# Packing with biased random-key genetic algorithms

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UNICAMP

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U. do Porto
Portugal

# Summary

- Metaheuristics and basic concepts of genetic algorithms
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
  - Encoding / Decoding
  - Initial population
  - Evolutionary mechanisms
  - Problem independent / problem dependent components
  - Multi-start strategy
  - Specifying a BRKGA
  - Application programming interface (API) for BRKGA
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing
- Concluding remarks

# Metaheuristics

Metaheuristics are heuristics to devise heuristics.

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Metaheuristics are high level procedures that coordinate simple heuristics, such as local search, to find solutions that are of better quality than those found by the simple heuristics alone.

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Metaheuristics are high level procedures that coordinate simple heuristics, such as local search, to find solutions that are of better quality than those found by the simple heuristics alone.

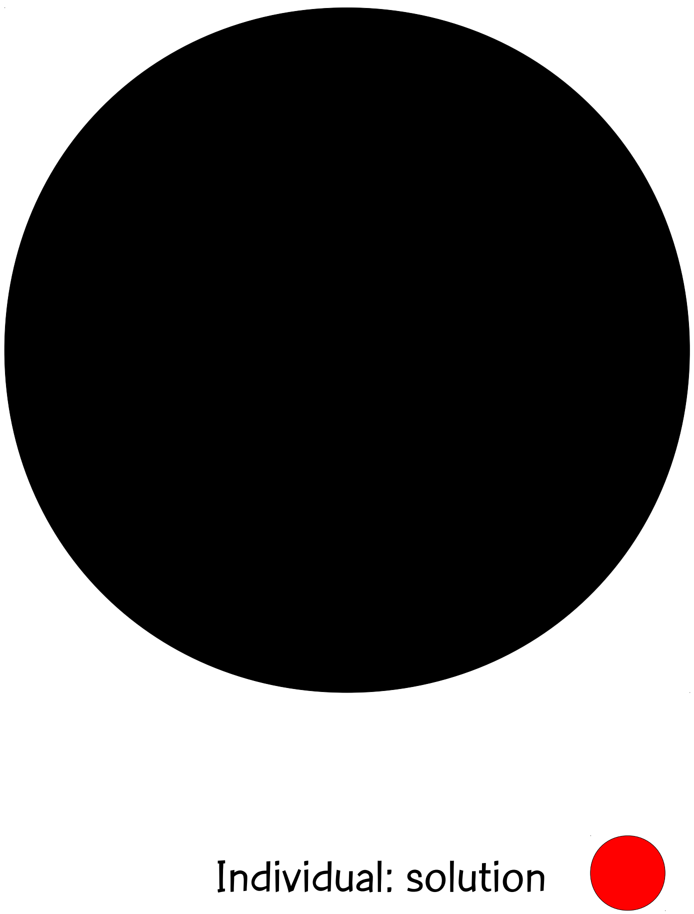
Examples: GRASP and C-GRASP, simulated annealing, genetic algorithms, tabu search, scatter search, ant colony optimization, variable neighborhood search, and biased random-key genetic algorithms (BRKGA).

# Genetic algorithms

# Genetic algorithms

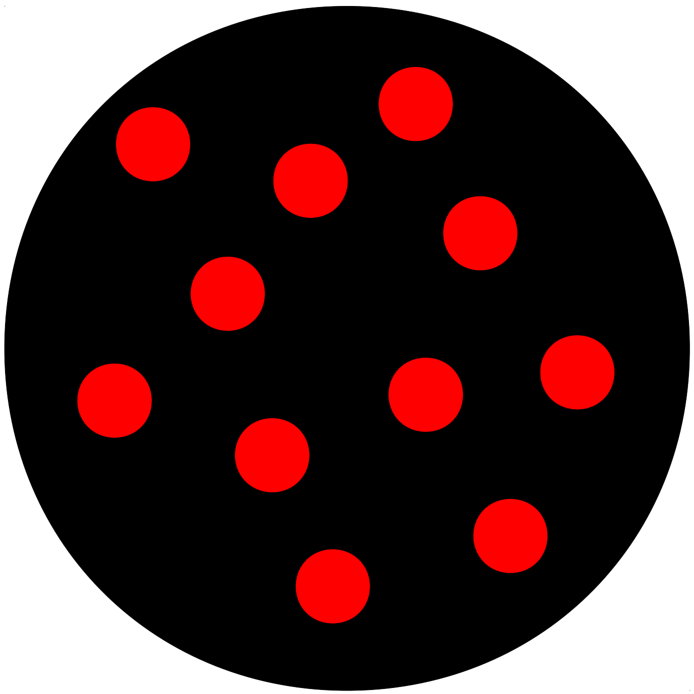
Holland (1975)

Adaptive methods that are used to solve search and optimization problems.



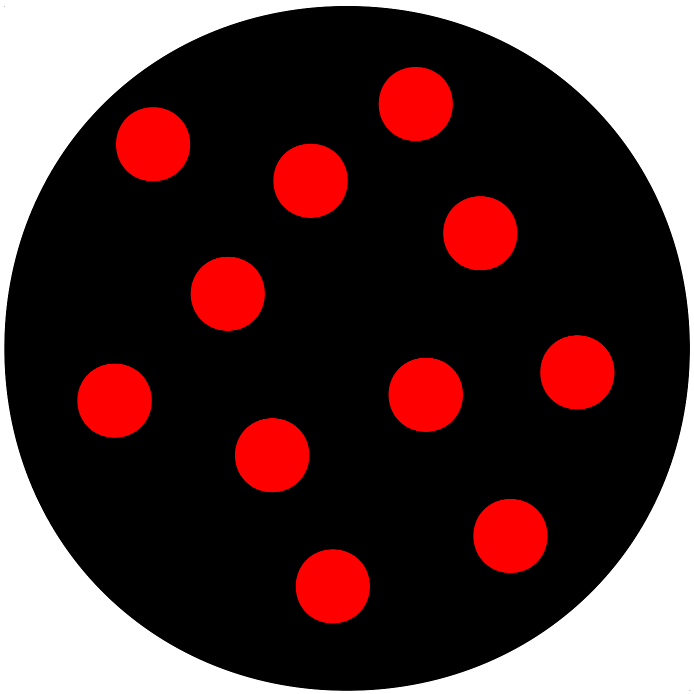
Individual: solution

# Genetic algorithms



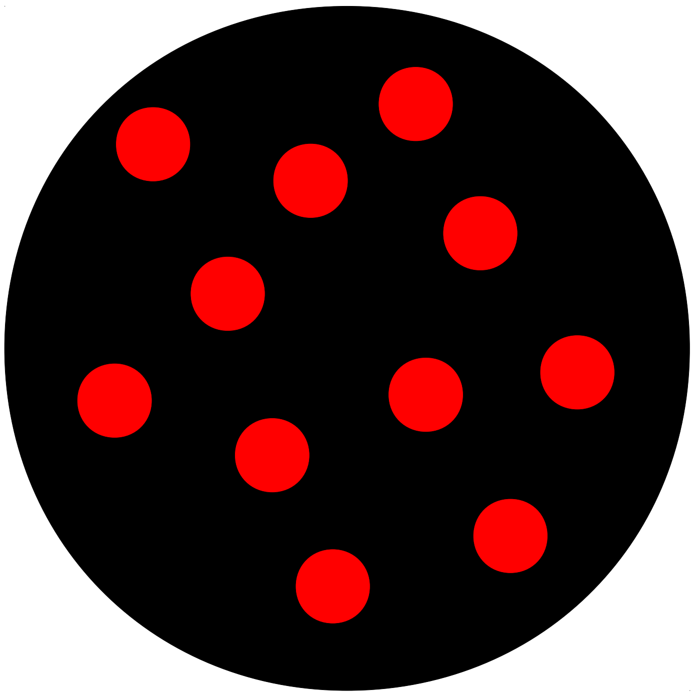
Individual: solution (chromosome = string of genes)
Population: set of fixed number of individuals

# Genetic algorithms



Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

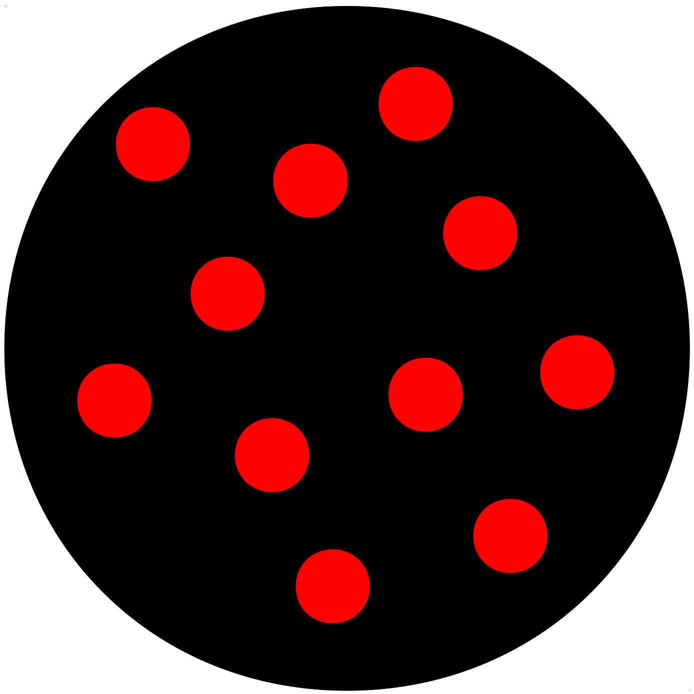
# Genetic algorithms



Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

# Genetic algorithms

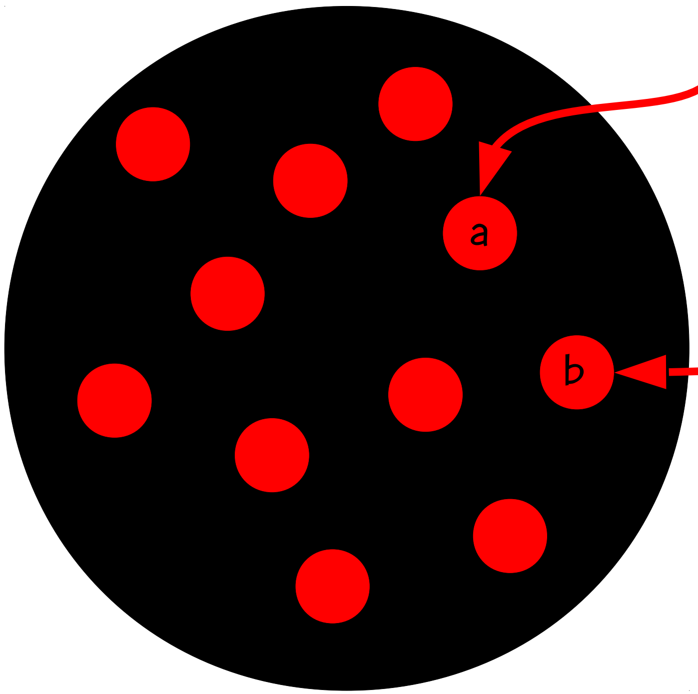


Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

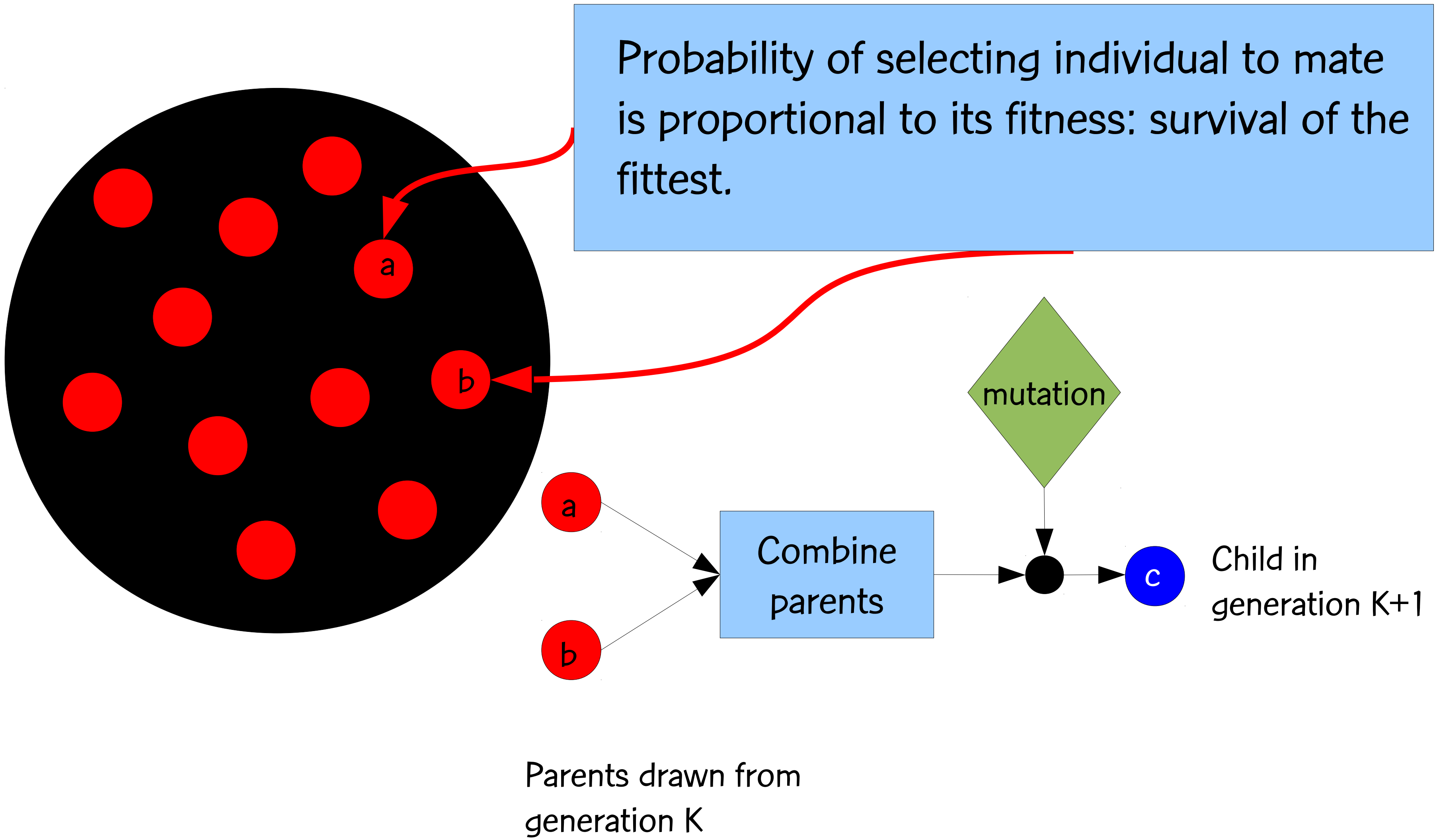
Individuals from one generation are combined to produce offspring that make up next generation.

# Genetic algorithms

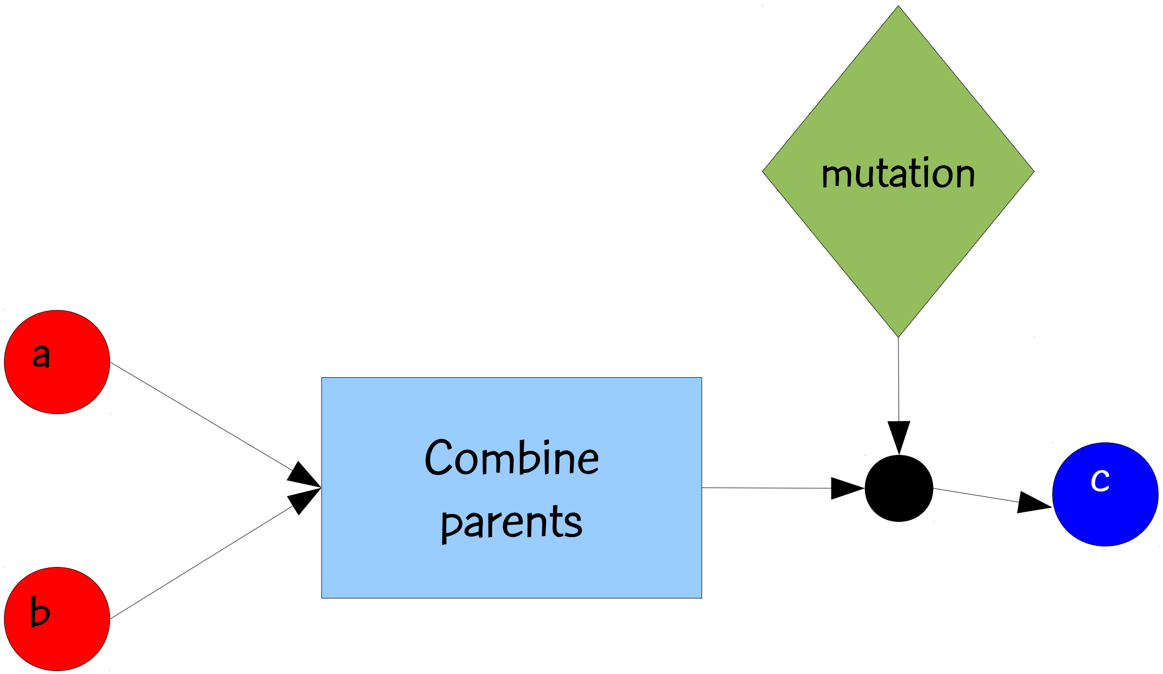



Probability of selecting individual to mate is proportional to its fitness: survival of the fittest.

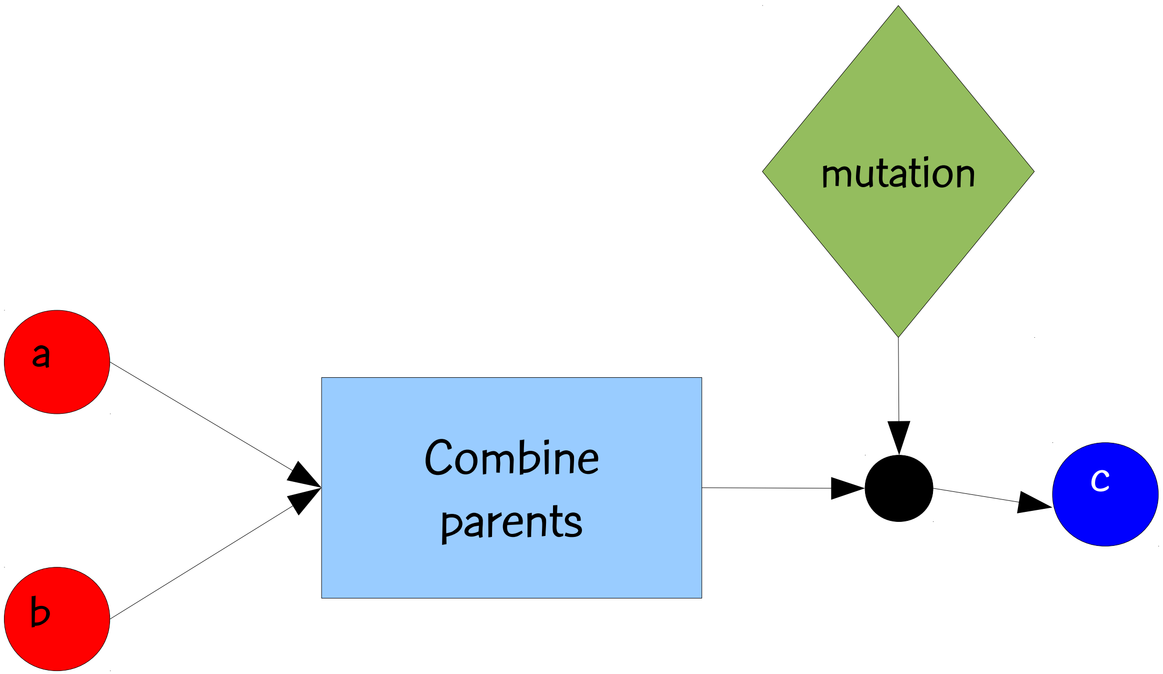
# Genetic algorithms



# Crossover and mutation



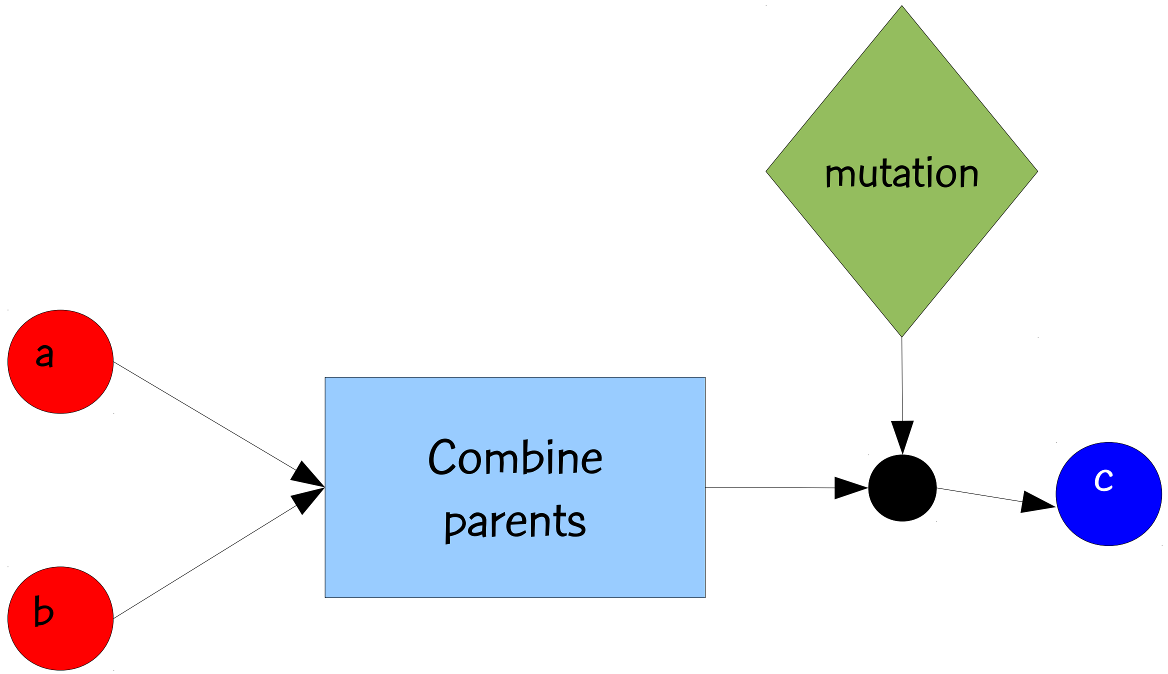
# Crossover and mutation




Crossover: Combines parents ... passing along to offspring characteristics of each parent ...

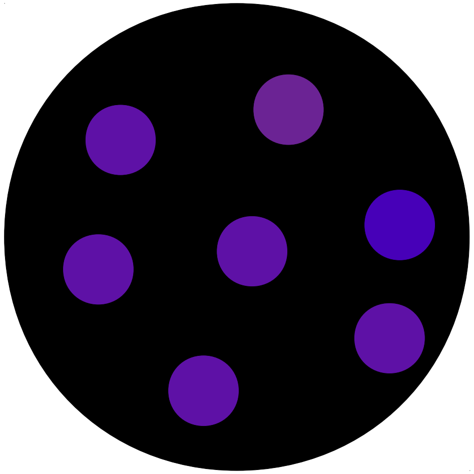
Intensification of search

# Crossover and mutation

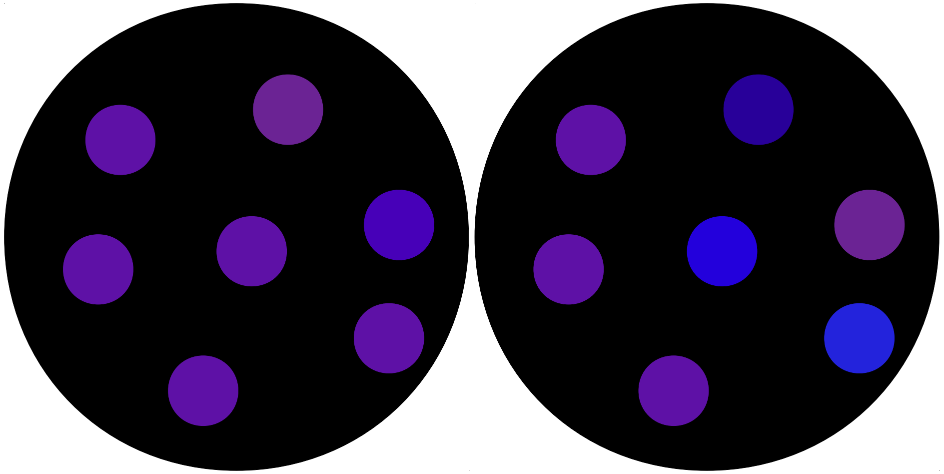



Mutation: Randomly changes chromosome of offspring ...
Driver of evolutionary process ...
Diversification of search

# Evolution of solutions



# Evolution of solutions



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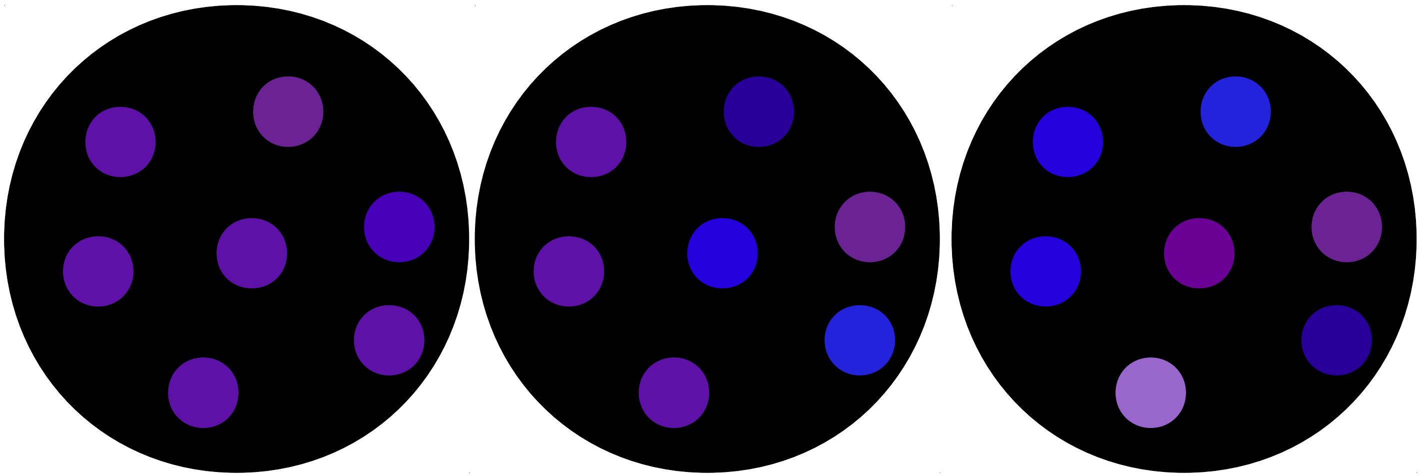

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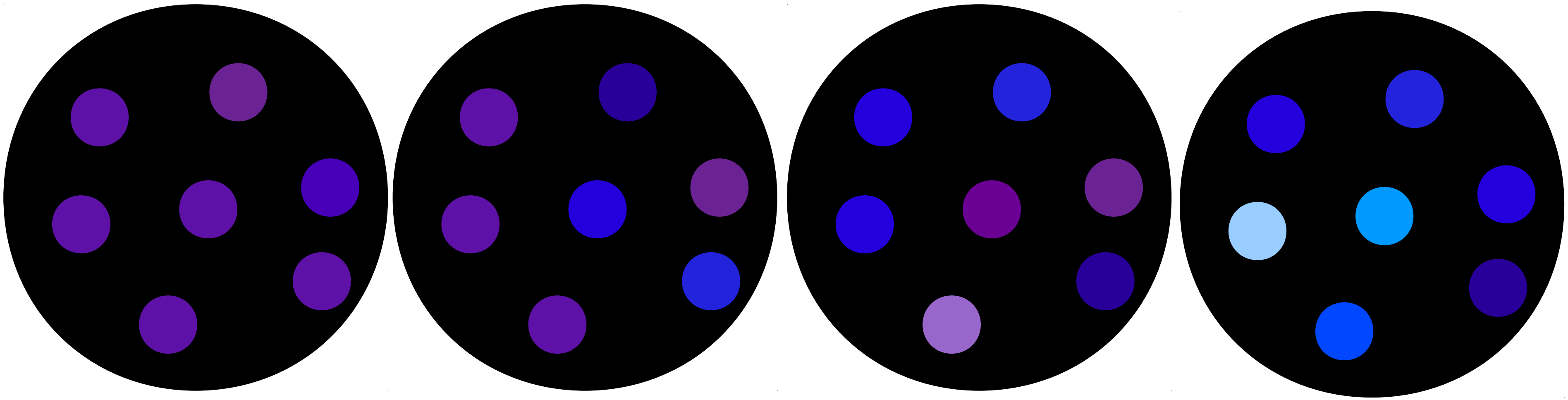

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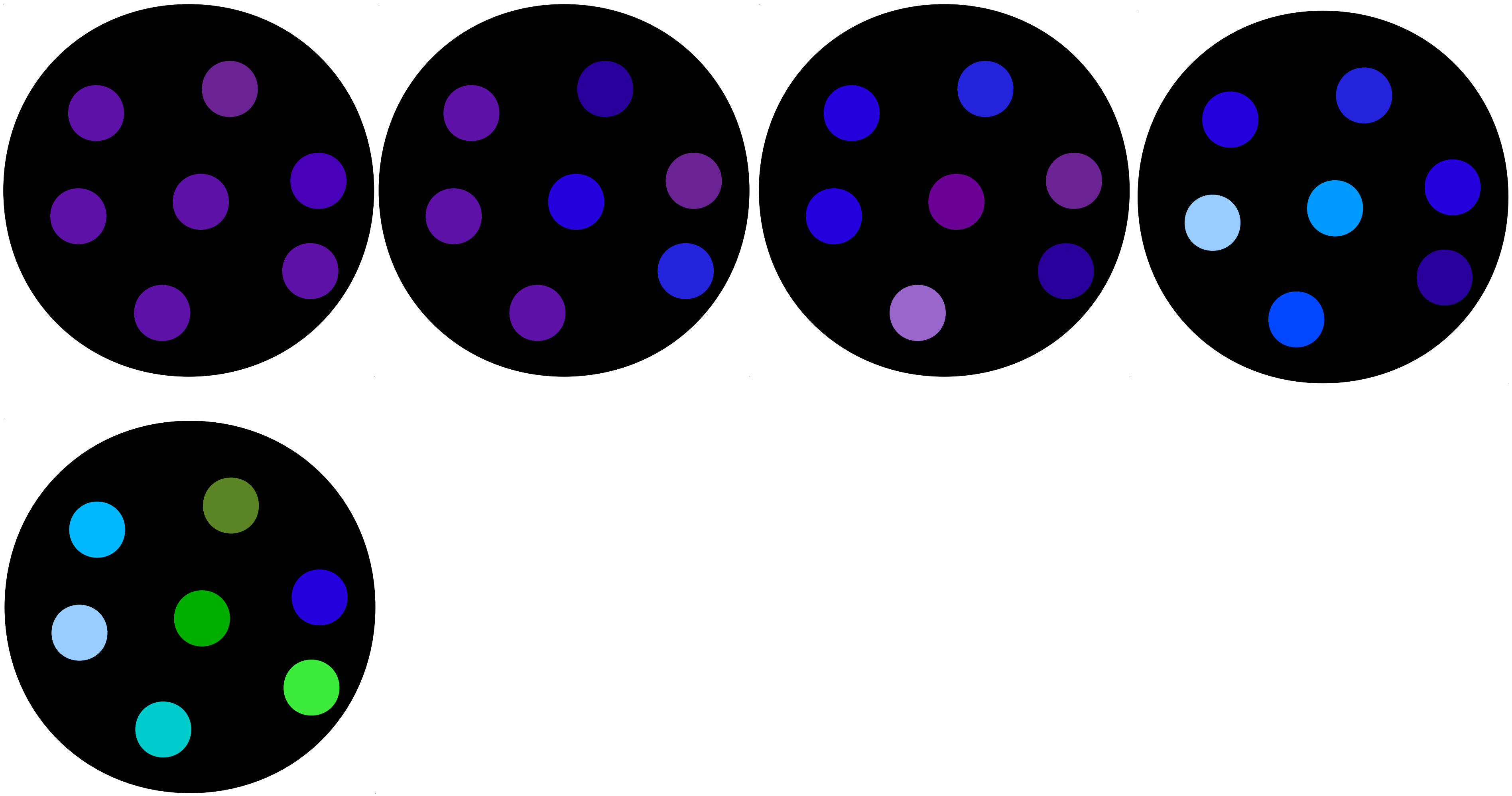

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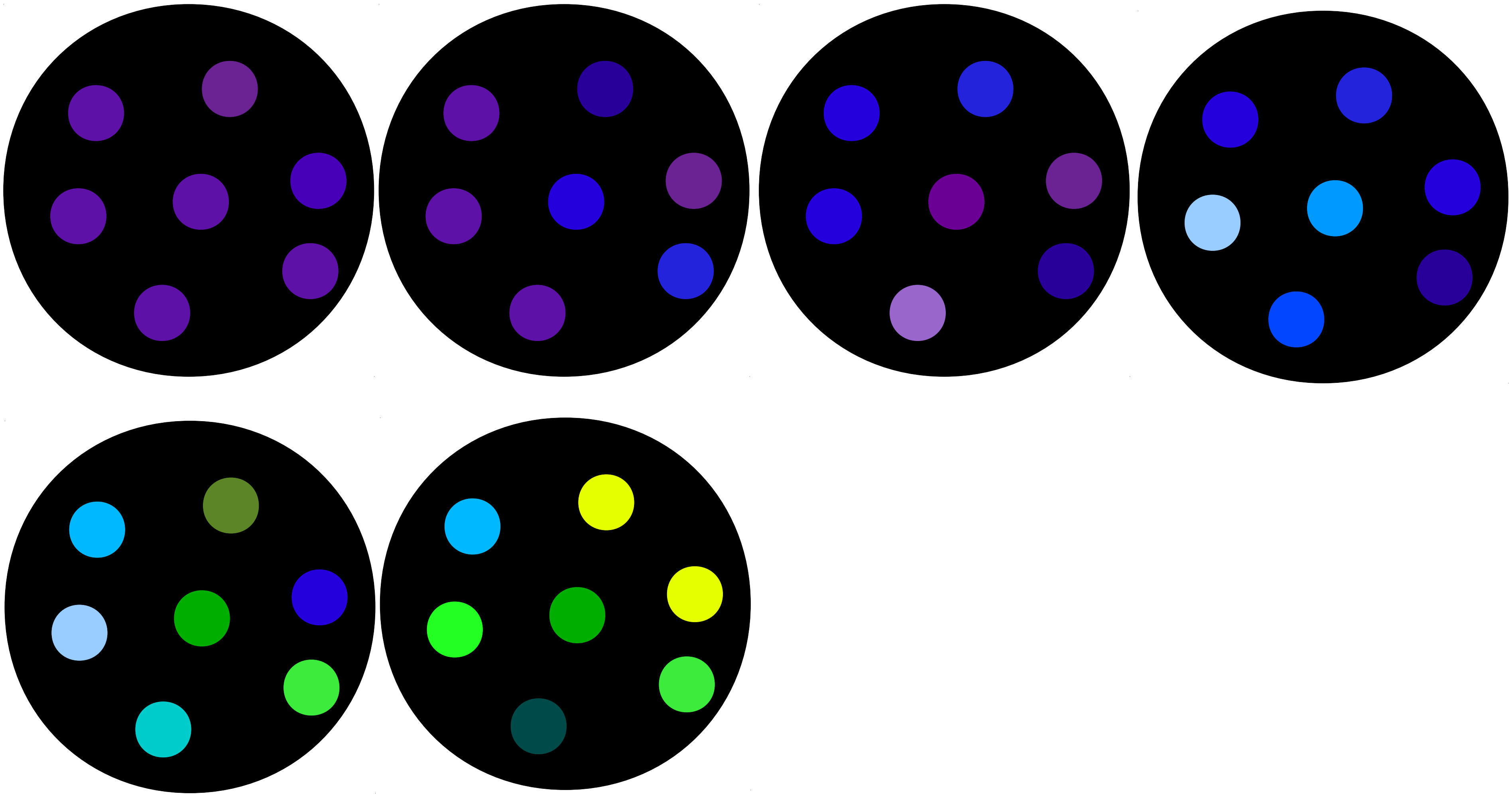

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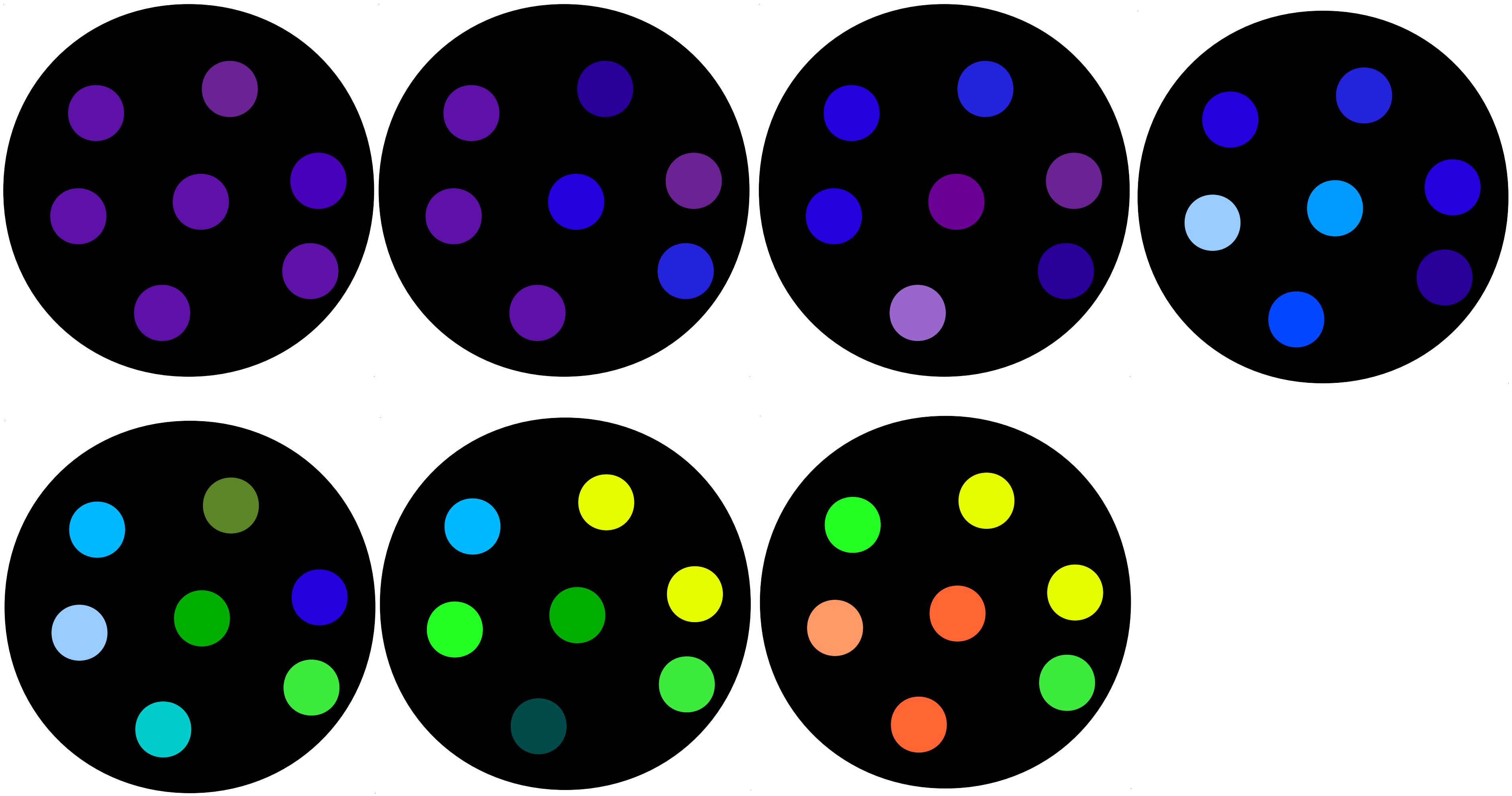

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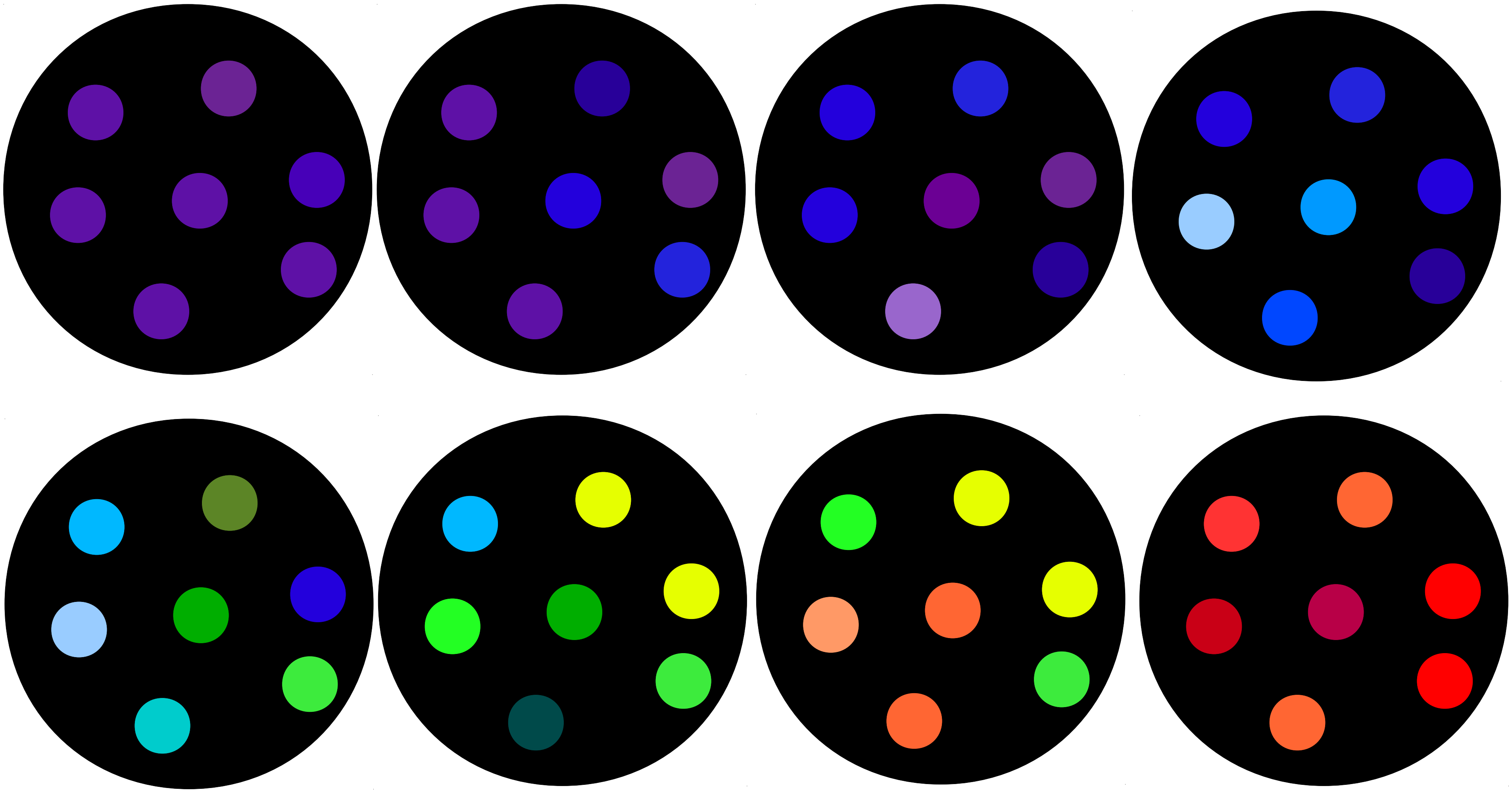
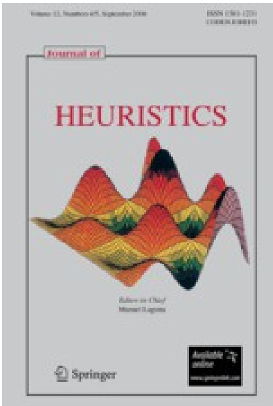


# Reference




J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:

http://mauricio.resende.info/doc/srkga.pdf

# Encoding solutions with random keys

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- Solutions of optimization problems can be **encoded** by random keys.

# Encoding with random keys

- A random key is a real random number in the continuous interval [0,1).
- A vector X of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.

# Encoding with random keys: Sequencing

## Encoding

$$
\begin{array}{ccccc}
[\;\; 1, & 2, & 3, & 4, & 5\;] \\
X = [\, 0.099, & 0.216, & 0.802, & 0.368, & 0.658\, ]
\end{array}
$$

# Encoding with random keys: Sequencing

## Encoding

$$[\quad 1, \quad 2, \quad 3, \quad 4, \quad 5 \,]$$

$$X = [\, 0.099, 0.216, 0.802, 0.368, 0.658 \,]$$

## Decode by sorting vector of random keys

$$[\quad 1, \quad 2, \quad 4, \quad 5, \quad 3 \,]$$

$$X = [\, 0.099, 0.216, 0.368, 0.658, 0.802 \,]$$

# Encoding with random keys: Sequencing

Therefore, the vector of random keys:

X = [ 0.099, 0.216, 0.802, 0.368, 0.658 ]

encodes the sequence: 1 – 2 – 4 – 5 – 3

# Encoding with random keys: Subset selection (select 3 of 5 elements)

Encoding

[ 1, 2, 3, 4, 5 ]

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Decode by sorting vector of random keys

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# Encoding with random keys: Subset selection (select 3 of 5 elements)

Therefore, the vector of random keys:

X = [ 0.099, 0.216, 0.802, 0.368, 0.658 ]

encodes the subset: {1, 2, 4 }

# Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

## Encoding

$$[\quad 1, \quad 2, \quad 3, \quad 4, \quad 5 \mid 1, \quad 2, \quad 3, \quad 4, \quad 5\ ]$$

$$X = [\ 0.099, 0.216, 0.802, 0.368, 0.658 \mid 0.4634, 0.5611, 0.2752, 0.4874, 0.0348\ ]$$

# Encoding with random keys: Assigning integer weights $\in$ [0,10] to a subset of 3 of 5 elements

## Encoding

[ 1, 2, 3, 4, 5 | 1, 2, 3, 4, 5 ]

X = [ 0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348 ]

Decode by sorting the first 5 keys and assign as the weight the value $W_i = \textbf{floor} [ 10 X_{5+i} ] + 1$ to the 3 elements with smallest keys $X_i$, for $i = 1,\ldots,5$.

# Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

Therefore, the vector of random keys:

X = [ 0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348 ]

encodes the weight vector W = (5,6,–,5,–)

# Genetic algorithms and random keys

# GAs and random keys

- Introduced by Bean (1994) for sequencing problems.

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- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).

$$S = (\, 0.25,\ 0.19,\ 0.67,\ 0.05,\ 0.89 \,)$$

s(1) s(2) s(3) s(4) s(5)

# GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).
- Sorting random keys results in a sequencing order.

$$S = (\, 0.25,\ 0.19,\ 0.67,\ 0.05,\ 0.89 \,)$$
$$\quad s(1) \quad s(2) \quad s(3) \quad s(4) \quad s(5)$$

$$S' = (\, 0.05,\ 0.19,\ 0.25,\ 0.67,\ 0.89 \,)$$
$$\quad s(4) \quad s(2) \quad s(1) \quad s(3) \quad s(5)$$

Sequence: 4 – 2 – 1 – 3 – 5

# GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)

a = ( 0.25, 0.19, 0.67, 0.05, 0.89 )
b = ( 0.63, 0.90, 0.76, 0.93, 0.08 )

# GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)
- For each gene, flip a biased coin to choose which parent passes the allele (key, or value of gene) to the child.

a = ( 0.25, 0.19, 0.67, 0.05, 0.89 )
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c = ( 0.25 )

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c = ( 0.25, 0.90 )

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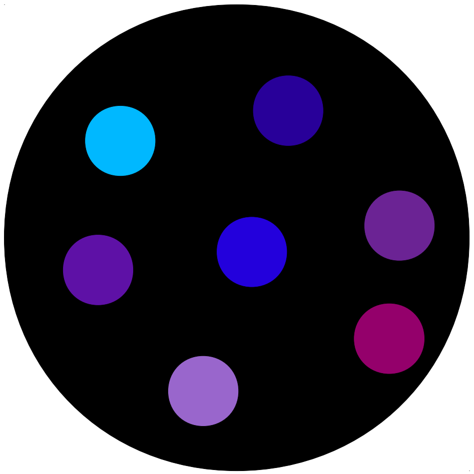
If every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

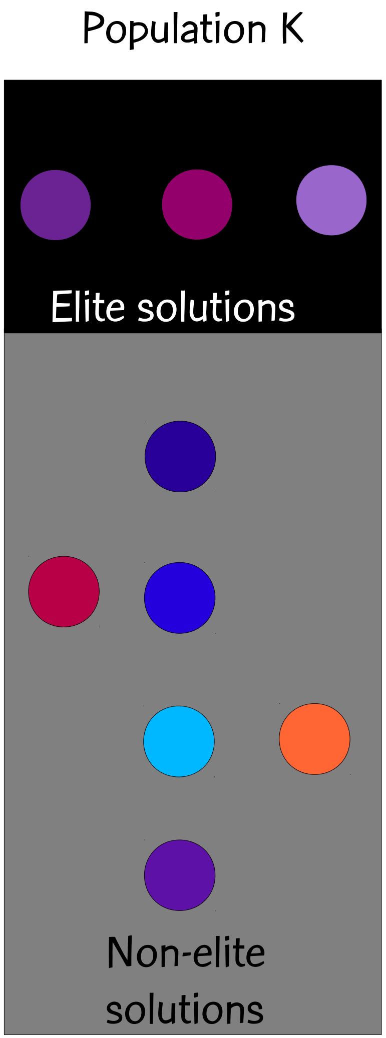
# GAs and random keys

Initial population is made up of P random-key vectors, each with N keys, each having a value generated uniformly at random in the interval [0,1).



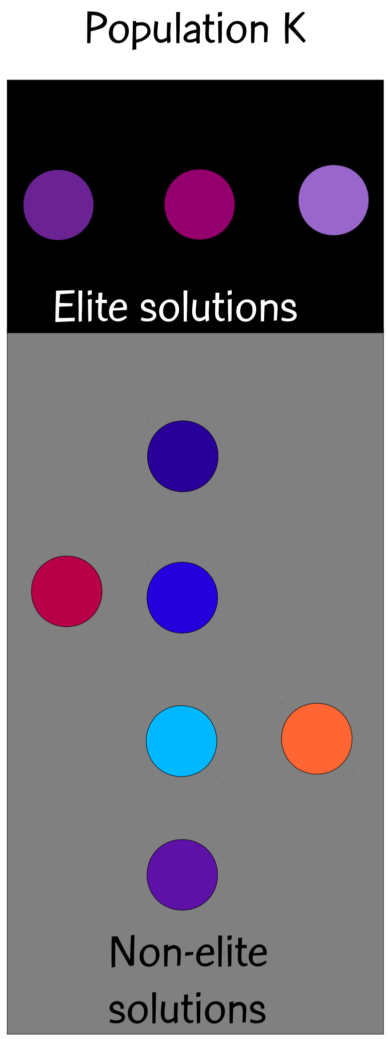
# GAs and random keys

At the K-th generation,
compute the cost of each
solution ...



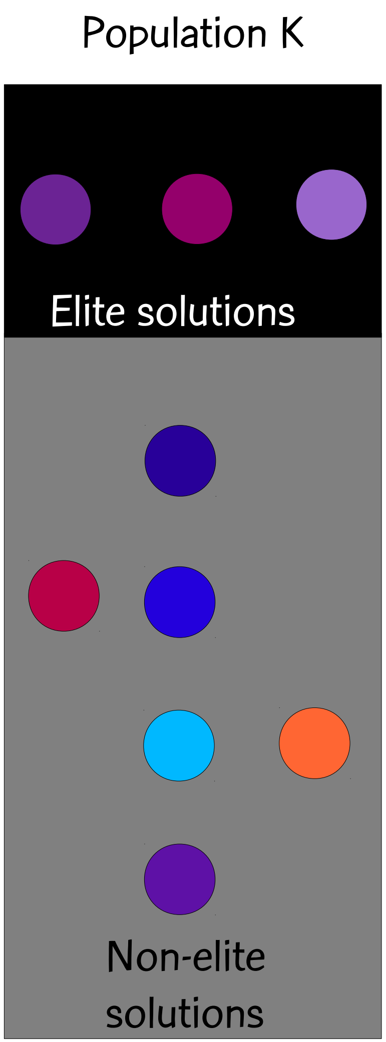
# GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets:



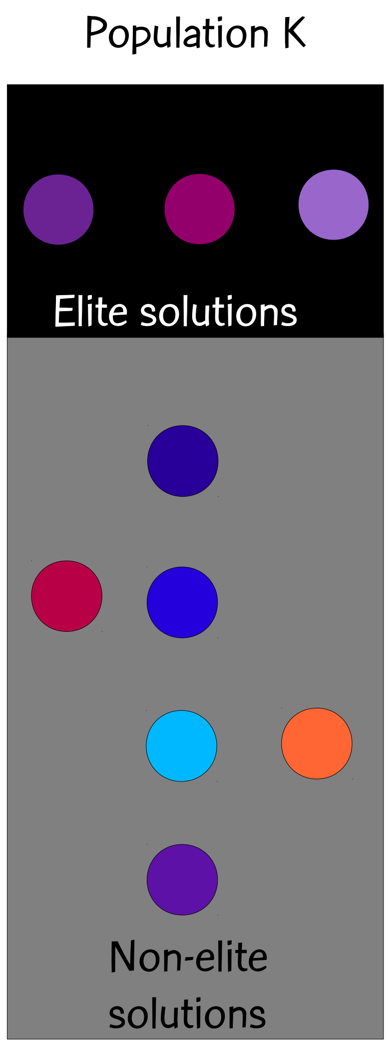
# GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions.



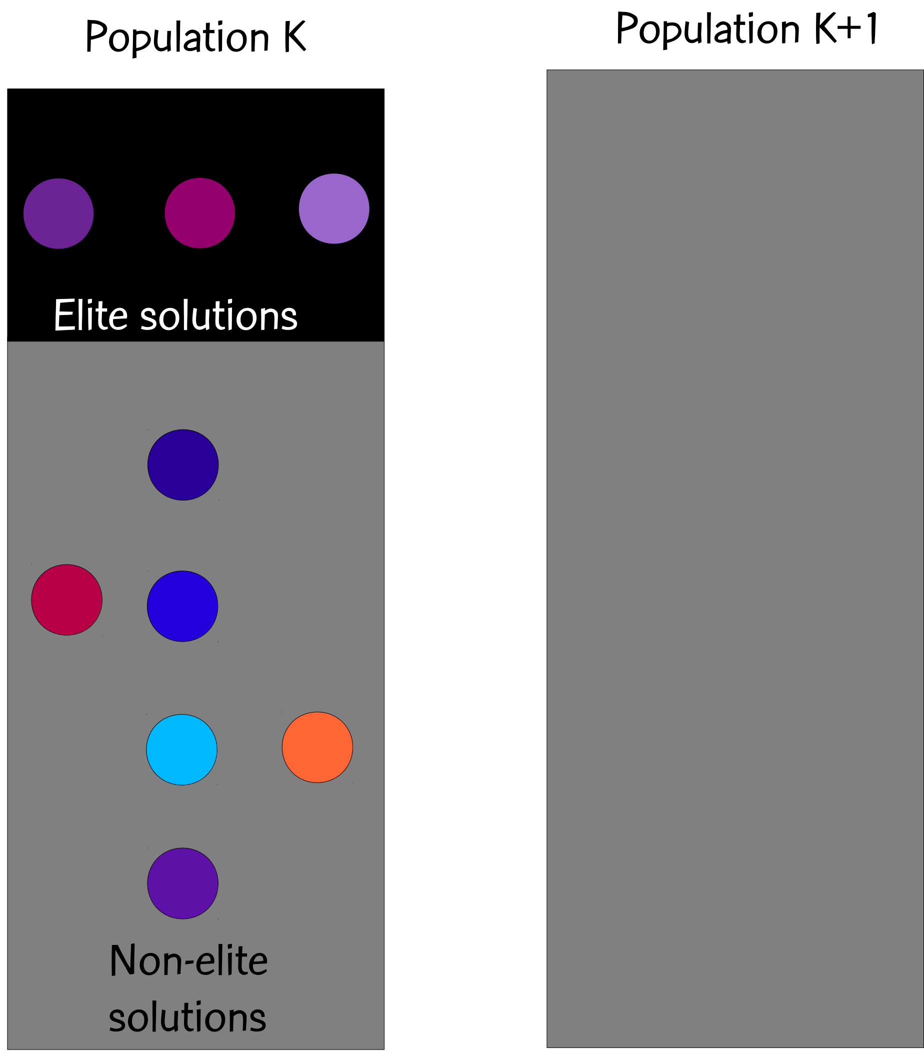
# GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.



# GAs and random keys

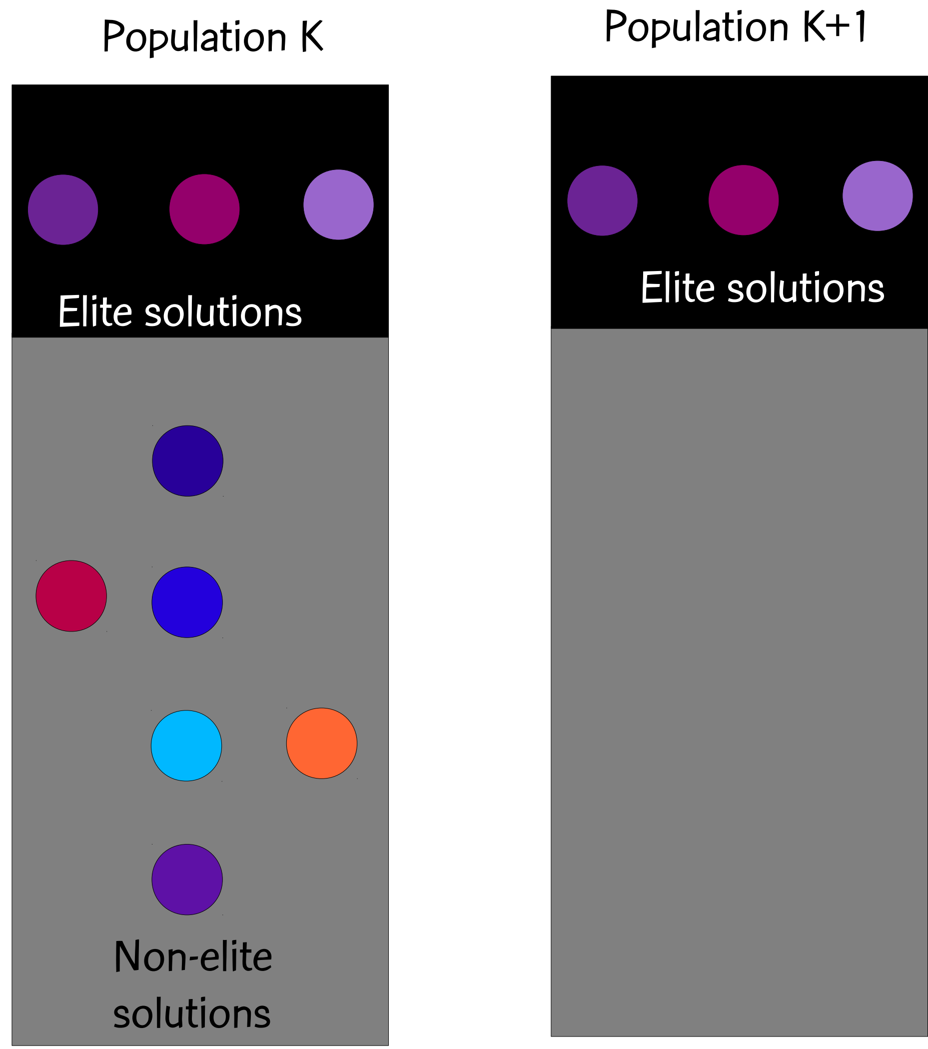
## Evolutionary dynamics



# GAs and random keys

## Evolutionary dynamics

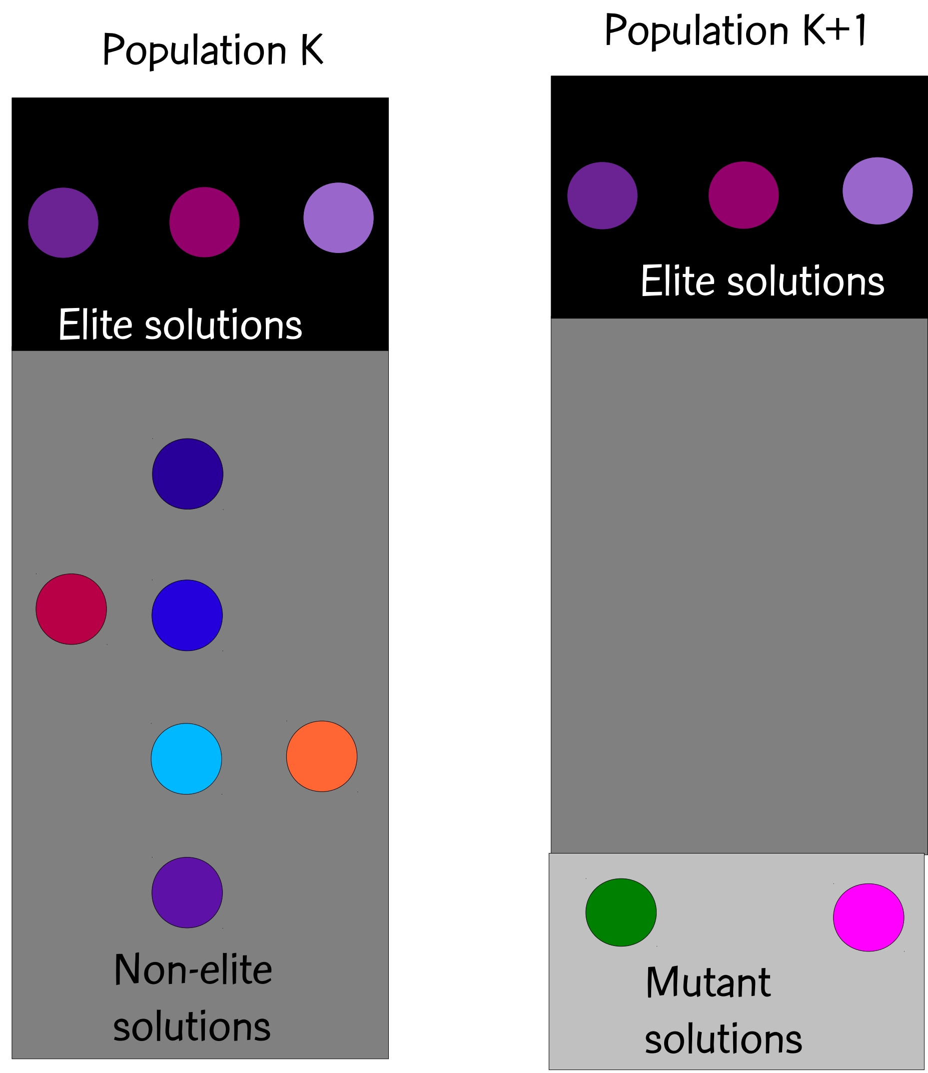
– Copy elite solutions from population K to population K+1



# GAs and random keys

## Evolutionary dynamics

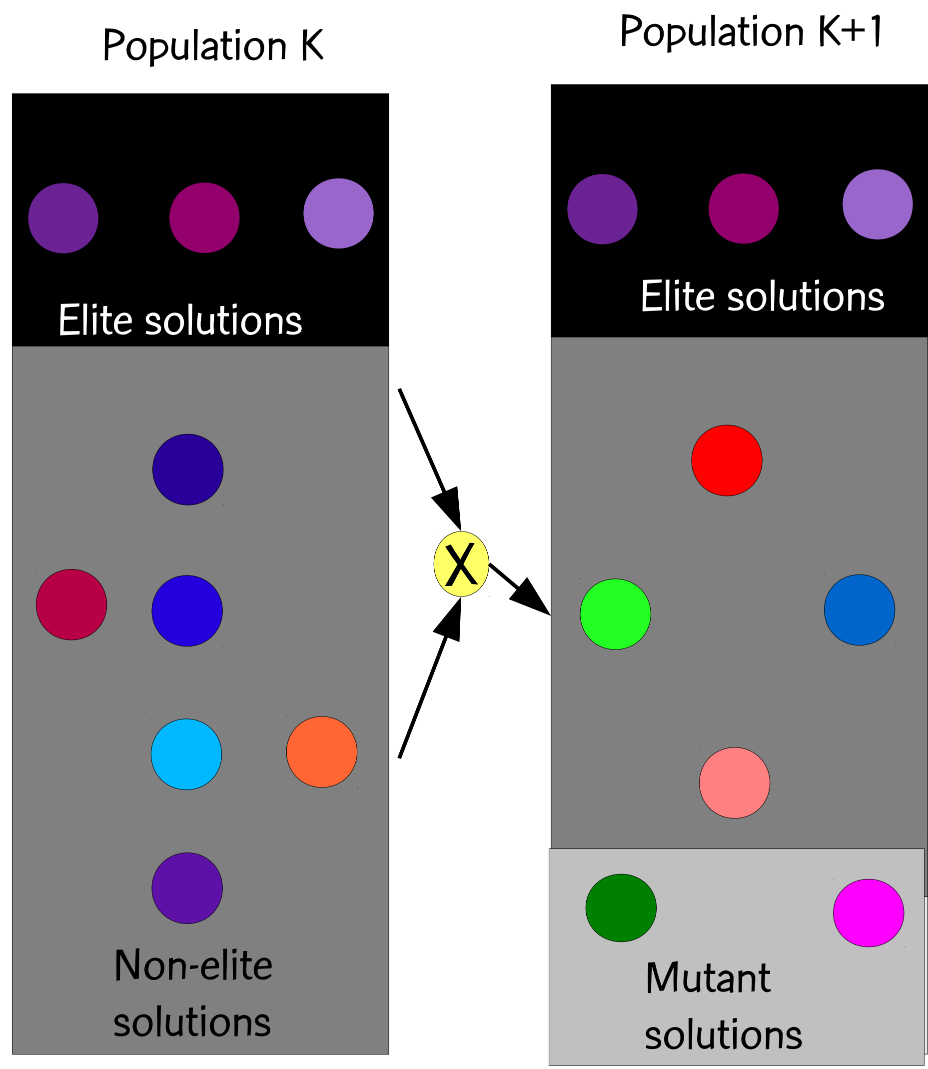
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1



# GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.



# Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).

# Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.

# How RKGA & BRKGA differ

**RKGA**

both parents chosen at random from entire population

**BRKGA**

# How RKGA & BRKGA differ

**RKGA**

both parents chosen at random from entire population

**BRKGA**

both parents chosen at random but one parent chosen from population of elite solutions

# How RKGA & BRKGA differ

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both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

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# How RKGA & BRKGA differ

**RKGA**

both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

**BRKGA**

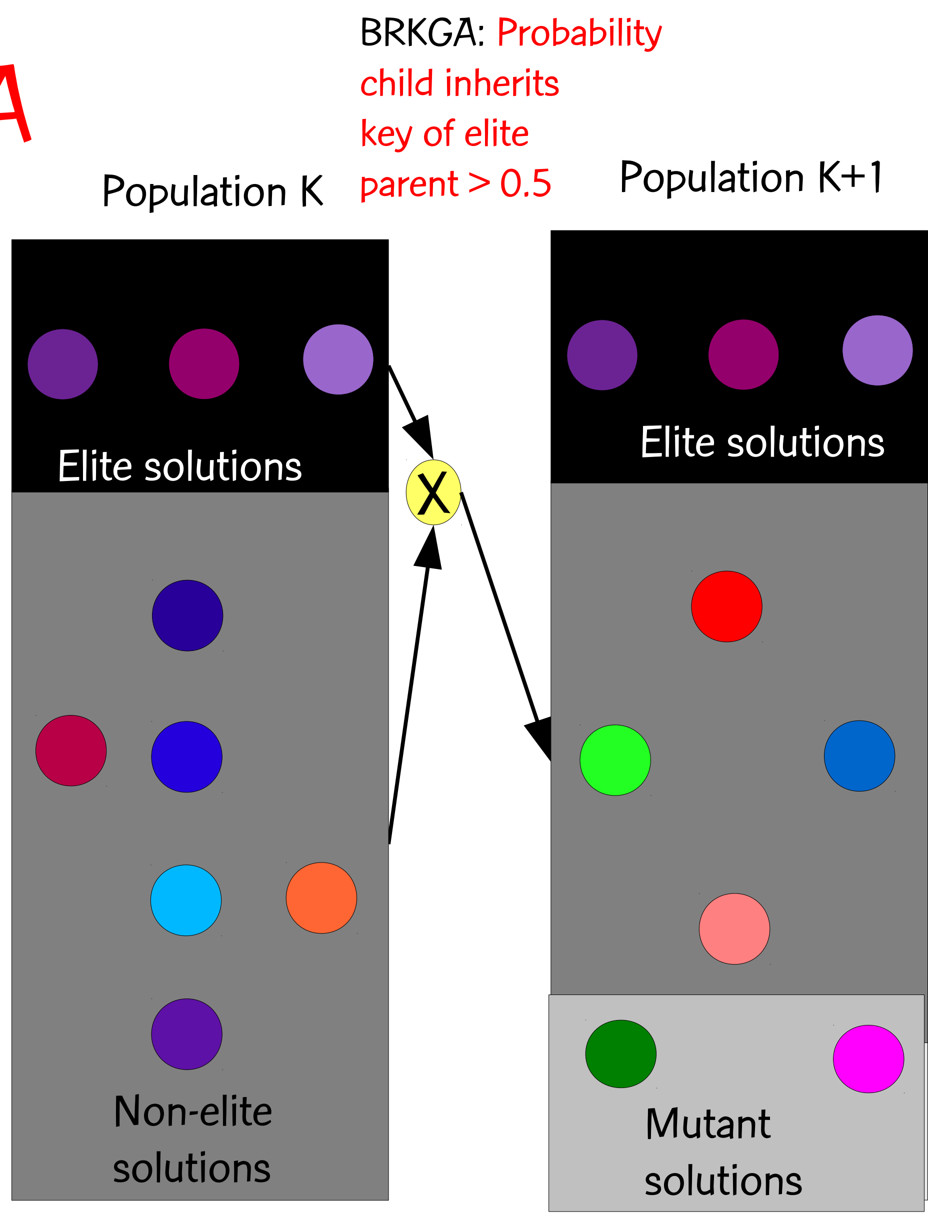
both parents chosen at random but one parent chosen from population of elite solutions

best fit parent is parent A in parametrized uniform crossover

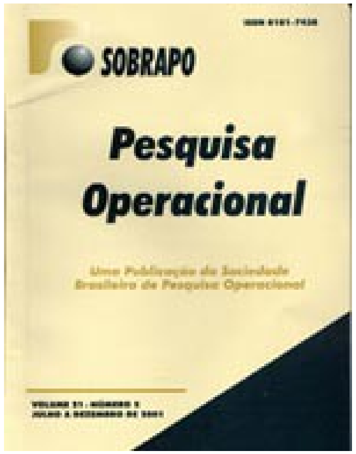
# Biased random key GA

## Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.
  - BIASED RANDOM-KEY GA: Mate elite solution with other solution of population K to produce child in population K+1. Mates are chosen at random.



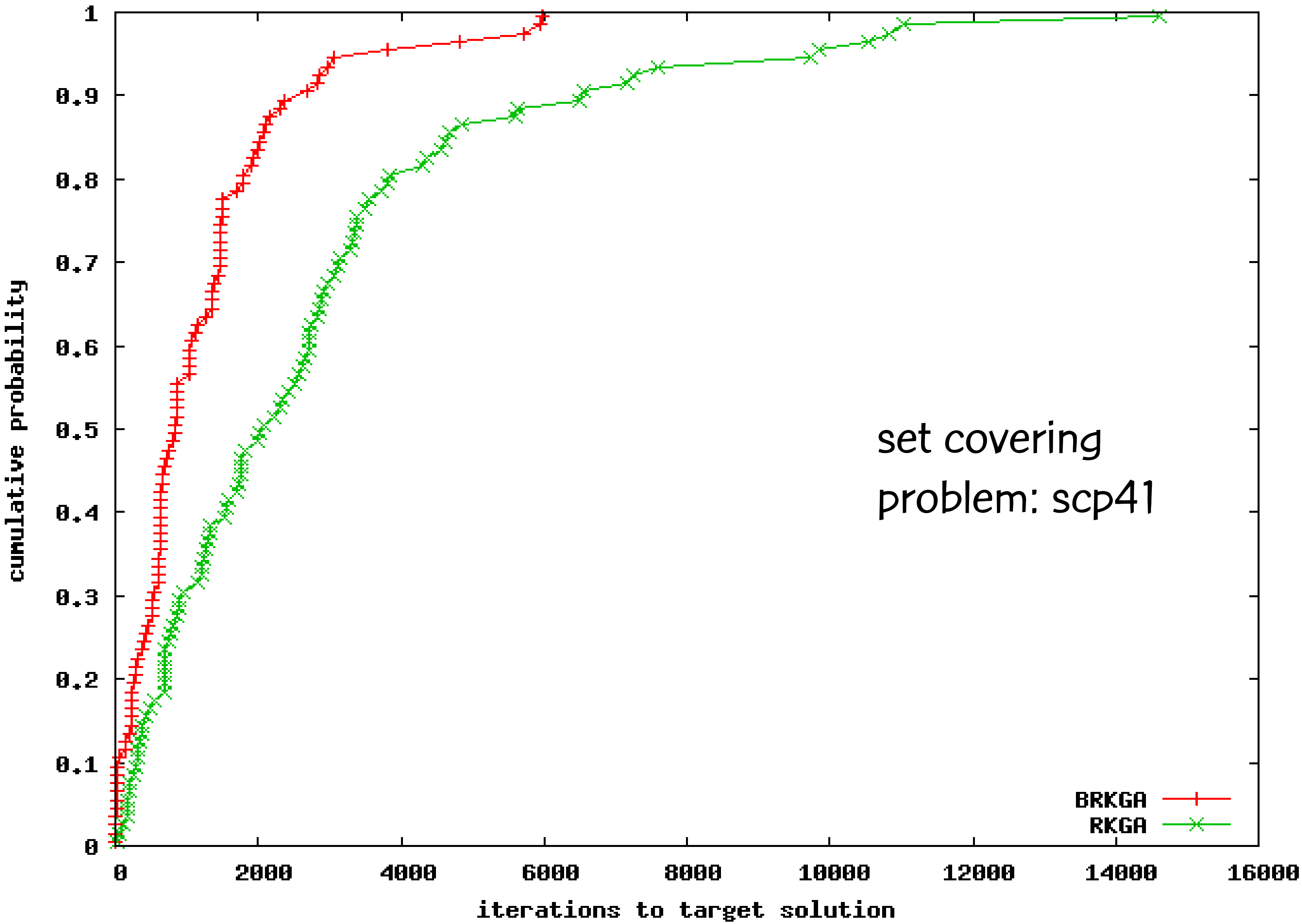
# Paper comparing BRKGA and Bean's Method

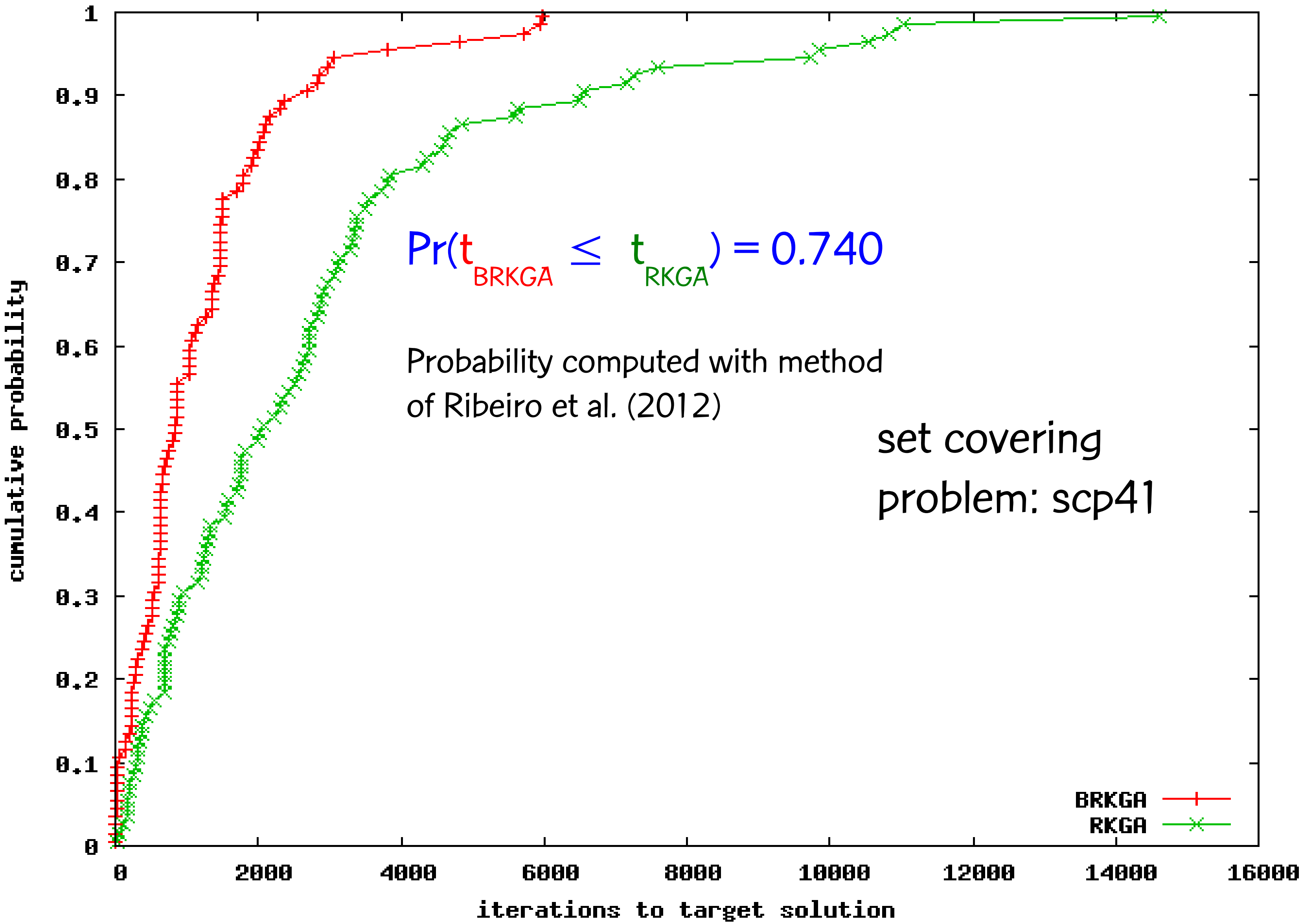


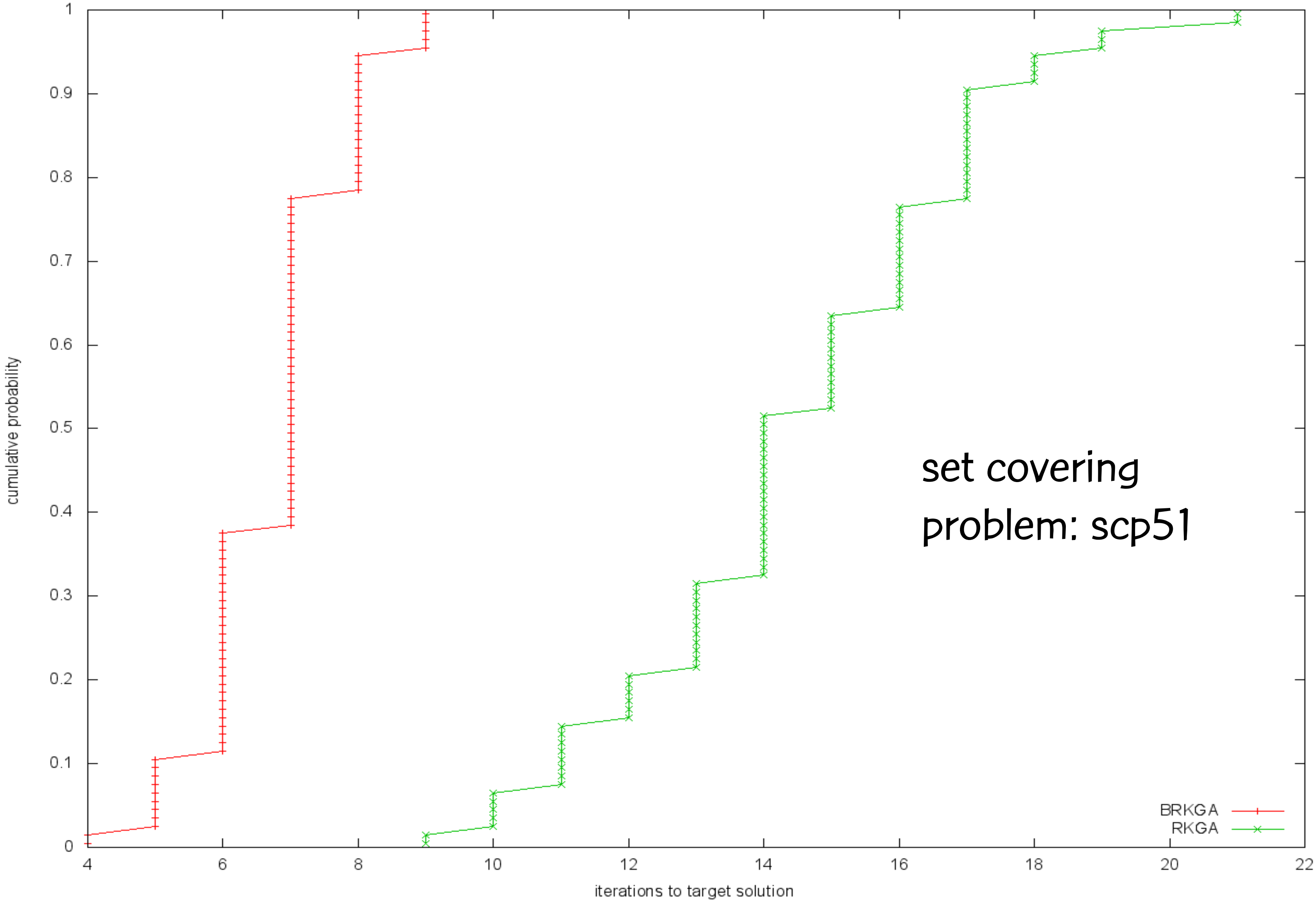

Gonçalves, R., and Toso,

"An experimental comparison of biased and unbiased random-key genetic algorithms",

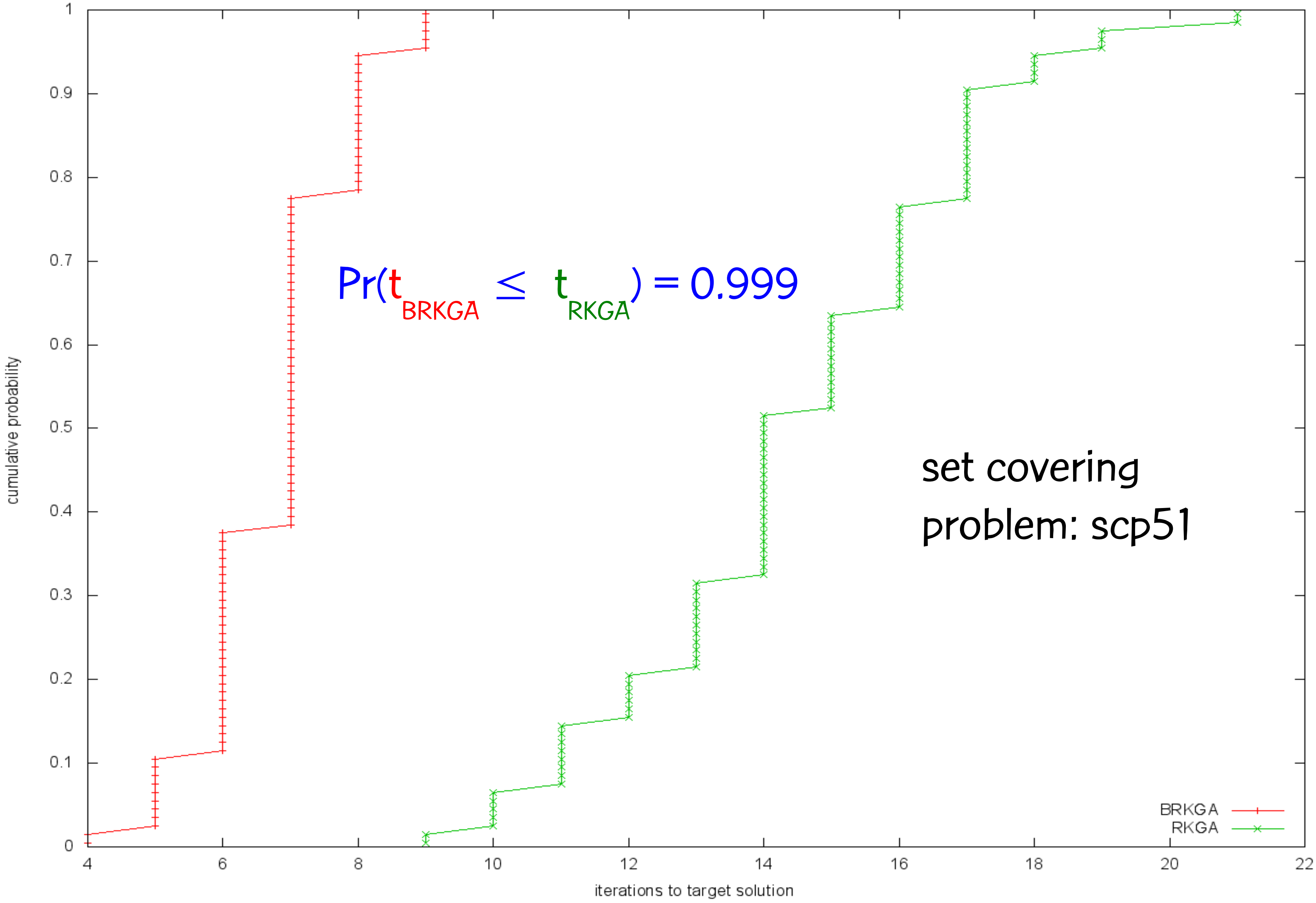
Pesquisa Operacional, vol. 34, pp. 143-164, 2014.


cumulative probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2000
4000
6000
8000
10000
12000
14000
16000
iterations to target solution
set covering
problem: scp41
BRKGA
RKGA

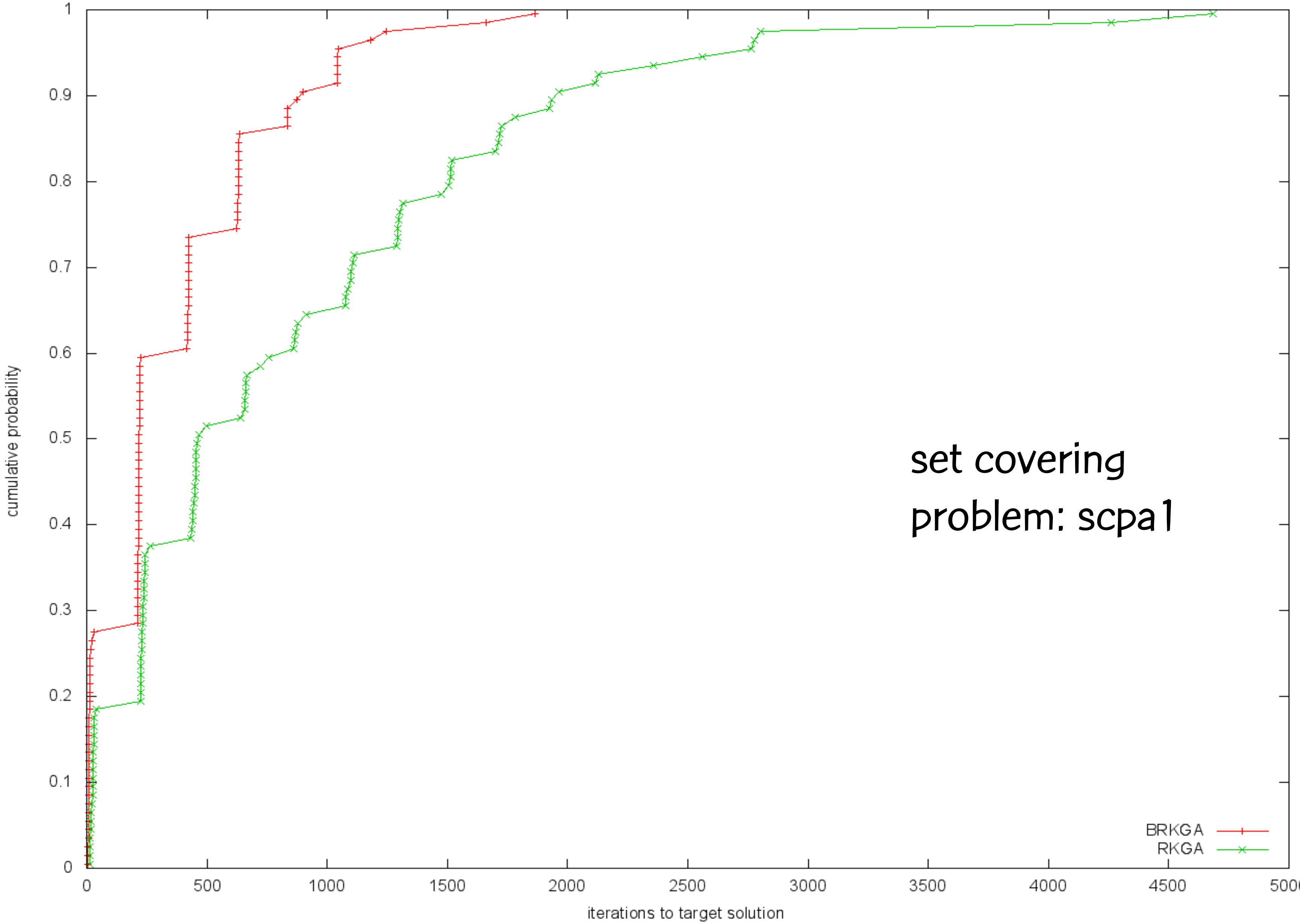

Pr(t_BRKGA ≤ t_RKGA) = 0.740
Probability computed with method
of Ribeiro et al. (2012)
set covering
problem: scp41
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2000
4000
6000
8000
10000
12000
14000
16000



cumulative probability
iterations to target solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
4
6
8
10
12
14
16
18
20
22
BRKGA
RKGA
set covering
problem: scp51

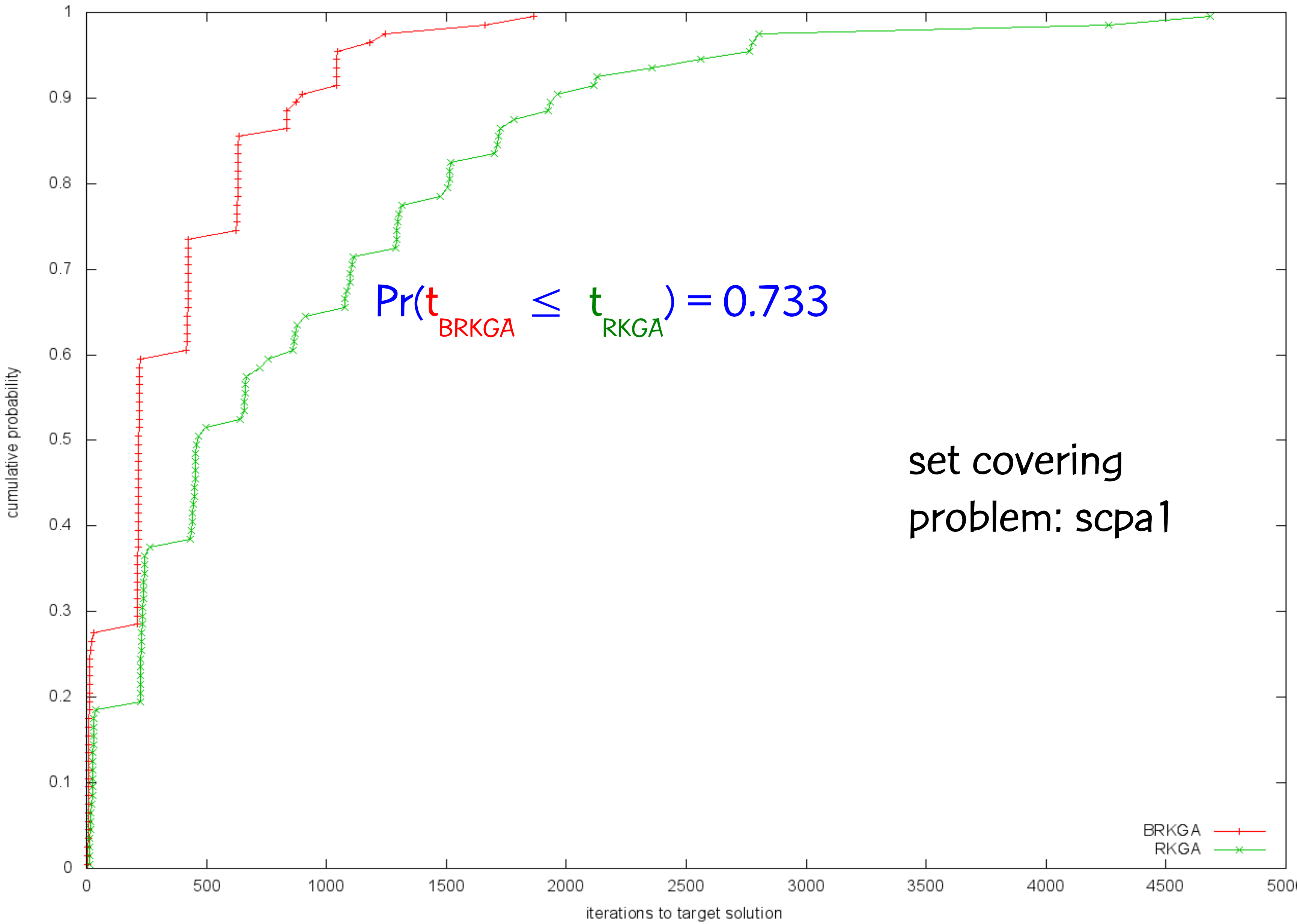


$\Pr(t_{BRKGA} \leq t_{RKGA}) = 0.999$
set covering
problem: scp51
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
4
6
8
10
12
14
16
18
20
22

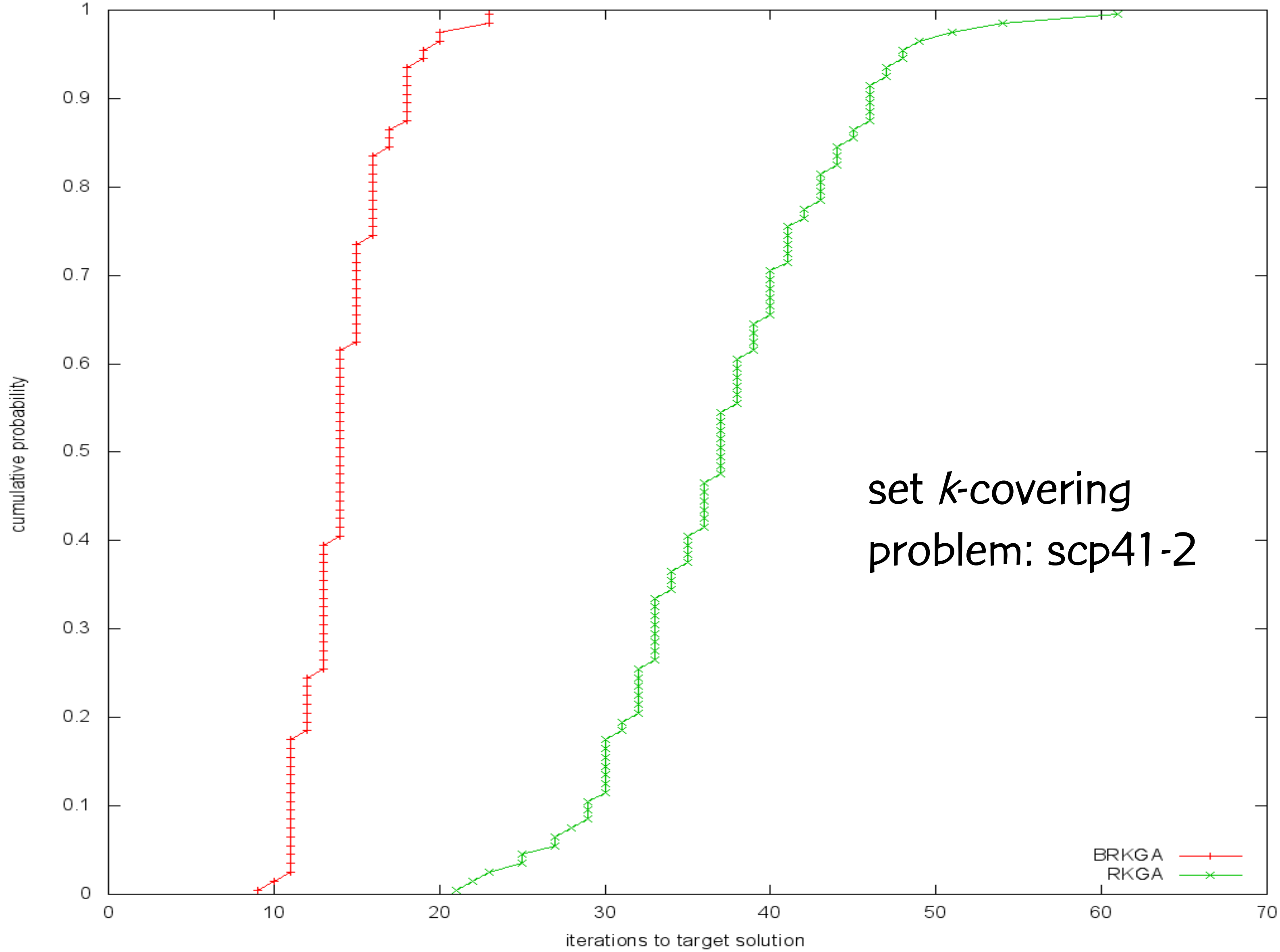

cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
set covering
problem: scpa1



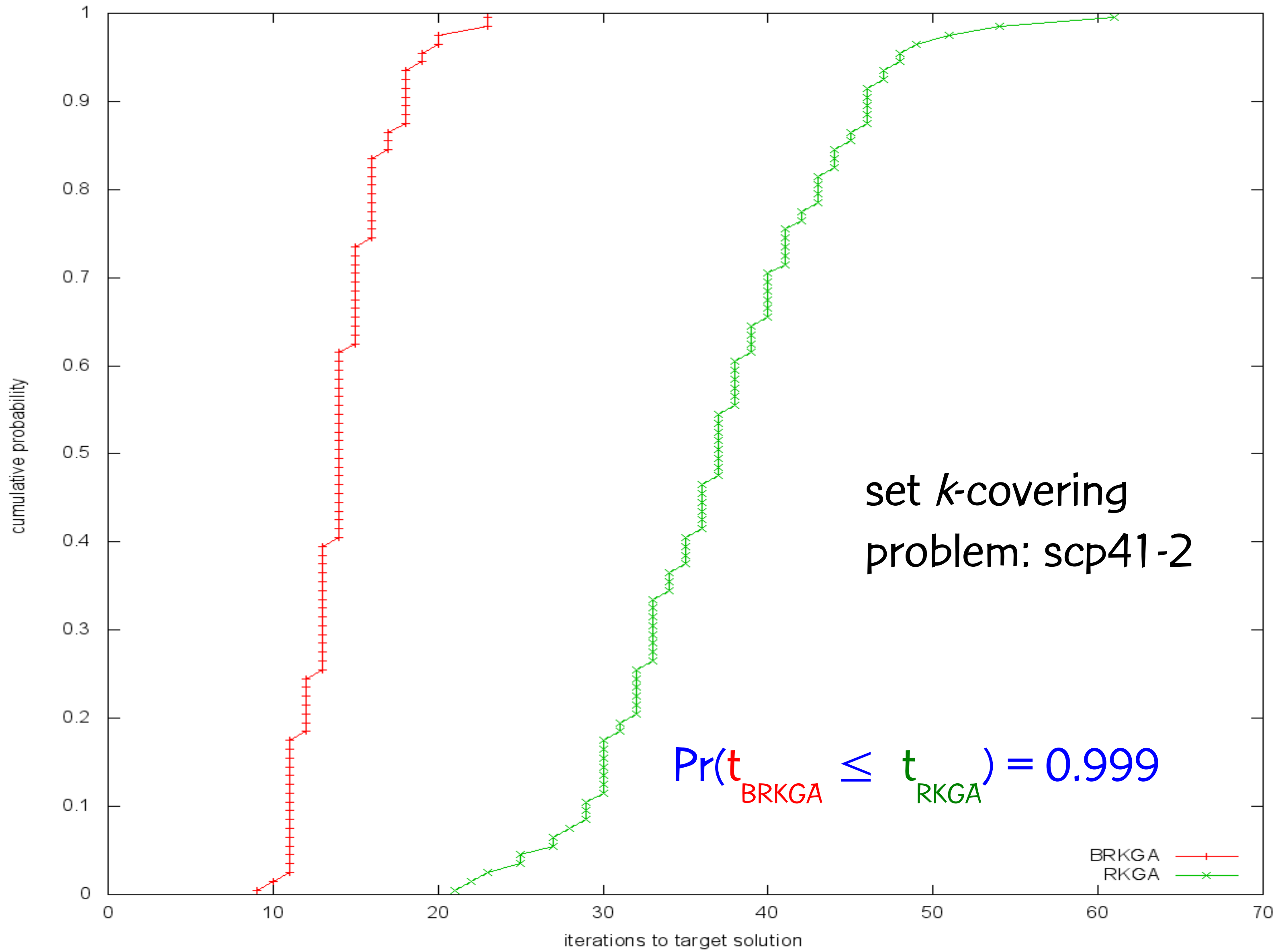
amazon



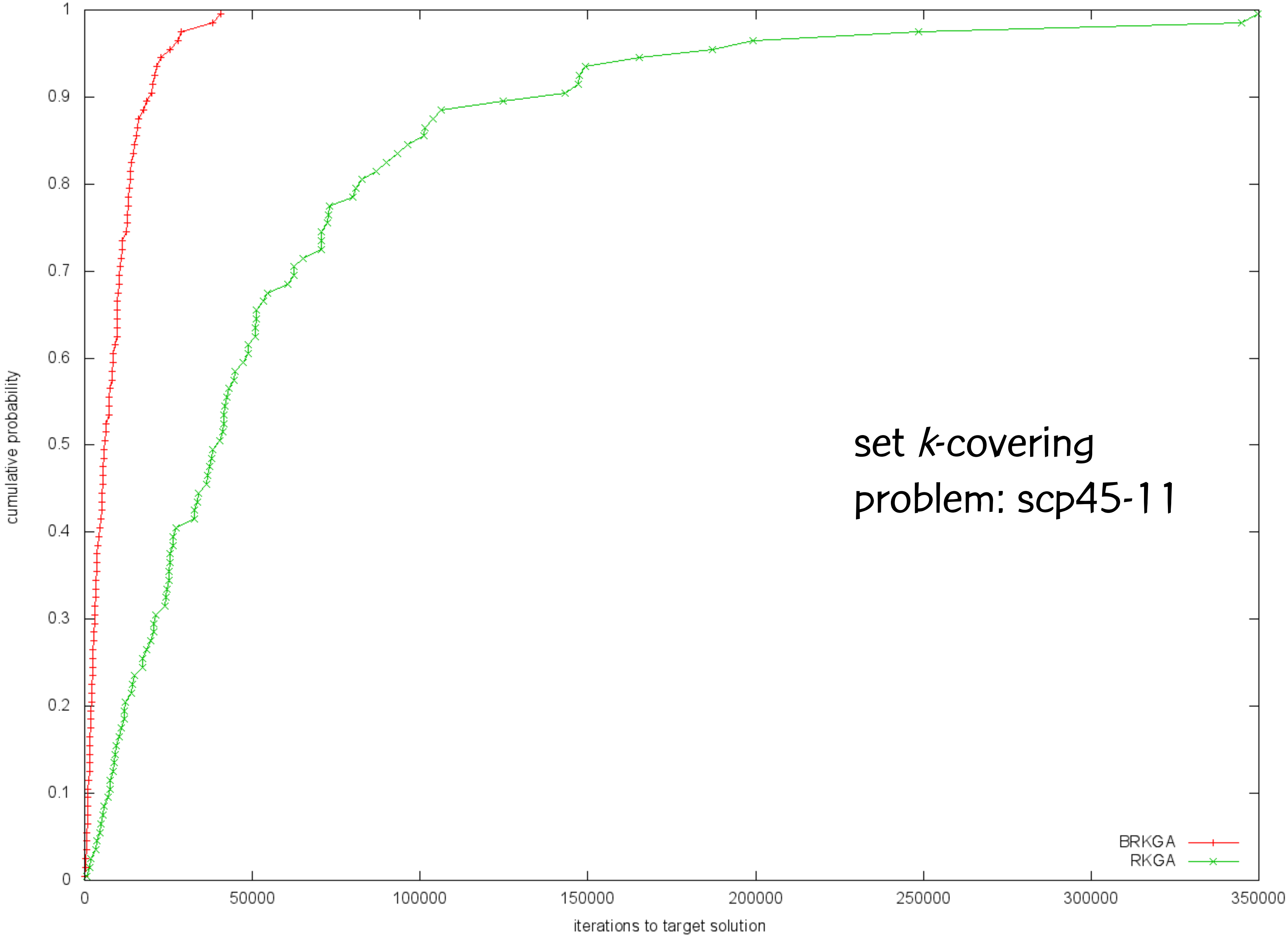
cumulative probability
iterations to target solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
BRKGA
RKGA
$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.733$
set covering
problem: scpa1

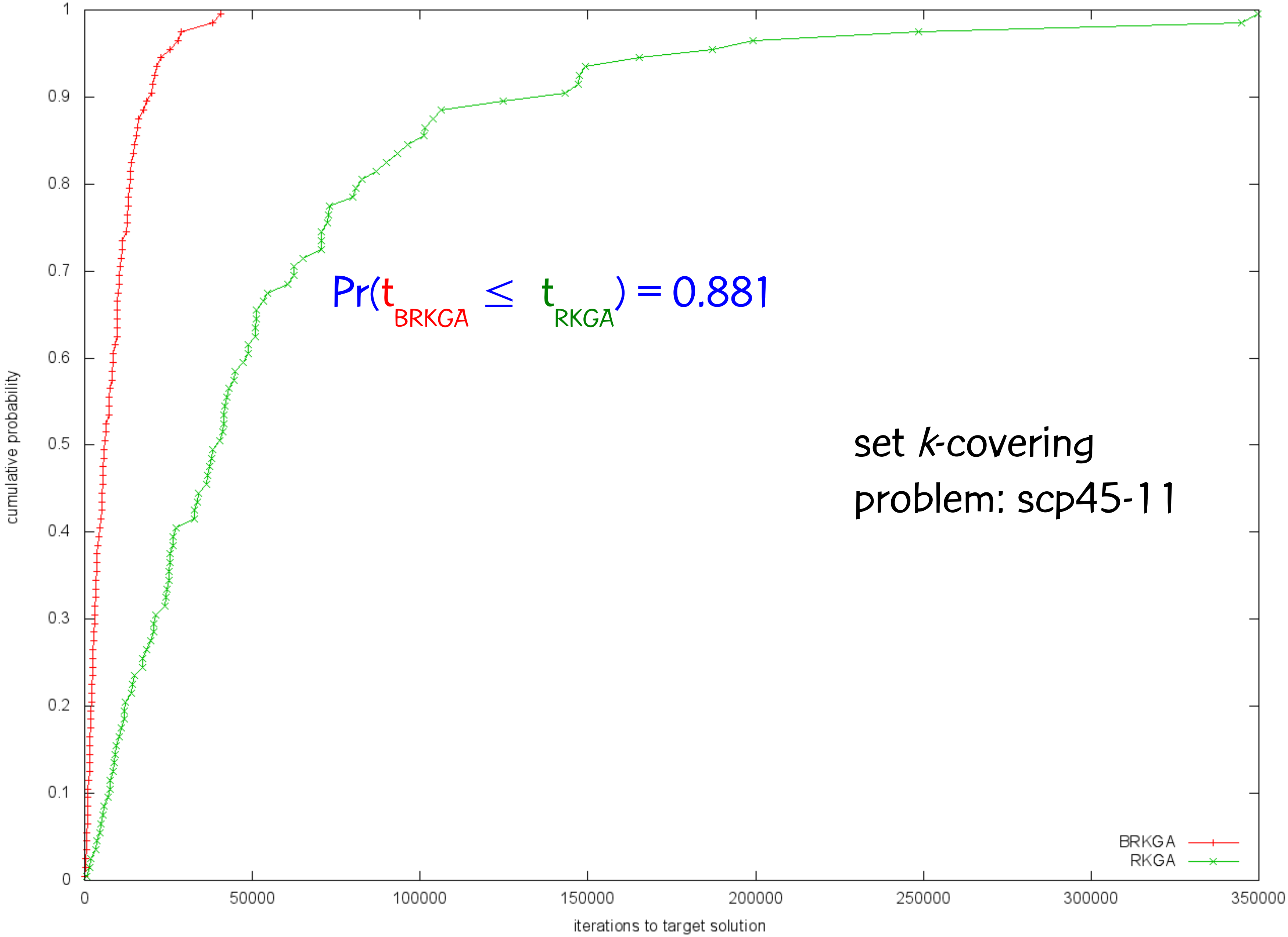


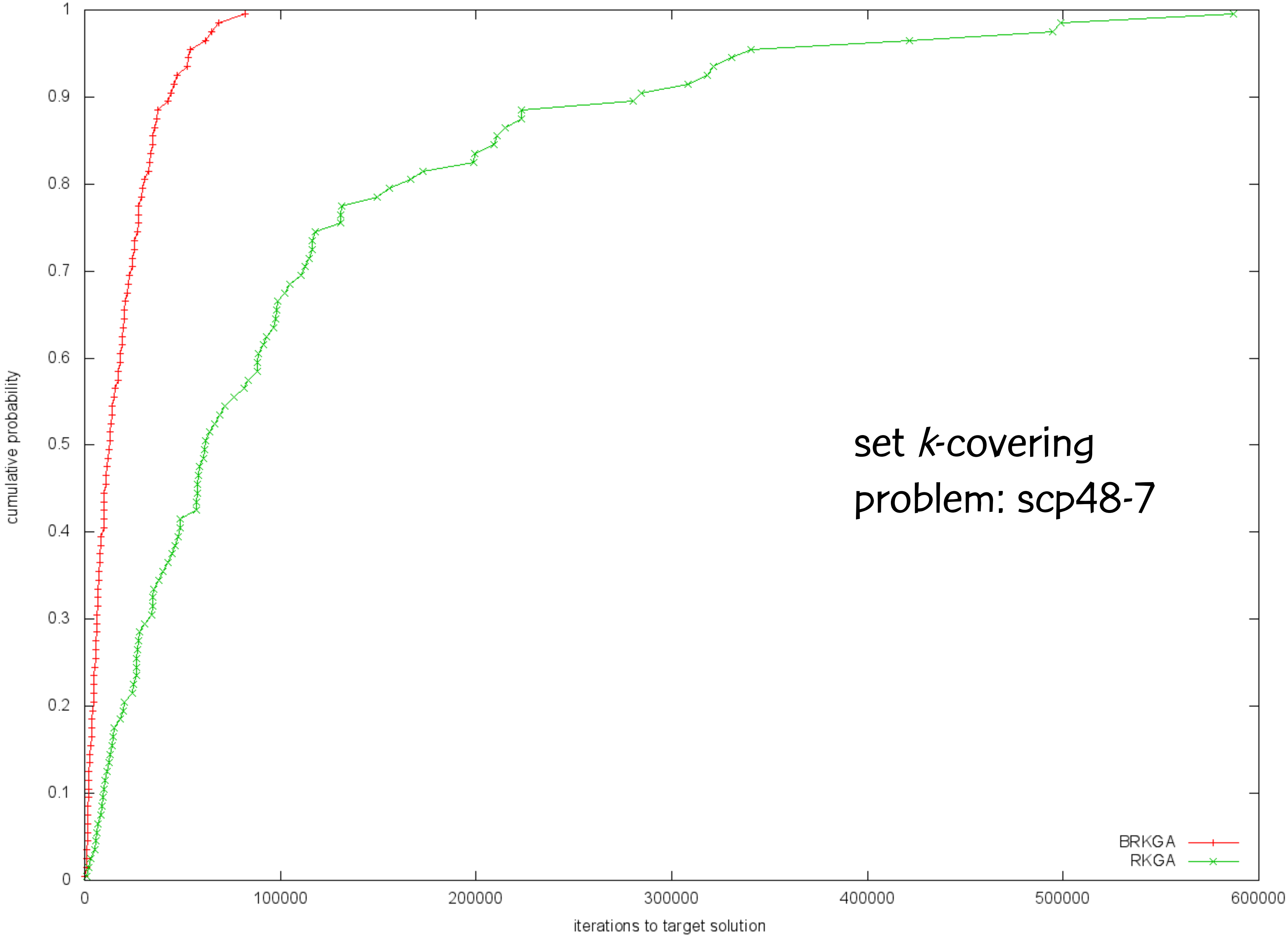
cumulative probability
iterations to target solution
BRKGA
RKGA
set k-covering problem: scp41-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70

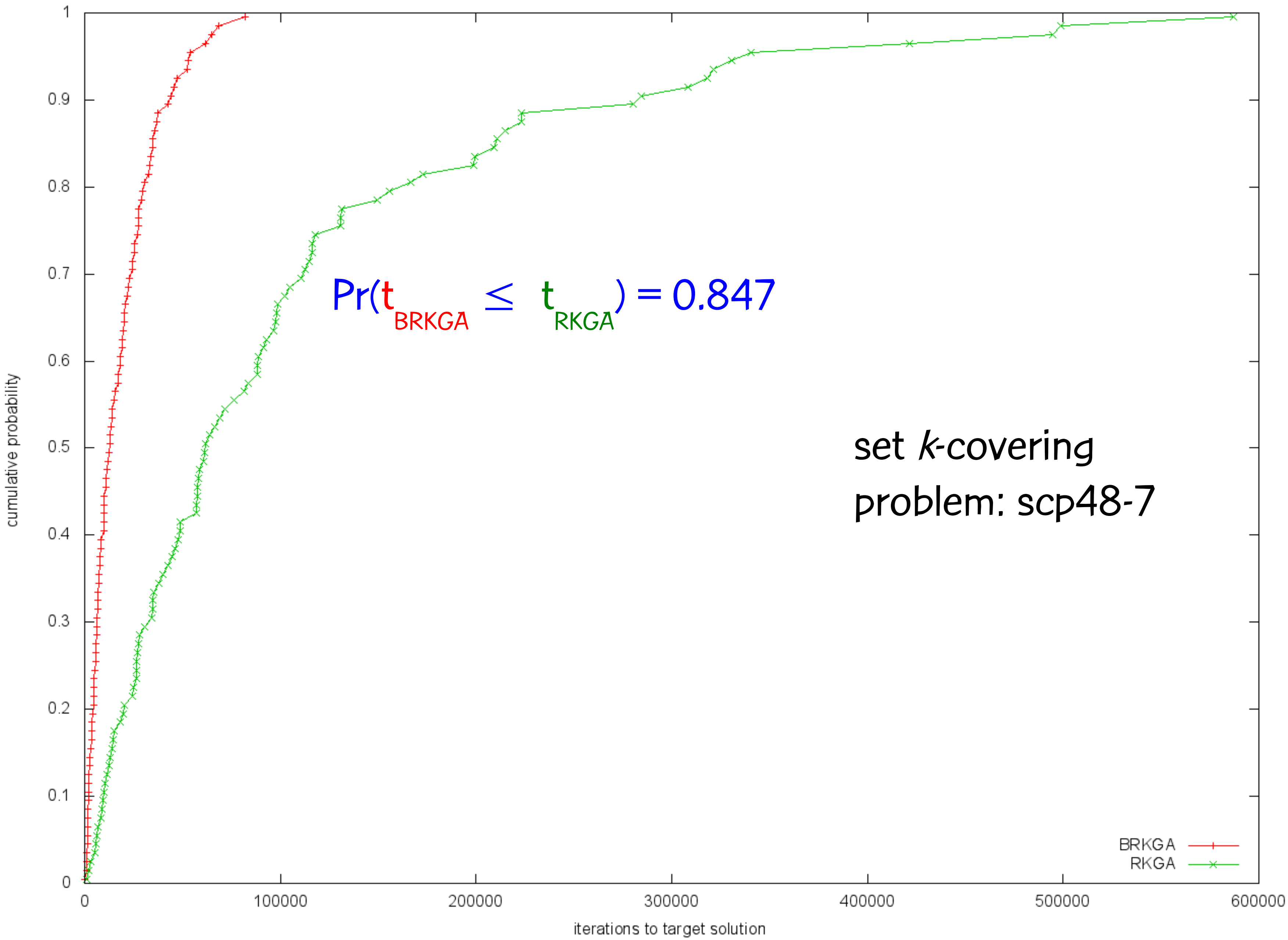


cumulative probability
iterations to target solution
BRKGA
RKGA
set k-covering
problem: scp41-2
Pr(t BRKGA ≤ t RKGA) = 0.999


set *k*-covering problem: scp45-11
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50000
100000
150000
200000
250000
300000
350000


$\Pr(t_{BRKGA} \leq t_{RKGA}) = 0.881$
set *k*-covering problem: scp45-11
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50000
100000
150000
200000
250000
300000
350000


set *k*-covering
problem: scp48-7
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
100000
200000
300000
400000
500000
600000



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
cumulative probability
0
100000
200000
300000
400000
500000
600000
iterations to target solution
BRKGA
RKGA
$Pr(t_{BRKGA} \le t_{RKGA}) = 0.847$
set *k*-covering
problem: scp48-7

# Observations

- Random method: keys are randomly generated so solutions are always vectors of random keys

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- Elitist strategy: best solutions are passed without change from one generation to the next (incumbent is kept)

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- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent $> 0.5$ Not so in the RKGA of Bean.

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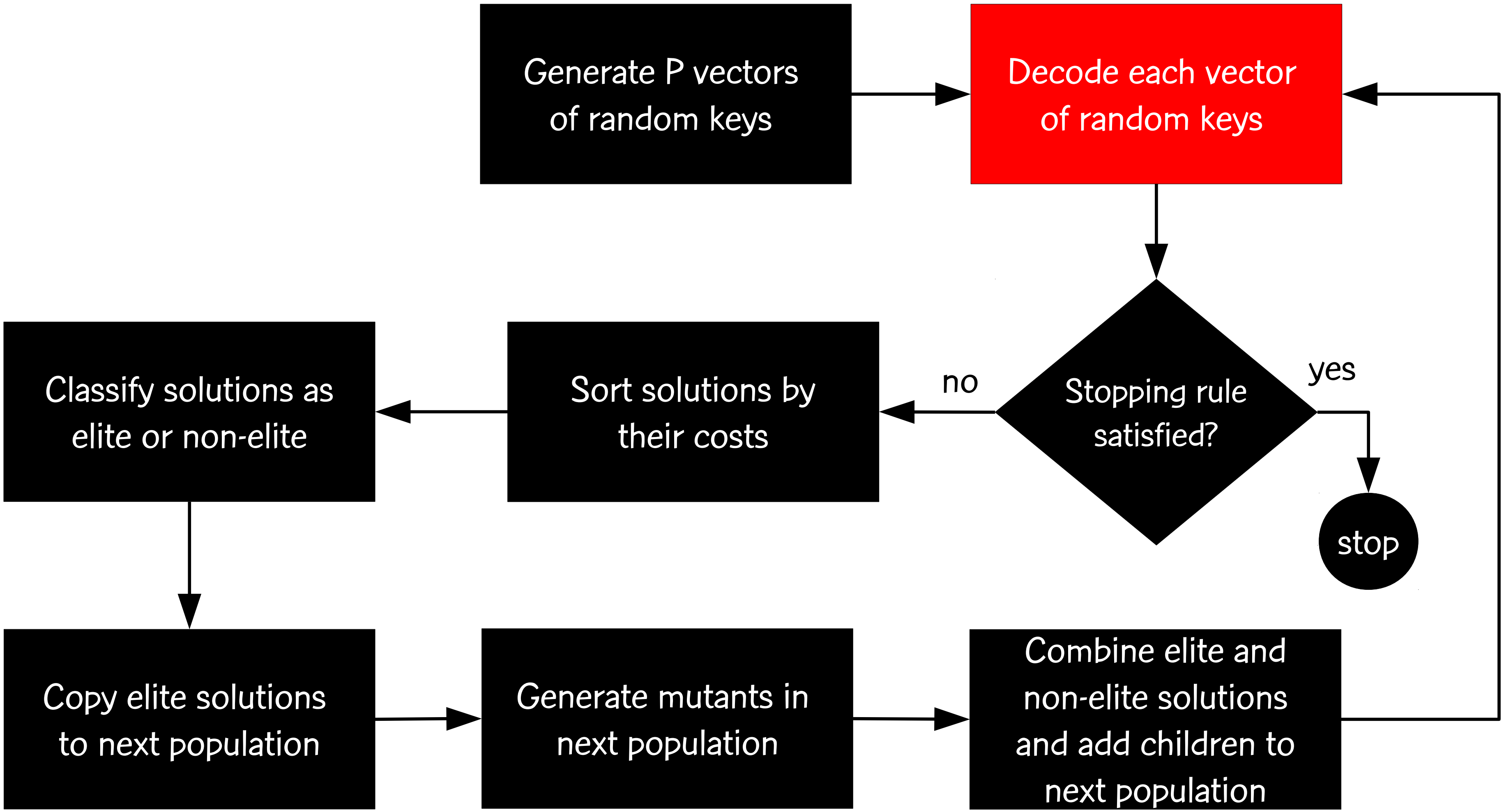
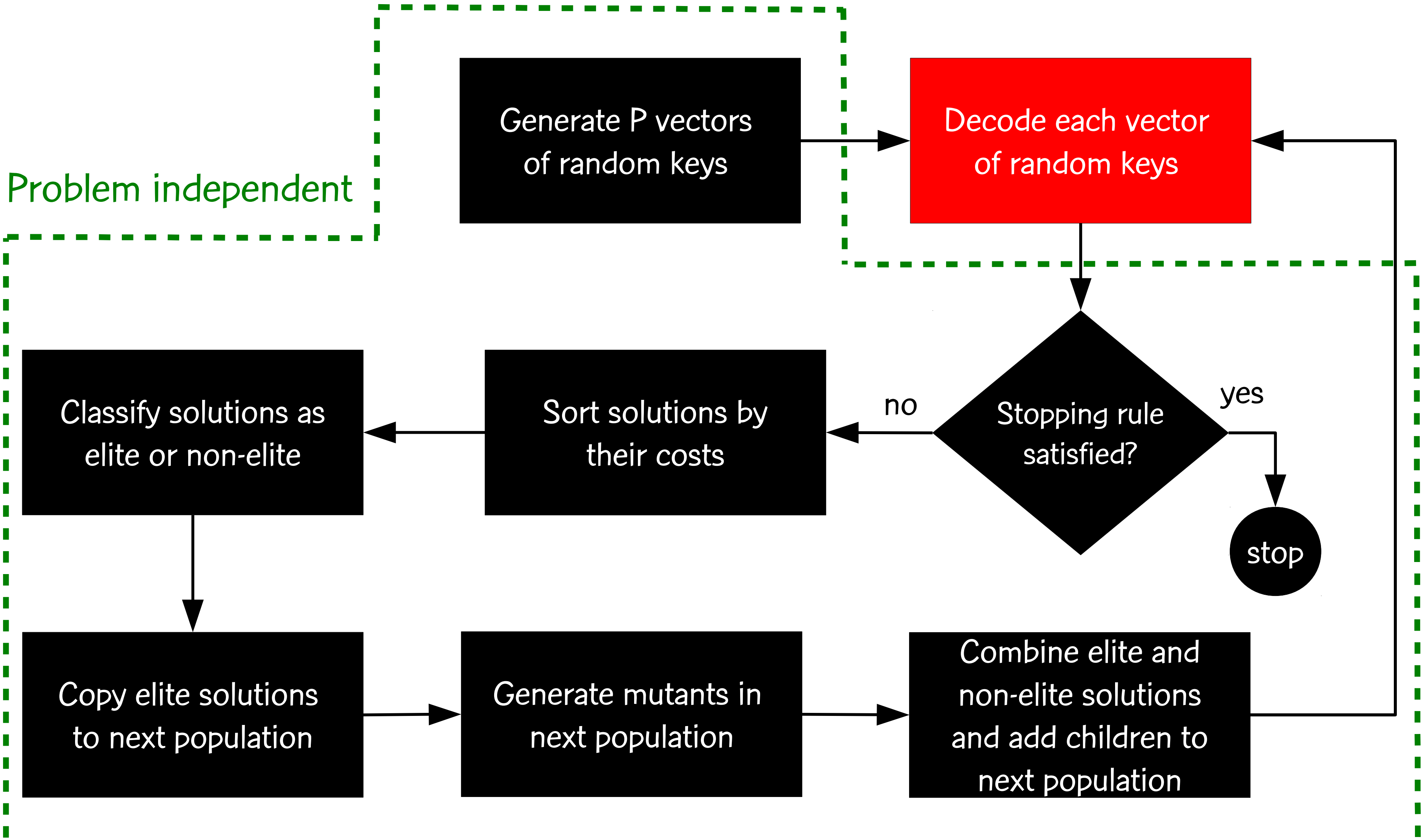
- Elitist strategy: best solutions are passed without change from one generation to the next (incumbent is kept)
- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent $> 0.5$ Not so in the RKGA of Bean.
- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs … help escape local optima)

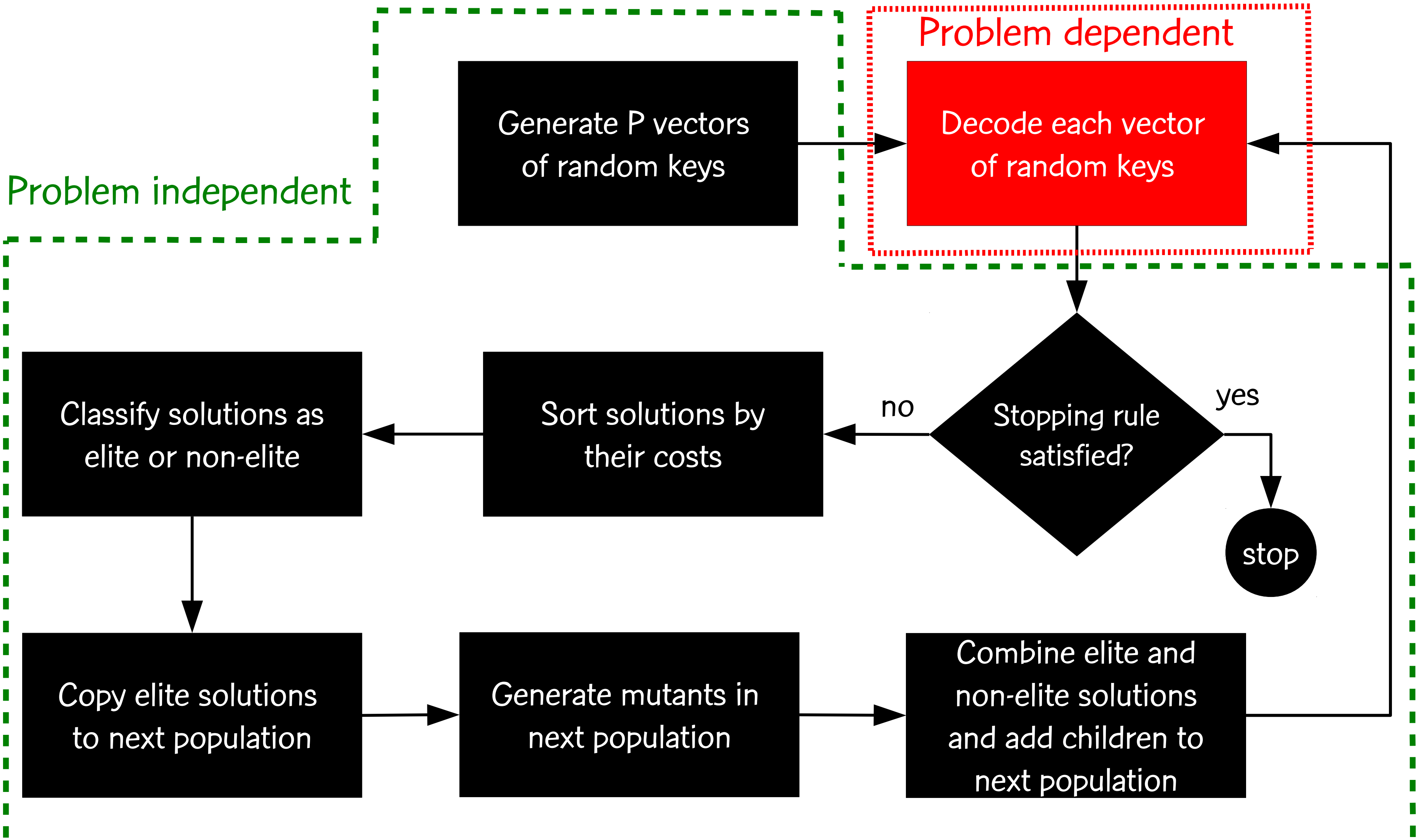
# Framework for biased random-key genetic algorithms



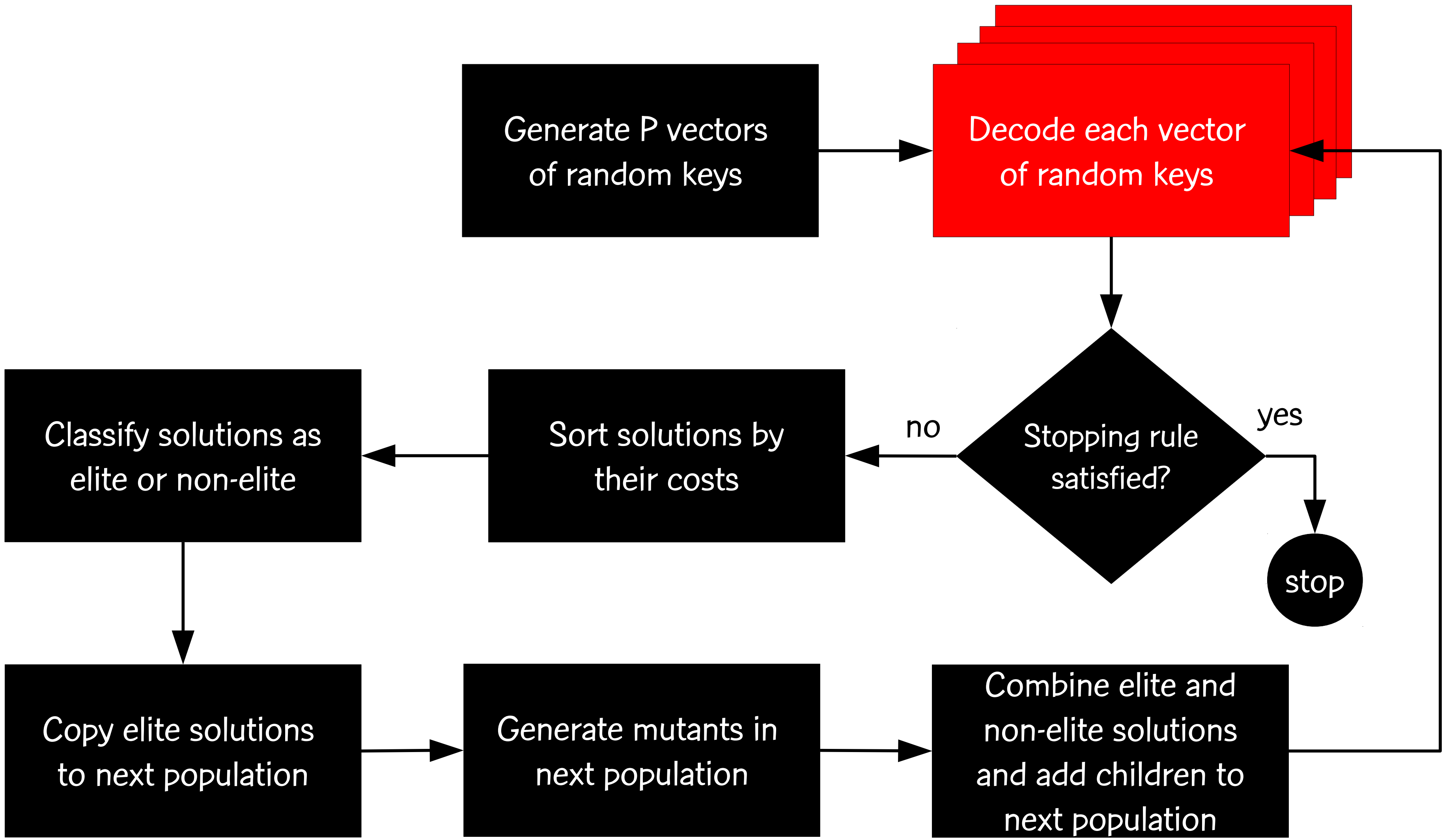
# Framework for biased random-key genetic algorithms



# Framework for biased random-key genetic algorithms



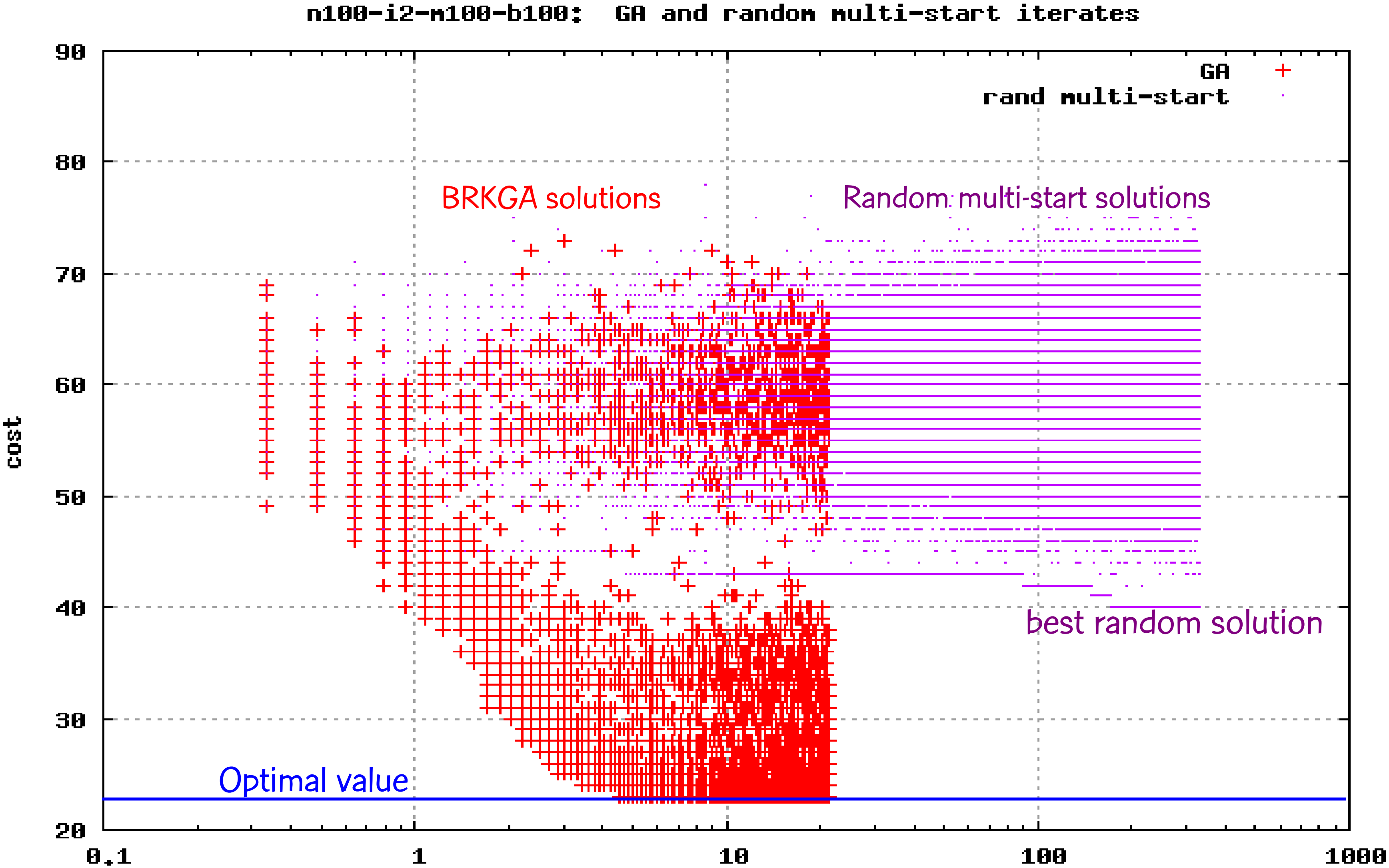
# Decoding of random key vectors can be done in parallel



# Is a BRKGA any different from applying the decoder to random keys?

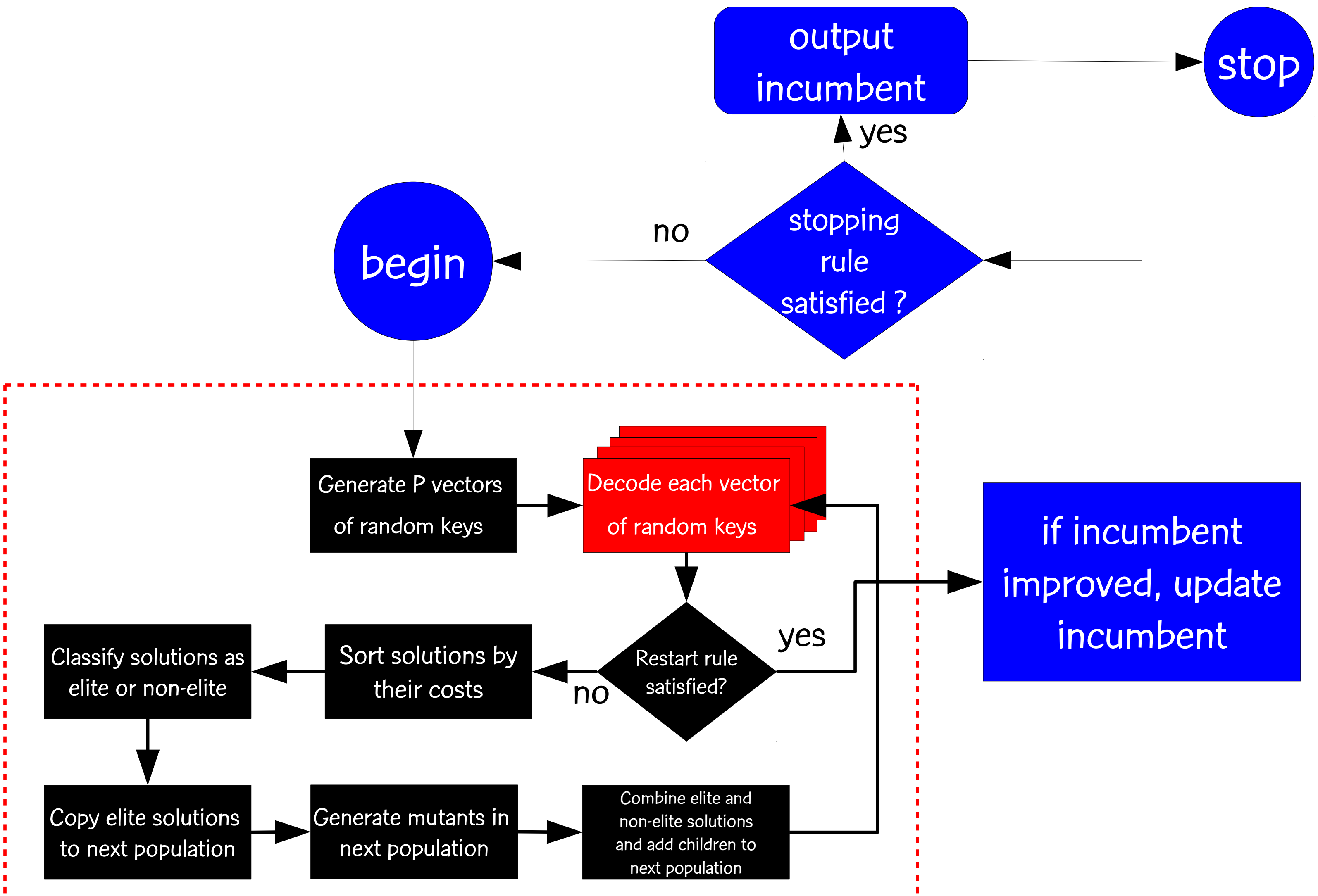
- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to P–1
- Each iteration, best solution is maintained in elite set and P–1 random key vectors are generated as mutants … no mating is done since population already has P individuals

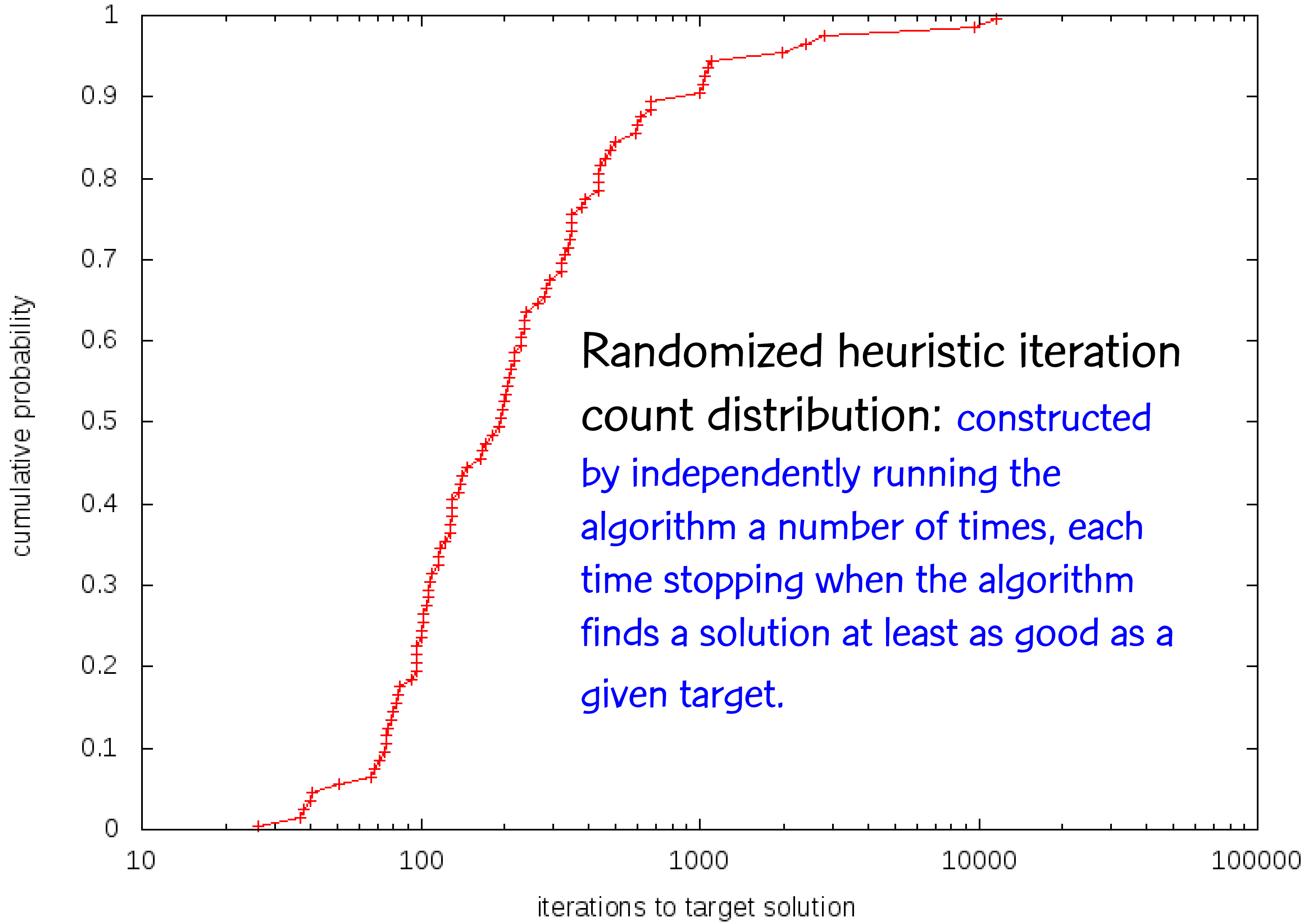
solution



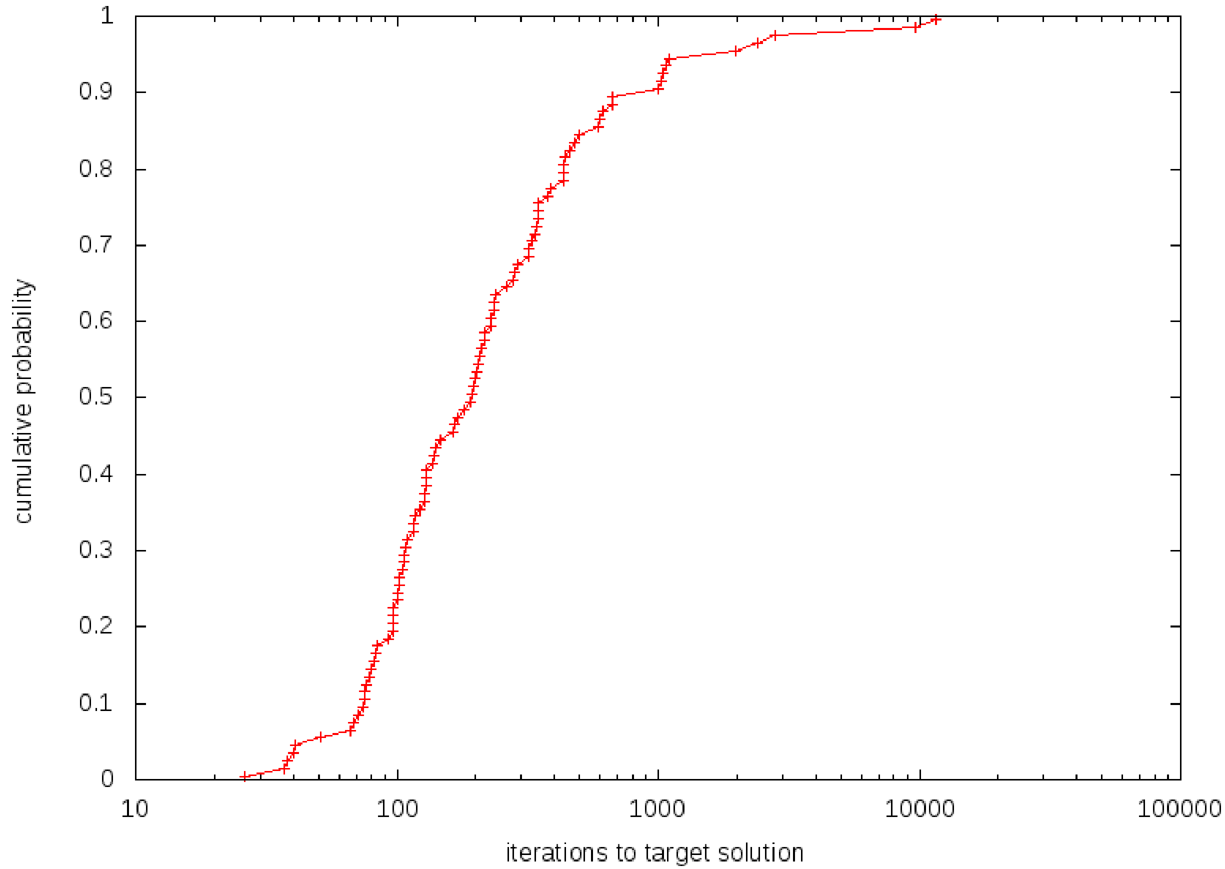

Time (ibm t41 secs)

# BRKGA in multi-start strategy

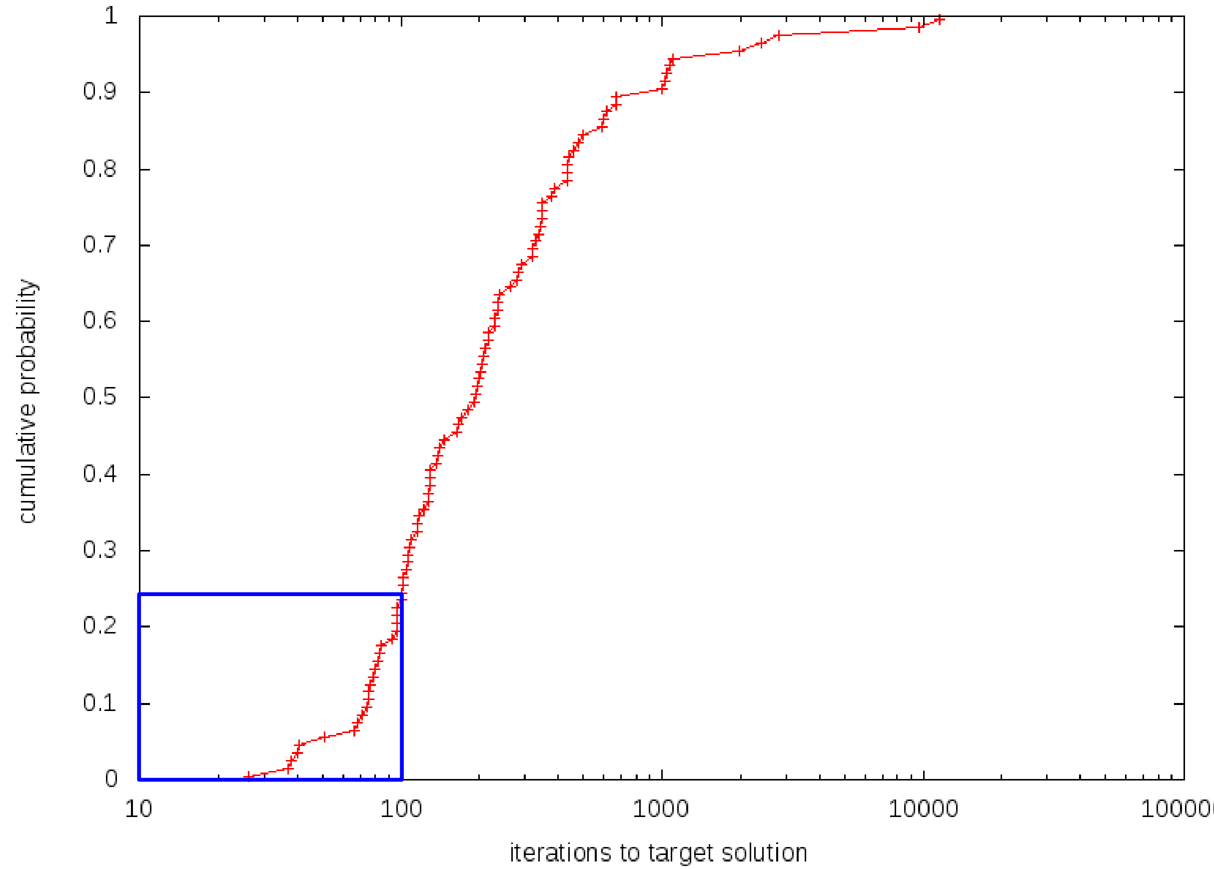




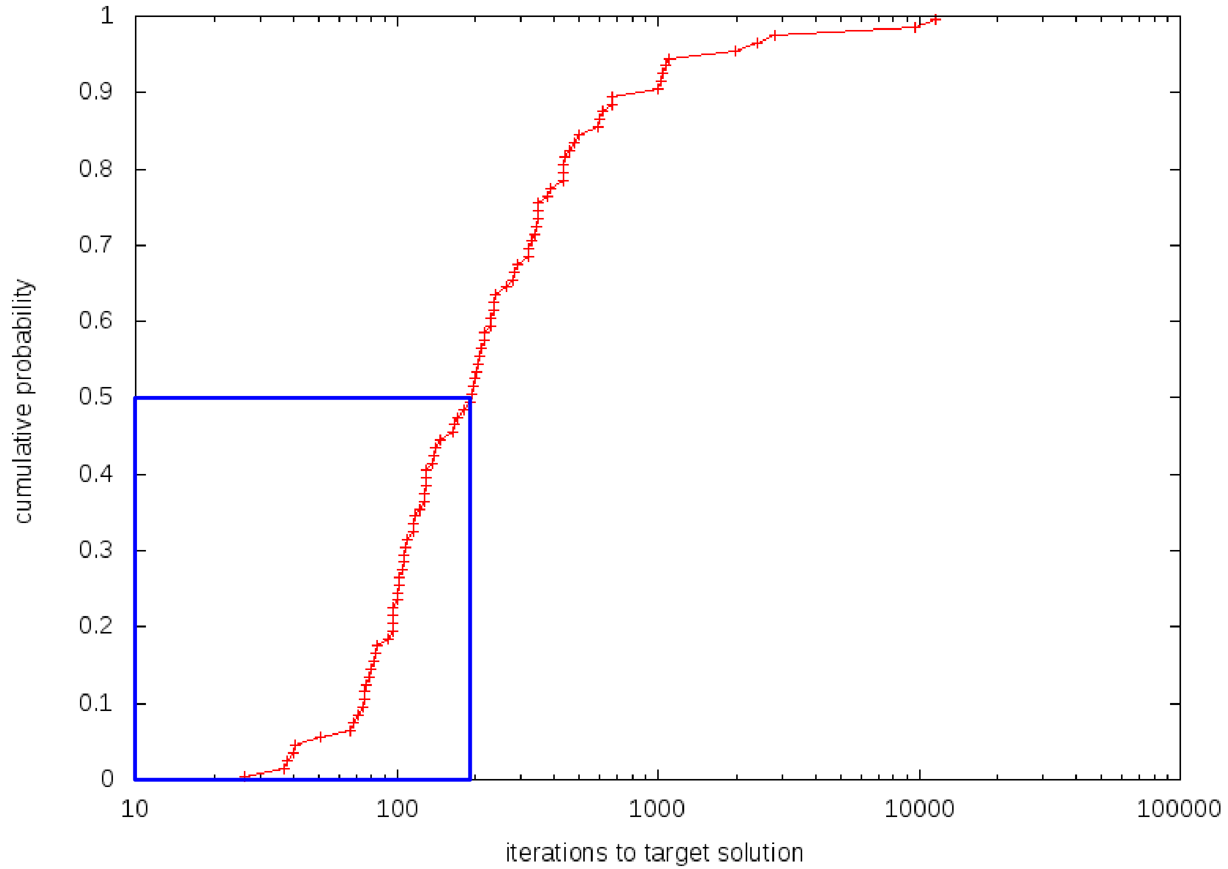

Randomized heuristic iteration count distribution: constructed by independently running the algorithm a number of times, each time stopping when the algorithm finds a solution at least as good as a given target.



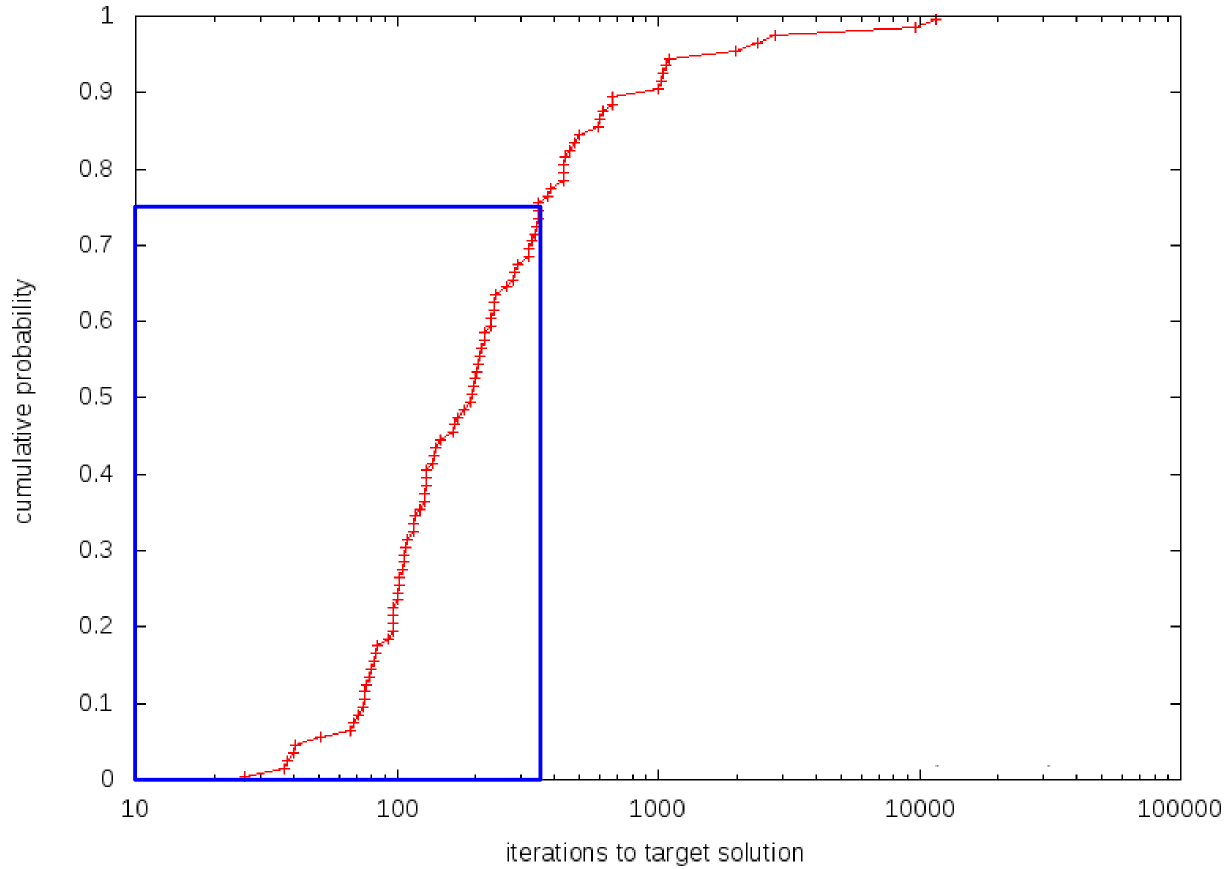

In most of the independent runs, the algorithm finds the target solution in relatively few iterations:



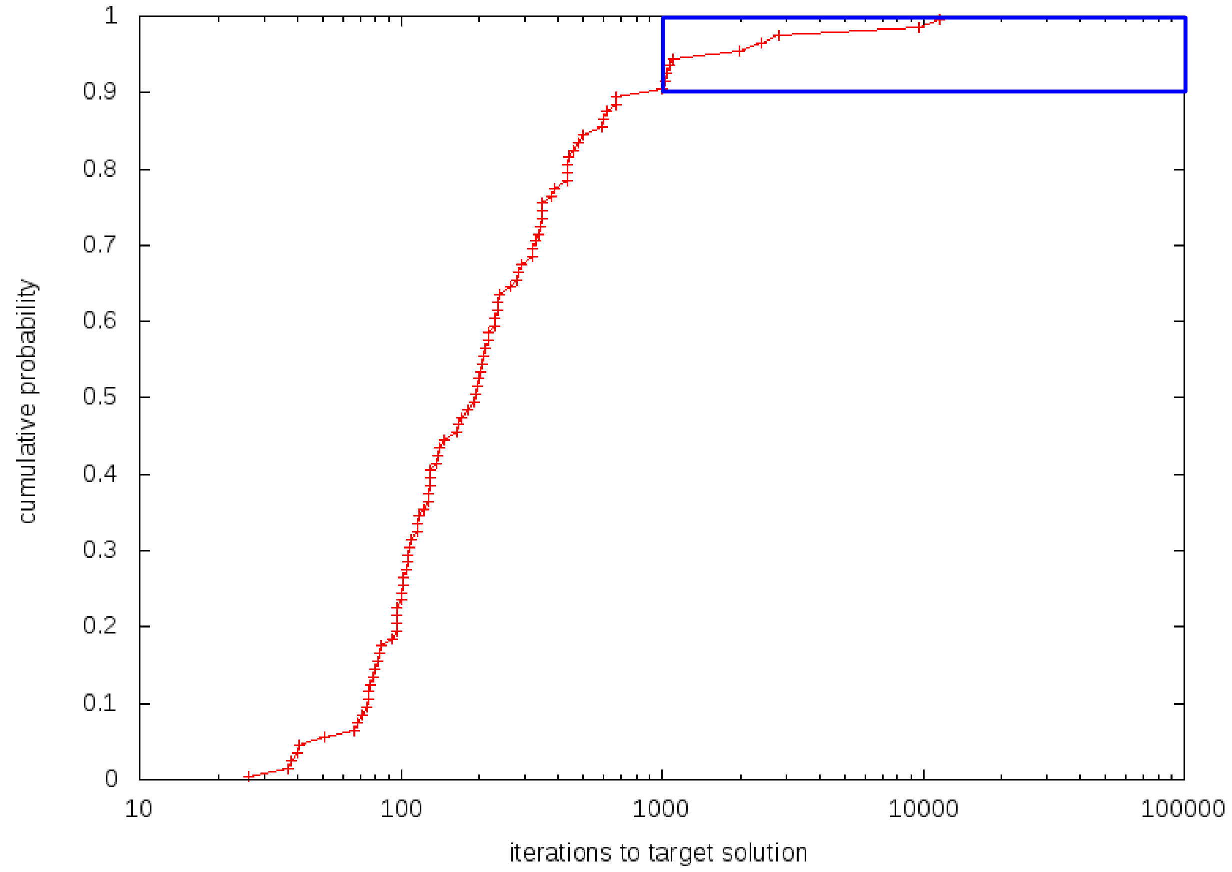

In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 25% of the runs take fewer than 101 iterations



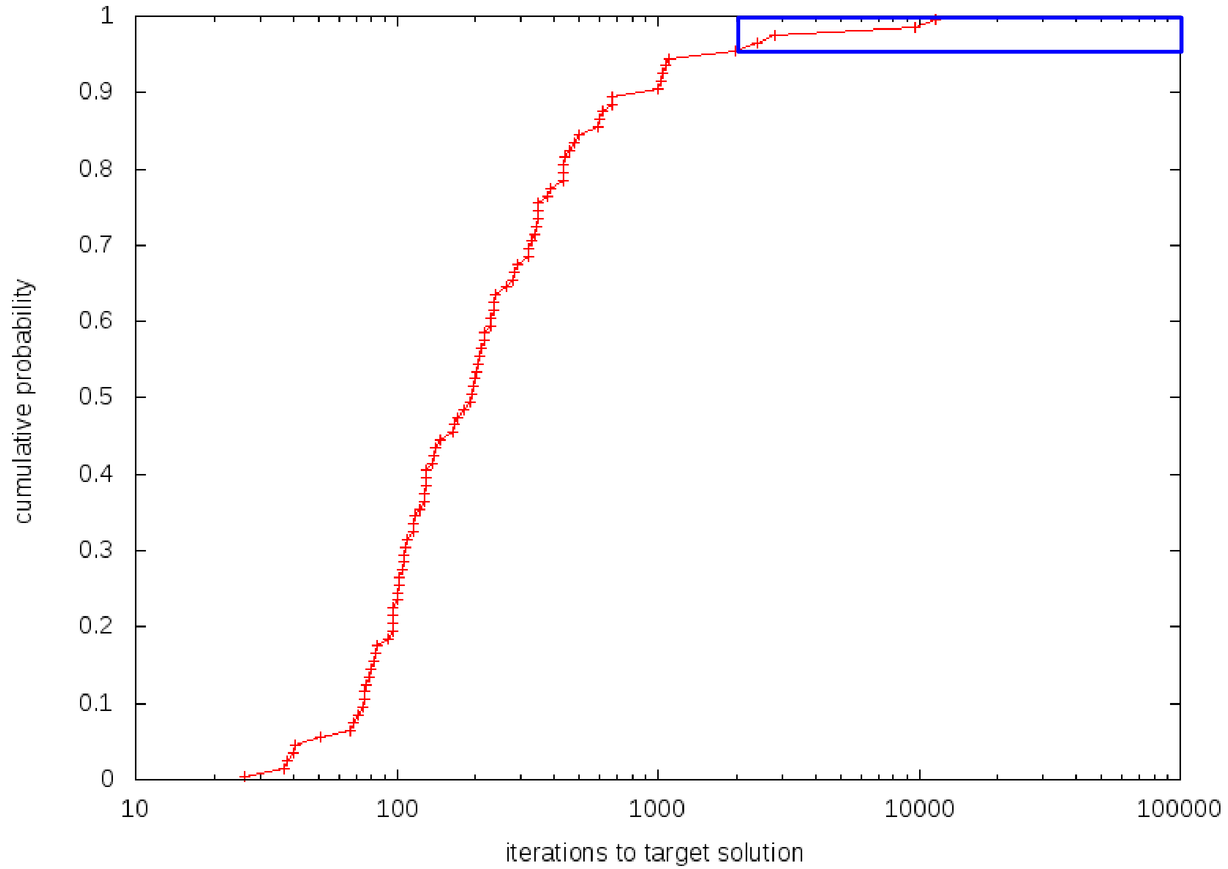

In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 50% of the runs take fewer than 192 iterations




In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 75% of the runs take fewer than 345 iterations

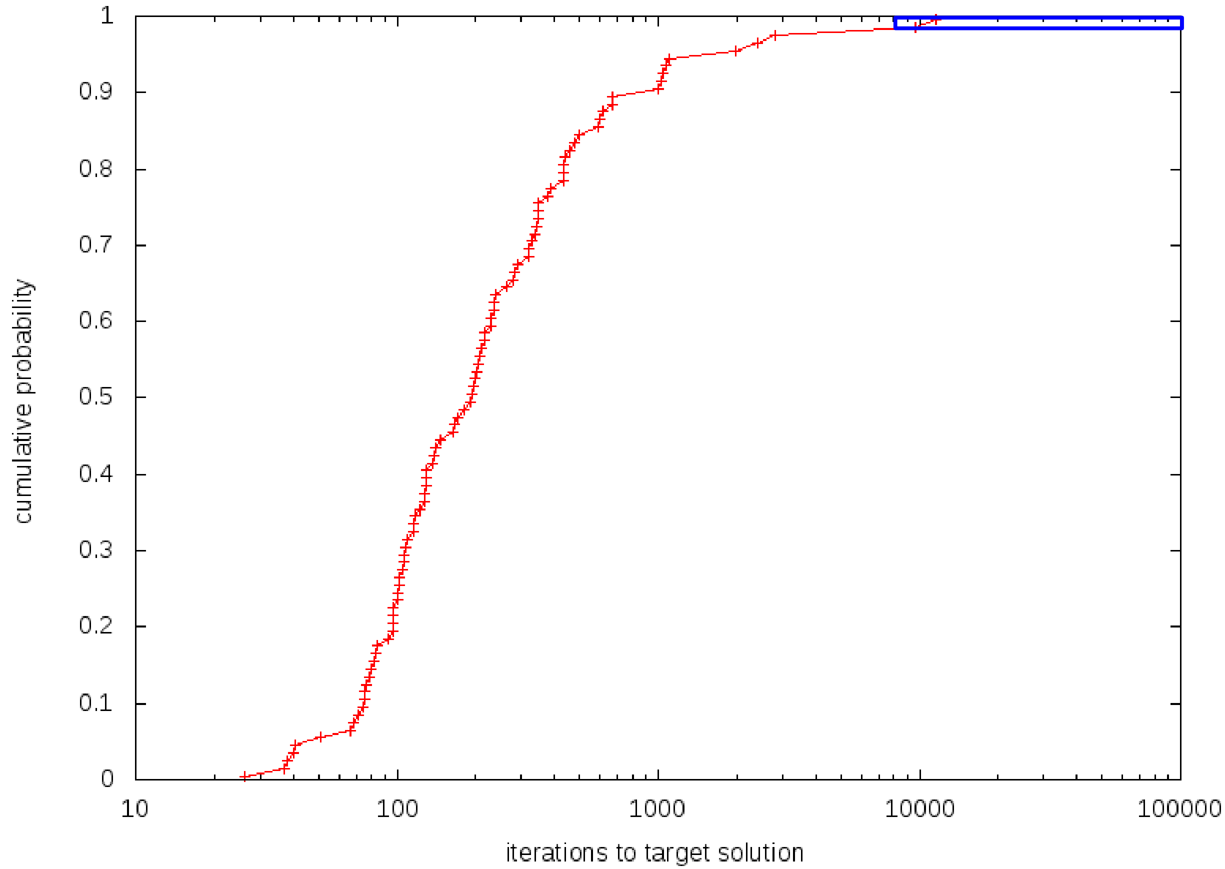



However, some runs take much longer: 10% of the runs take over 1000 iterations

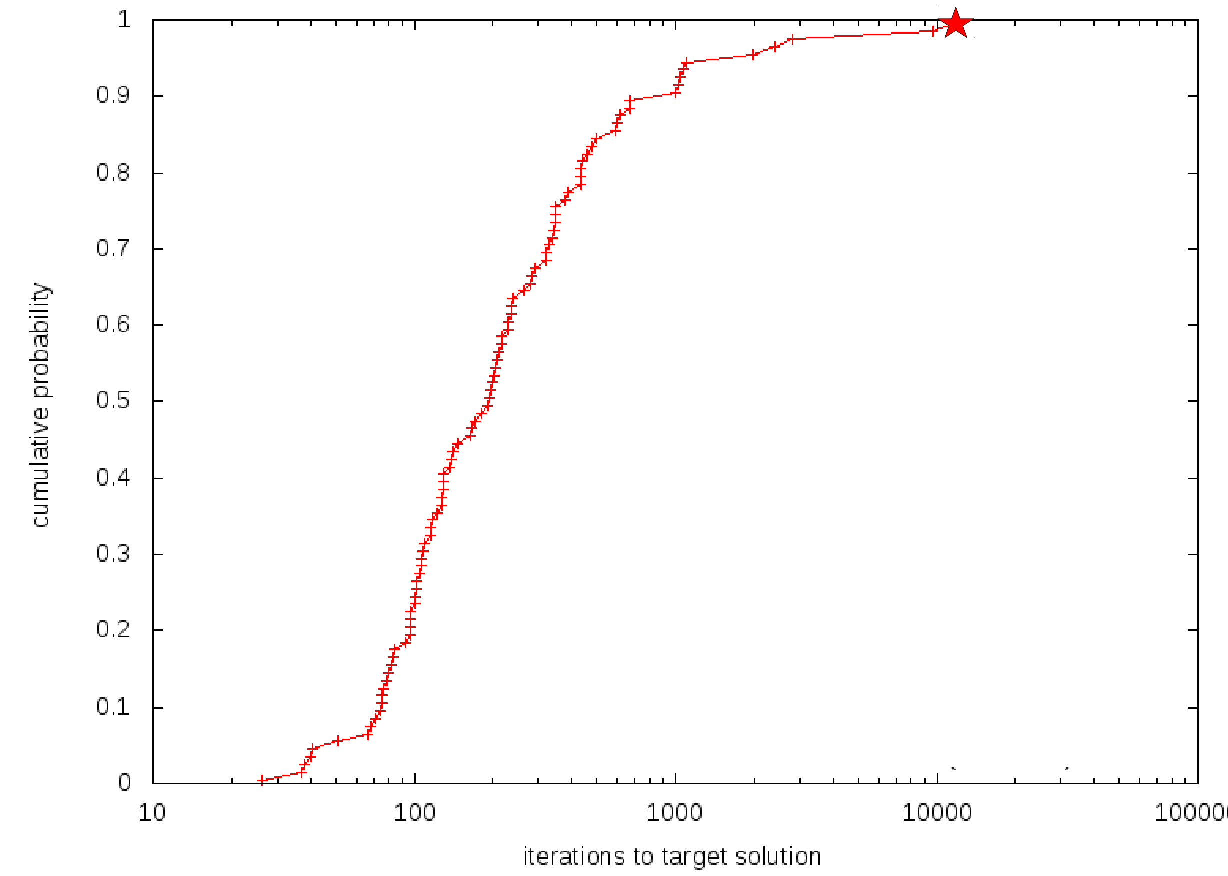



However, some runs take much longer: 5% of the runs take over 2000 iterations

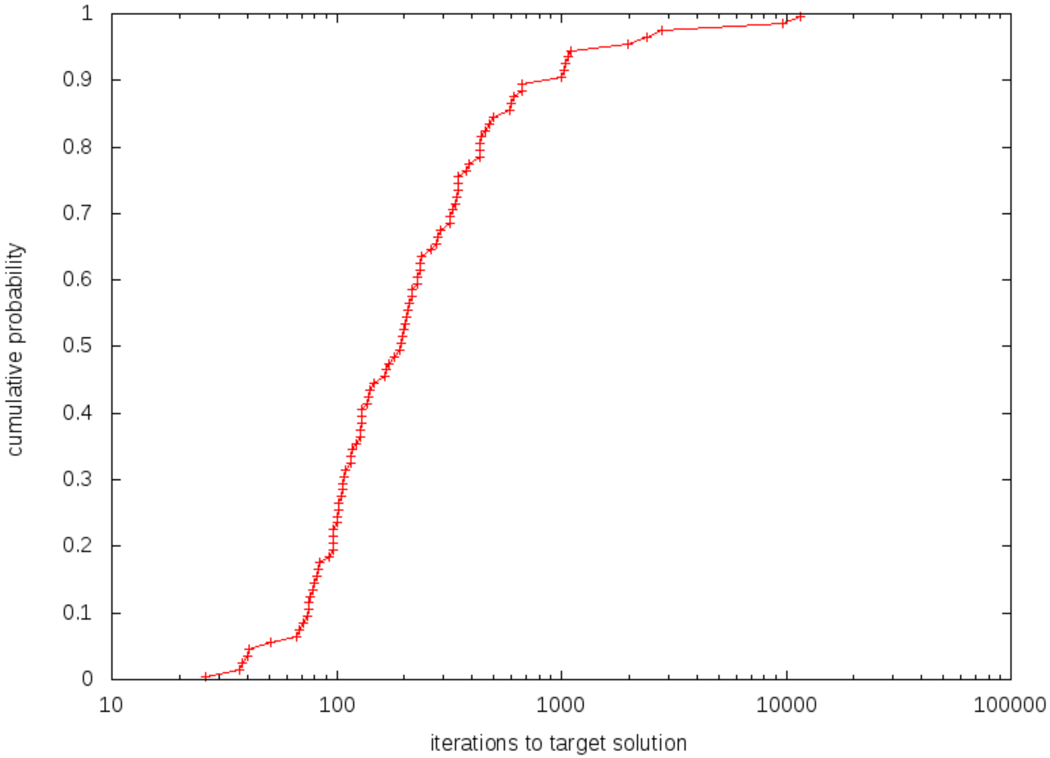

amazon



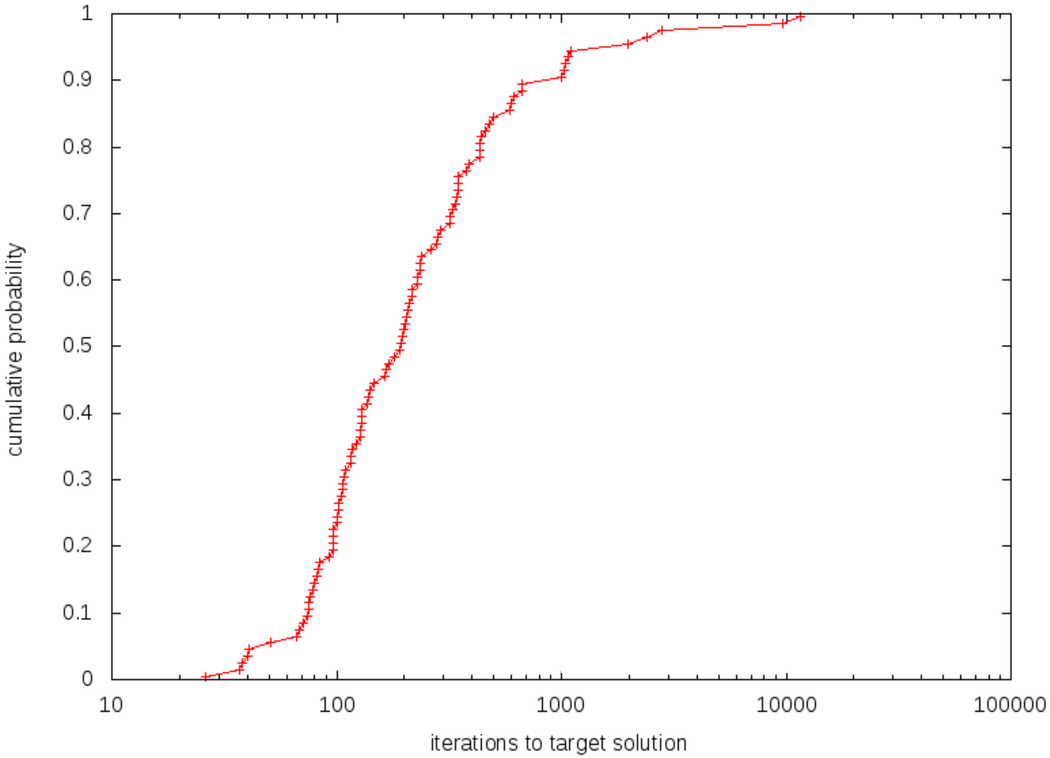

However, some runs take much longer: 2% of the runs take over 9715 iterations




However, some runs take much longer: the longest run took 11607 iterations



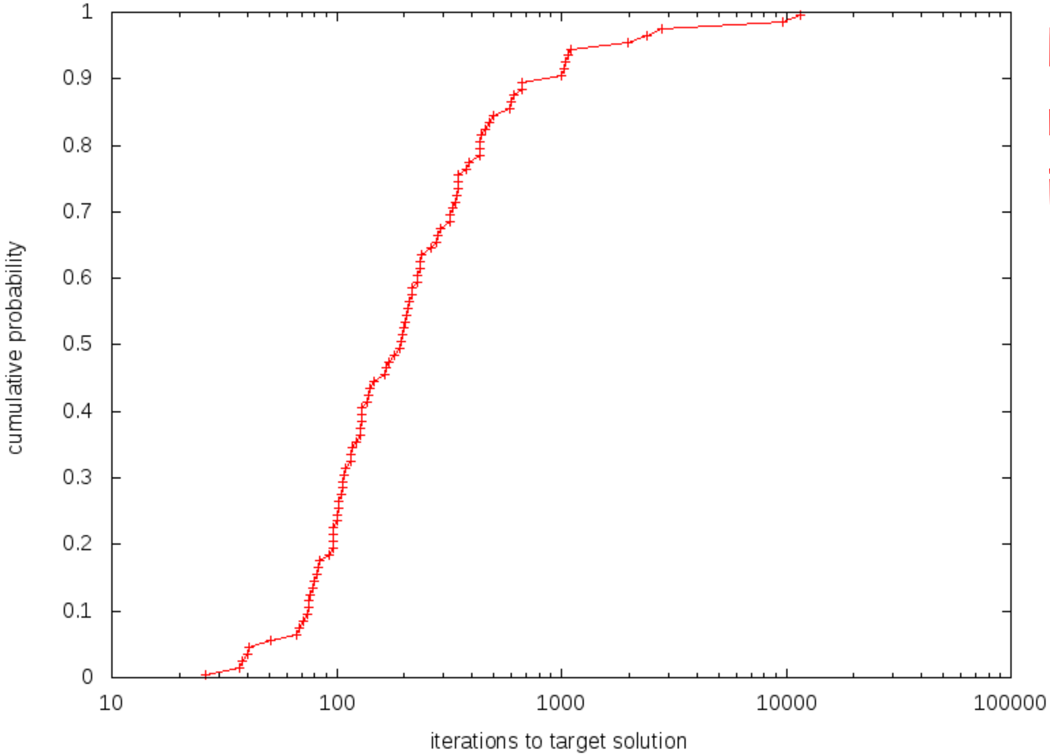

Probability that algorithm will take over 345 iterations: 25% = 1/4



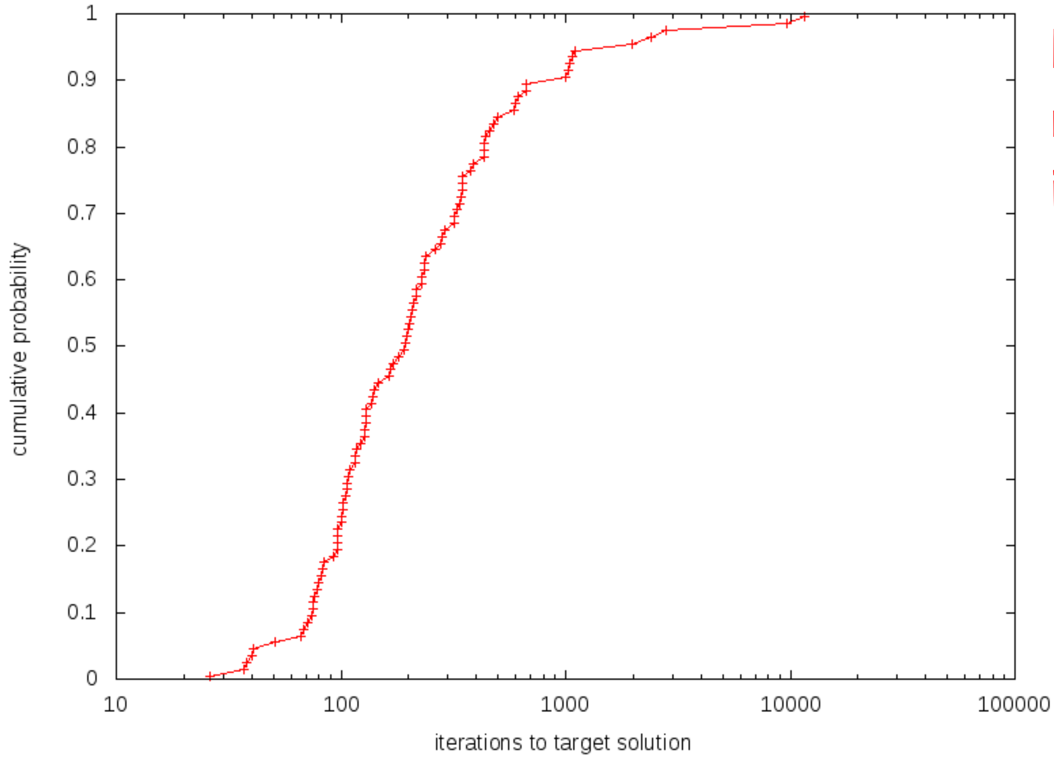

Probability that algorithm will take over 345 iterations: 25% = 1/4

By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: 25% = 1/4

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 X probability of taking over 690 iterations given it took over 345 = ¼ x ¼ = $1/4^2$

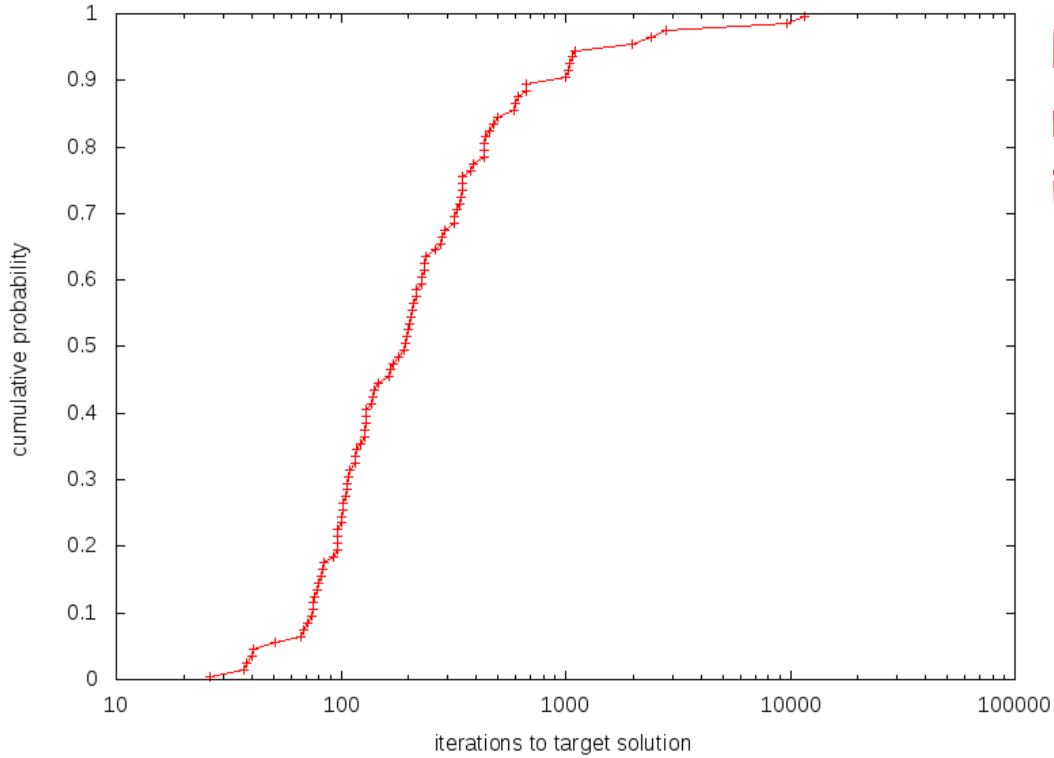



Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$




Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong$ 0.0977%




Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong$ 0.0977%

This is much less than the 5% probability that the algorithm without restart will take over 2000 iterations.

# Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S = \{\tau_1, \tau_2, \tau_3, \ldots\}$ which define epochs $\tau_1$, $\tau_1+\tau_2$, $\tau_1+\tau_2+\tau_3$, ... when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$, where $\tau^*$ is a constant.

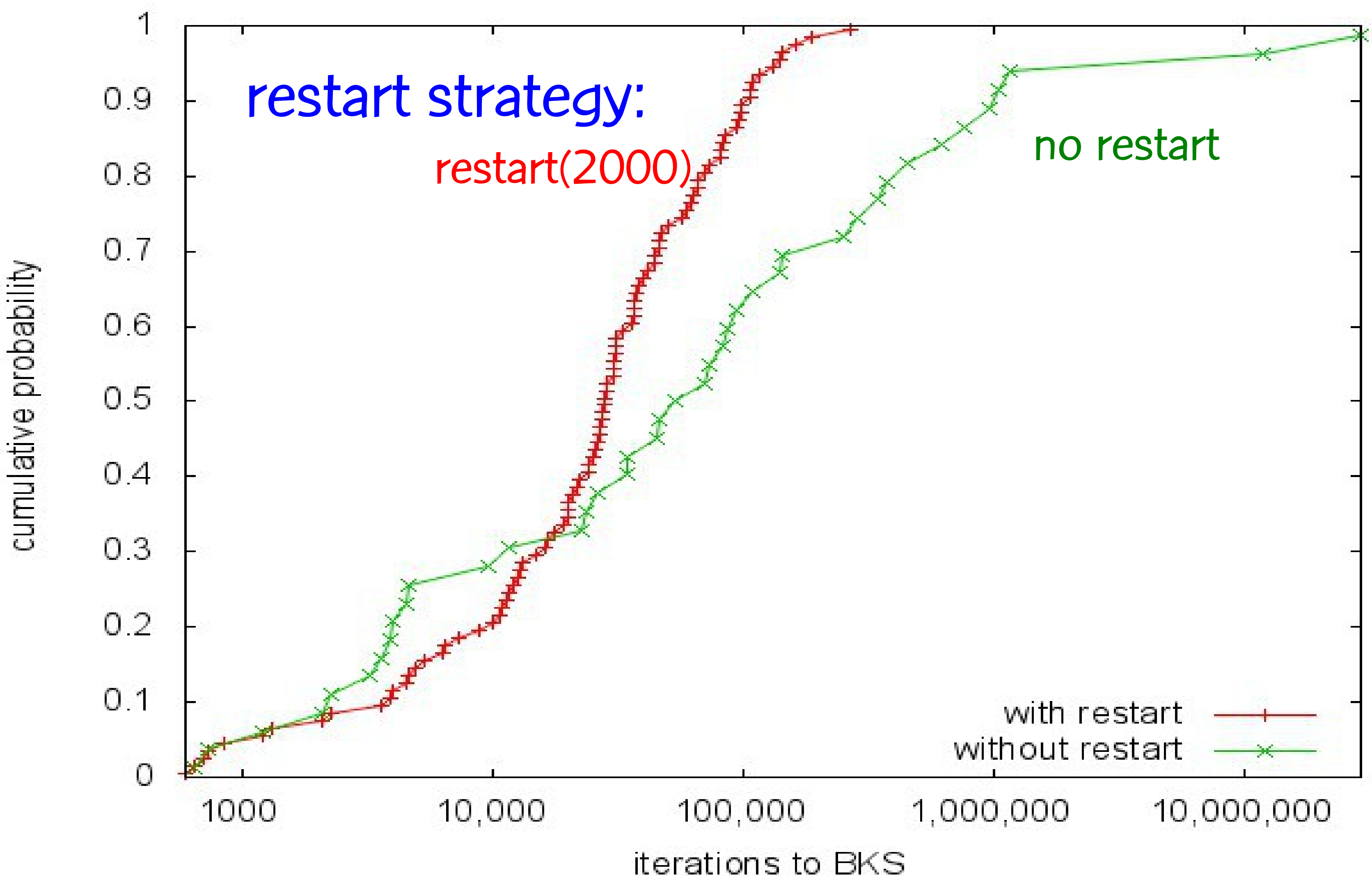
# Restart strategy for BRKGA

- Recall the restart strategy of Luby et al. where equal time intervals $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$ pass between restarts.
- Strategy requires $\tau^*$ as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
  - choosing $\tau^*$ too small: restart variant may take long to converge
  - choosing $\tau^*$ too big: restart variant may become like no-restart variant

# Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy restart(K)

# Example of restart strategy for BRKGA: Load balancing



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# Specifying a BRKGA

# Specifying a biased random-key GA

- Encoding is always done the same way, i.e. with a vector of N random-keys (parameter N must be specified)

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- Parameters

# Specifying a biased random-key GA

## Parameters:

– Size of population
– Size of elite partition
– Size of mutant set
– Child inheritance probability
– Restart strategy parameter
– Stopping criterion

# Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N, say N or 2N
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- Size of mutant set
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# Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N, say N or 2N
- Size of elite partition: 15-25% of population
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- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

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## Parameters:

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- Stopping criterion

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- Stopping criterion

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## Parameters:

- Size of population: a function of N, say N or 2N
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- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N, say 2N or 10N
- Stopping criterion: e.g. time, # generations, solution quality, # generations without improvement

# brkgaAPI: A C++ API for BRKGA

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.

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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
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  - evolutionary dynamics

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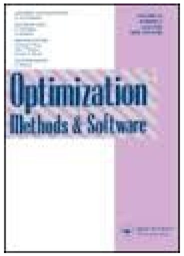
  - population management
  - evolutionary dynamics
- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.

# brkgaAPI: A C++ API for BRKGA

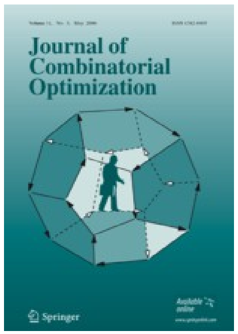



Paper: Rodrigo F. Toso and M.G.C.R.,
"A C++ Application Programming Interface for Biased Random-Key Genetic Algorithms,"
Optimization Methods & Software, vol. 30, pp. 81-93, 2015.

Software: http://mauricio.resende.info/src/brkgaAPI

# An example BRKGA: Packing weighted rectangles

# Reference




J.F. Gonçalves and M.G.C.R., "A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal packing problem," Journal of Combinatorial Optimization, vol. 22, pp. 180-201, 2011.

Tech report:
http://mauricio.resende.info/doc/pack2d.pdf

# Constrained orthogonal packing

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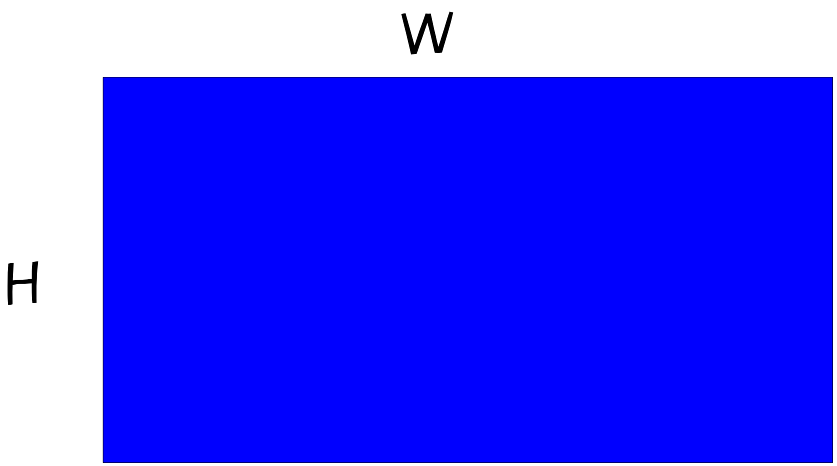
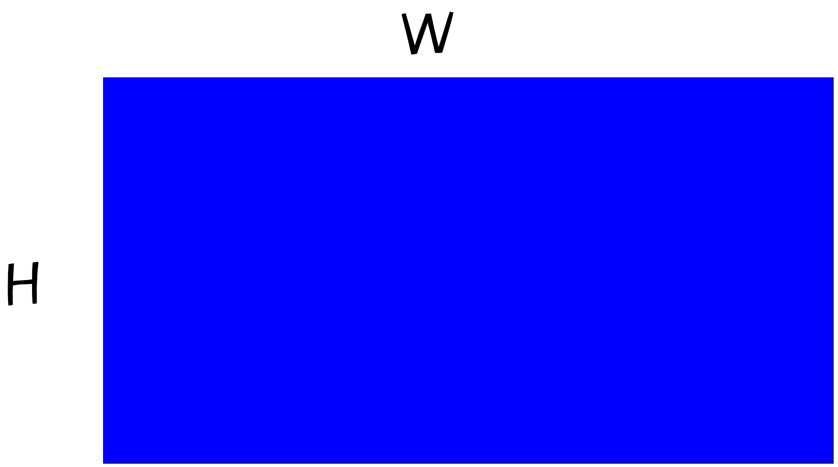


# Constrained orthogonal packing

- Given a large planar stock rectangle (W, H) of width W and height H;
- Given N smaller rectangle types (w[i], h[i]), i = 1,...,N, each of width w[i], height h[i], and value v[i];



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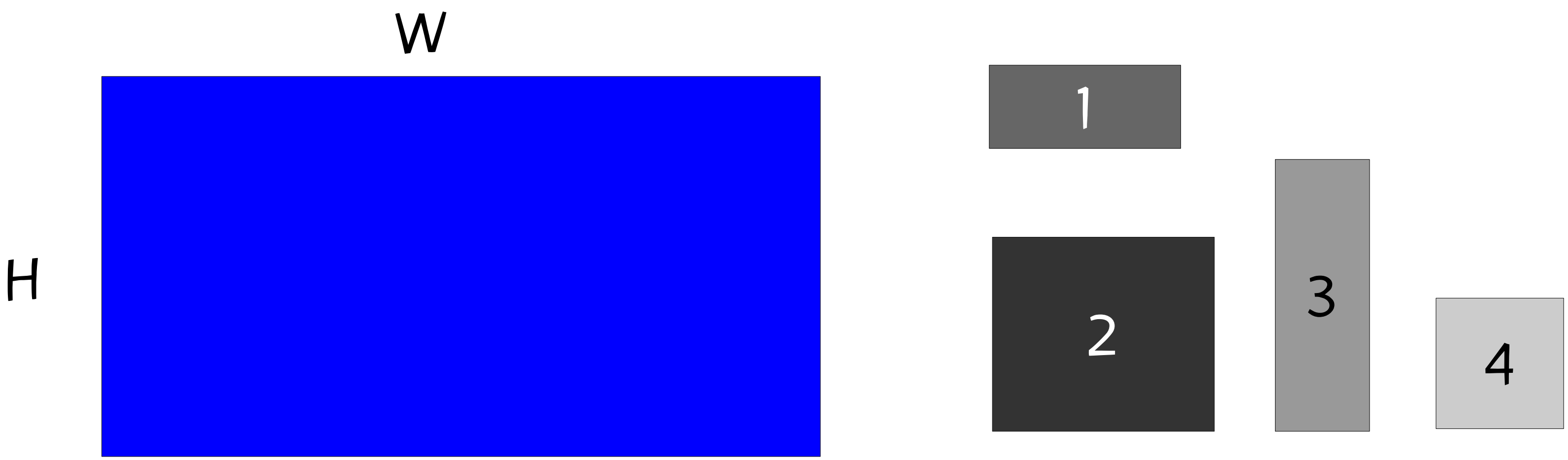
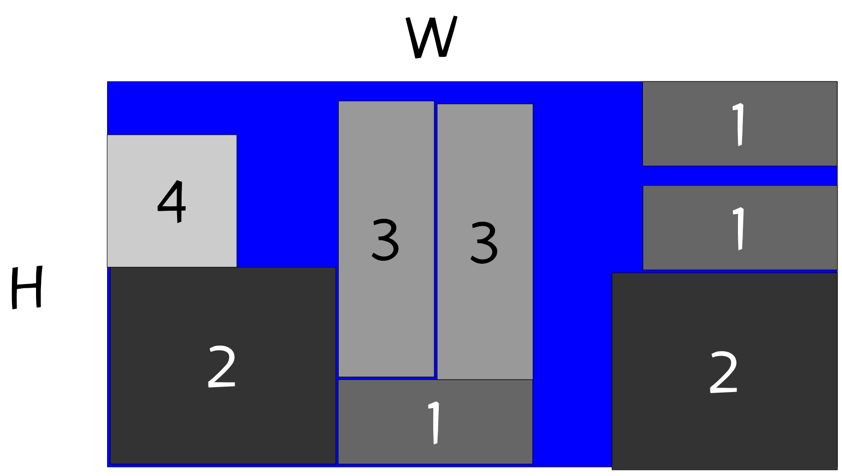


# Constrained orthogonal packing

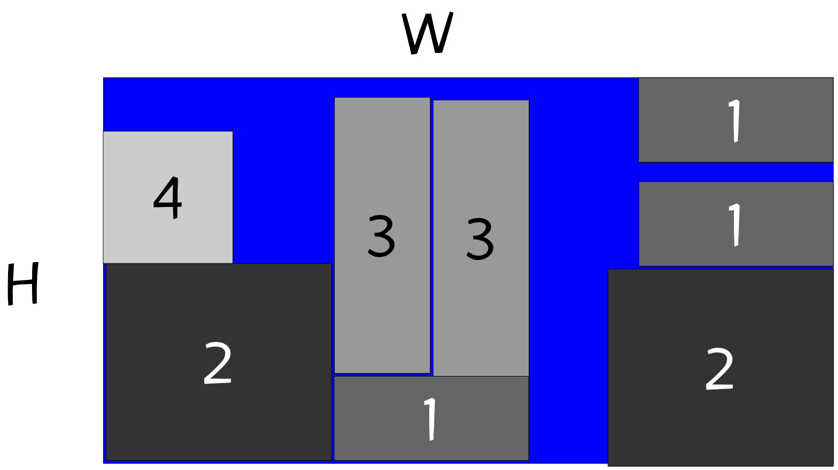
- r[i] rectangles of type i = 1, ..., N are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;



# Constrained orthogonal packing

- r[i] rectangles of type i = 1, ..., N are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;
- For i = 1, ..., N, we require that:

$$0 \leq P[i] \leq r[i] \leq Q[i]$$



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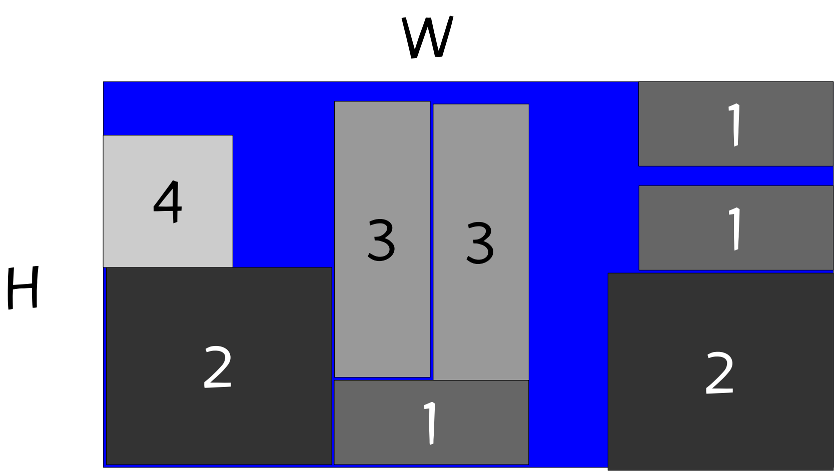
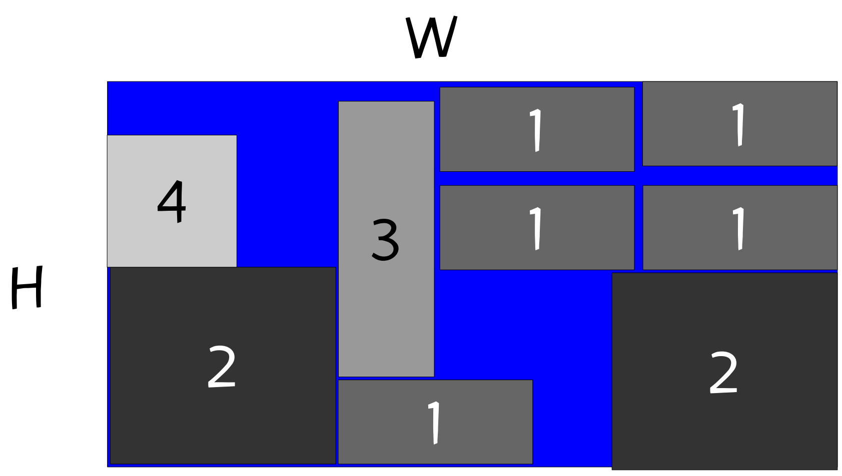



Suppose $5 \leq r[1] \leq 12$

# Constrained orthogonal packing

- r[i] rectangles of type i = 1, ..., N are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;
- For i = 1, ..., N, we require that:

$$0 \leq P[i] \leq r[i] \leq Q[i]$$

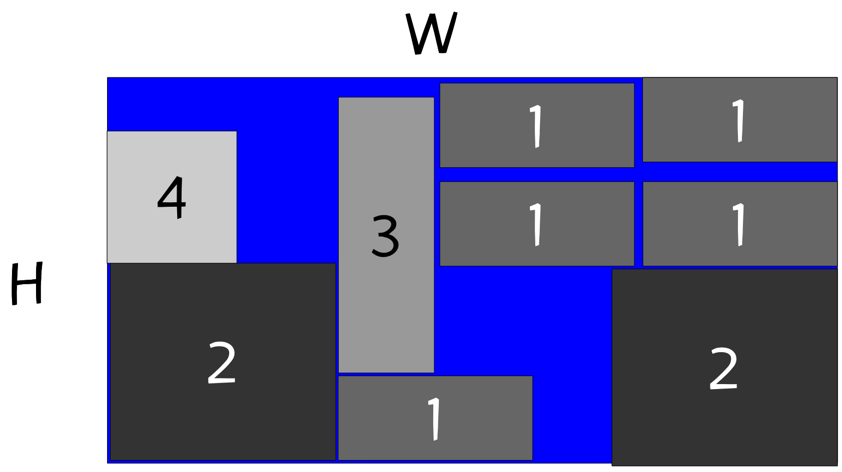



Suppose $5 \leq r[1] \leq 12$

# Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

$$v[1]\ r[1] + v[2]\ r[2] + \cdots + v[N]\ r[N]$$



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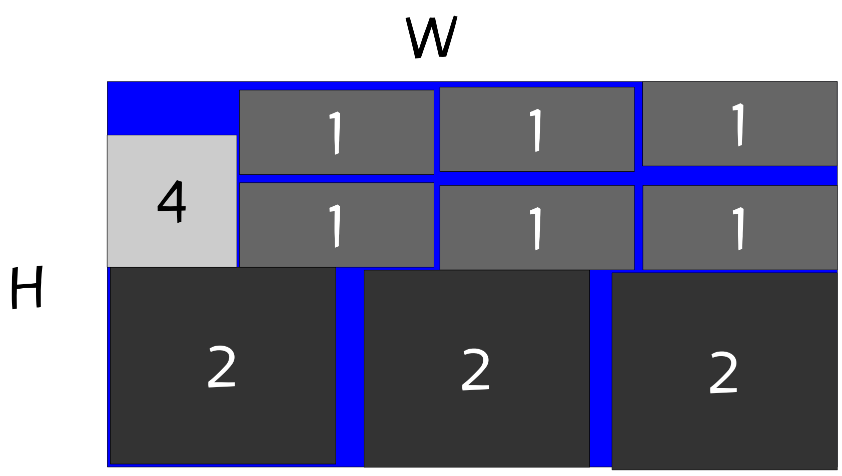

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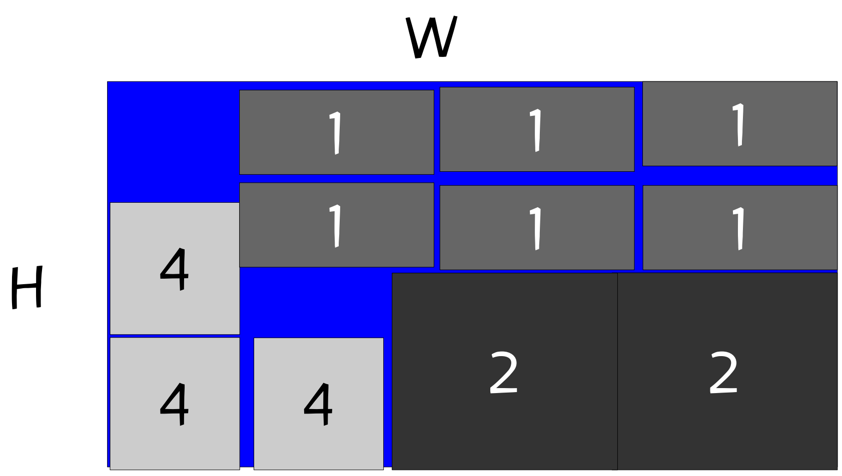

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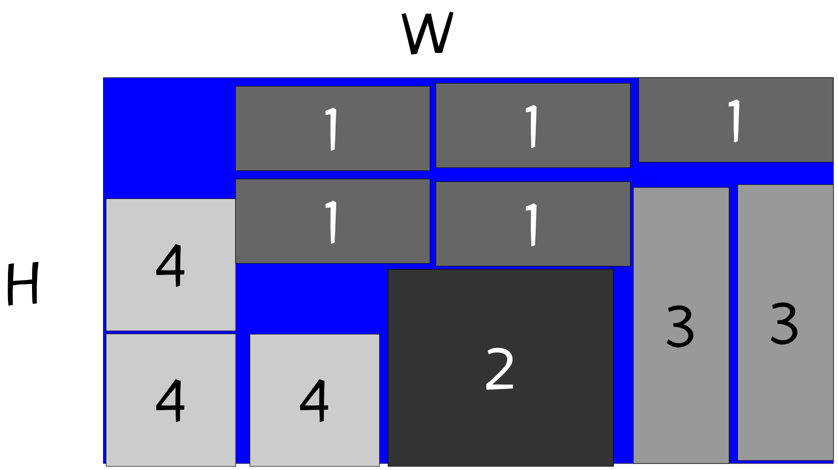
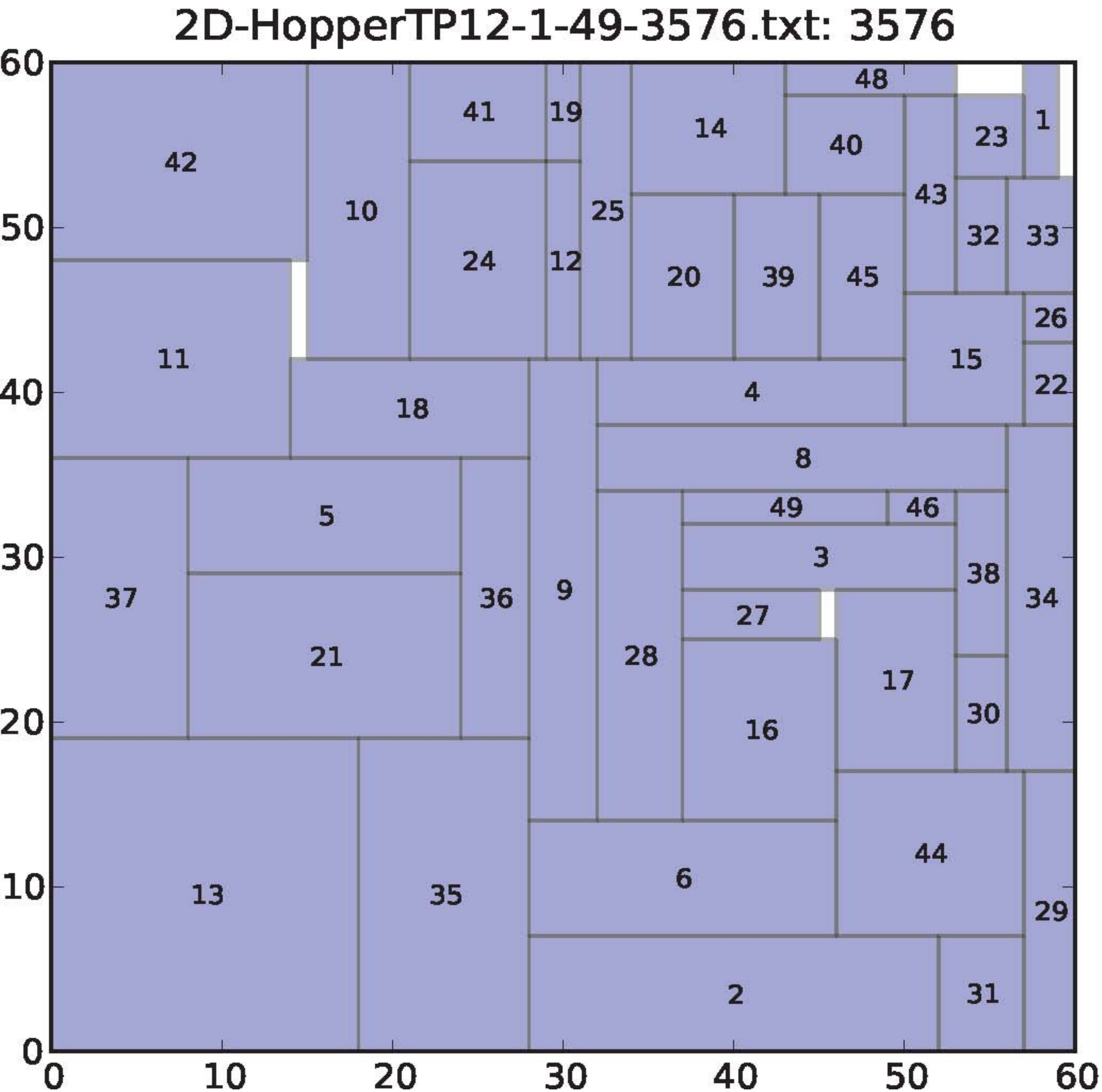


# Applications

Problem arises in several production processes, e.g.

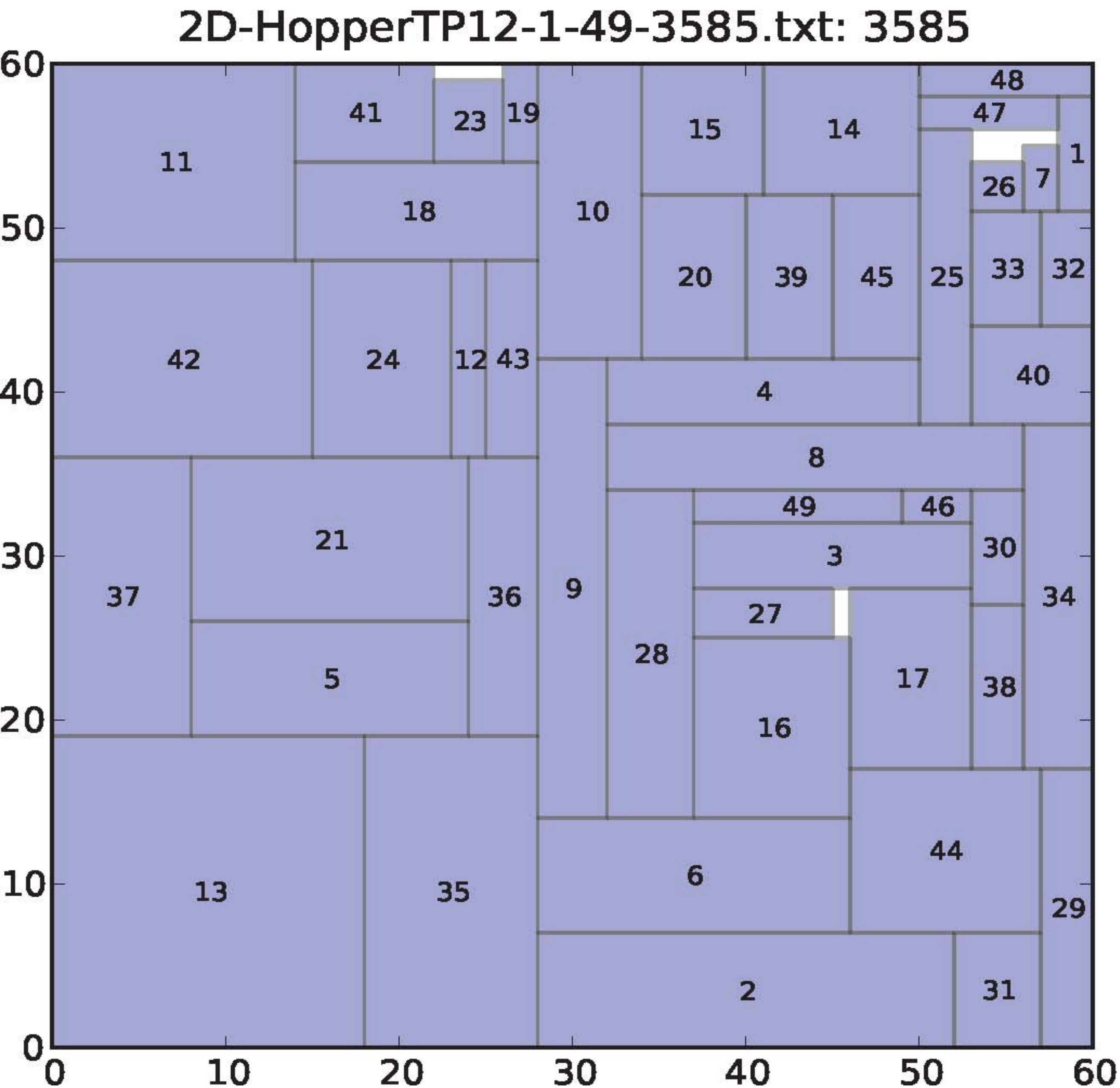
- Textile
- Glass
- Wood
- Paper

where rectangular figures are cut from large rectangular sheets of materials.



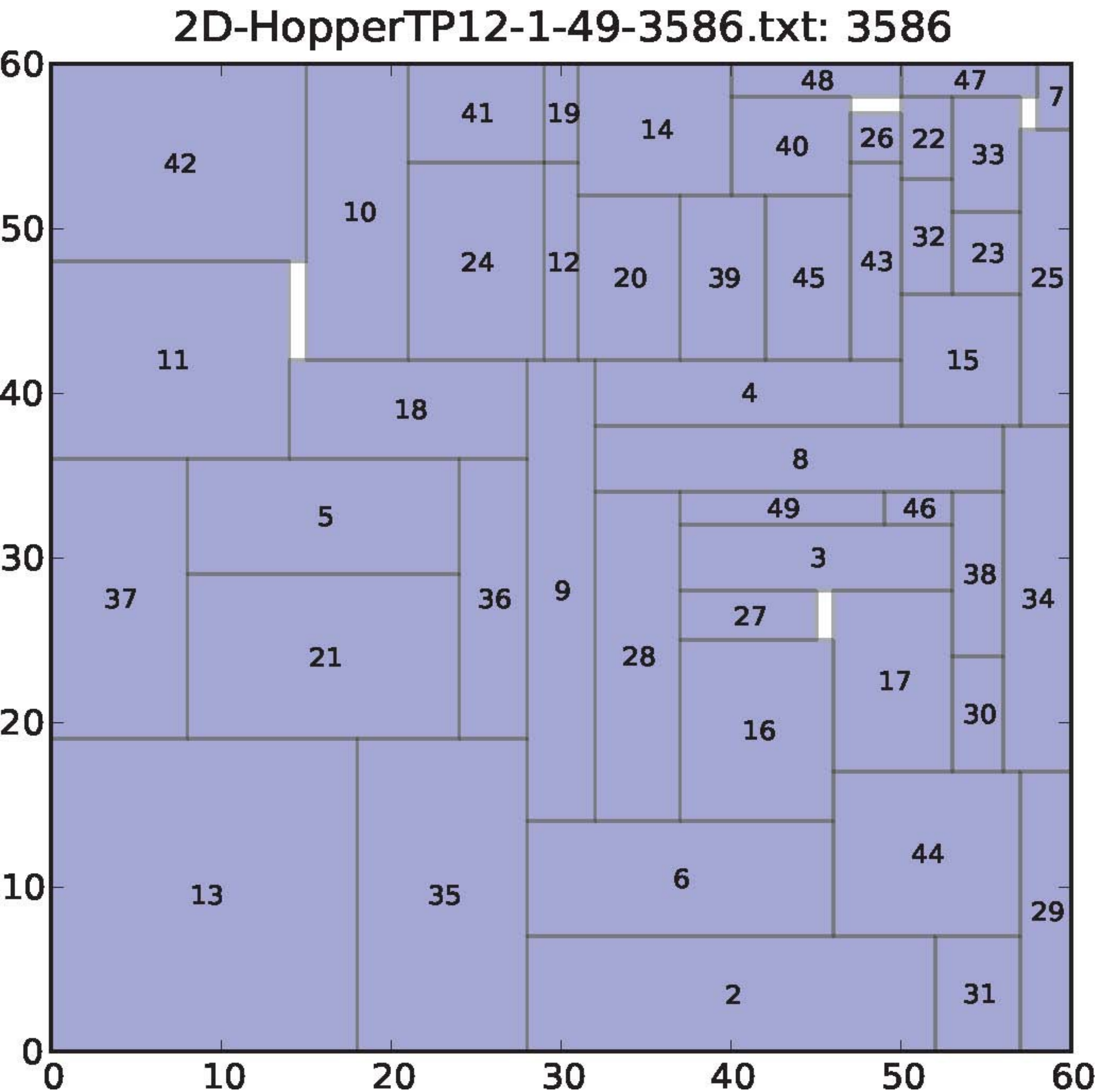

Hopper & Turton, 2001
Instance 4-1 60 x 60
Value: 3576

Previous best: 3580 by a Tabu Search heuristic
(Alvarez-Valdes et al., 2007)



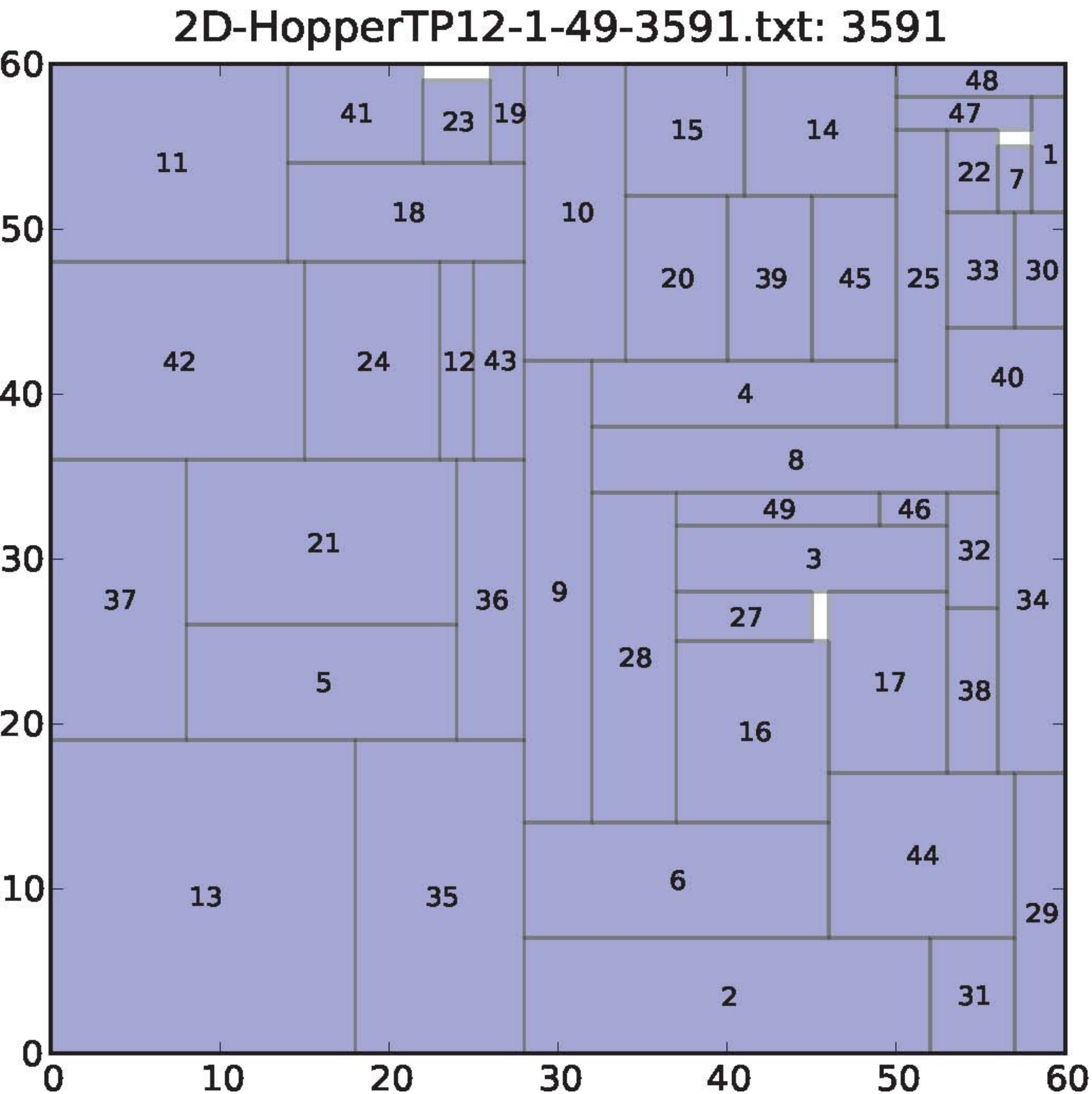

Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3585

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)



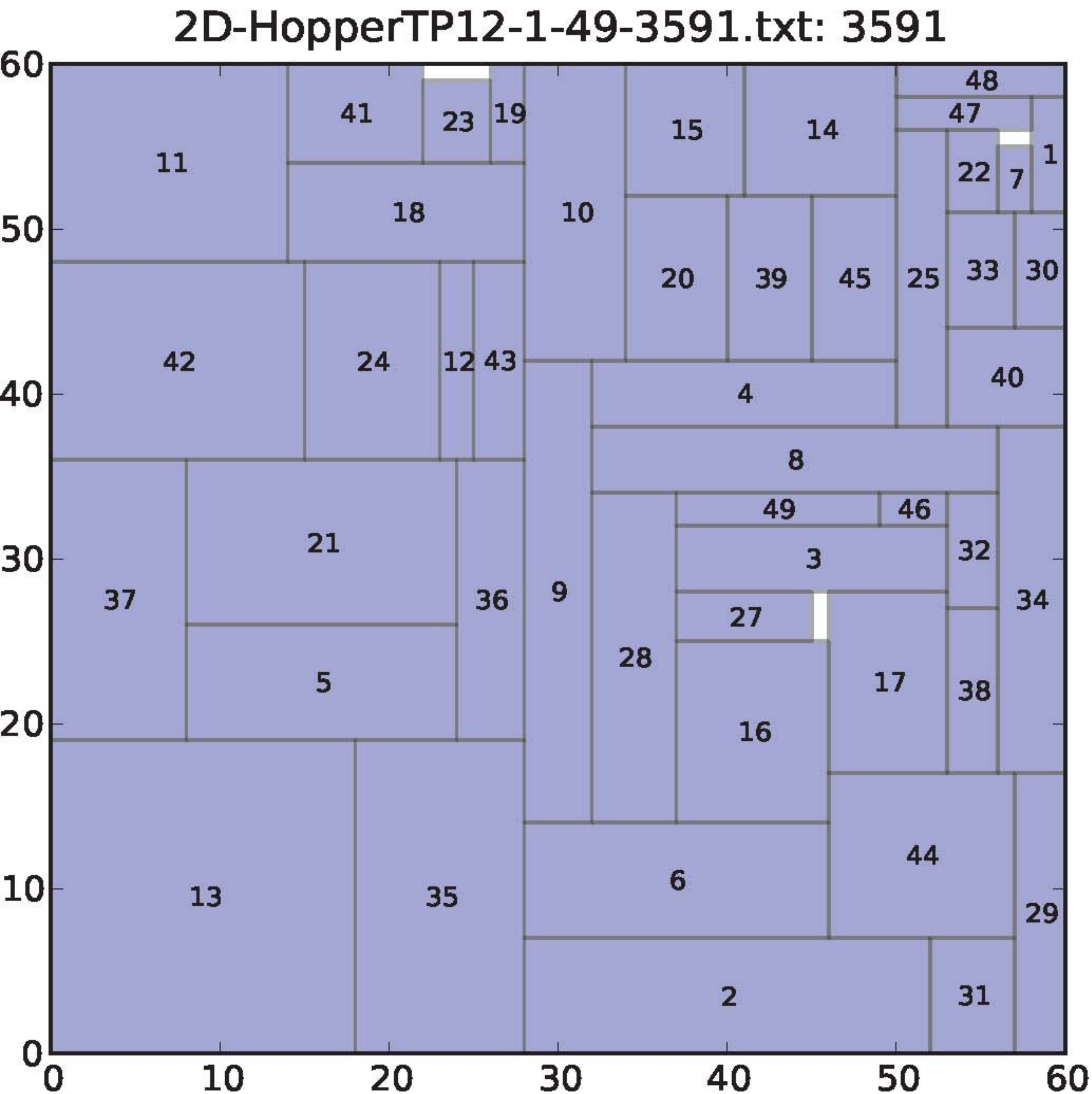

Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3586

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)




Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3591

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)




Hopper & Turton, 2001

Instance 4-2 60 x 60

Value: 3591

New best known solution!

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

# BRKGA for constrained 2-dim orthogonal packing

# Encoding

- Solutions are encoded as vectors X of

$$2N' = 2 \{ Q[1] + Q[2] + \cdots + Q[N] \}$$

random keys, where Q[i] is the maximum number of rectangles of type i (for i = 1, ..., N) that can be packed.

- X = ( X[1], ..., X[N'], X[N'+1], ..., X[2N'] )

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$\underbrace{\text{X[1], ..., X[N']}}_{\text{Rectangle type packing sequence (RTPS)}}$

Rectangle type
packing sequence
(RTPS)

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  random keys, where Q[i] is the maximum number of rectangles of type i (for i = 1, ..., N) that can be packed.

- $X = ( \underbrace{X[1], \ldots, X[N'],}_{\text{Rectangle type packing sequence (RTPS)}} \quad \underbrace{X[N'+1], \ldots, X[2N']}_{\text{Vector of placement procedures (VPP)}} )$

# Decoding

- Simple heuristic to pack rectangles:
  - Make Q[i] copies of rectangle i, for i = 1, ..., N.
  - Order the $N' = Q[1] + Q[2] + \cdots + Q[N]$ rectangles in some way.
  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order.

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  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order. **Use the last N' keys of X to determine which heuristic to use. If k[N'+i] > 0.5 use LB, else use BL.**

# Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
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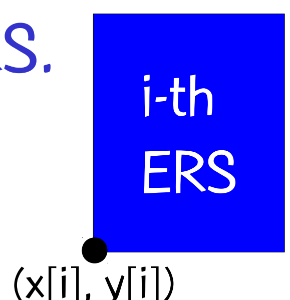

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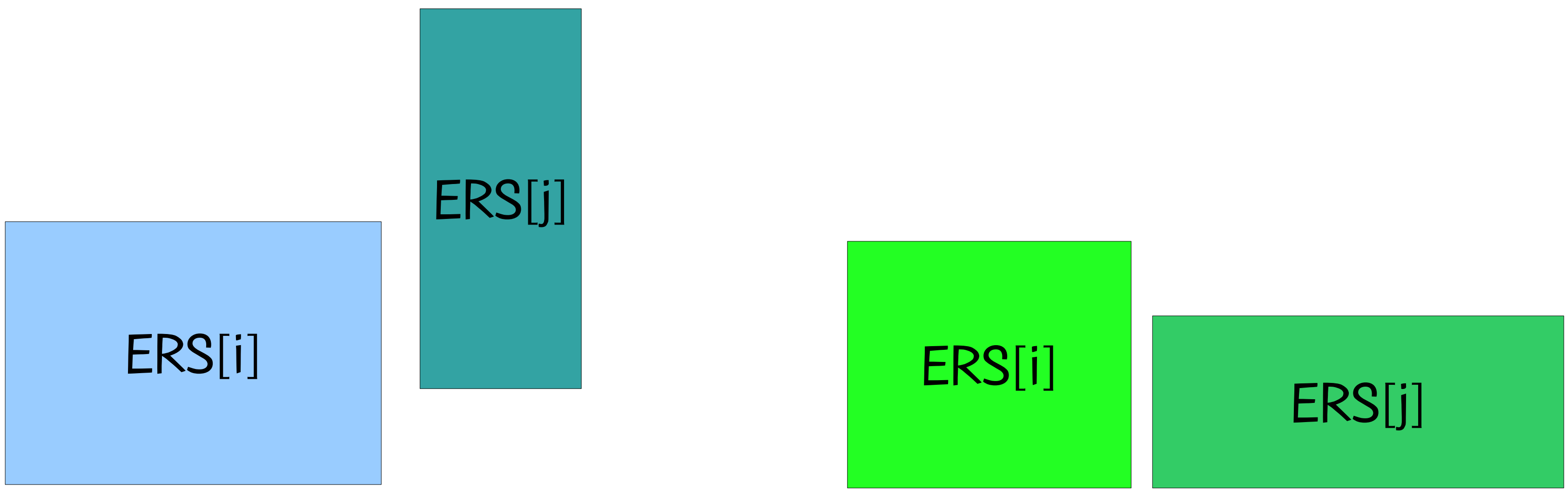
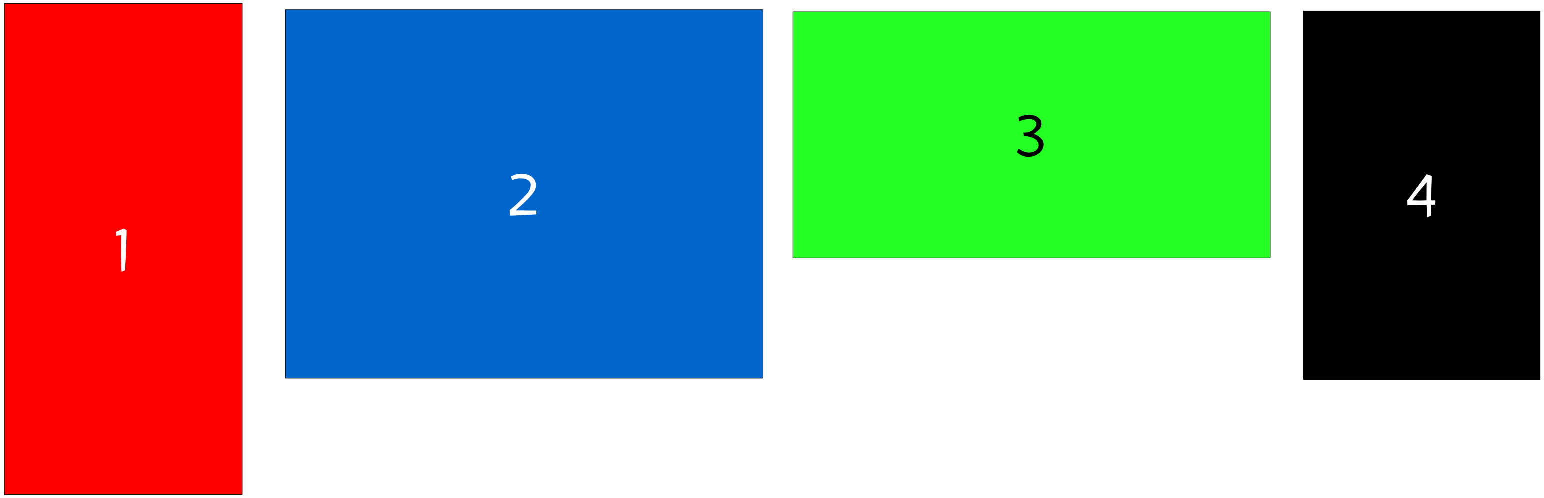
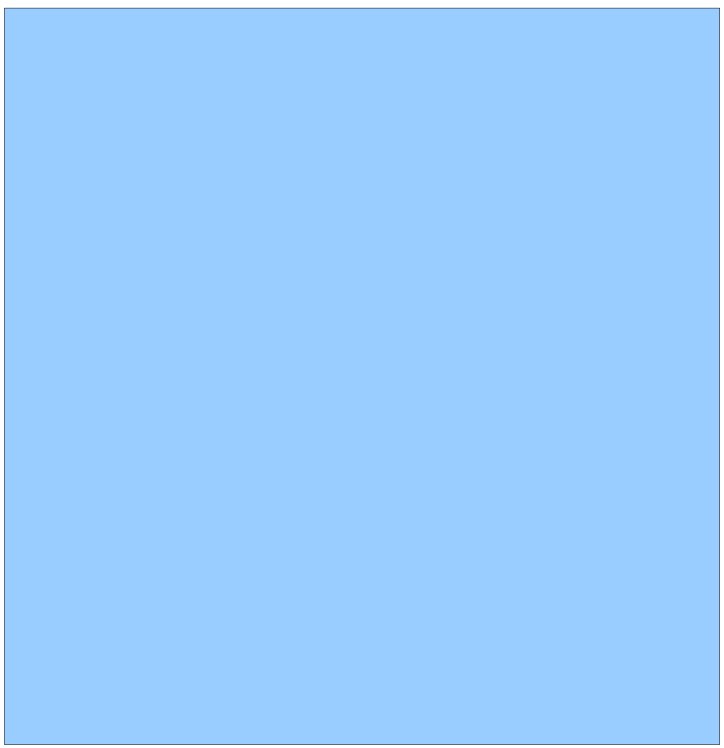



ERS[i] < ERS[j]

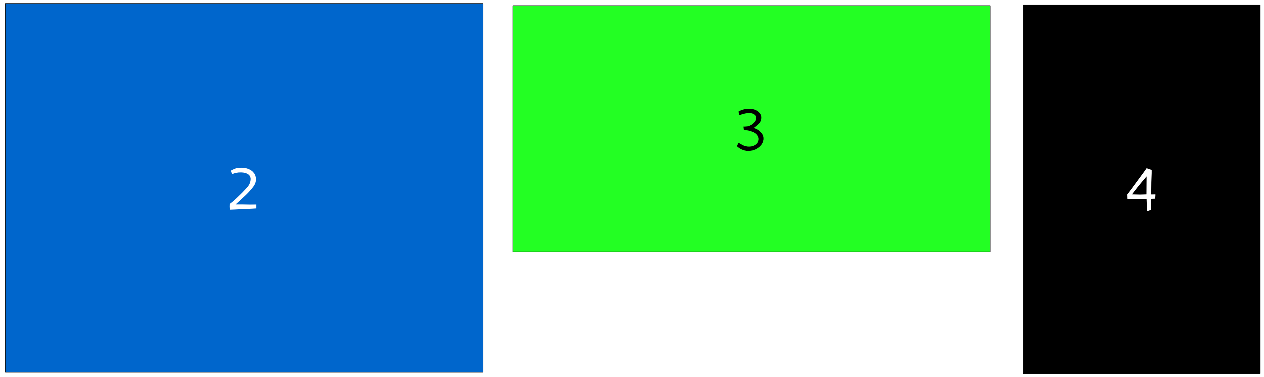


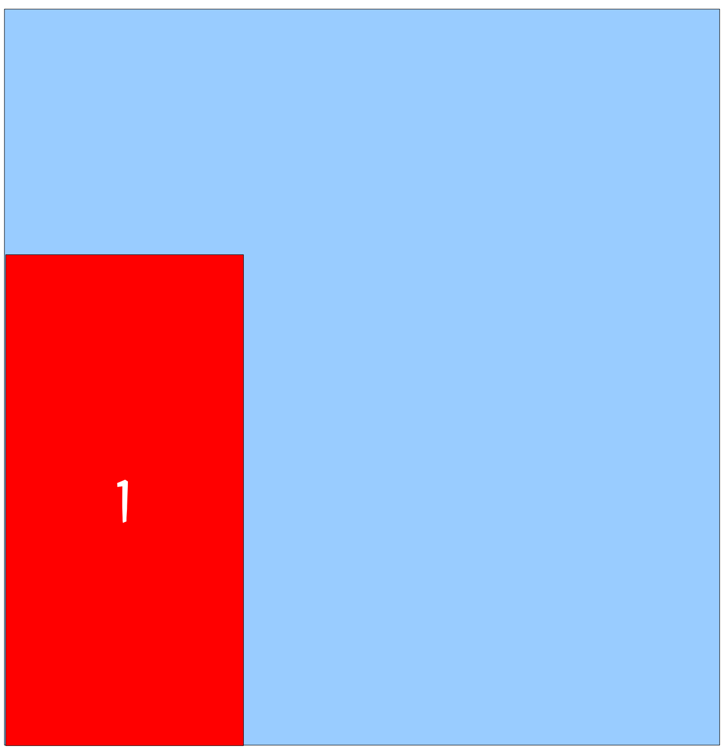



BL can run into problems even on small instances (Liu & Teng, 1999).

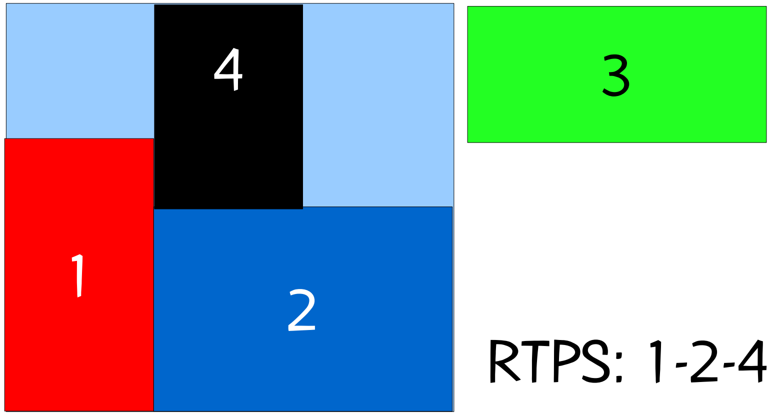
Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.

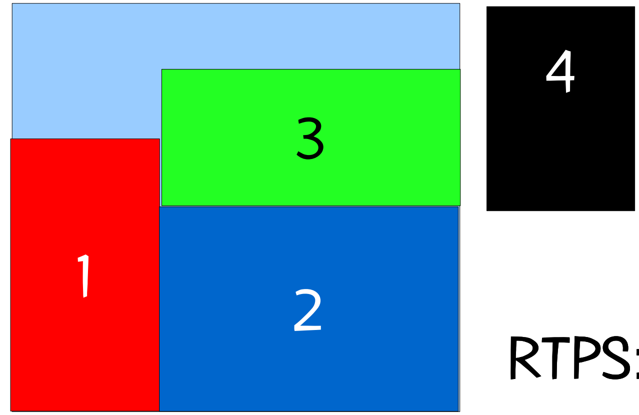


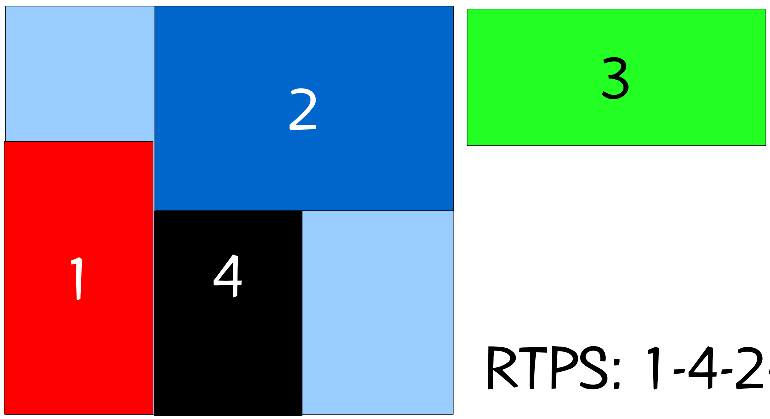

We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



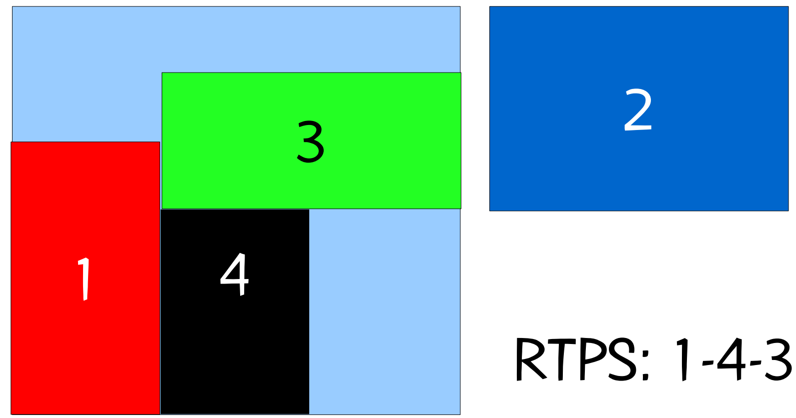

RTPS: 1-2-4-3



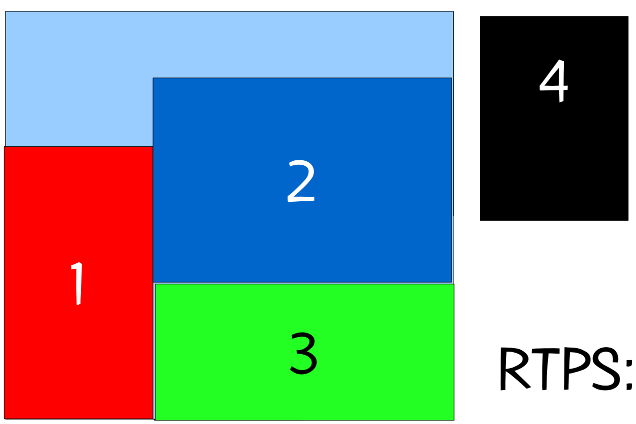

RTPS: 1-2-3-4



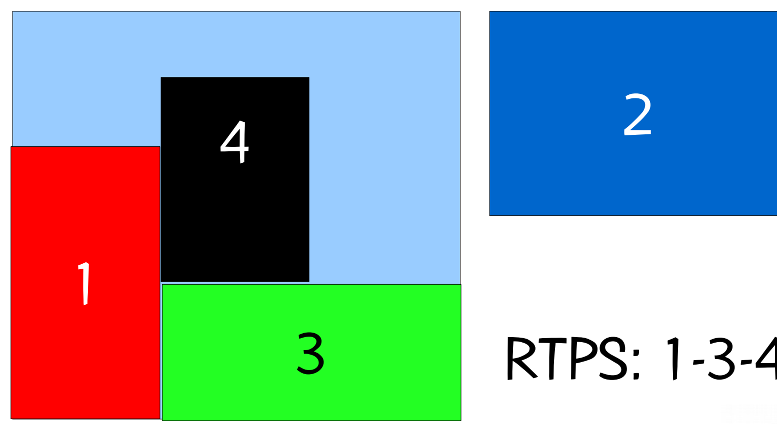

RTPS: 1-4-2-3



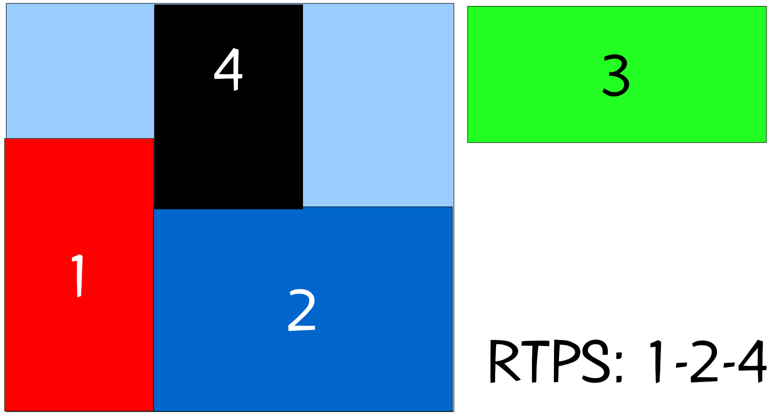

RTPS: 1-4-3-2



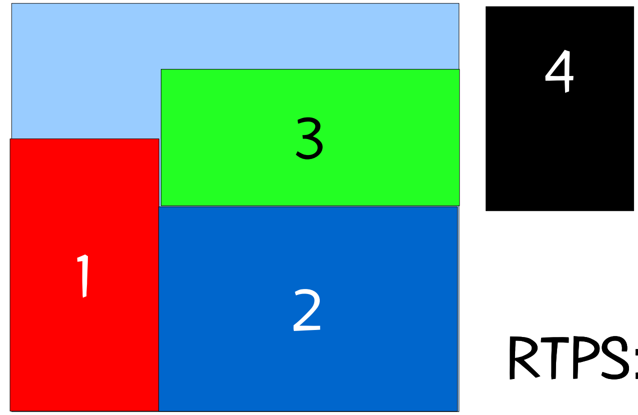

RTPS: 1-3-2-4




RTPS: 1-3-4-2

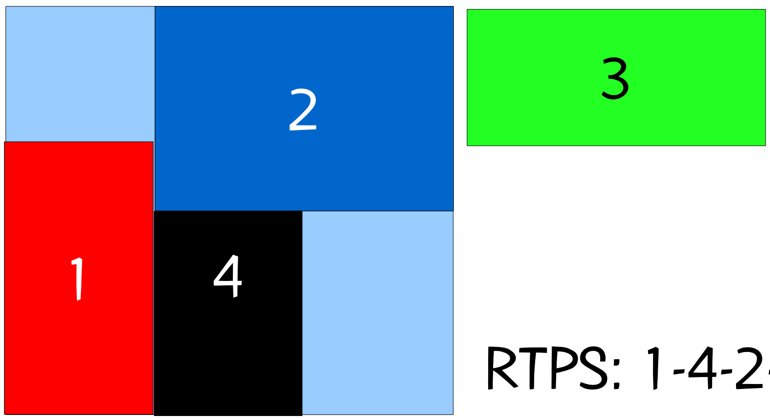



RTPS: 1-2-4-3

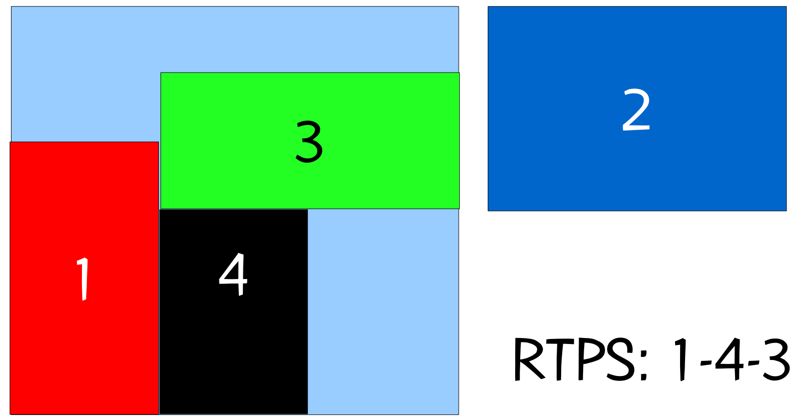



RTPS: 1-2-3-4

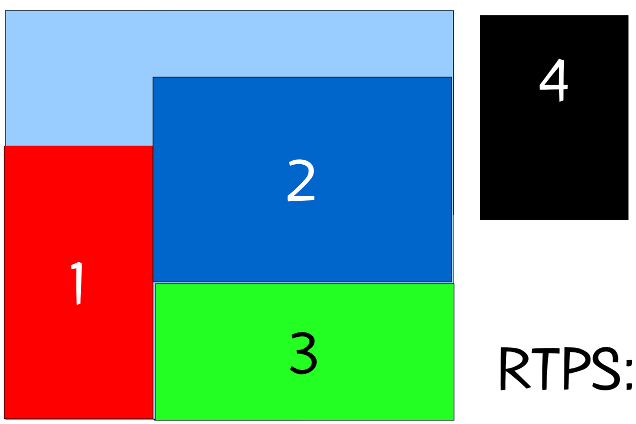
Similar infeasibilities are observed if 2, 3, or 4 is the first rectangle in the RTPS.



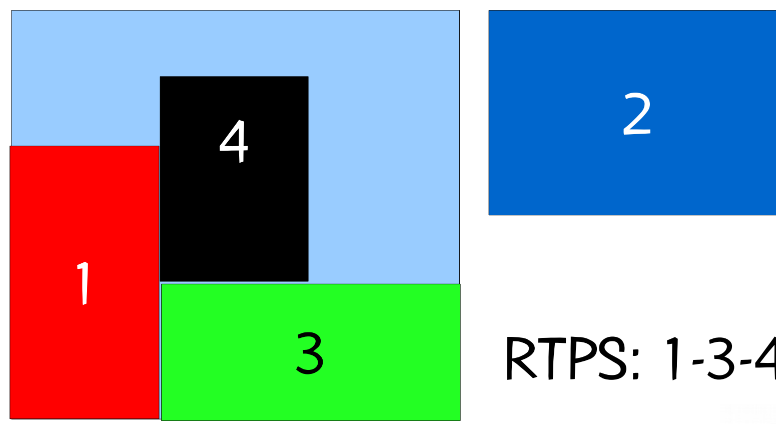

RTPS: 1-4-2-3




RTPS: 1-4-3-2




RTPS: 1-3-2-4

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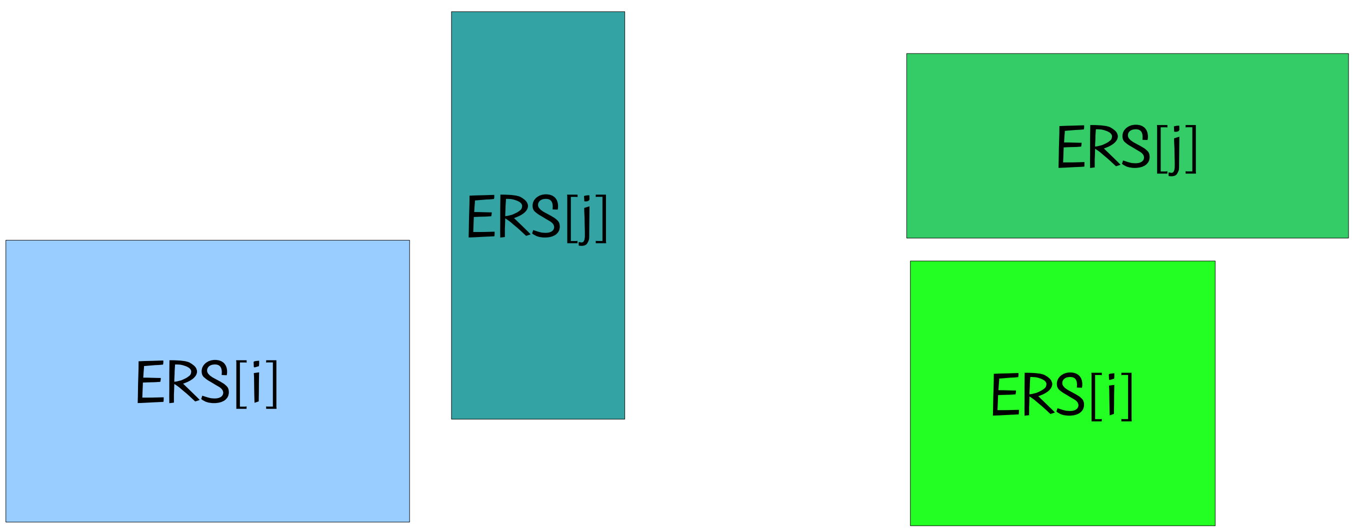
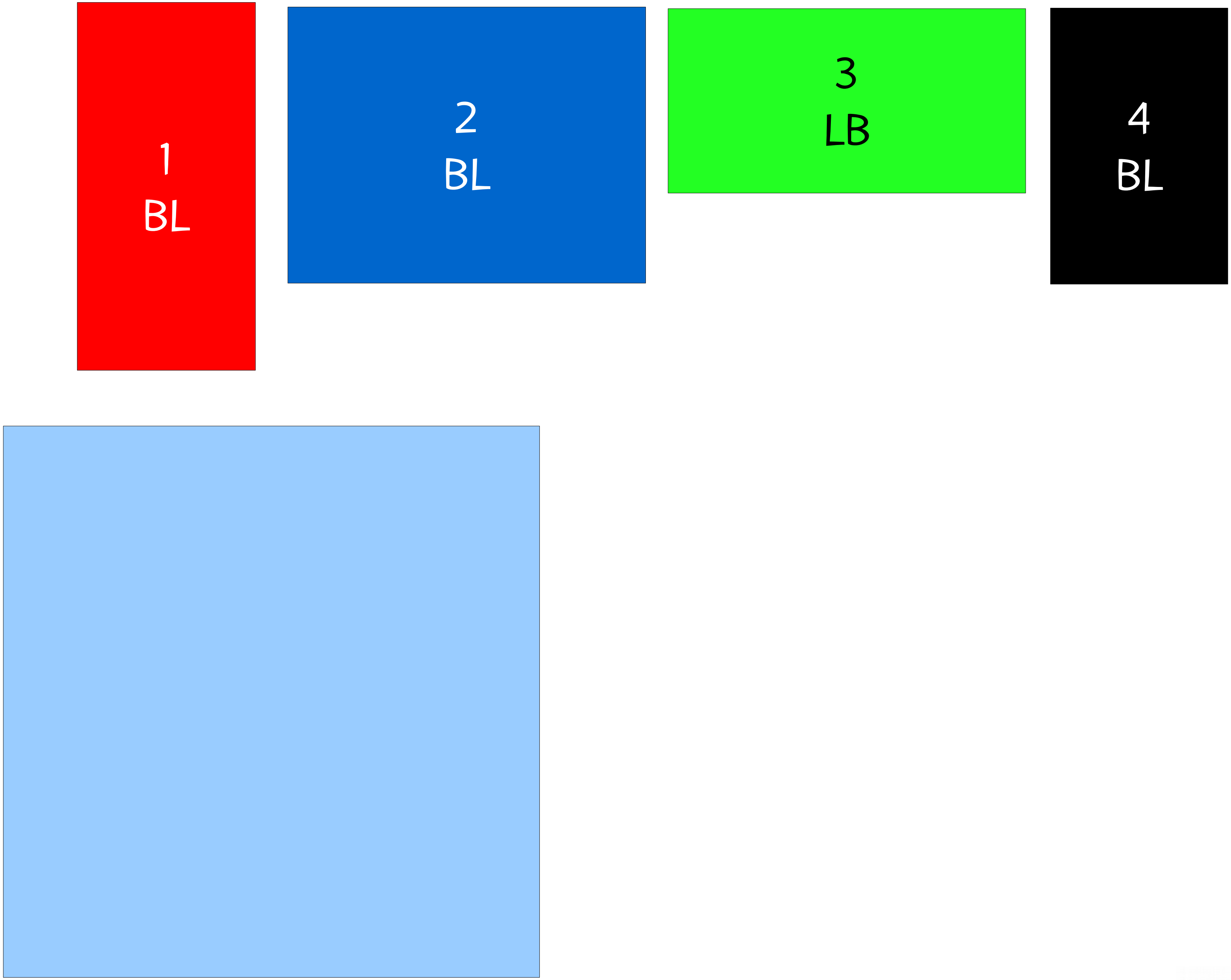
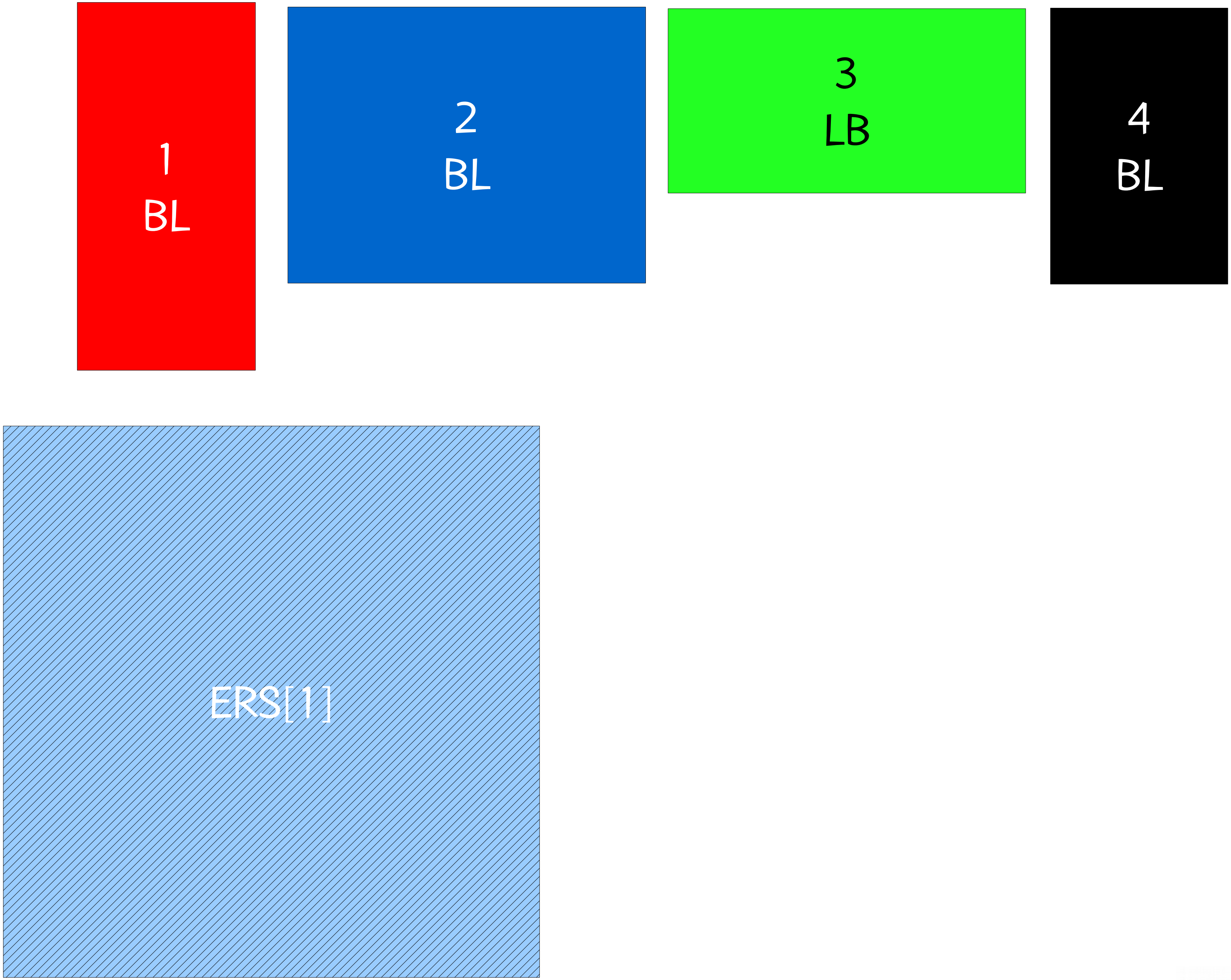
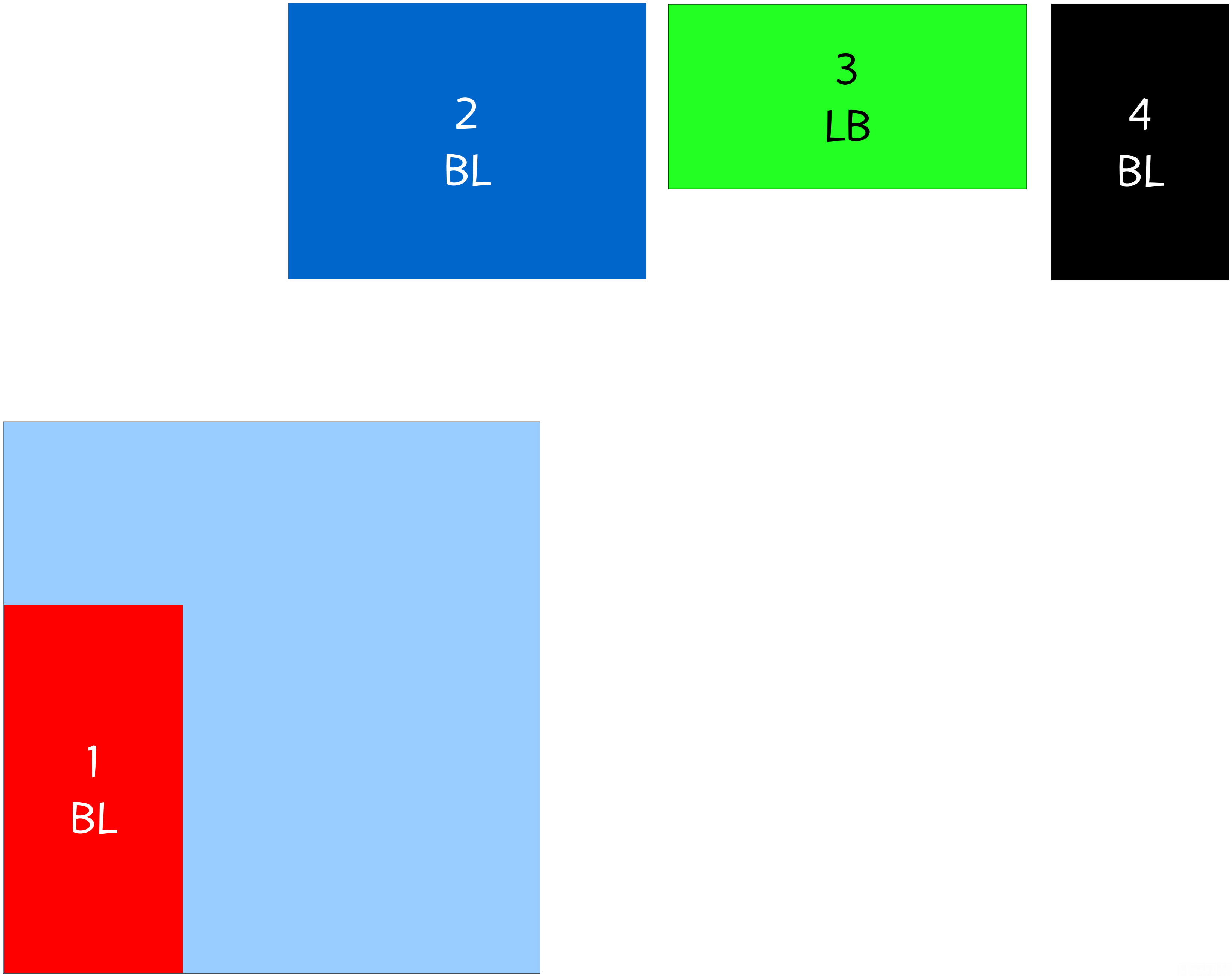
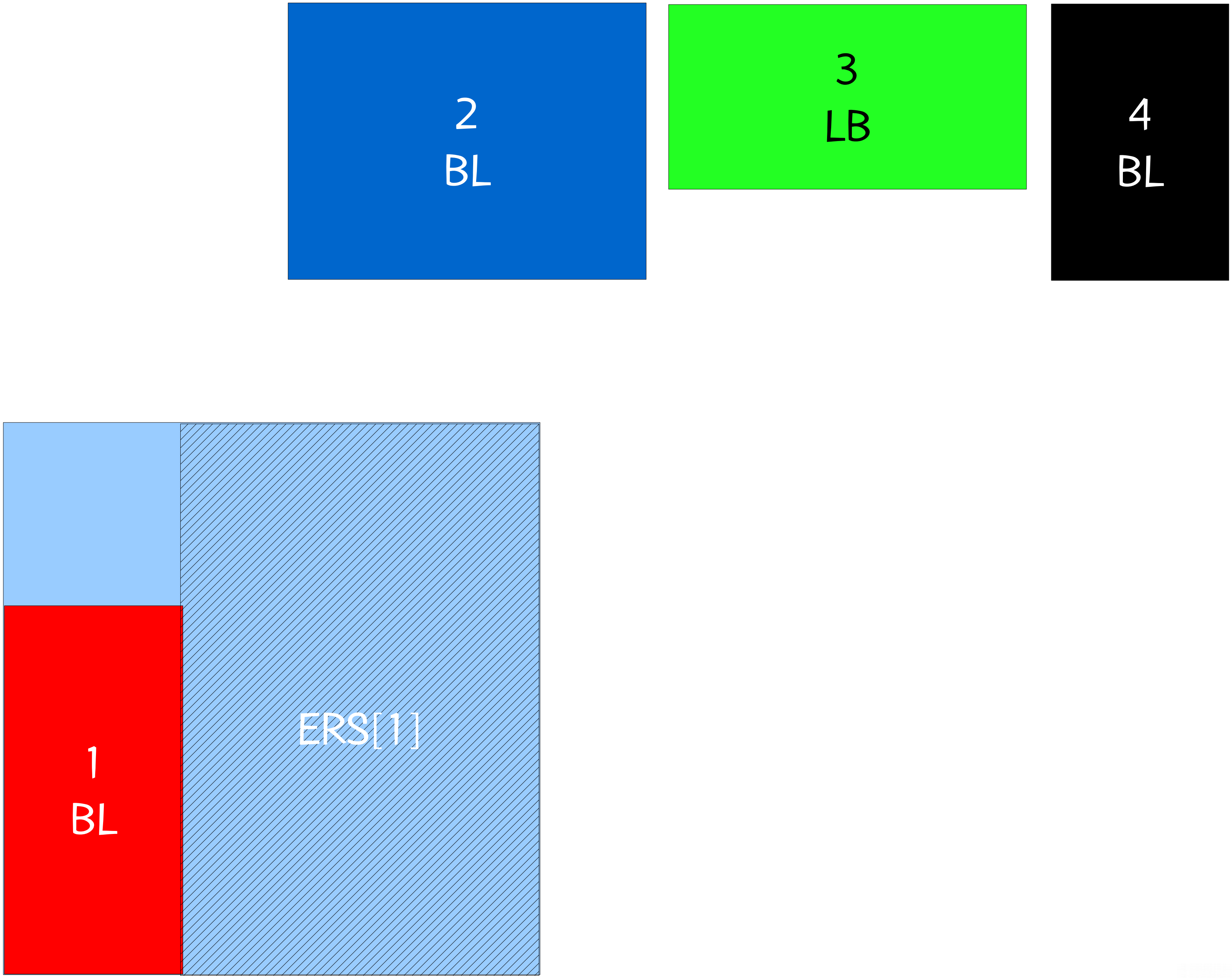


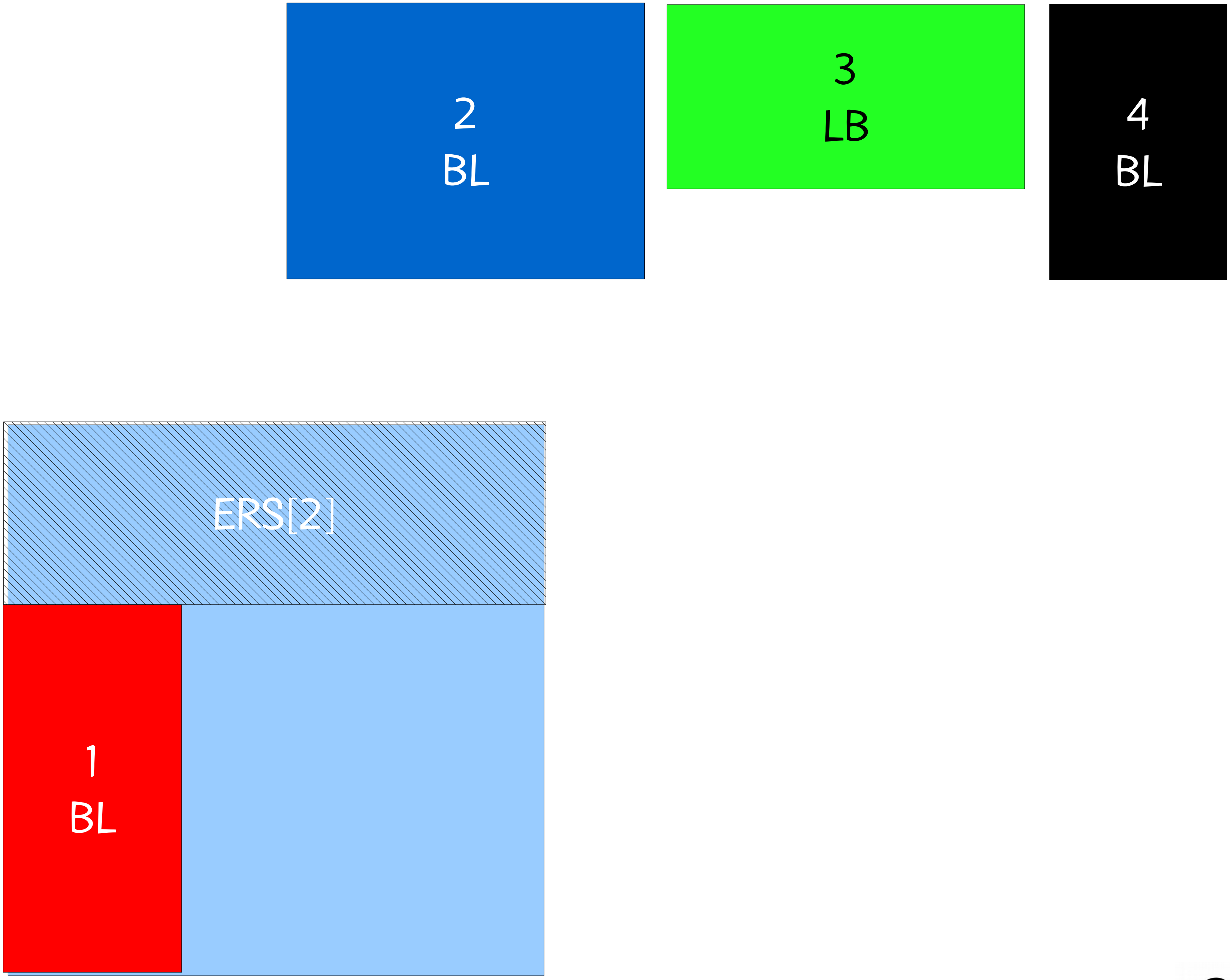

ERS[i] < ERS[j]

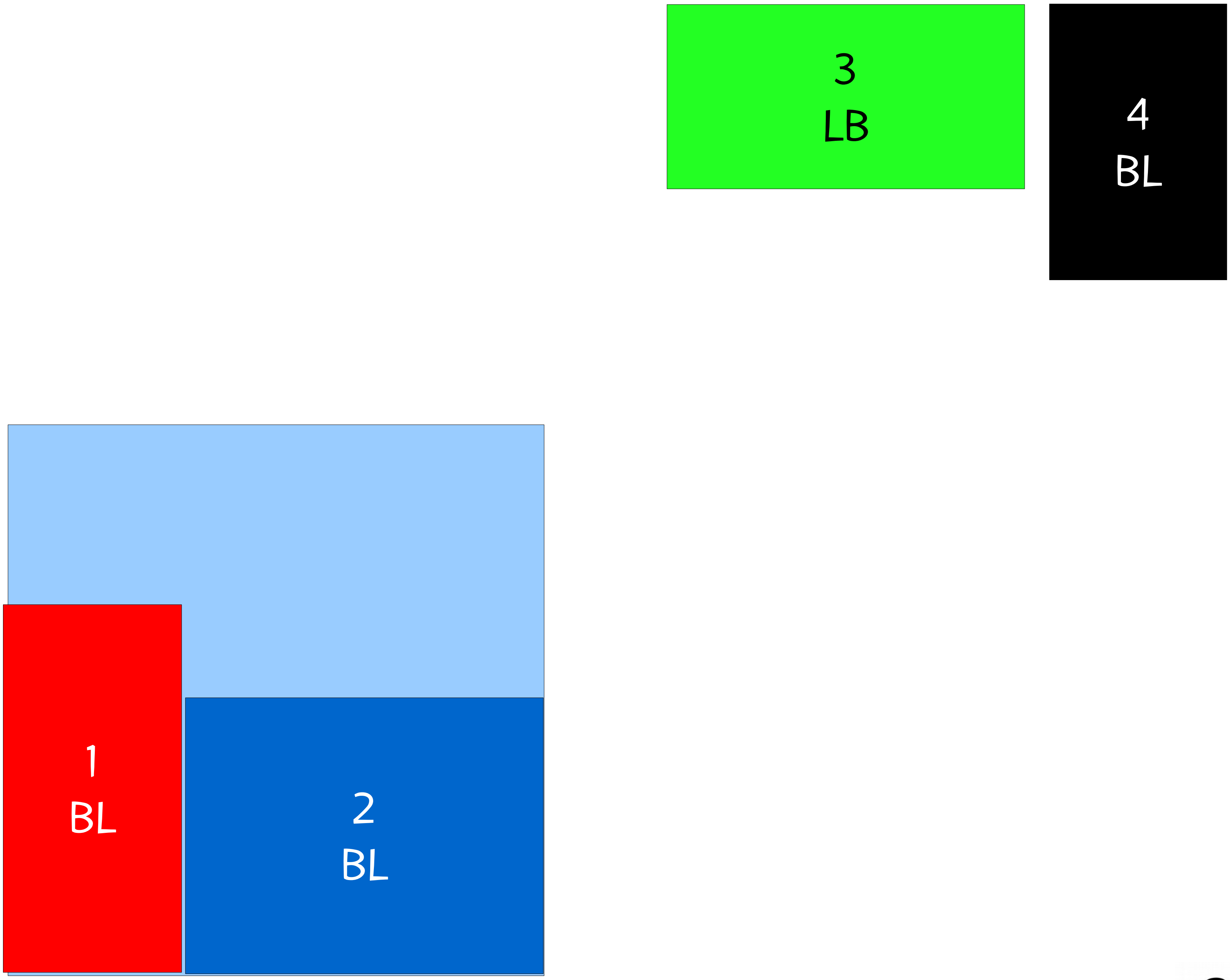

1
BL
2
BL
3
LB
4
BL

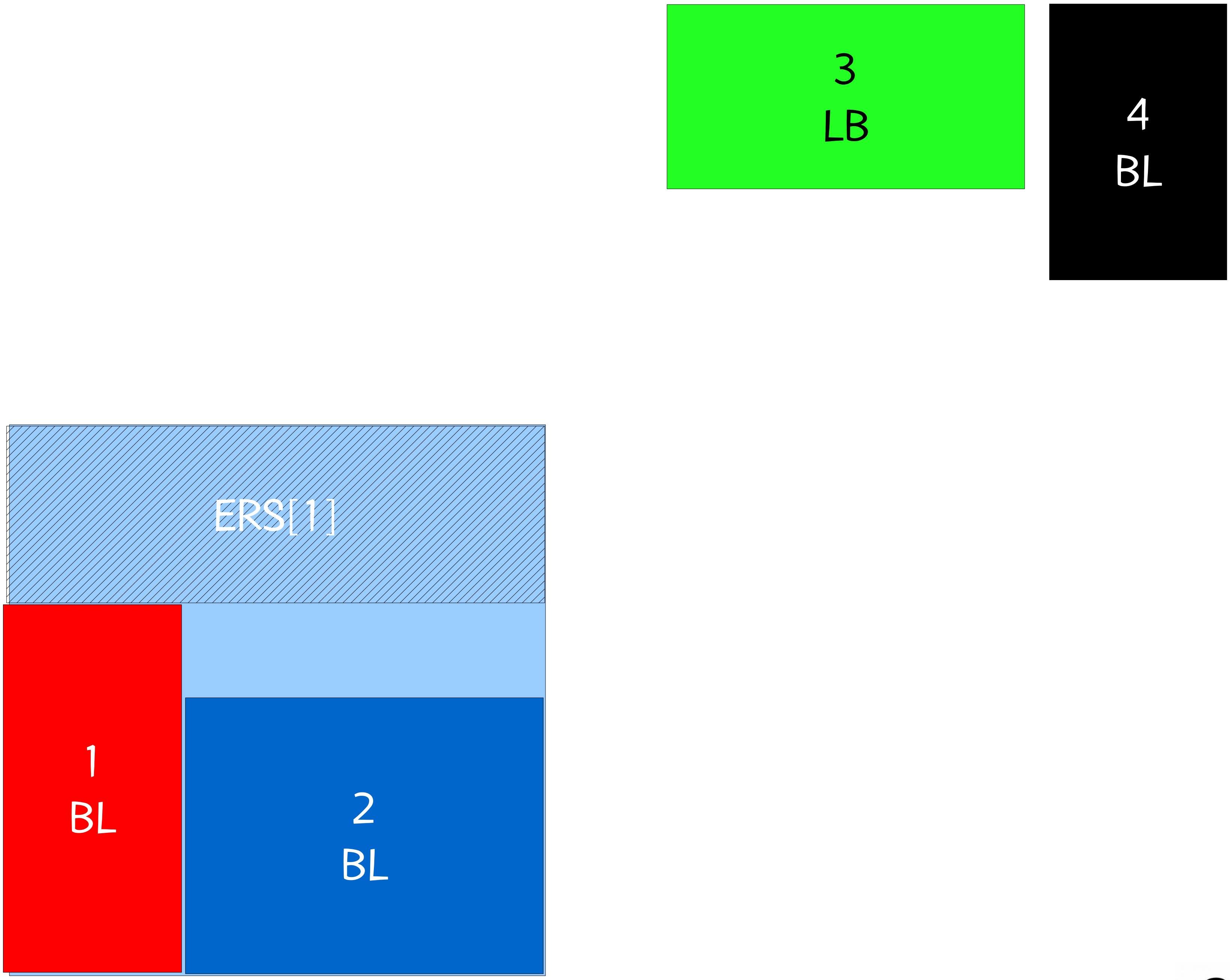

1
BL
2
BL
3
LB
4
BL
ERS[1]

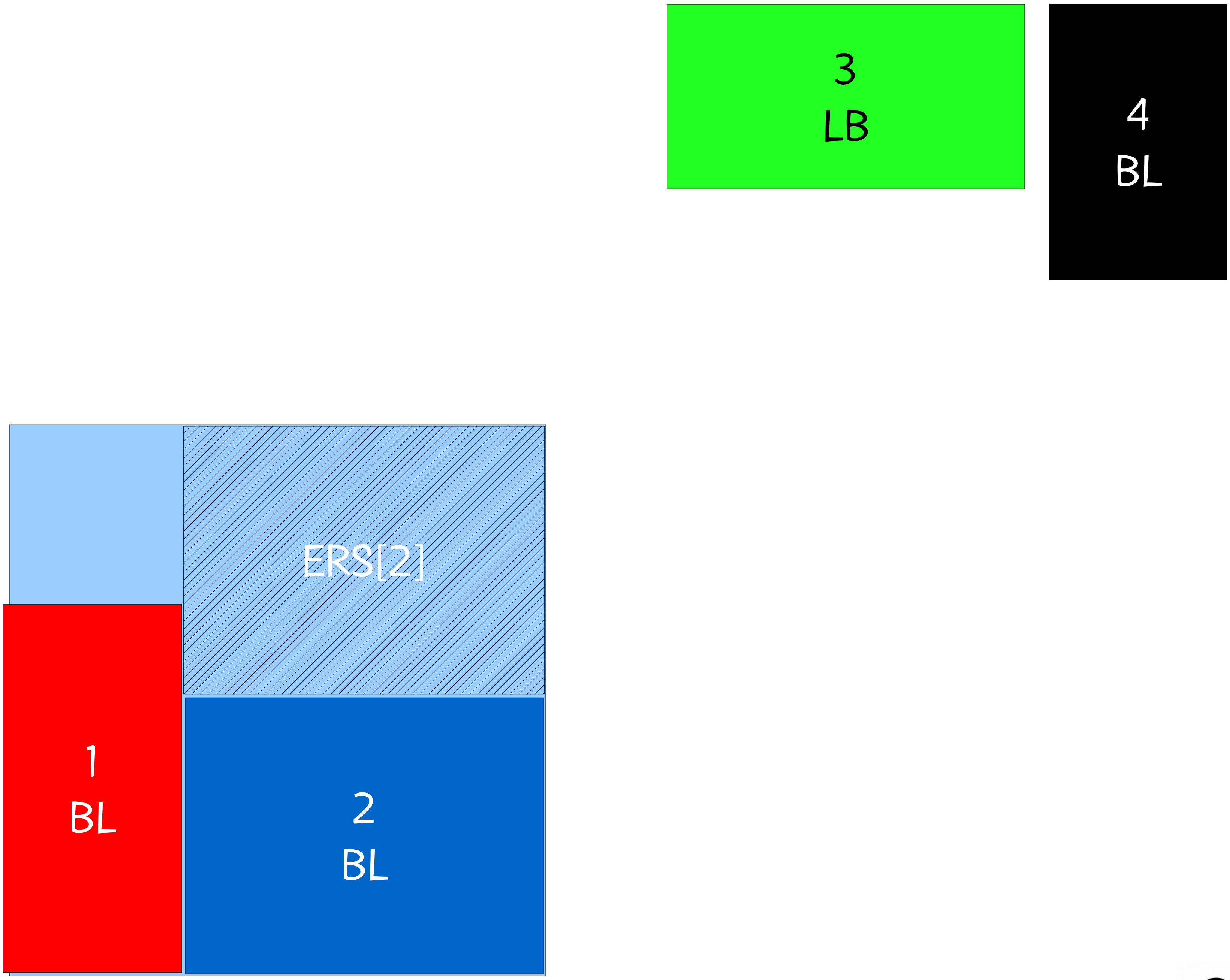

2
BL
3
LB
4
BL
1
BL

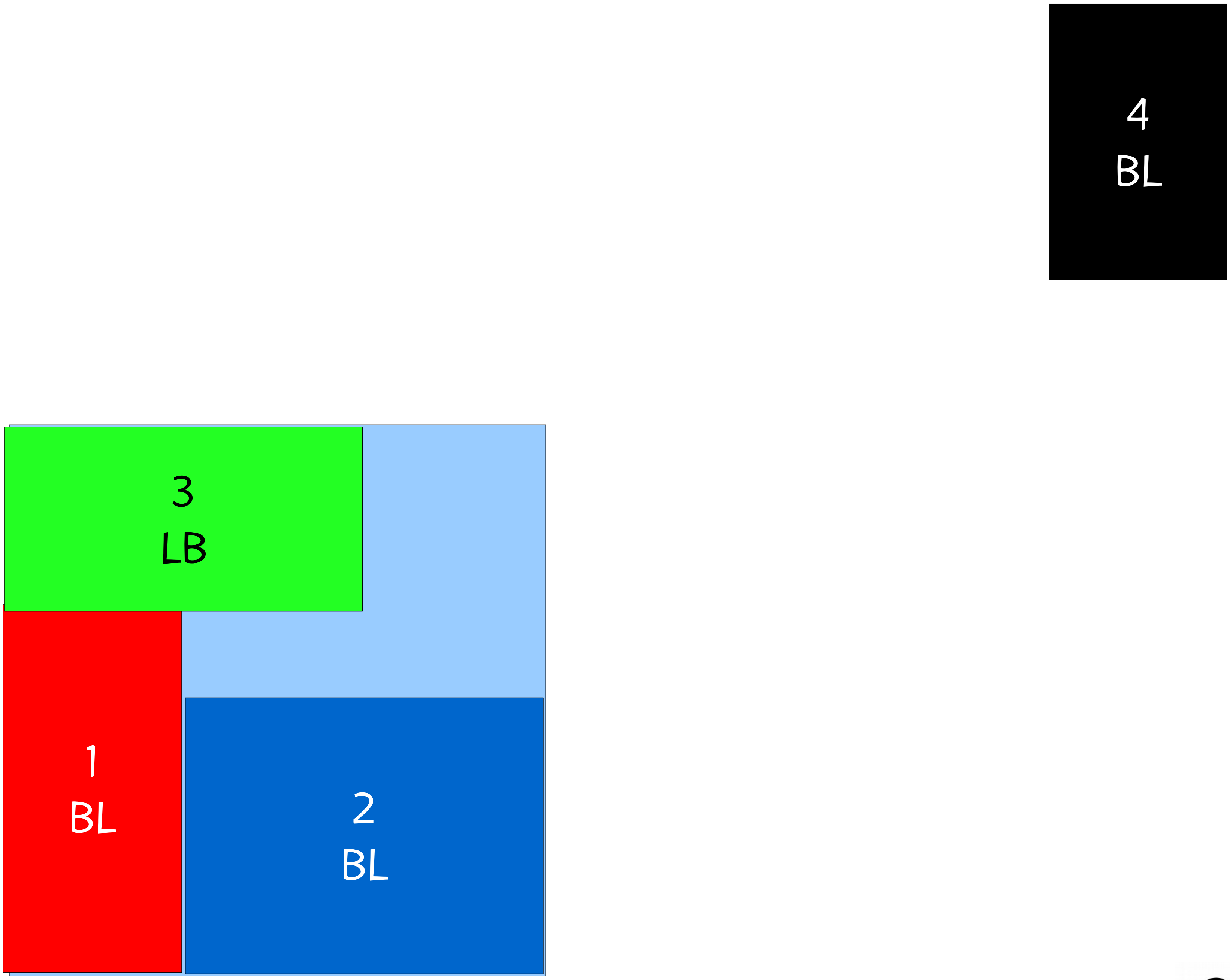

2
BL
3
LB
4
BL
1
BL
ERS[1]

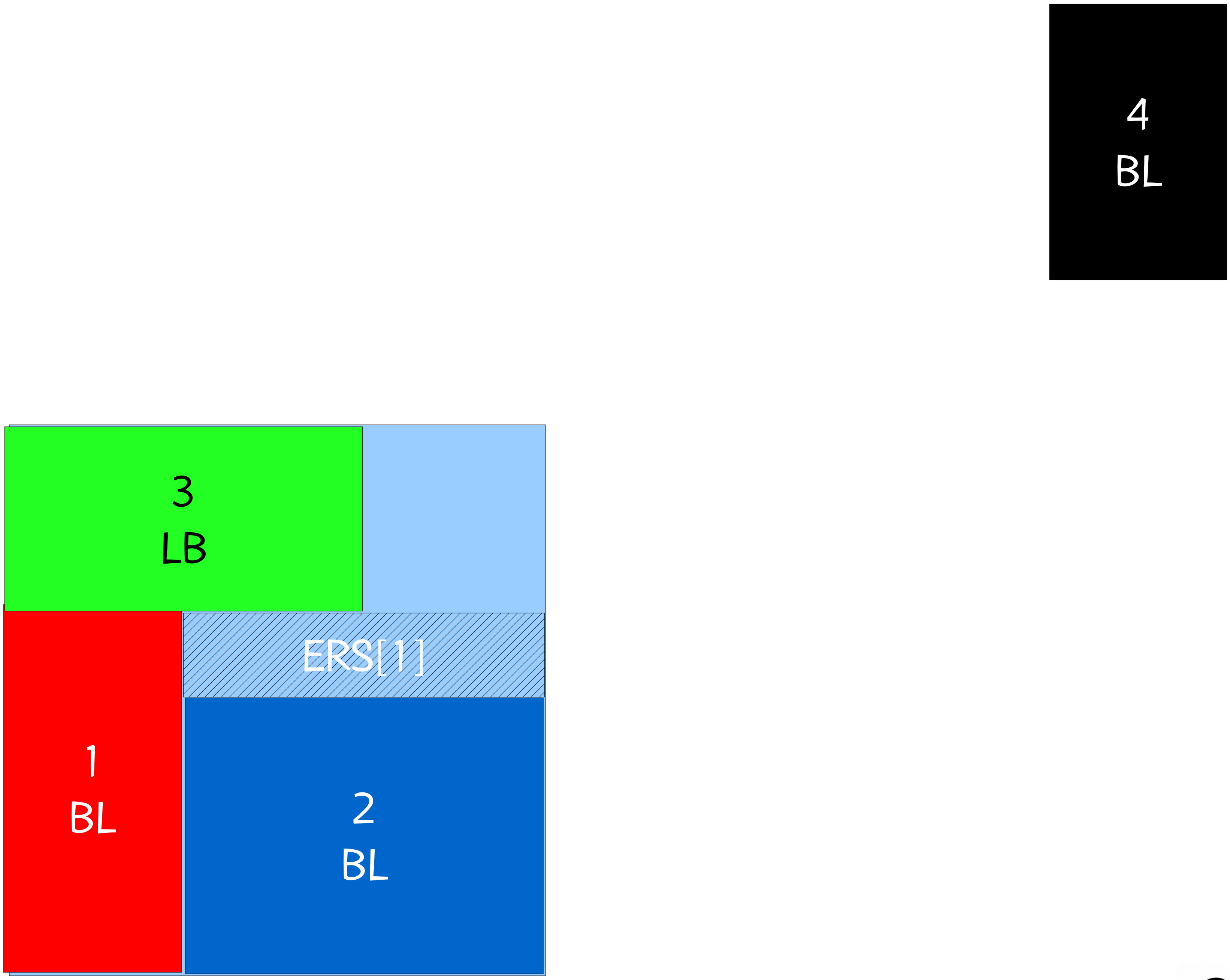

2
BL
3
LB
4
BL
ERS[2]
1
BL

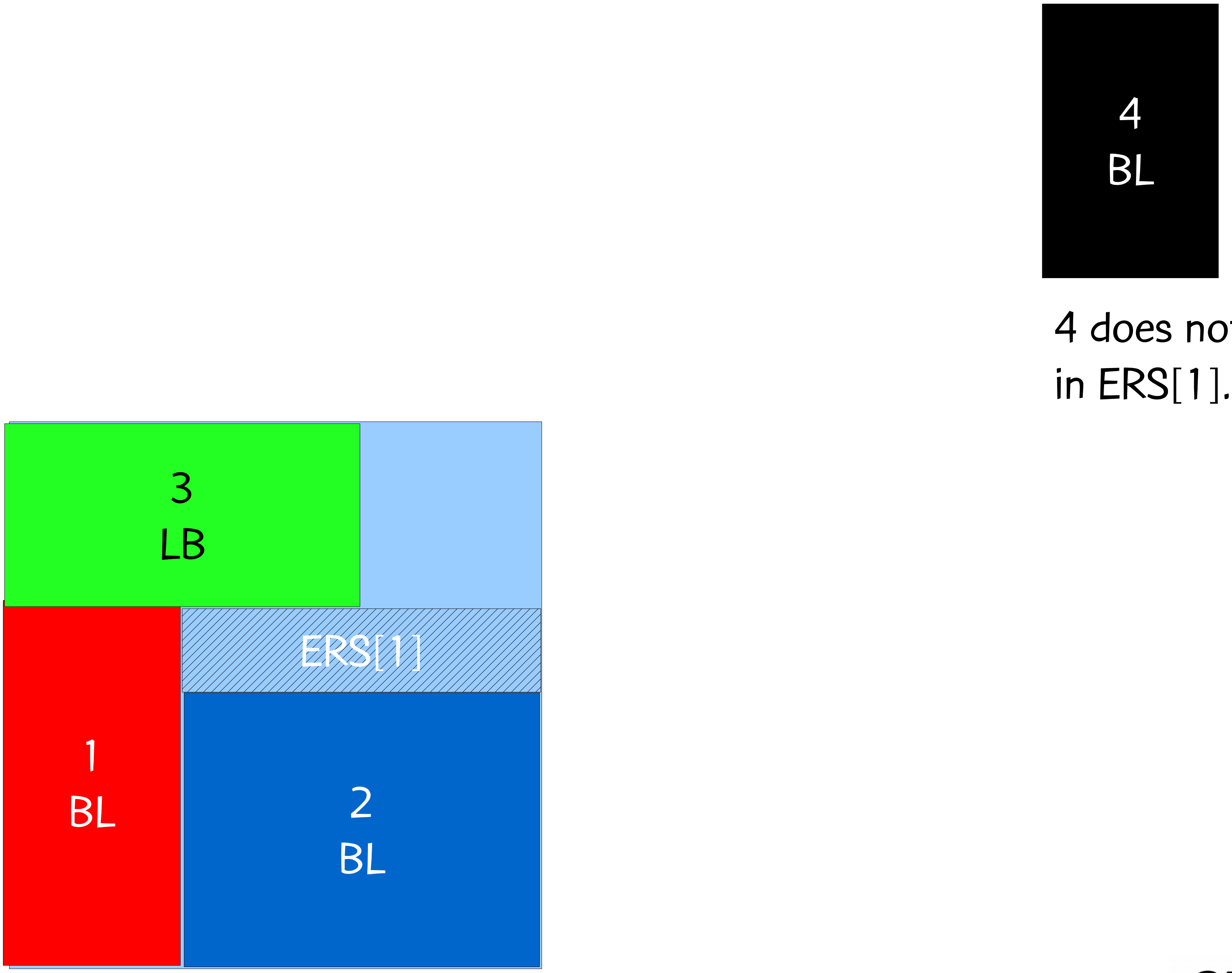

3
LB
4
BL
1
BL
2
BL


3
LB
4
BL
ERS[1]
1
BL
2
BL

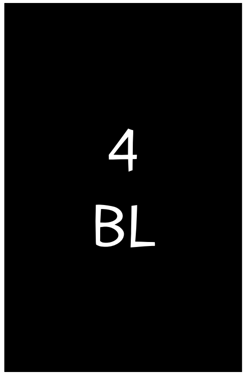

3
LB
4
BL
ERS[2]
1
BL
2
BL


4
BL
3
LB
1
BL
2
BL

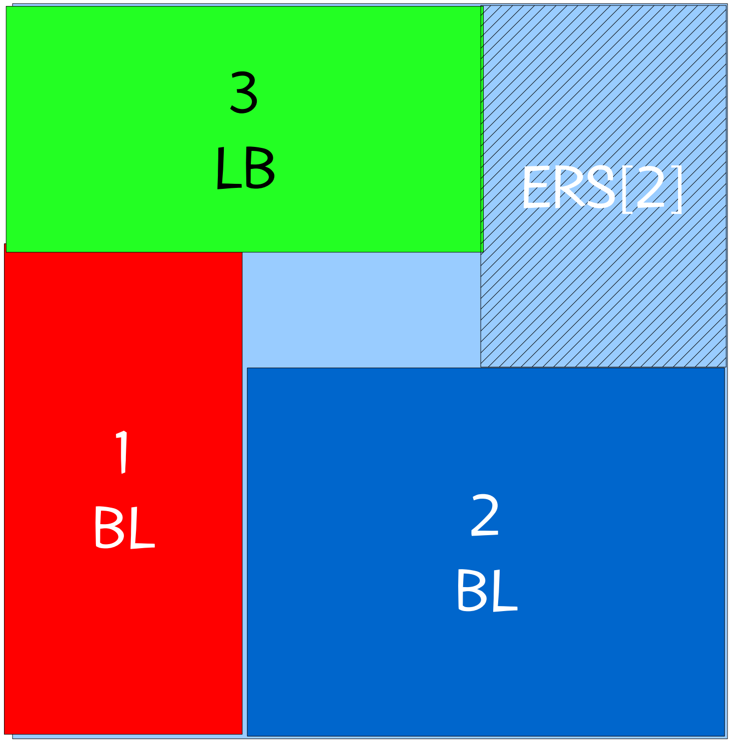

4
BL
3
LB
ERS[1]
1
BL
2
BL

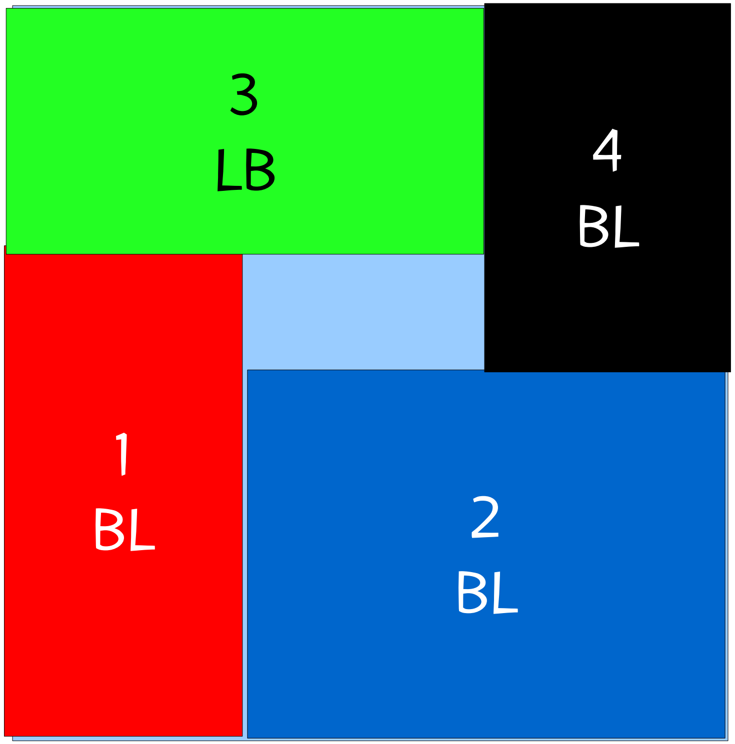



4 does not fit in ERS[1].




4 does fit
in ERS[2].






Optimal solution!

# Experimental results

# Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:

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  - TABU: tabu search of Alvarez-Valdes et al. (2007)

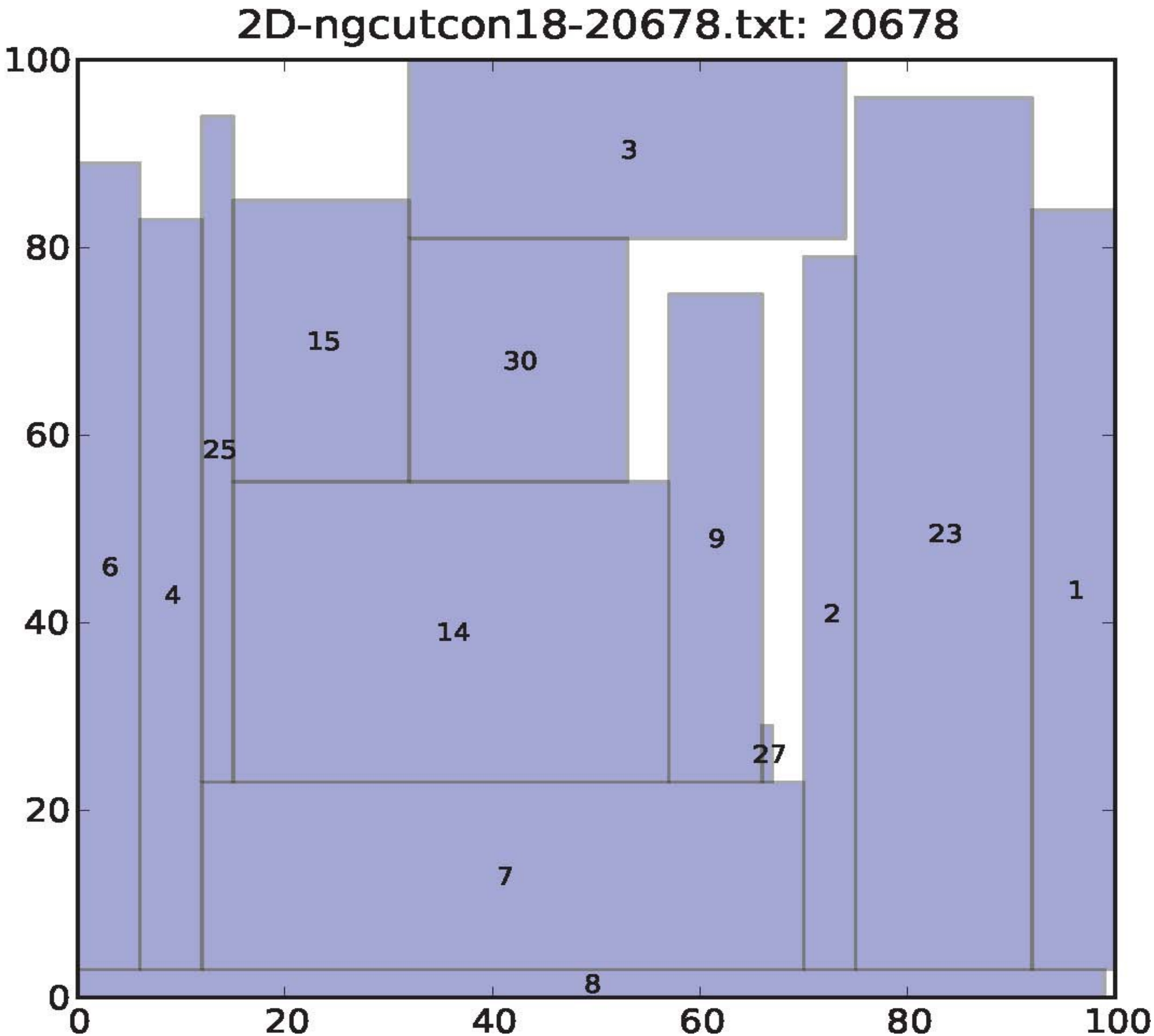
# Number of best solutions / total instances

| Problem | PH | GA | GRASP | TABU | BRKGA BL-LB-L-4NR |
|---|---|---|---|---|---|
| From literature (optimal) | 13/21 | **21/21** | 18/21 | **21/21** | **21/21** |
| Large random* | 0/21 | 0/21 | 5/21 | 8/21 | **20/21** |
| Zero-waste | | | 5/31 | 17/31 | **30/31** |
| Doubly constrained | 11/21 | | 12/21 | 17/21 | **19/21** |

* For large random: number of best average solutions / total instance classes

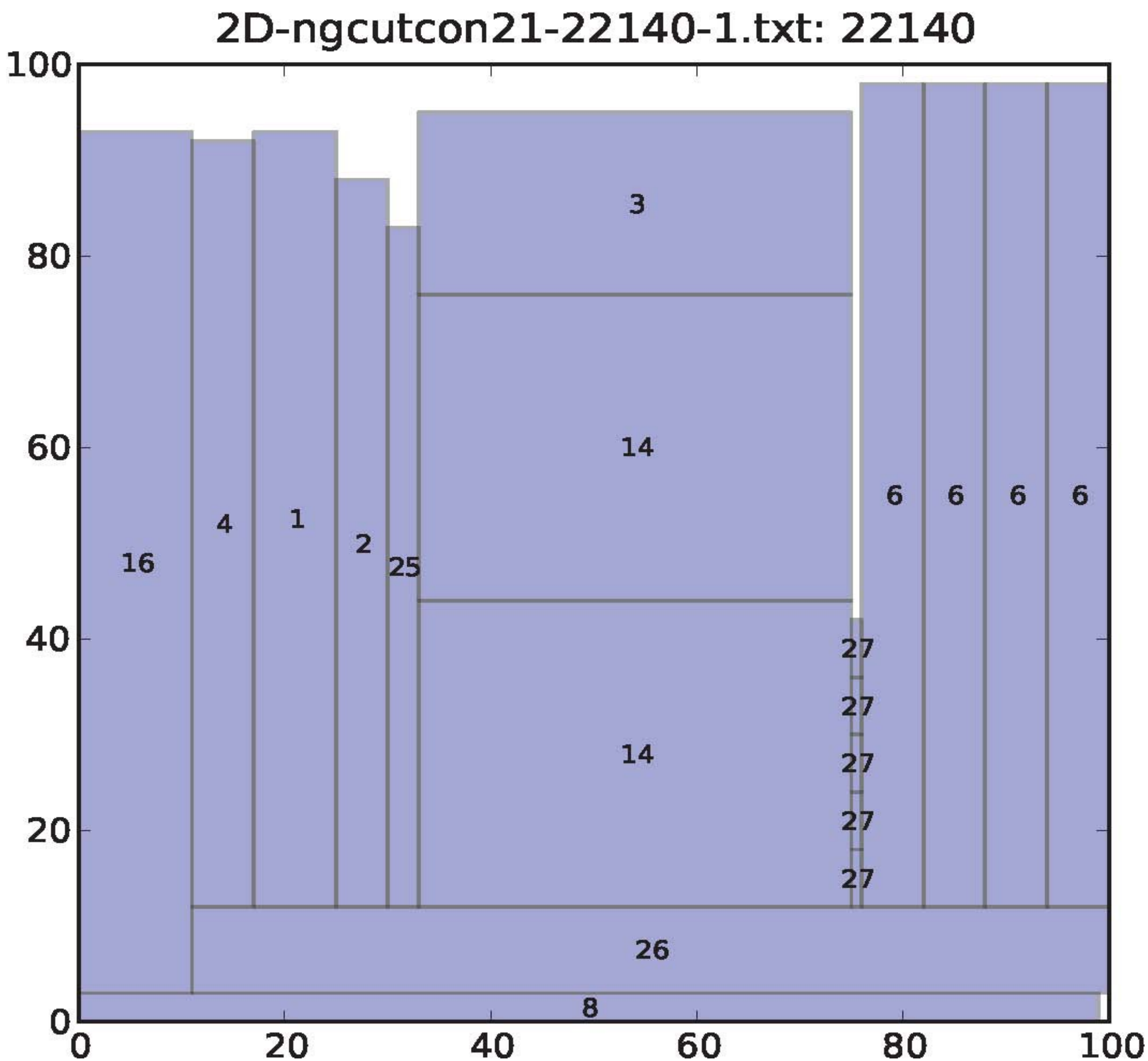
# Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

| Problem | Min solution time (secs) | Avg solution time (secs) | Max solution time (secs) |
|---|---|---|---|
| From literature (optimal) | 0.00 | 0.05 | 0.55 |
| Large random | 1.78 | 23.85 | 72.70 |
| Zero-waste | 0.01 | 82.21 | 808.03 |
| Doubly constrained | 0.00 | 1.16 | 16.87 |




New BKS for a 100 x100 doubly constrained instance of Fekete & Schepers (1997) of value **20678.** Previous best was **19657** by tabu search of Alvarez-Valdes et al., (2007).

30 types
30 rectangles




New BKS for a 100 x 100 doubly constrained instance Fekete & Schepers (1997) of value **22140**.

Previous BKS was **22011** by tabu search of Alvarez-Valdes et al. (2007).

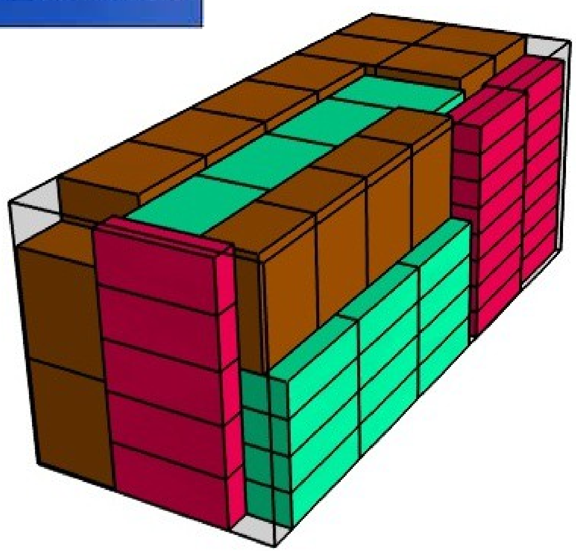
29 types
97 rectangles

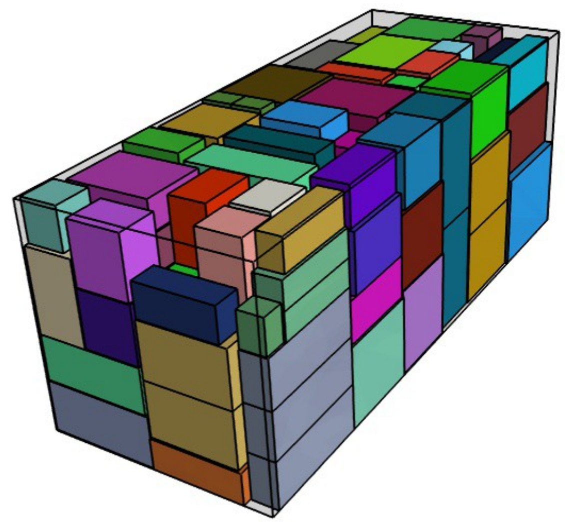
# Some remarks


We have extended this to 3D packing:

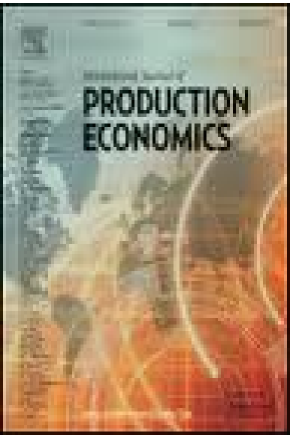
J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers & Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: http://mauricio.resende.info/doc/brkga-pack3d.pdf





# 3D bin packing

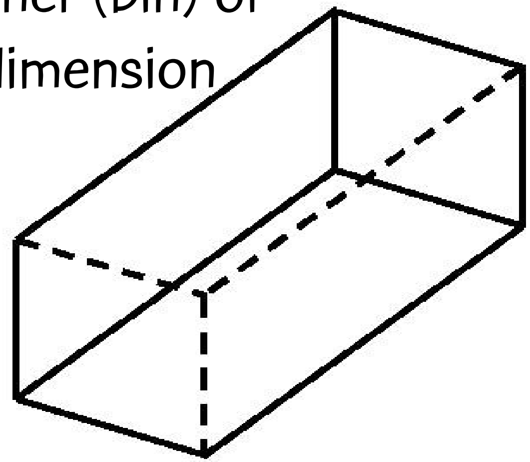


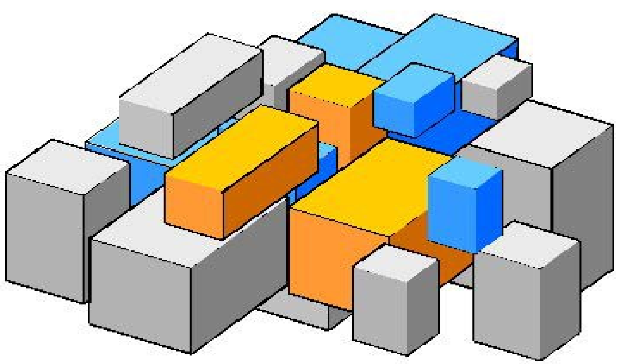

J.F. Gonçalves and M.G.C.R., "A biased random-key genetic algorithm for 2D and 3D bin packing problems," International J. of Production Economics, vol. 15, pp. 500–510, 2013.

http://mauricio.resende.info/doc/brkga-binpacking.pdf

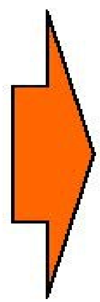
# 3D bin packing problem

Container (bin) of fixed dimension

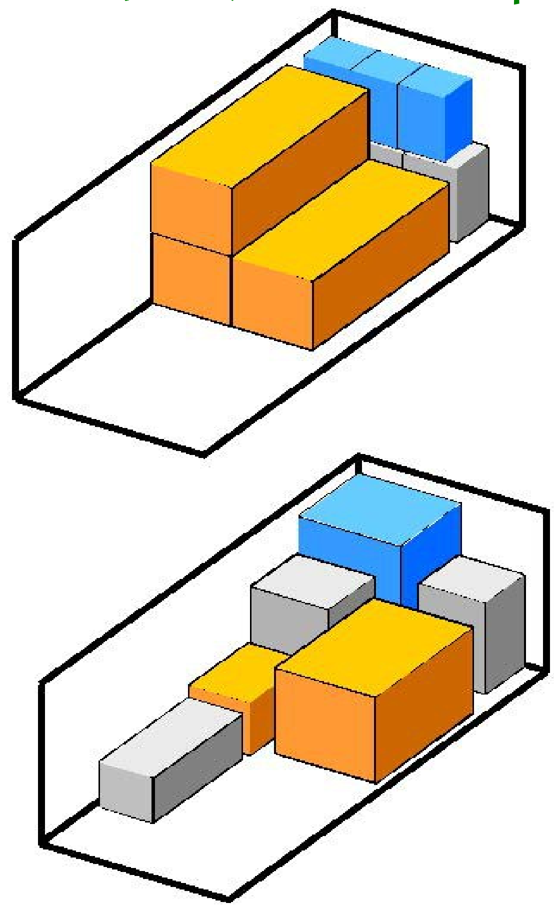




Boxes of different dimensions



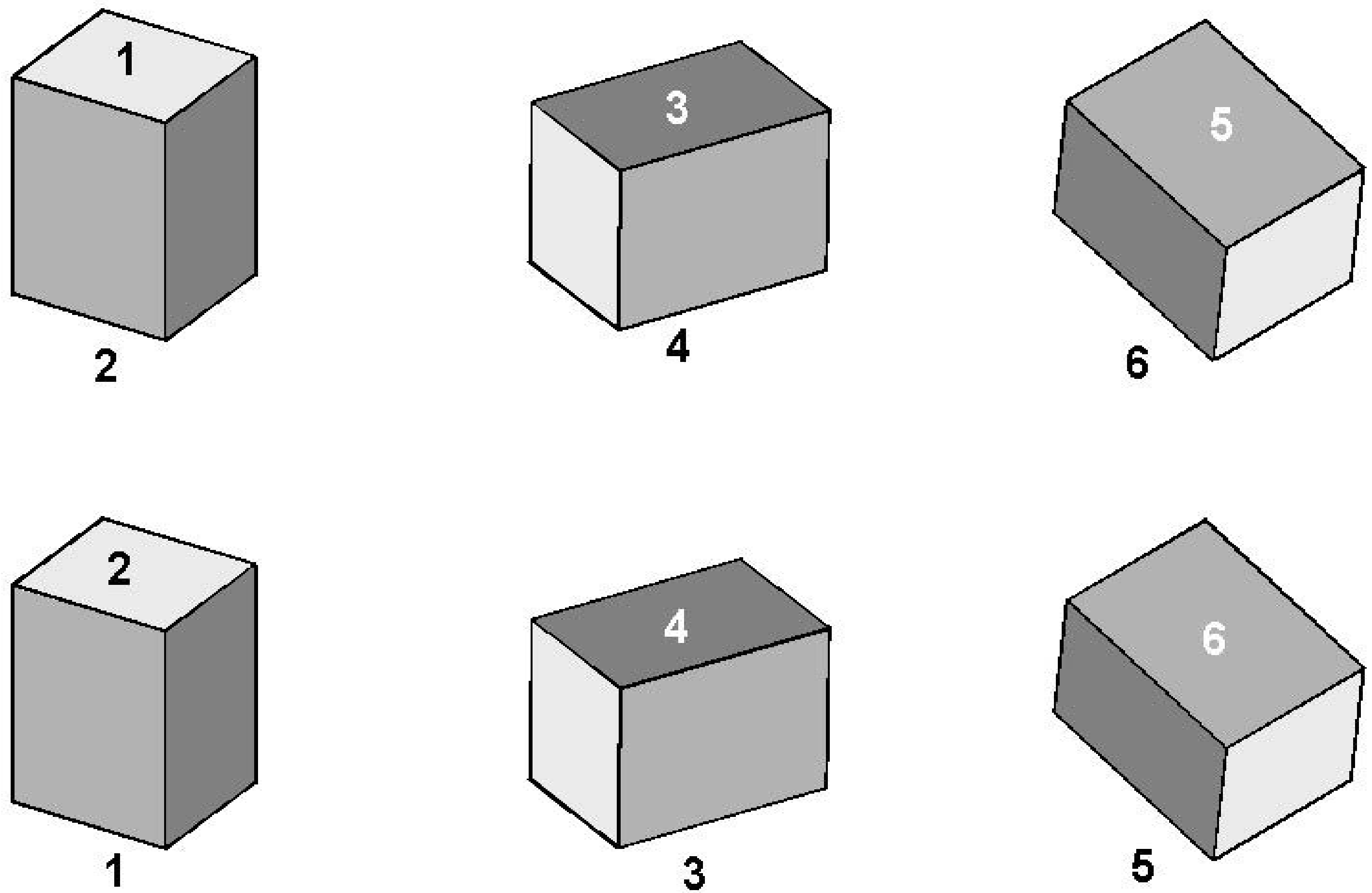
Minimize number of containers (bins) needed to pack all boxes



# 3D bin packing constraints

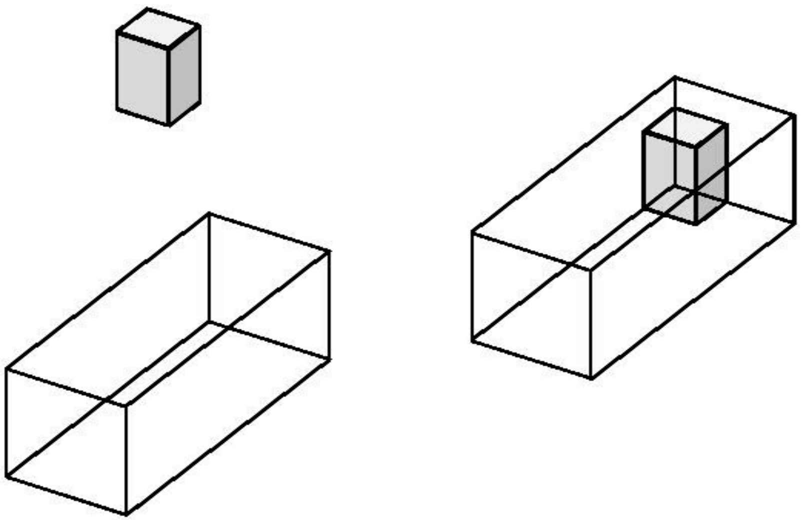
- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)

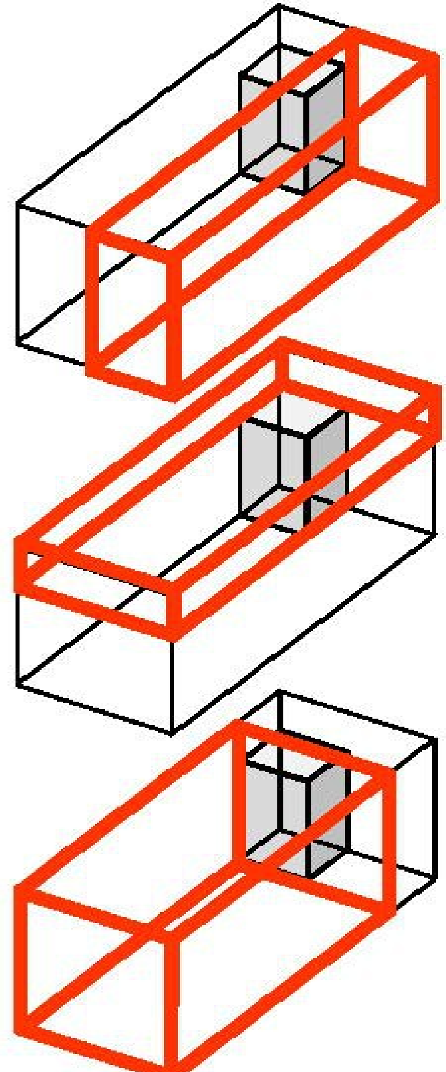
# Six possible orientations for each box



# Difference process - DP

(Lai & Chan, 1997)





When box is placed in container ...

use DP to keep track of maximal free spaces

# Encoding

Solutions are encoded as vectors of 3n random keys, where n is the number of boxed to be packed.

$$X = (\underbrace{x_1, x_2, \ldots, x_n}_{\text{Box packing sequence}}, \underbrace{x_{n+1}, x_{n+2}, \ldots, x_{2n}}_{\text{Placement heuristic}}, \underbrace{x_{2n+1}, x_{2n+2}, \ldots, x_{3n}}_{\text{Box orientation}})$$

# Decoding

1) Sort first n keys of X to produce sequence boxes will be packed;
2) Use second n keys of X to determine which placement heuristic to use (back-bottom-left or back-left-bottom):
   - if $x_{n+i} < ½$ then use back-bottom-left to pack i-th box
   - if $x_{n+i} \geq ½$ then use back-left-bottom to pack i-th box
3) Use third n keys of X to determine which of six orientations to use when packing box:
   - $x_{2n+i} \in [0,1/6)$: orientation 1;
   - $x_{2n+i} \in [1/6,2/6)$: orientation 2; ...
   - $x_{2n+i} \in [5/6,1]$: orientation 6.

# Decoding

For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation

# Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness: aNB
- aNB = NB + ( LeastLoad / BinVolume ), where
  - LeastLoad is load on least loaded bin
  - BinVolume is volume of bin: H x W x L

# Experiment

- Parameters:
  - population size: $p = 30n$
  - size of elite partition: $p_e = .10p$
  - number of of mutans: $p_m = .15p$
  - crossover probability: 0.7
  - stopping criterion: 300 generations

# Experiment

- Instances:
  - 320 instances of Martello et al. (2000)
  - generator is available at http://www.diku.dk/~pisinger/codes/html
  - 8 classes
  - 40 instances per class
  - 10 instances for each value of $n \in \{50, 100, 150, 200)$

# Experiment

- We compare BRKGA with:
  - TS3, the tabu search of Lodi et al. (2002)
  - GLS, the guided local search of Faroe et al. (2003)
  - TS2PACK, the tabu search of Crainic et al. (2009)
  - GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)

# Summary

| Class | Bin size | BRKGA | GRASP | TS3 | TS2PACK | GLS |
|---|---|---|---|---|---|---|
| 1 | $100^3$ | **127.3** | **127.3** | 127.9 | 128.2 | 128.3 |
| 2 | $100^3$ | **125.5** | 125.8 | 126.8 | | |
| 3 | $100^3$ | **126.5** | 126.9 | 127.5 | | |
| 4 | $100^3$ | 294.0 | 294.0 | 294.0 | **293.9** | 294.2 |
| 5 | $100^3$ | **70.4** | 70.5 | 71.4 | 71.0 | 70.8 |
| 6 | $10^3$ | **95.0** | 95.4 | 96.1 | 95.8 | 96.0 |
| 7 | $40^3$ | **58.2** | 59.4 | 60.0 | 59.0 | 59.0 |
| 8 | $100^3$ | **80.9** | 82.0 | 82.6 | 81.9 | 81.9 |
| Sum(rows 1, 4-8): | | **725.8** | 728.6 | 732.0 | 729.8 | 730.2 |
| Sum(rows 1-8): | | **977.8** | 981.3 | 986.3 | | |

# Concluding remarks

- Reviewed BRKGA framework
- Applied framework to
  - 2D/3D packing to maximize value packed
  - 2D/3D bin packing to minimize number of bins
- All decoders were simple heuristics
- BRKGA "learned" how to "operate" the heuristics
- In all cases, several new best known solutions were produced

# Other applications of BRKGA

## Telecommunications

- Survey (R., 2012)
- Weight setting in OSPF routing (Ericsson et al., 2002; Buriol et al., 2005; Reis et al., 2011)
- Survivable network design (Andrade et al., 2006; Buriol et al., 2007; Ruiz et al., 2015; Andrade et al., 2015)
- Facility location (Breslau et al., 2011; Morán-Mirabal et al., 2013; Duarte et al., 2014; Stefanello et al., 2015)
- Routing & wavelength assignment (Noronha et al., 2011)

# Other applications of BRKGA

## Scheduling

- Job-shop scheduling (Gonçalves et al., 2005; Gonçalves & R., 2014 )
- Project scheduling (Gonçalves et al., 2008; 2009; 2011)
- Survey of project scheduling (Gonçalves et al., 2014)
- Field technician scheduling (Damm et al., 2015)

# Other applications of BRKGA

## Manufacturing and facility layout

- Manufacturing cell formation (Gonçalves & R., 2004)
- Minimization of open stacks (Gonçalves et al., 2014)
- Unequal area facility layout (Gonçalves & R., 2014)
- Minimization of tool switches (Chaves et al., 2014)

# Other applications of BRKGA

## Algorithm engineering

- Automatic tuning of parameters (Festa et al., 2010; Morán-Mirabal et al., 2013)
- Benchmarking (Gonçalves et al., 2014)
- Extensions of BRKGA (Lucena et al., 2014)
- Application programming interface (Toso et al., 2015)

# Other applications of BRKGA

## Clustering, covering, and packing

- 2D/3D orthogonal packing (Gonçalves & R., 2011; 2012)
- 2D/3D bin packing (R. et al., 2012)
- Steiner triple covering (Lucena et al., 2014)
- Overlapping correlation clustering (Andrade et al., 2014)
- Winner determination in combinatorial auctions (Andrade et al., 2014)

# Other applications of BRKGA

## Routing

- Capacitated arc routing (Martinez et al., 2011)
- K-interconnected multi-depot multi-TSP (Andrade et al., 2013)
- Family TSP (Morán-Mirabal et al., 2014)
- Capacitated VRP for blood sample collection (Grasas et al., 2014)

# Other applications of BRKGA

## Toll setting in road networks

- Road congestion minimization (Buriol et al., 2009; 2010; Stefanello et al., 2015)

# Other applications of BRKGA

## Continuous global optimization

- Bound-constrained GO (Silva et al., 2012)
- Nonlinearly-constrained GO (Silva et al., 2013)
- Python/C++ library for bound-constained GO (Silva et al., 2013)
- Finding multiple roots of system of nonlinear equations (Silva et al., 2014)

# Thanks!

These slides and all of the papers cited in this lecture can be downloaded from my homepage:

http://mauricio.resende.info