRKGA for

## constrained 2-dim

## orthogonal packing

## Encoding

- Solutions are encoded as vectors $X$ of

$$
2 N^{\prime}=2\{\mathrm{Q}[1]+\mathrm{Q}[2]+\cdots+\mathrm{Q}[\mathrm{~N}]\}
$$

random keys, where $\mathrm{Q}[\mathrm{i}]$ is the maximum number of rectangles of type $i$ (for $i=1, \ldots, N$ ) that can be packed.

- $X=\left(X[1], \ldots, X\left[N^{\prime}\right], \quad X\left[N^{\prime}+1\right], \ldots, X\left[2 N^{\prime}\right]\right)$


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Rectangle type
packing sequence
(RTPS)

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Rectangle type
packing sequence
(RTPS)
$\left.X\left[N^{\prime}+1\right], \ldots, X\left[2 N^{\prime}\right]\right)$

Vector of placement procedures (VPP)

## Decoding

- Simple heuristic to pack rectangles:
- Make Q[i] copies of rectangle i , for $\mathrm{i}=1, \ldots, \mathrm{~N}$.
- Order the $\mathrm{N}^{\prime}=\mathrm{Q}[1]+\mathrm{Q}[2]+\cdots+\mathrm{Q}[\mathrm{N}]$ rectangles in some way.
- Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or leftbottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order.


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- Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or leftbottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order. Use the last N ' keys of X to determine which heuristic to use. If $k\left[N^{\prime}+i\right]>0.5$ use LB, else use BL.


## Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
- Let (x[i], y[i]) be the coordinates of the bottom left corner of the $i$-th ERS.


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> i-th
> ERS

## Decoding

- If BL is used, ERSs are ordered such that $\operatorname{ERS}[i]<\operatorname{ERS}[j]$ if $y[i]<y[j]$ or $y[i]=y[j]$ and $x[i]<x[j]$.

$\operatorname{ERS}[i]<\operatorname{ERS}[j]$



# BL can run into problems even on small instances (Liu \& Teng, 1999). 

Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.


> We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



## Decoding

- If LB is used, ERSs are ordered such that $E R S[i]<E R S[j]$ if $x[i]<x[j]$ or $x[i]=x[j]$ and $y[i]<y[j]$.





## ERS:









## 4 does not fit in ERS[1].



4 does fit in $\mathrm{ERS}[2]$.



Optimal solution!

## Experimental results

## Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:


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## Design

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- PH: population-based heuristic of Beasley (2004)
- GA: genetic algorithm of Hadjiconsantinou \& Iori (2007)
- GRASP: greedy randomized adaptive search procedure of Alvarez-Valdes et al. (2005)
- TABU: tabu search of Alvarez-Valdes et al. (2007)


## Number of best solutions / total instances

| Problem | PH | GA | GRASP | TABU | BRKGA <br> BLLLBL-ANR |
| :--- | :--- | :--- | :--- | :--- | :--- |
| From <br> literature <br> (optimal) | $13 / 21$ | $\mathbf{2 1 / 2 1}$ | $18 / 21$ | $\mathbf{2 1 / 2 1}$ | $\mathbf{2 1 / 2 1}$ |
| Large <br> random | $0 / 21$ | $0 / 21$ | $5 / 21$ | $8 / 21$ | $\mathbf{2 0 / 2 1}$ |
| Zero-waste |  | $5 / 31$ | $17 / 31$ | $\mathbf{3 0 / 3 1}$ |  |
| Doubly <br> constrained | $11 / 21$ |  | $12 / 21$ | $17 / 21$ | $\mathbf{1 9 / 2 1}$ |

[^0]Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

| Problem | Min solution <br> time | (secs) solution | Avg solut solution <br> time |
| :--- | :--- | :--- | :--- | :--- |
| (secs) |  |  |  | | (ime (secs) |
| :--- |

2D-ngcutcon18-20678.txt: 20678



New BKS for a 100 $\times 100$ doubly constrained instance Fekete \& Schepers (1997) of value 22140.

Previous BKS was 22011 by tabu search of AlvarezValdes et al. (2007).

29 types
97 rectangles

## Some remarks



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We have extended this to 3D packing:
J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers \& Operations Research, vol. 29, pp. 179-190,

Tech report: http://mauricio.resende.info/doc/brkga-pack3d.pdf


BRKGA

## 3D bin packing

PRODUCTION ECONOMICS
J.F. Gonçalves and R., "A biased random-key genetic algorithm for 2D and 3D bin packing problems," International J. of Production Economics, vol. 15, pp. 500-510, 2013.
http://mauricio.resende.info/doc/brkga-binpacking.pdf

## 3D bin packing problem

Container (bin) of


Minimize number of containers


Boxes of different dimensions

## 3D bin packing constraints

- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)


## Six possible orientations for each box



## Difference process - DP

 (Lai \& Chan, 1997)

When box is placed in container ... use DP to keep track of maximal free spaces

## Encoding

Solutions are encoded as vectors of $3 n$ random keys, where n is the number of boxed to be packed.

$$
X=\left(x_{1}, x_{2}, \ldots, x_{n}, x_{n+1}, x_{n+2}, \ldots, x_{2 n}, x_{2 n+1}, x_{2 n+2}, \ldots, x_{3 n}\right)
$$

Box packing sequence
Placement heuristic
Box orientation

## Decoding

1) Sort first $n$ keys of $X$ to produce sequence boxes will be packed;
2) Use second $n$ keys of $X$ to determine which placement heuristic to use (back-bottom-left or back-left-bottom):

- if $x_{n+i}<1 / 2$ then use back-bottom-left to pack $i$-th box
- if $x_{n+i} \geq 1 / 2$ then use back-left-bottom to pack i-th box

3) Use third $n$ keys of $X$ to determine which of six orientations to use when packing box:

- $x_{2 n+i} \in[0,1 / 6)$ : orientation 1 ;
- $x_{2 n+i} \in[1 / 6,2 / 6)$ : orientation $2 ; \ldots$
- $x_{2 n+i} \in[5 / 6,1]$ : orientation 6 .


## Decoding

## For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation


## Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness: aNB
$-\mathrm{aNB}=\mathrm{NB}+($ LeastLoad $/$ BinVolume $)$, where
* LeastLoad is load on least loaded bin
x BinVolume is volume of bin: $\mathrm{H} \times \mathrm{W} \times \mathrm{L}$


## Experiment

- Parameters:
- population size: $p=30 n$
- size of elite partition: $\mathrm{p}_{\mathrm{e}}=.10 \mathrm{p}$
- number of of mutans: $p_{m}=.15 p$
- crossover probability: 0.7
- stopping criterion: 300 generations


## Experiment

- Instances:
- 320 instances of Martello et al. (2000)
- generator is available at http://www.diku.dk/~pisinger/codes/html
- 8 classes
- 40 instances per class
- 10 instances for each value of $n \in\{50,100,150$, 200)


## Experiment

- We compare BRKGA with:
- TS3, the tabu search of Lodi et al. (2002)
- GLS, the guided local search of Faroe et al. (2003)
- TS2PACK, the tabu search of Crainic et al. (2009)
- GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)


## Summary

| Class | Bin size | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | $100^{3}$ | 127.3 | 127.3 | 127.9 | 128.2 | 128.3 |
| 2 | $100^{3}$ | 125.5 | 125.8 | 126.8 |  |  |
| 3 | $100^{3}$ | 126.5 | 126.9 | 127.5 |  |  |
| 4 | $100^{3}$ | 294.0 | 294.0 | 294.0 | 293.9 | 294.2 |
| 5 | $100^{3}$ | 70.4 | 70.5 | 71.4 | 71.0 | 70.8 |
| 6 | $10^{3}$ | 95.0 | 95.4 | 96.1 | 95.8 | 96.0 |
| 7 | $40^{3}$ | 58.2 | 59.4 | 60.0 | 59.0 | 59.0 |
| 8 | $100^{3}$ | 80.9 | 82.0 | 82.6 | 81.9 | 81.9 |
| Sum(rows 1, 4-8): | 725.8 | 728.6 | 732.0 | 729.8 | 730.2 |  |
| Sum(rows 1-8): | 977.8 | 981.3 | 986.3 |  |  |  |

# The unequal area facility layout problem 


J.F. Gonçalves \& R., "A biased random-key genetic algorithm for the unequal area facility layout problem," European J. of Operational Research, vol. 246, pp. 86-107, 2015

## Unequal area facility layout



## Unequal area facility layout



## Unequal area facility layout



## Unequal area facility layout

We consider two types of problems

- In the constrained type, we are given the rectangular floor dimensions $\mathrm{W} \times \mathrm{H}$.
- In the unconstrained type, we assume the floor space can include all the facilities laid out horizontally or vertically at their maximum horizontal or vertical dimensions, i.e.

$$
(W, H)=\left(\sum_{i=1}^{N}\left(A_{i} \times R\right)^{1 / 2}, \sum_{i=1}^{N}\left(A_{i} \times R\right)^{1 / 2}\right)
$$

## Unequal area facility layout

Of all feasible layouts, find one that minimizes
$\sum_{i=1}^{N} \sum_{j=1}^{N} f_{i, j} \times c_{i, j} \times d_{i, j}$
where

- $f_{i, j}$ is the flow between facilities $i$ and $j\left(f_{i, i}=0\right)$
- $c_{i, j}$ is the cost per unit distance between i and j
- $d_{i, j}=\left|x_{i}-x_{j}\right|+\left|y_{i}-y_{j}\right|$ is the rectilinear distance between ( $\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{j}}$ ) and ( $\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{j}}$ )


## Unequal area facility layout

Of all feasible layouts, find one that minimizes
$\sum_{i=1}^{N} \sum_{j=1}^{N} f_{i, j} \times c_{i, j} \times d_{i, j}$

## quadratic assignment problem (QAP)

where

- $f_{i, j}$ is the flow between facilities $i$ and $j\left(f_{i, i}=0\right)$
- $\mathrm{c}_{\mathrm{i}, \mathrm{j}}$ is the cost per unit distance between i and j
- $d_{i, j}=\left|x_{i}-x_{j}\right|+\left|y_{i}-y_{j}\right|$ is the rectilinear distance between ( $\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}$ ) and ( $\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{j}}$ )


## Unequal area facility layout

Of all feasible layouts, find one that minimizes
$\sum_{i=1}^{N} \sum_{j=1}^{N} f_{i, j} \times c_{i, j} \times d_{i, j}$

Besides rectilinear ( R ) distance metric, we also deal with Euclidean ( E ), and Squared Euclidean (SE) in paper.
where

- $f_{i, j}$ is the flow between facilities i and $\mathrm{j}\left(\mathrm{f}_{\mathrm{i}, \mathrm{i}}=0\right)$
- $c_{i, j}$ is the cost per unit distance between i and j
- $d_{i, j}=\left|x_{i}-x_{j}\right|+\left|y_{i}-y_{j}\right|$ is the rectilinear distance between ( $\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}$ ) and ( $\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{j}}$ )

Dunker62 (3685136.02)


## Dunker62

New best known
solution: 3.68 E 6
Previous best known
solution: 3.81E6
TS-BST (McKendall Jr. \& Hajobyan, 2010)

```
L125B (943140.07)
```

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BRKGA

L125B

New best known solution: 9.43E5

Previous best known solution: 1.01E6 TS-BST (McKendall Jr. \& Hajobyan, 2010)

## BRKGA for the

## unequal area facility

 layout problem
## Encoding

Solutions are encoded with a vector of random keys of length $2 N+2$

$$
X=\left(X_{1}, \ldots, X_{N}, X_{N+1}, \ldots, X_{2 N}, X_{2 N+1}, X_{2 N+2}\right)
$$

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$$

Facility placement sequence

## Encoding

Solutions are encoded with a vector of random keys of length $2 N+2$

$$
X=\left(X_{1}, \ldots, X_{N}, X_{N+1}, \ldots, X_{2 N}, X_{2 N+1}, X_{2 N+2}\right)
$$

Facility aspect ratios

## Encoding

Solutions are encoded with a vector of random keys of length $2 N+2$

$$
X=\left(X_{1}, \ldots, X_{N}, X_{N+1}, \ldots, X_{2 N}, X_{2 N+1}, X_{2 N+2}\right)
$$

Facility placement sequence
Facility aspect ratios
( $x, y$ ) coordinates of the first facility to be placed

## Decoding

1. Use $X_{1}, \ldots, X_{N}$ to determine the sequence in which the facilities are placed on the floor space
2. Use $X_{N+1}, \ldots, X_{2 N}$ to determine the aspect ratio of each facility
3. Use $X_{2 N+1}, X_{2 N+2}$ to determine the ( $x, y$ ) coordinates of the first facility to be placed on the floor space
4. Use results of (1)-(3) with placement heuristic to place all the facilities on the floor space
5. Evaluate fitness of solution

## Decoder: Step 1

Use $X_{1}, \ldots, X_{N}$ to determine the sequence in which the facilities are placed on the floor space:

Simply sort the key values $X_{1}, \ldots, X_{N}$ to determine the indices of the permutation of the facilities.

## Decoder: Step 2

Use $X_{N+1}, \ldots, X_{2 N}$ to determine the aspect ratio of each facility:

Aspect ratio of facility i is
$\operatorname{FAR}_{i}=(1 / R)+X_{N+i} \times(R-(1 / R))$,
where $R$ is the given maximum facility aspect ratio.

## Decoder: Step 2

Use $X_{N+1}, \ldots, X_{2 N}$ to determine the aspect ratio of each facility:

Aspect ratio of facility i is
$F A R_{i}=(1 / R)+X_{N+i} \times(R-(1 / R))$,
where $R$ is the given maximum facility aspect ratio.

$$
\begin{aligned}
& w_{i}=\left(A_{i} \times F A R_{i}\right)^{1 / 2} \text { and } \\
& h_{i}=A_{i} / w_{i}
\end{aligned}
$$

## Decoder: Step 3

Use $X_{2 N+1}, X_{2 N+2}$ to determine the ( $x, y$ ) coordinates of the first facility to be placed on the floor space.

$$
\begin{aligned}
& x=\left(w_{i} / 2\right)+X_{2 N+1} \times\left(W-w_{i}\right) \\
& y=\left(h_{i} / 2\right)+X_{2 N+2} \times\left(H-h_{i}\right)
\end{aligned}
$$

## Decoder: Step 4 Makes use of empty maximal-spaces (EMS)

1

```
2
```


a) Facilities to be placed and the initial empty maximal-space (the floor space)

## Floor space

## 1



c) Empty maximal-spaces after placing facility 2.

## Decoder: Step 4 when placing a facility we only consider

 EMSs where the facility fits. This way we avoid overlapping.
## Floor space


a) Facilities to be placed and the initial empty maximal-space (the floor space)
b) Empty maximal-spaces after placing facility 1 .

## Decoder: Step 4 EmSs are generated and kept track of with the Difference Process (DP) of Lai and Chan (1997).

## Floor space


a) Facilities to be placed and the initial empty maximal-space (the floor space)
b) Empty maximal-spaces after placing facility 1 .

c) Empty maximal-spaces after placing facility 2.

## Decoder: Step 4 Recall that in the unconstrained case the

 floor space can include all facilities laid out horizontally or vertically.Floor space
1

a) Facilities to be placed and the initial empty maximal-space (the floor space)

b) Empty maximal-spaces after placing facility 1.
c) Empty maximal-spaces after placing facility 2.

Decoder: Step 4 For each EMS in which the facility fits, we compute the incremental cost associated with placing the facility in that EMS and then place it in the least-cost EMS.

Compute positions that minimize cost of placing


Decoder: Step 4 For each EMS in which the facility fits, we compute the incremental cost associated with placing the facility in that EMS and then place it in the least-cost EMS.

Compute positions that minimize cost of placing $\left(x_{U}, y_{U}\right) \quad$ facility i in each available $\operatorname{EMS}\left\{\left(\mathrm{x}_{\mathrm{L}}, \mathrm{y}_{\mathrm{L}}\right),\left(\mathrm{x}_{\mathrm{U}}, \mathrm{y}_{U}\right)\right\}$ w.r.t. all already-placed facilities $K$ :

EMS
$\left(x_{L}, y_{L}\right)$
Instead of solving this directly with a NLP solver we propose a different approach.
$\min \sum_{k \in K} c_{i, k} \times f_{i, k} \times d_{i, k}$ subject to:

$$
\begin{aligned}
& x_{L}+w_{i} / 2 \leq x_{i} \leq x_{U} \quad w_{i} / 2 \\
& y_{L}+h_{i} / 2 \leq y_{i} \leq y_{U} \quad h_{i} / 2
\end{aligned}
$$

Decoder: Step 4 For each EMS in which the facility fits, we compute the incremental cost associated with placing the facility in that EMS and then place it in the least-cost EMS.

Find the unconstrained optimum (UO) using $\left(x_{U}, y_{U}\right) \quad$ a method described in Heragu (1997):

EMS
$\left(x_{L}, y_{L}\right)$

$$
\min \sum_{k \in K} c_{i, k} \times f_{i, k} \times d_{i, k}
$$

If there is no flow between facility $i$ and the already laid-out facilities, then UO is assumed to be geometric center of all laidout facilities.

Tentatively place facility i in the geometric center of each EMS in which it fits.

Decoder: Step 4 For each EMS in which the facility fits, we place the facility in the center of the EMS and move it as close as possible to the UO and compute the objective.

## Initial position



[^1]

## Experimental results - Unconstrained

We compare our BRKGA with eight algorithms:

1) Hierarchical approach with clusters (HA-C) of Tam and Li (1991)
2) GA with slicing tree structure (GA-STS) of Kado (1996)
3) Genetic programming algorithm (GP-STS) of GarcesPerez et al. (1996)

## Experimental results - Unconstrained

We compare our BRKGA with eight algorithms:
4) GA with tree-structured genotype representation (GA-TSG) of Schnecke and Vornberger (1997)
5) Tabu search with slicing tree (TSaST) of Scholtz et al. (2009)
6) Commercial solver from Engineering Optimization Software (VIP-PLANOPT) based on algorithms of Mir and Imam (1996, 2001) and Imam and Mir (1998)

## Experimental results - Unconstrained

We compare our BRKGA with eight algorithms:
7) Tabu search with boundary search technique (TSBST) of McKendall Jr. and Hakobyan (2010)
8) The MIP solver from Gurobi Optimization (Gurobi) version 5.5.

## Experimental results - Unconstrained

## Benchmark instances:

- Seven L instances of Imam and Mir (1993, 1998), Mir and Imam (1996, 2001), and VIP-PLANOPT $(2006,2010)$ with 20 to 125 facilities
- Dunker62 instance of Dunker et al. (2003) with 62 facilities
- Eight TL instances of Tam and Li (1991) with 5 to 30 instances
- 100 random (RND) instances with known optimal with 10 to 100 facilities of Gonçalves \& R. (2014)


## Experimental results - Unconstrained

## Computational setup:

- BRKGA coded in C++
- Experiments run on a computer with an Intel Xeon E52630 processor at 2.30 GHz and 16 GB of RAM running Linux O.S. (Fedora, release 18)
- BRKGA parameters
- Population size: $\mathrm{p}=100 \times \mathrm{N}$
- Elite population: min ( $0.25 \times \mathrm{p}, 50$ )
- Mutation population: $0.25 \times \mathrm{p}$
- Inheritance probability: 0.70
- Stopping rule: 50 generations


## Experimental results - Unconstrained

## VIP-PLANOPT

TSaST
TS-BST
BRKGA
Dataset Cost Time Cost Time Cost Time Cost Time \%Impr

| L20 | 1.13 E 3 | 0.3 | - | - | 1.15 E 3 | 10351.9 | 1.13 E 3 | 0.5 | 1.86 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| L28 | 6.45 E 3 | 1.5 | - | - | - | - | 6.01 E | 1.0 | 6.72 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| L50 | 7.82 E 4 | 7.0 | - | - | $7.13 E 4$ | 7626.5 | 6.94 E 4 | 6.3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2.65 |  |  |  |  |  |  |  |  |


| L75 | 3.44 E 4 | 13.0 | - | - | - | - | 3.15 E 4 | 11.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8.47 |  |  |  |  |  |  |  |  |


| L100 | 5.38 E 5 | 14.0 | - | - | $4.97 E 5$ | 11397.2 | 4.79 E 5 | 57.0 | 3.60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

L125A 2.89E5 110.0 $\quad$ - $\quad$ - $\quad$ - $\quad$ - $\quad 2.57$ E5 83.611 .05
L125B 1.08E6 $70.0 \quad-\quad-\quad 1.01 \mathrm{E} 6 \quad 9250.3 \quad 9.43 \mathrm{E} 5118.7 \quad 6.51$
$\begin{array}{llllllllll}\text { Dunker62 } & \text { 3.94E6 } & \text { 4996.0 } & 3.87 E 6 & 252.0 & 3.81 E 6 & 7304.1 & 3.69 E 6 & 9.1 & 3.35\end{array}$
Times are in seconds

## Experimental results - Unconstrained

HA-C GA-STS GP-STS GA-TSG TSaST
BRKGA

Dataset Cost Cost Cost Cost Cost Time Cost Time \%lmpr

| TL05 | 247 | 228 | 226 | 214 | 213.5 | 2.3 | 210.1 | 0.035 | 1.60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TLO6 | 514 | 361 | 384 | 327 | 348.8 | 3.0 | 345.0 | 0.049 | $(5.51)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL07 | 559 | 596 | 568 | 629 | 562.9 | 2.5 | 549.7 | 0.060 | 1.67 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL08 | 839 | 878 | 878 | 833 | 810.4 | 4.7 | 799.1 | 0.080 | 1.40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL12 | 3162 | 3283 | 3220 | 3164 | 3054.2 | 12.5 | 2920.5 | 0.162 | 4.38 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL15 | 5862 | 7384 | 7510 | 6813 | 6615.8 | 17.0 | 6395.4 | 0.251 | (9.10) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL20 | 16393 | 14033 | 13190 | 13198.4 | 50.0 | 9892.4 | 0.443 | 25.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| TL30 | - | 41095 | 39018 | 25358 | 33721.5 | 95.4 | 31454.2 | 1.132 | 6.72 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Experimental results - Unconstrained (\% deviation from optimum)
Each dataset consists of 10 instances, each with known optimum.

|  | Gurobi |  |  | BRKGA |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Dataset | Time | Avg \% Dev | Max \% Dev | Time | Avg \% Dev | Max \% Dev |
| RND10 | 3600 | 0.21 | 1.66 | 1.76 | 0.00 | 0.00 |
| RND20 | 3600 | 0.01 | 0.12 | 6.13 | 0.00 | 0.00 |
| RND30 | 3600 | 0.32 | 2.14 | 15.00 | 0.00 | 0.00 |
| RND40 | 3600 | 2.37 | 7.10 | 28.67 | 0.00 | 0.00 |
| RND50 | 3600 | 3.99 | 9.30 | 48.30 | 0.11 | 1.12 |
| RND60 | 3600 | 16.65 | 29.73 | 72.86 | 0.02 | 0.15 |
| RND70 | 3600 | 12.21 | 22.70 | 102.90 | 1.44 | 5.29 |
| RND80 | 3600 | 22.31 | 50.97 | 143.37 | 3.31 | 7.10 |
| RND90 | 3600 | 36.11 | 52.99 | 186.87 | 6.00 | 9.09 |
| RND100 | 3600 | 101.78 | 235.31 | 235.84 | 7.36 | 10.97 |



RND instance

$1^{\text {st }}$ generation: 530404.76
$50^{\text {th }}$ generation: 478910.09


## L100

$1^{\text {st }}$ generation: 530404.76
$50^{\text {th }}$ generation: 478910.09


## L100

$1^{\text {st }}$ generation: 530404.76
$50^{\text {th }}$ generation: 478910.09


## L100

$1^{\text {st }}$ generation: 530404.76
$50^{\text {th }}$ generation: 478910.09


## L100

L100 (478910.09)


## L100

New best known
Solution: 4.79E5

Previous best known
Solution: 4.97E5
TS-BST (McKendall Jr. \& Hajobyan, 2010)



New best known Solution: 549.7

Previous best known Solution: 559.0 HA-C (Tam and Li, 1991)

TL12 (2920.47)


New best known Solution: 2920.5

Previous best known Solution: 3054.2 TSaST (Scholtz et al., 2009)

TL2O (9892.38)


TL20

New best known Solution: 9892.4

Previous best known Solution: 13190.0 GA-TSG (Schnecke and Vornberger, 1997)


## Other applications of BRKGA

## Telecommunications

- Weight setting in OSPF routing (Ericsson et al., 2002; Buriol et al., 2005; Reis et al., 201 1)
- Survivable network design (Andrade et al., 2006; Buriol et al., 2007; Ruiz et al., 2015; Andrade et al., 2015)
- Facility location (Breslau et al., 2011; Morán-Mirabal et al., 2013; Duarte et al., 2014; Stefanello et al., 2015)
- Routing $\mathcal{E}$ wavelength assignment (Noronha et al., 2011)
- Assignment of virtual machines to datacenters (Stefanello et al., 2015)
- Design of wireless backhaul network (Andrade et al., 2015)
- Cloud resource management (Heilig et al., 2015)


## Other applications of BRKGA

## Scheduling

- Job-shop scheduling (Gonçalves et al., 2005; Gonçalves \& R., 2014 )
- Project scheduling (Gonçalves et al., 2008; 2009; 2011)
- Survey of project scheduling (Gonçalves et al., 2014)
- Field technician scheduling (Damm et al., 2015)
- Scheduling divisible loads (Brandão et al., 2015)
- Scheduling Earth observations with agile satellite (Tangpattanakul et al., 2013)
- Multi-user Earth observation scheduling (Tangpattanakul et al., 2015)


## Other applications of BRKGA

## Manufacturing and facility layout

- Assembly line balancing (Gonc̣alves \& Almeida, 2002, )
- Manufacturing cell formation (Gonçalves \& R., 2004)
- Assembly line worker assignment and balancing (Moreira et al., 2012)
- Minimization of open stacks (Gonçalves et al., 2014)
- Minimization of tool switches (Chaves et al., 2014)
- Unequal area facility layout (Gonçalves \& R., 2015)


## Other applications of BRKGA

Algorithm engineering

- Automatic tuning of parameters (Festa et al., 2010; MoránMirabal et al., 2013)
- Benchmarking (Gonçalves et al., 2014)
- Extensions of BRKGA (Lucena et al., 2014)
- Application programming interface (Toso et al., 2015)


## Other applications of BRKGA

Clustering, covering, and packing

- 2D/3D orthogonal packing (Gonçalves \& R., 2011; 2012)
- 2D/3D bin packing (Gonçalves and R., 2013)
- Multi-objective 3D container loading (Zheng et al., 2014)
- Steiner triple covering (R. et al., 2014)
- Overlapping correlation clustering (Andrade et al., 2014)
- Winner determination in combinatorial auctions (Andrade et al., 2014)


## Other applications of BRKGA

## Routing

- Capacitated arc routing (Martinez et al., 2011)
- K-interconnected multi-depot multi-TSP (Andrade et al., 2013)
- Family TSP (Morán-Mirabal et al., 2014)
- Capacitated VRP for blood sample collection (Grasas et al., 2014)


## Other applications of BRKGA

Graphs and Trees

- Stochastic Steiner tree (Hokama et al., 2014)
- Capacitated minimum spanning tree (Ruiz et al., 2015)
- Maximum cardinality quasi-clique (Pinto et al., 2015)


## Other applications of BRKGA

Toll setting in road networks

- Road congestion minimization (Buriol et al., 2009; 2010; Stefanello et al., 2015)


## Other applications of BRKGA

Continuous global optimization

- Bound-constrained GO (Silva et al., 2012)
- Nonlinearly-constrained GO (Silva et al., 2013)
- Python/C++ library for bound-constained GO (Silva et al., 2013)
- Finding multiple roots of system of nonlinear equations (Silva et al., 2014)


These slides and all of the papers cited in this lecture can be downloaded from my homepage:
http://mauricio.resende.info


[^0]:    * For large random: number of best average solutions / total instance classes

[^1]:    Minimum cost position

