A biased random-key genetic algorithm for a prize-collecting directed Steiner forest network design problem

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Joint work with

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Summary

- Prize collecting directed k-hop Steiner forest (PCk-HSF) problem
- Wireless backhaul network planning as a PCk-HSF problem with additional constraints
- Biased random-key genetic algorithm (BRKGA) for wireless backhaul network planning focusing on the decoder
- Application of the BRKGA to a “real” instance of wireless backhaul network planning
- Concluding remarks

Prize collecting directed Steiner Forest

- Let $G = (V,E)$ be a given directed graph
- Let V_r , V_s , and V_d partition the node set, i.e.

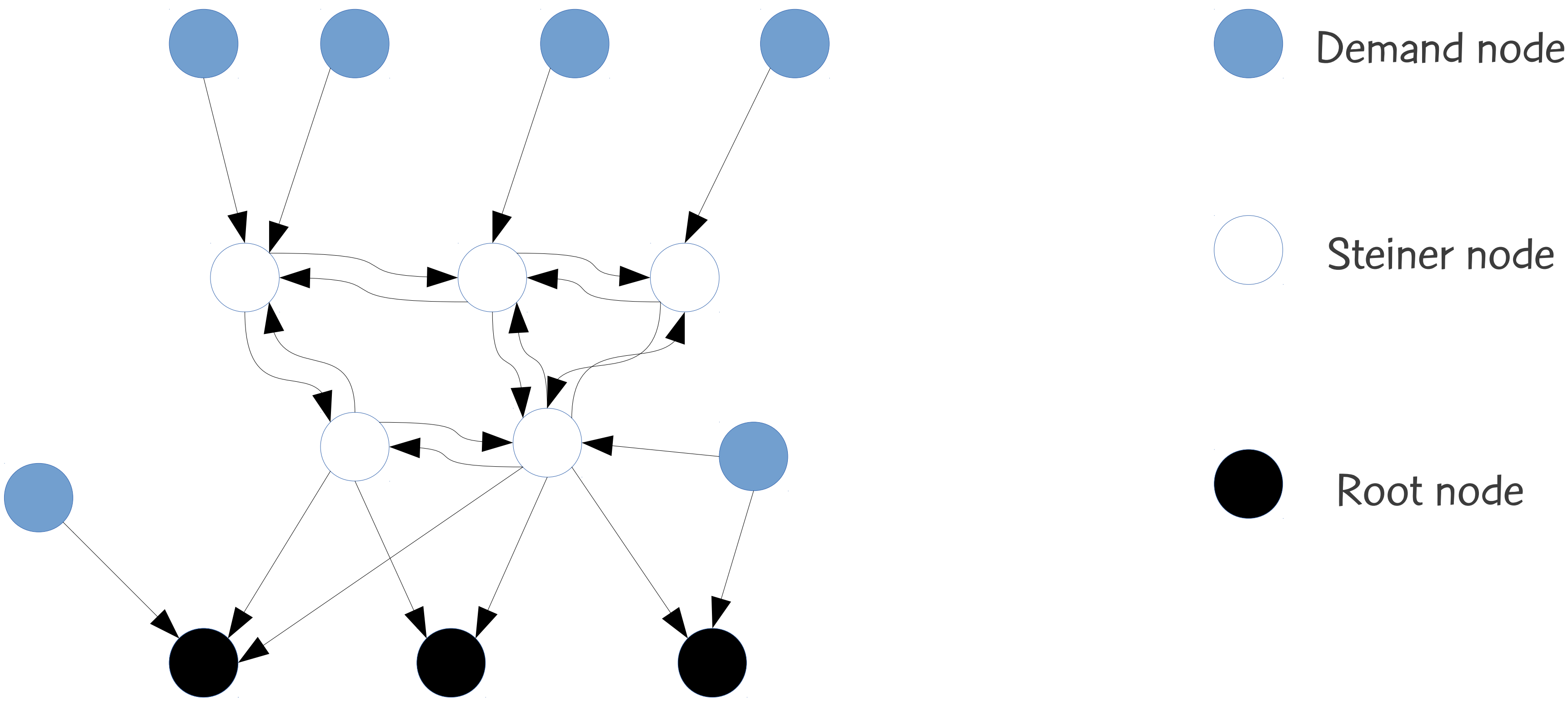
$$V_r \cup V_s \cup V_d = V \quad \text{and} \quad V_r \cap V_s \cap V_d = \emptyset$$

  - V_r – set of root nodes
  - V_s – set of Steiner nodes
  - V_d – set of demand nodes

Prize collecting directed Steiner Forest

- Let $G = (V,E)$ be a given directed graph
- Arcs are directed
  - From demand nodes to Steiner nodes and root nodes
  - From Steiner nodes to root nodes
- For each pair of nodes $v, u \in V_s$
  - $(u,v) \in E$
  - $(v,u) \in E$

Prize collecting directed Steiner Forest

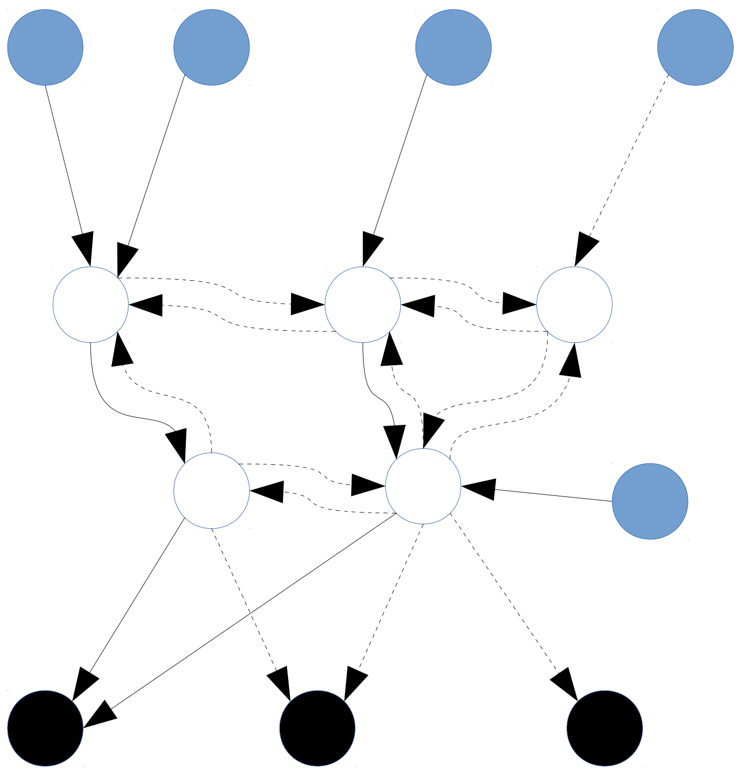


Prize collecting directed Steiner Forest

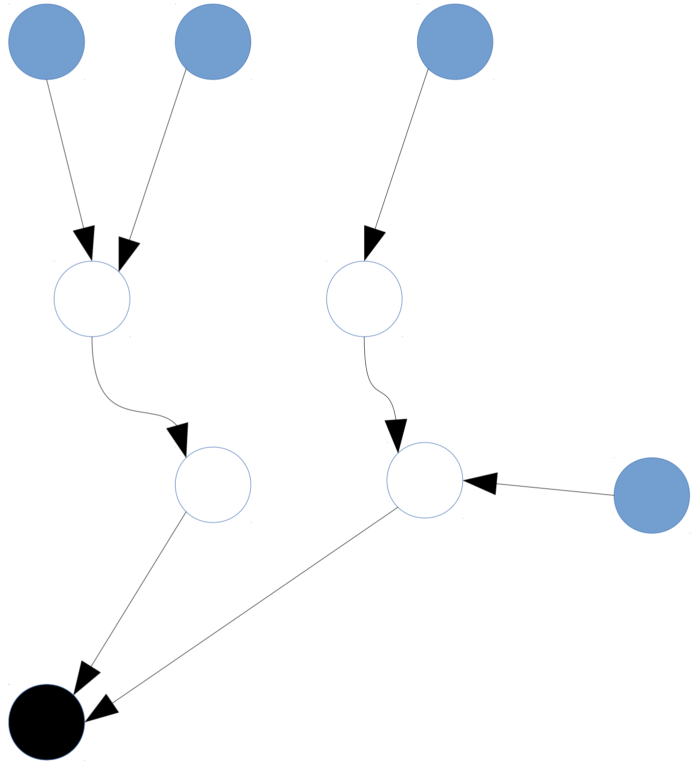
- A Directed Steiner Tree $T = (V[T] \subseteq V, E[T] \subseteq E)$ with root in $r \in V_r[T]$ is a loopless connected subgraph of G with a unique root r.
  - For each demand node $u \in V[T]$ there is a unique path in T from u to r.

Prize collecting directed Steiner Forest

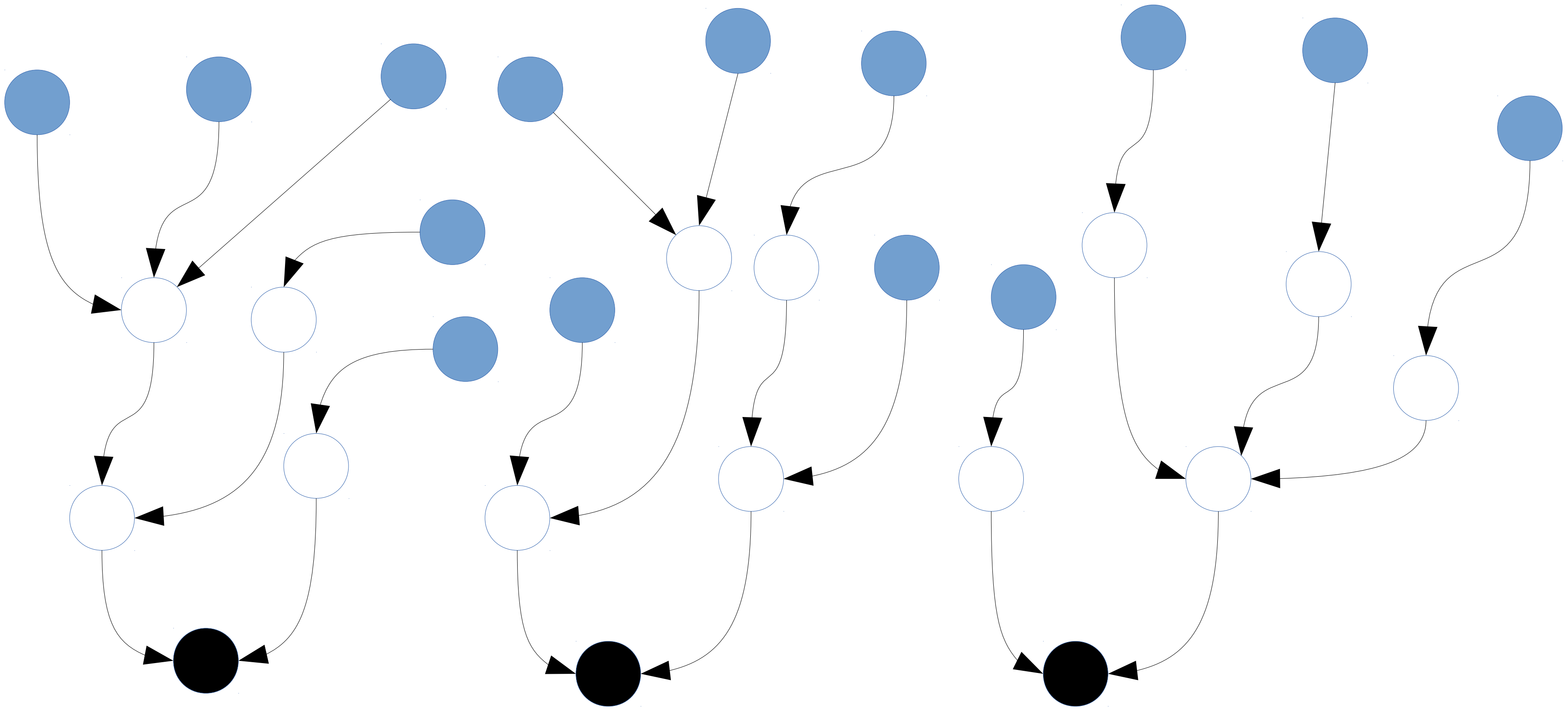
- A k-Hop Directed Steiner Tree is a directed Steiner tree (connected subgraph of G with a unique root r) such that:
  - Any path from a demand node u to the root r has no more than k+2 nodes, including u and r.
- A k-Hop Directed Steiner Forest is a collection of disjoint k-Hop Directed Steiner Trees



Acyclic directed graph



2-Hop Directed Steiner tree



2-Hop Directed Steiner Forest

Prize collecting directed Steiner Forest

- A demand function $d: V_d \rightarrow \mathbb{R}^+$ gives the prize to be collected from each demand node if it is a leaf on the tree.
- A cost function $c: V_s \rightarrow \mathbb{R}^+$ gives the cost of of each Steiner vertex.

Prize collecting directed Steiner Forest

- Given a directed graph $G = (V,E)$, vertex partition V_r , V_s , and V_d, hop parameter k, demand function d, and cost function c
- FIND: A k-hop directed Steiner forest F such that the profit

$$\text{sum} [v \in V_d[F]] \, d_v - \text{sum} [v \in V_s[F]] \, c_v$$

is maximized.

Wireless backhaul network planning

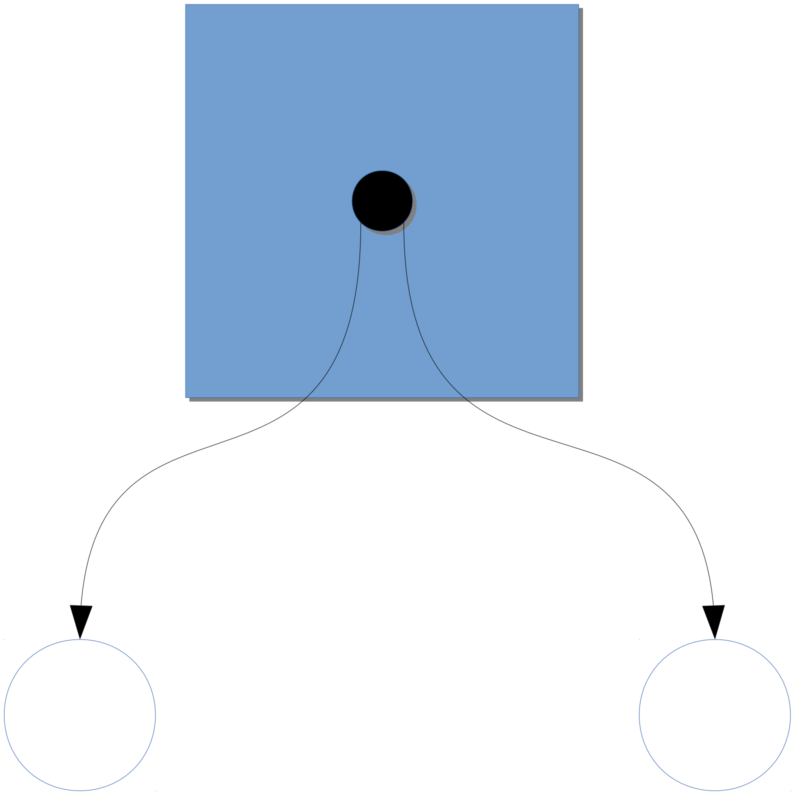
Given a geographical region where locations are represented as lat-long coordinates, where

- V_d is the set of nodes representing traffic origination points (demand points) in the region
- V_s is the set of nodes representing locations where equipment for traffic collection and routing is located (e.g. utility poles)
- V_r is the set of nodes representing fibered access points (FAP), e.g. remote terminals (RT), macrocell (MC), or central offices.

Wireless backhaul network planning

Constraints: Demand splitting

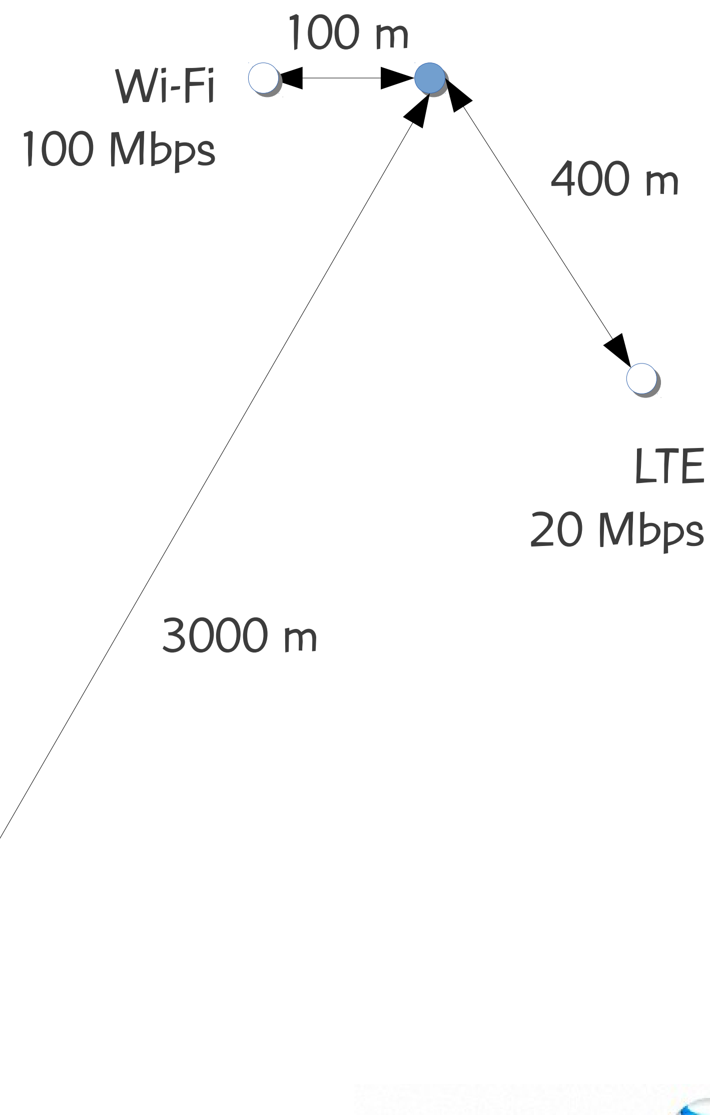
- Estimate for each block total demand is placed in center of block
- Block can be served by one or more antennae so demand can be split among them
- Because of this undirected cycles can be introduced resulting in a directed acyclic graph (DAG)



Wireless backhaul network planning

Constraints: Access equipment action radii & capacities

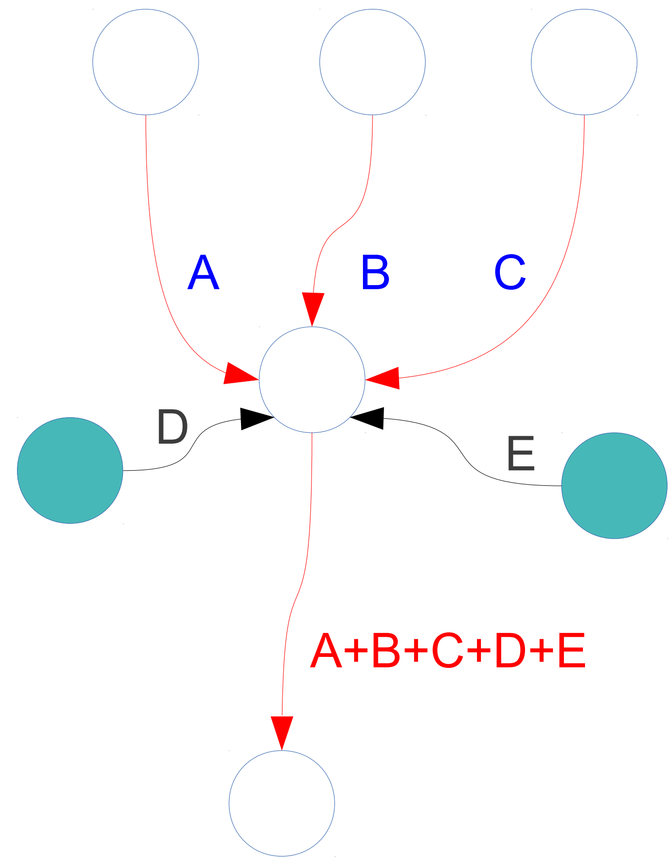
- Action radii
  - Wi-Fi: 100 m
  - LTE: 400 m
  - macrocell: 3 km
- Processing capacity
  - Wi-Fi: 100 Mbps
  - LTE: 20 Mbps
  - macrocell: 25 Mbps



Wireless backhaul network planning

Constraints: Retransmitter equipment capacity

- Action radii: 1 km
- Processing capacity
  - Flow that equipment receives from other wireless retransmitters plus flow it sends to other retransmitters is limited to at most 100 Mbps



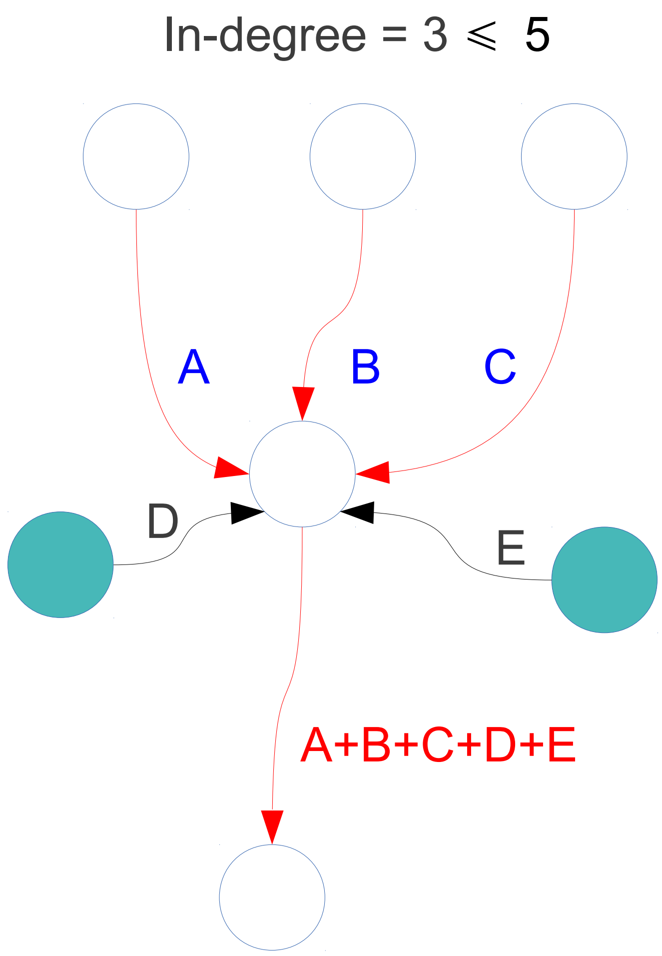
$A+B+C+A+B+C+D+E \leqslant 100$

Wireless backhaul network planning

Constraints: Retransmitter equipment capacity

Fan-in constraint

- A limited number (5) of neighboring transmission equipment can flow traffic into equipment



A+B+C+A+B+C+D+E ⩽100

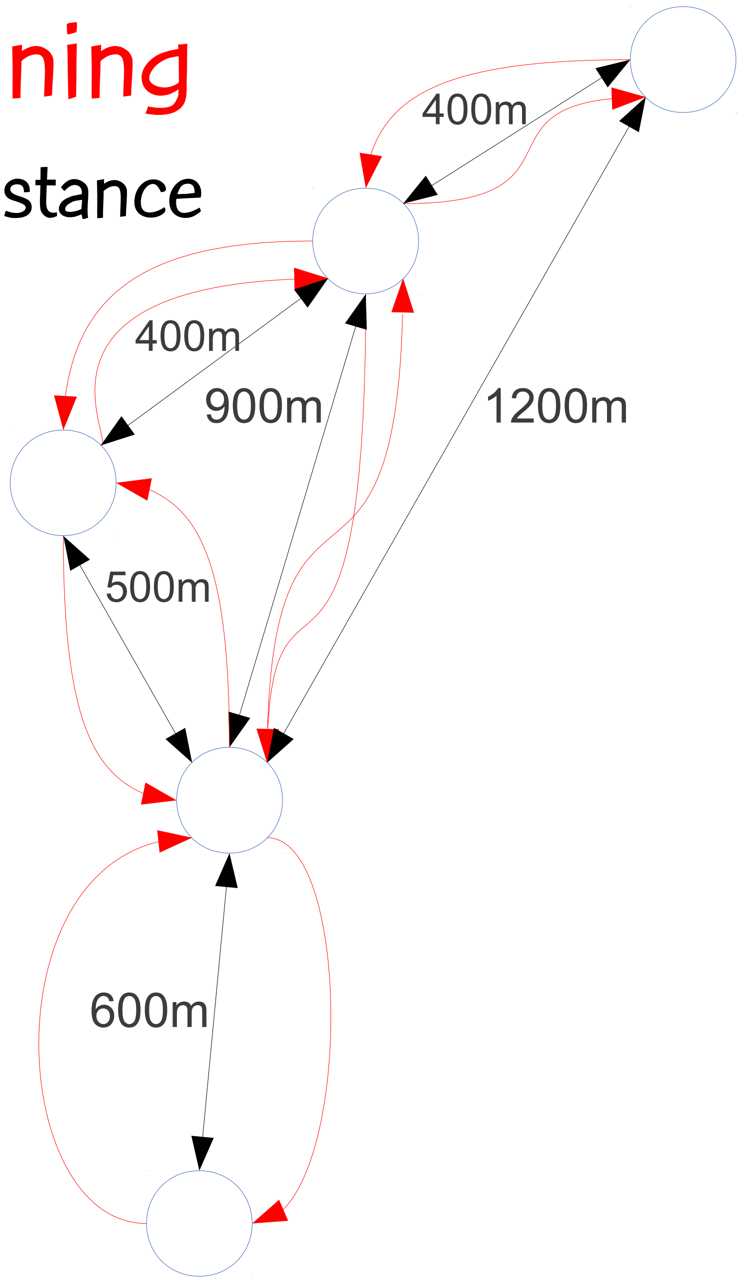
Wireless backhaul network planning

Constraints: Line of sight and minimum distance

PCSF Problem assumed, for each pair
$u, v \in V_s$ that $(u,v) \in E$ and $(v,u) \in E$

Not so in wireless backhaul planning:
$(u,v) \in E$ and $(v,u) \in E$ only if

- Poles u and v are within 1000 m of each other



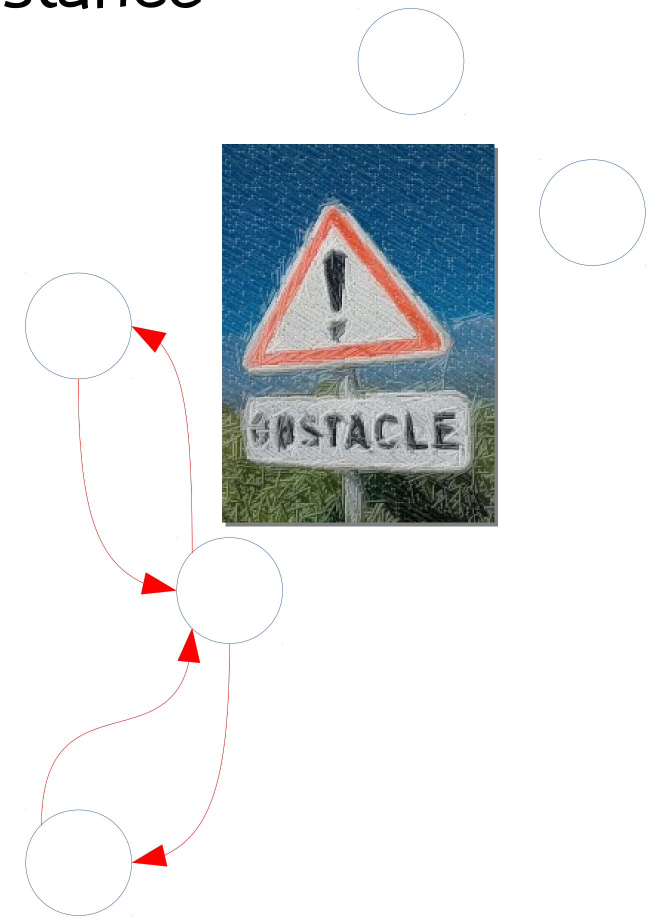
Wireless backhaul network planning

Constraints: Line of sight and minimum distance

PCSF Problem assumed, for each pair $u, v \in V_s$ that $(u,v) \in E$ and $(v,u) \in E$

Not so in wireless backhaul planning: $(u,v) \in E$ and $(v,u) \in E$ only if

- Poles u and v are within 1000 m of each other
- Poles u and v are in each other's line of sight

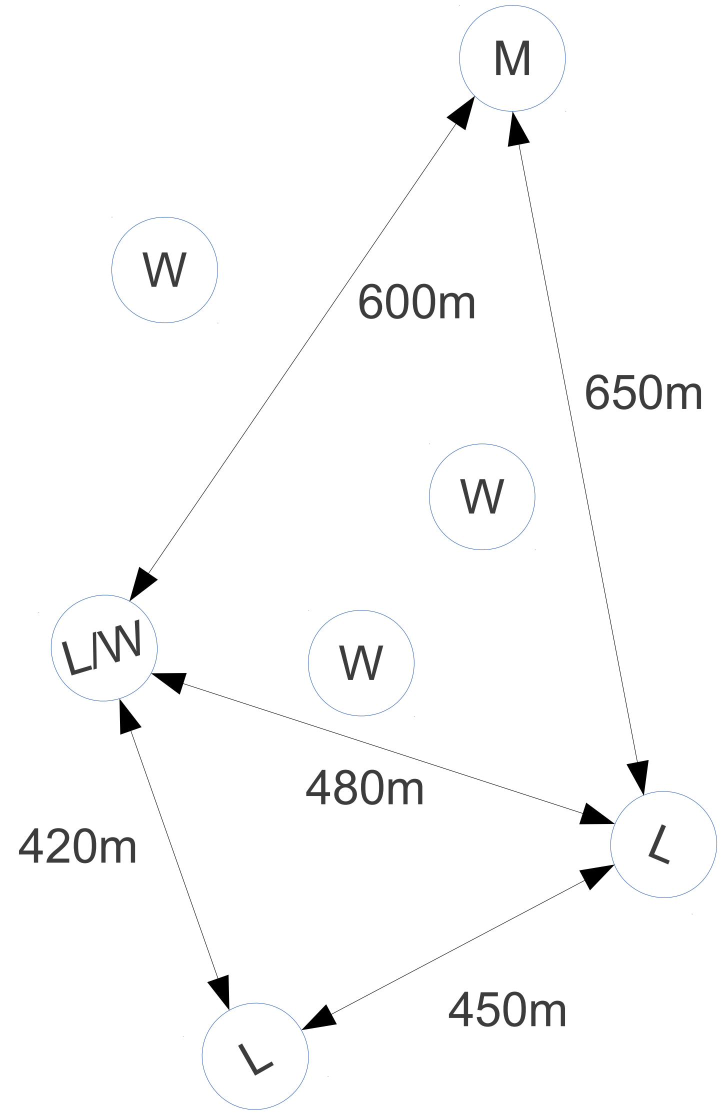


Wireless backhaul network planning

Constraints: Interference

LTE and MC use licensed spectrum and can interfere with each other.

- Pairs of LTE antennae must be separated by at least 400m
- LTE and macrocells by at least 500m
- No constraint on Wi-Fi exists since it uses non-licensed spectrum



Wireless backhaul network planning

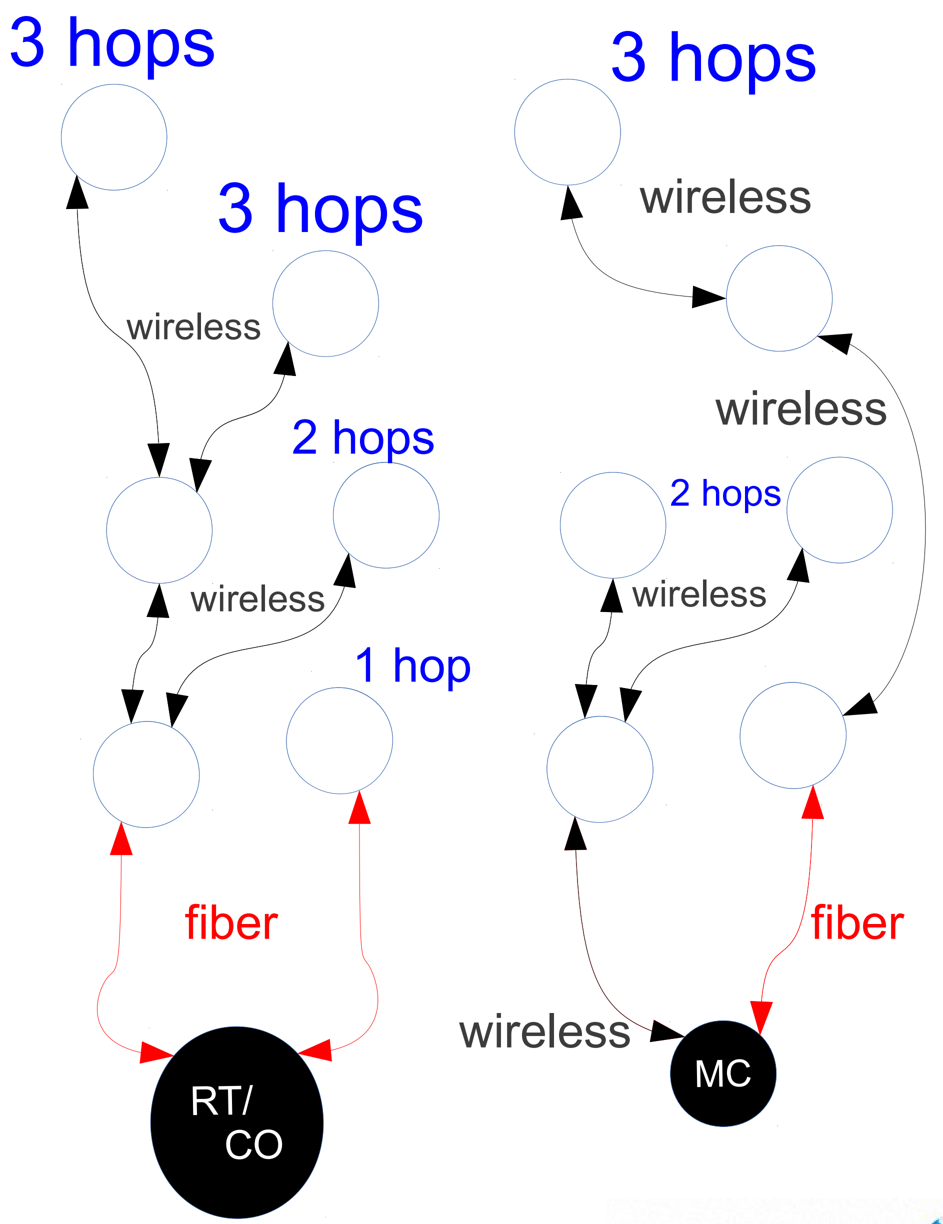
Constraints: k-Hops

First hop is from FAP (root) to pole

- If root is Central Office (CO) or RT link must use fiber
- If root is macrocell link can be fiber or wireless

All other links are wireless

Number of hops is limited to $k = 2$ or 3 (in case first link is fiber)

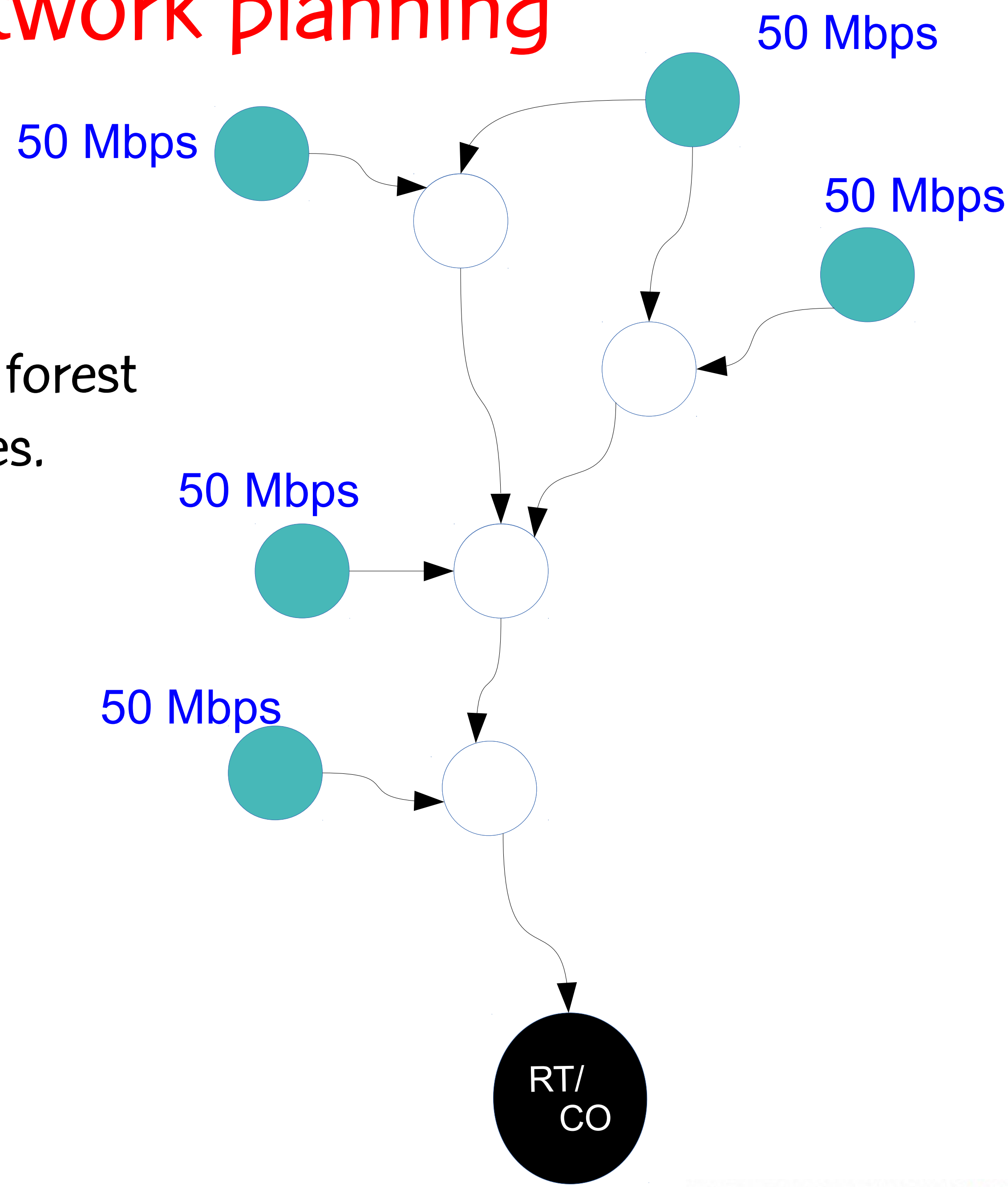


Wireless backhaul network planning

Constraints: Traffic flow

Traffic that is backhauled from demand points to root nodes of forest is limited by equipment capacities.

Only traffic that reaches roots is counted as revenue.



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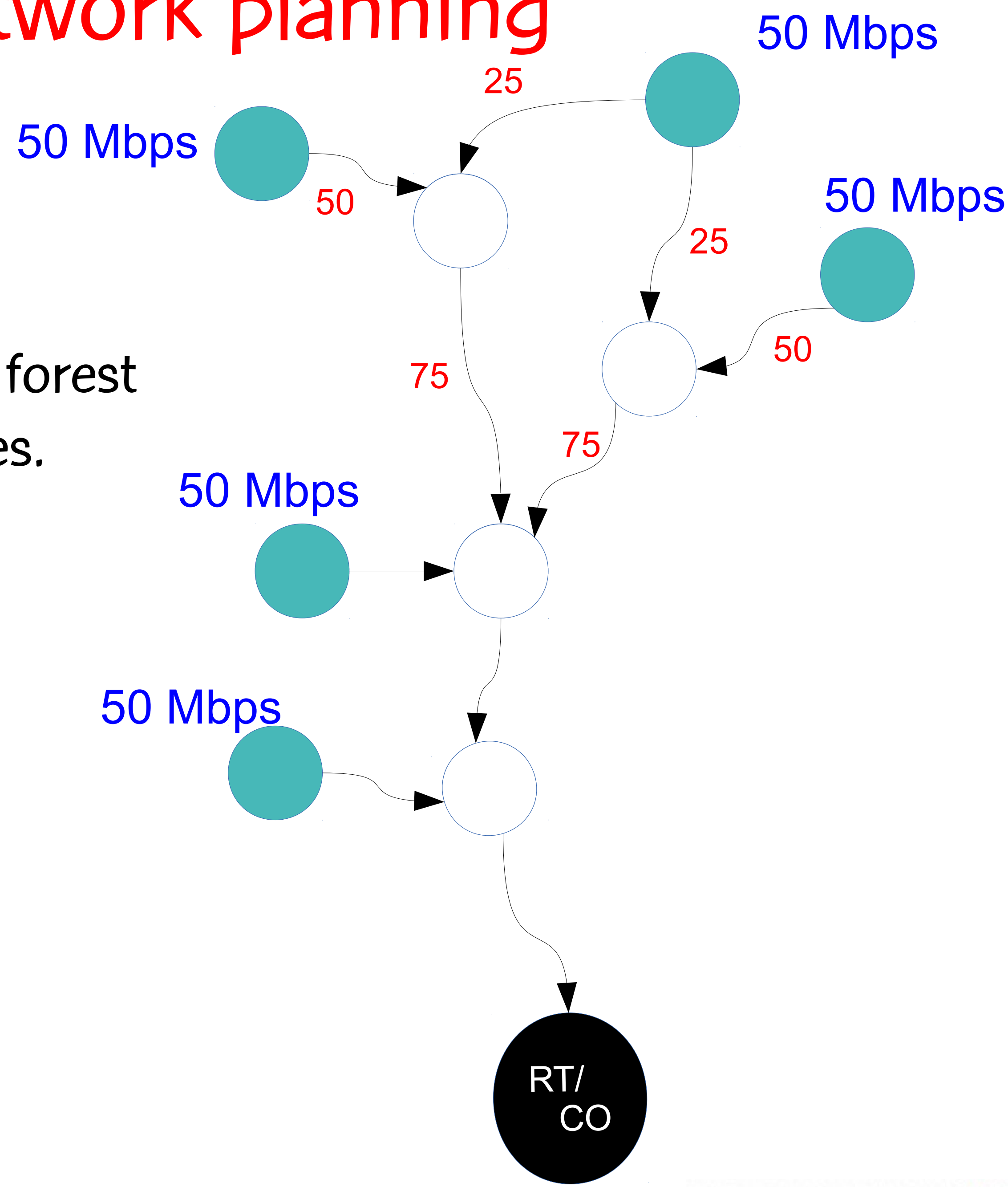

Wireless backhaul network planning

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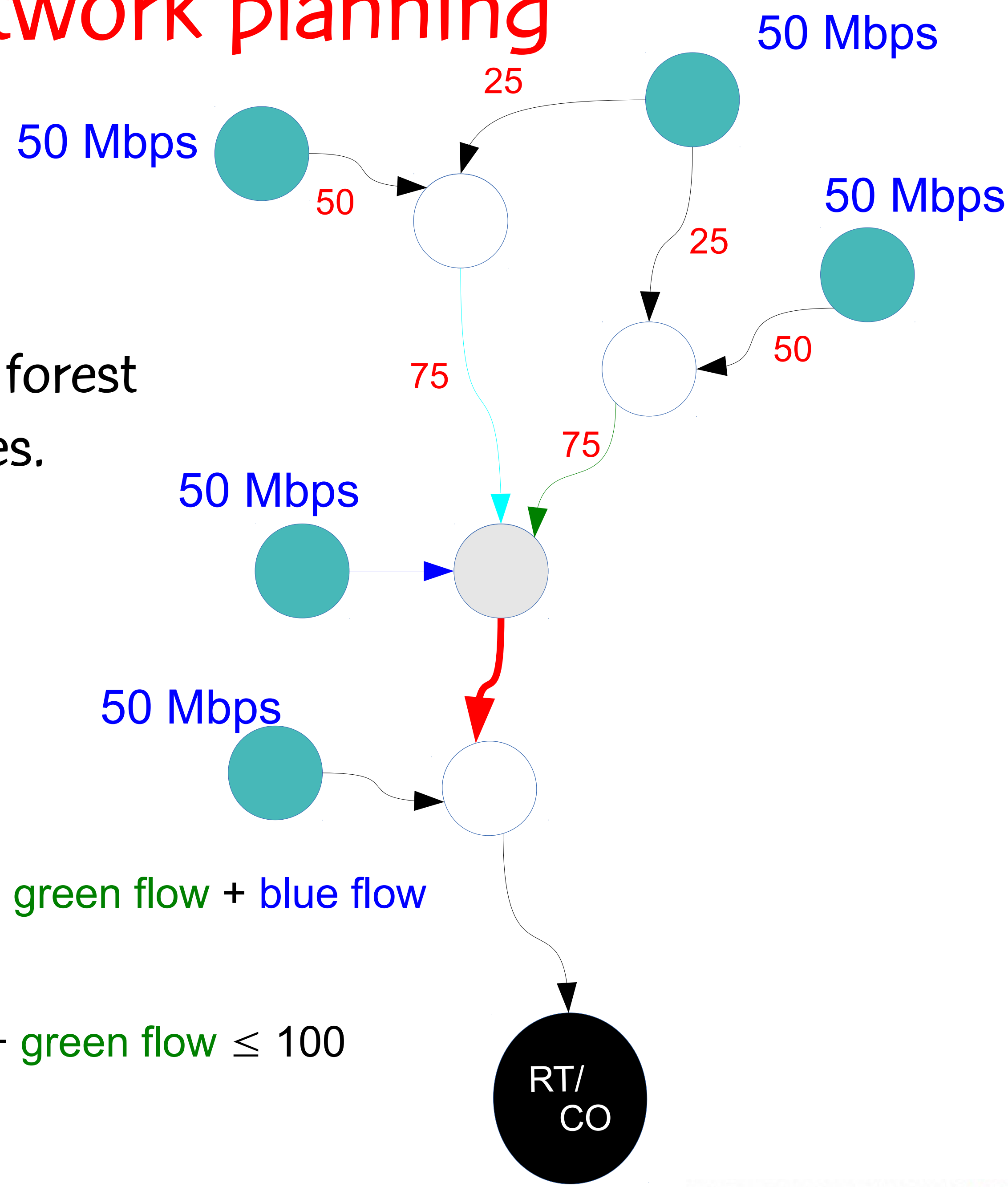

Flow conservation: red flow = cyan flow + green flow + blue flow

Capacity constraint: red flow + cyan flow + green flow $\leq$ 100

Wireless backhaul network planning

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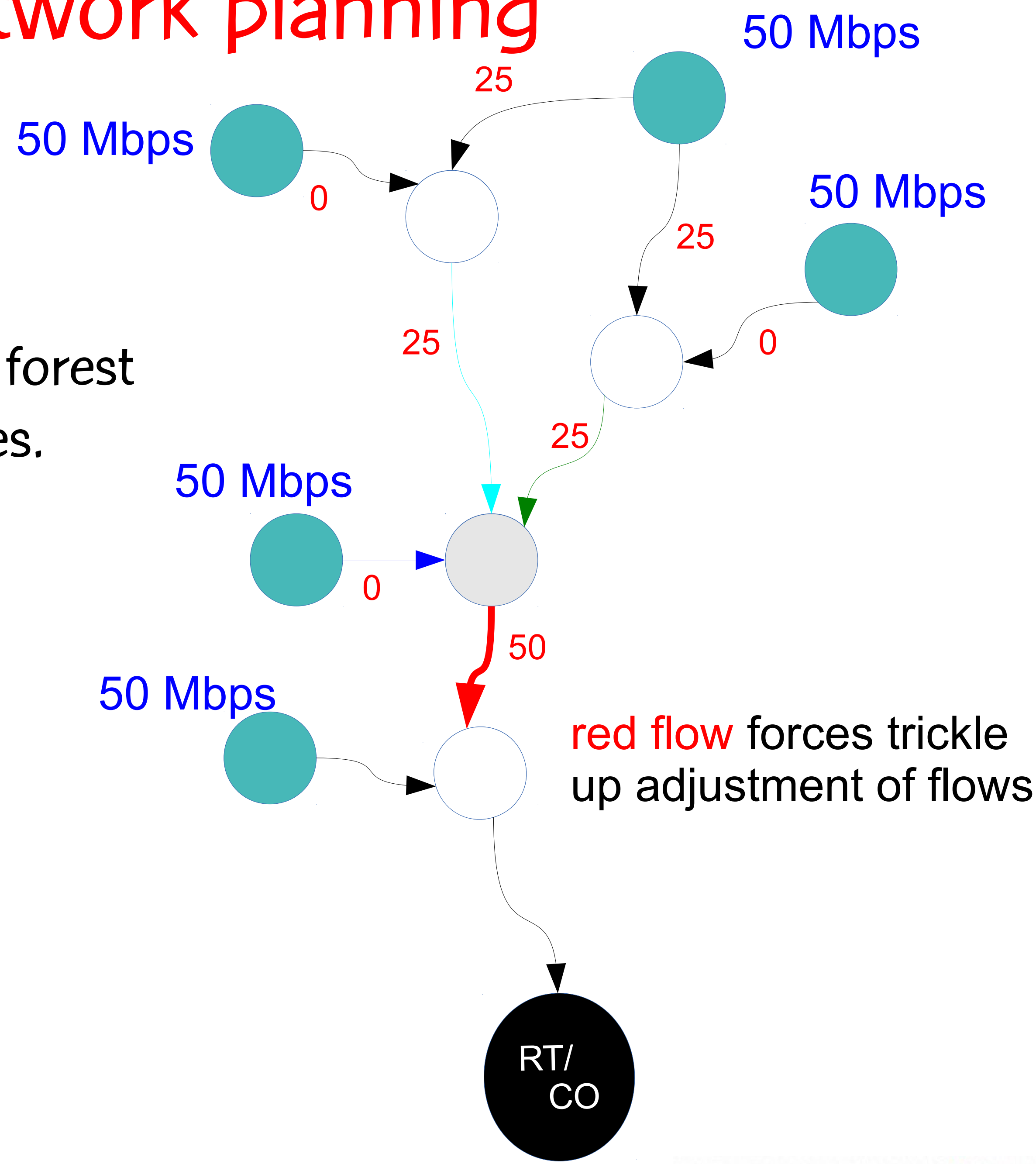

Wireless backhaul network planning

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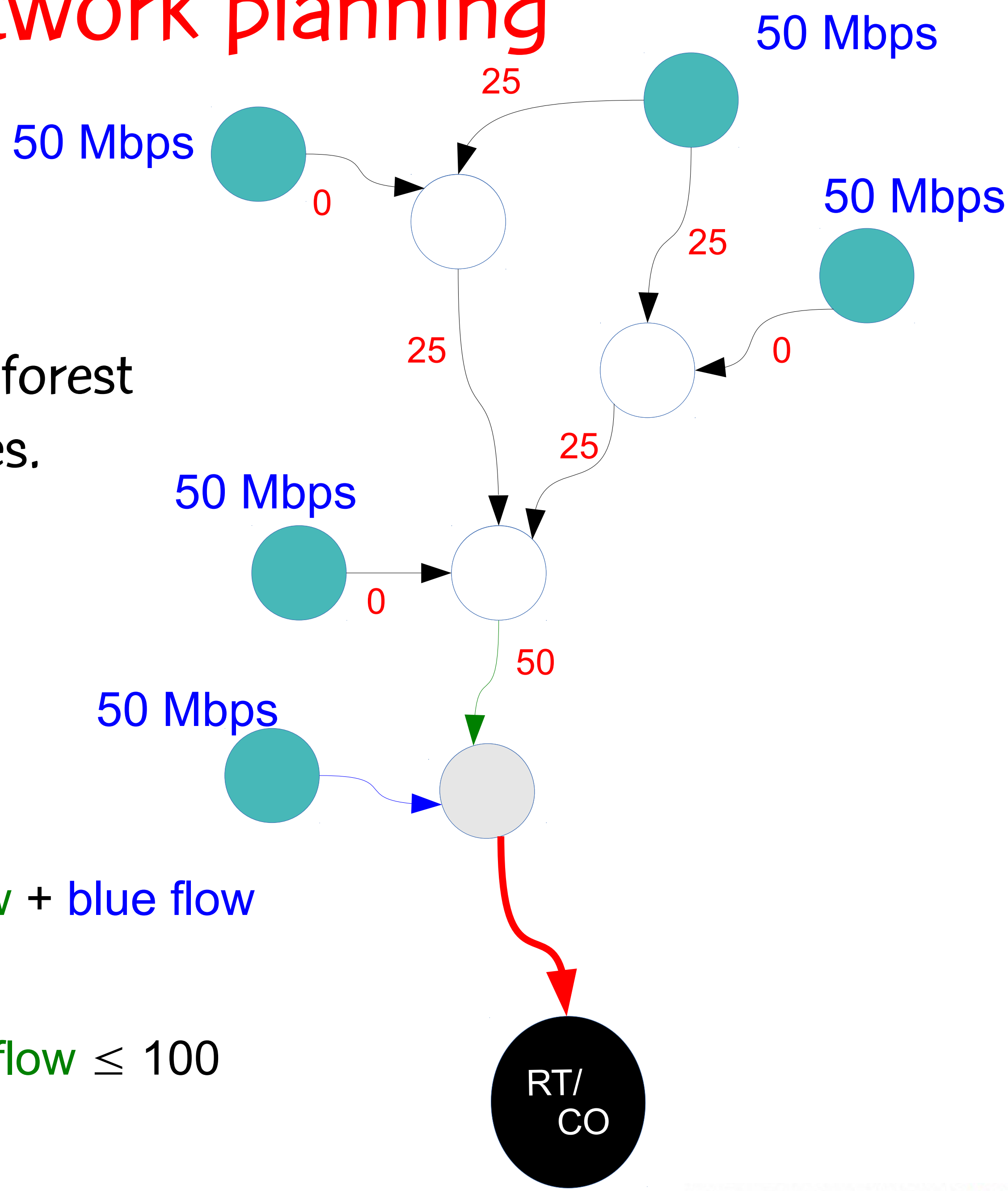
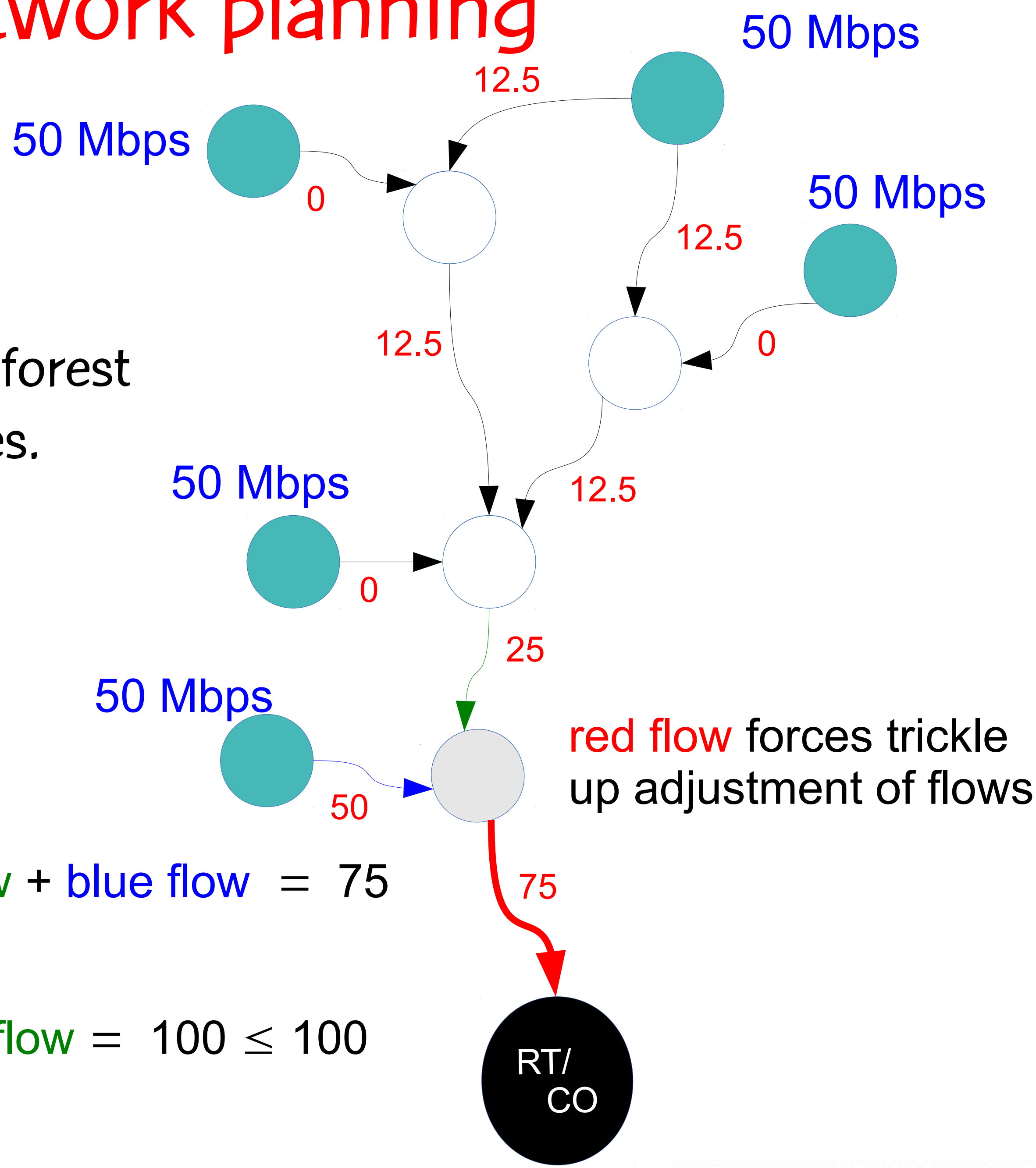


Flow conservation: red flow = green flow + blue flow

Capacity constraint: red flow + green flow $\leq$ 100

Wireless backhaul network planning

Constraints: Traffic flow



Traffic that is backhauled from demand points to root nodes of forest is limited by equipment capacities.

Only traffic that reaches roots is counted as revenue.

Flow conservation: red flow = green flow + blue flow $=$ 75

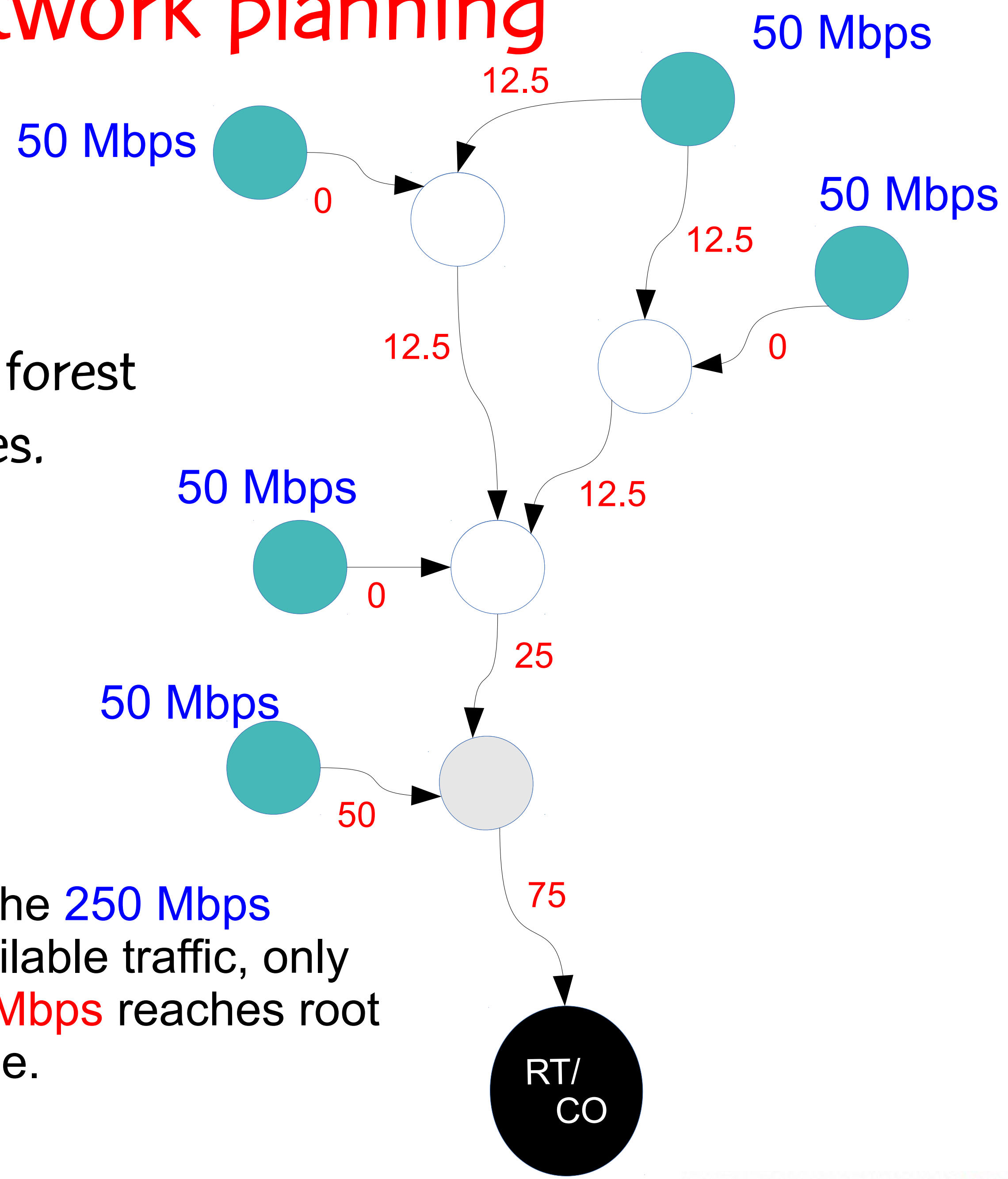
Capacity constraint: red flow + green flow $=$ 100 $\leq$ 100

Wireless backhaul network planning

Constraints: Traffic flow

Traffic that is backhauled from demand points to root nodes of forest is limited by equipment capacities.

Only traffic that reaches roots is counted as revenue.



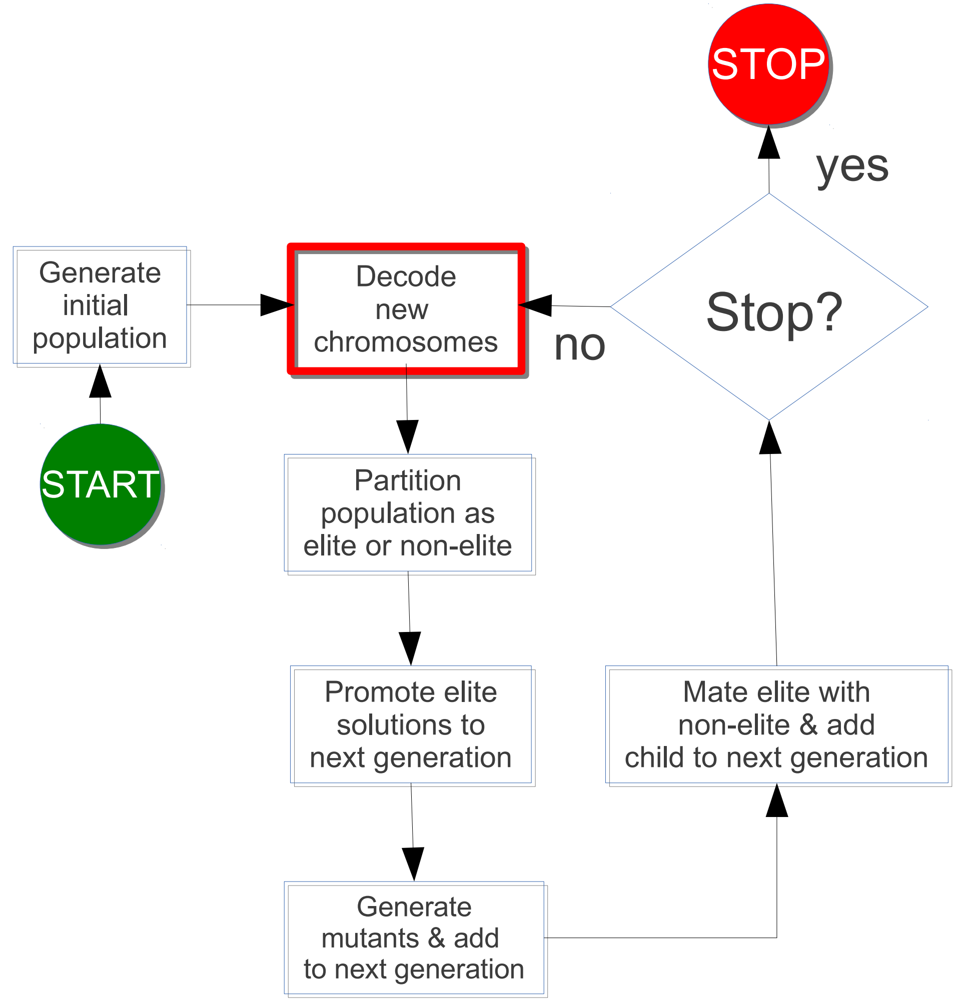
Of the 250 Mbps available traffic, only 75 Mbps reaches root node.

Biased random-key genetic algorithm

Gonçalves & Resende (J. of Heuristics, 2011)

Features of a BRKGA

- Fixed chromosome encoding using a vector of random keys
- Well-defined problem-independent evolutionary process for vectors of keys
  - parametrized uniform crossover
  - mutants in place of mutation
  - elitism
  - bias in selection of mates and in crossover
- Decoder is only problem-dependent component



Decoder for wireless backhaul planning

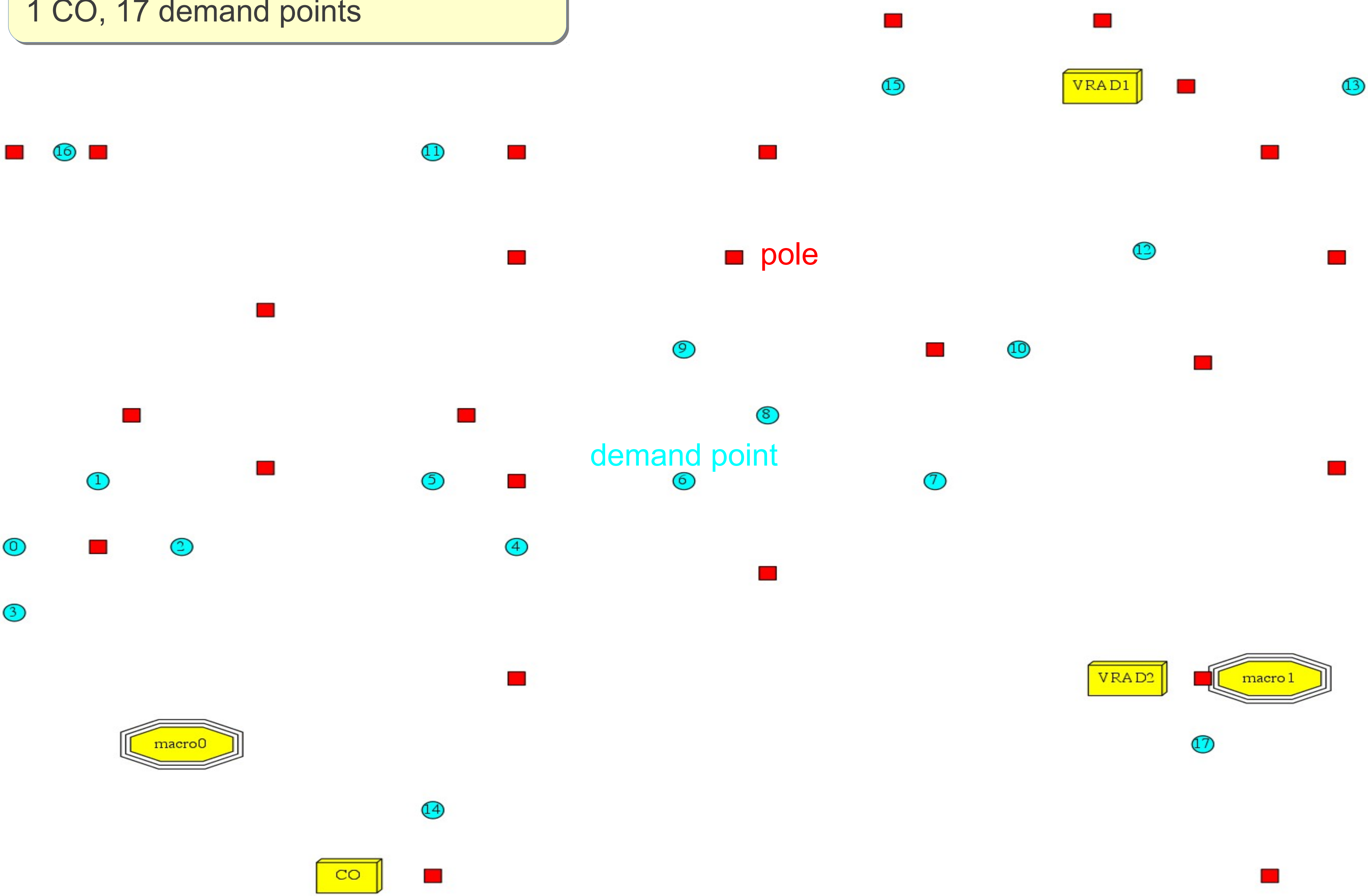
- Biased Random-Key Genetic Algorithm
  - Learn the best network layout and equipment placement
- A solution is encoded by a vector $x \in [0,1]^n$
  - Where $n = 5 \times \#$ of poles

Activation order	Activation parameters	Evaluation order	Neighborhood evaluation order	Minimum tree level of pole

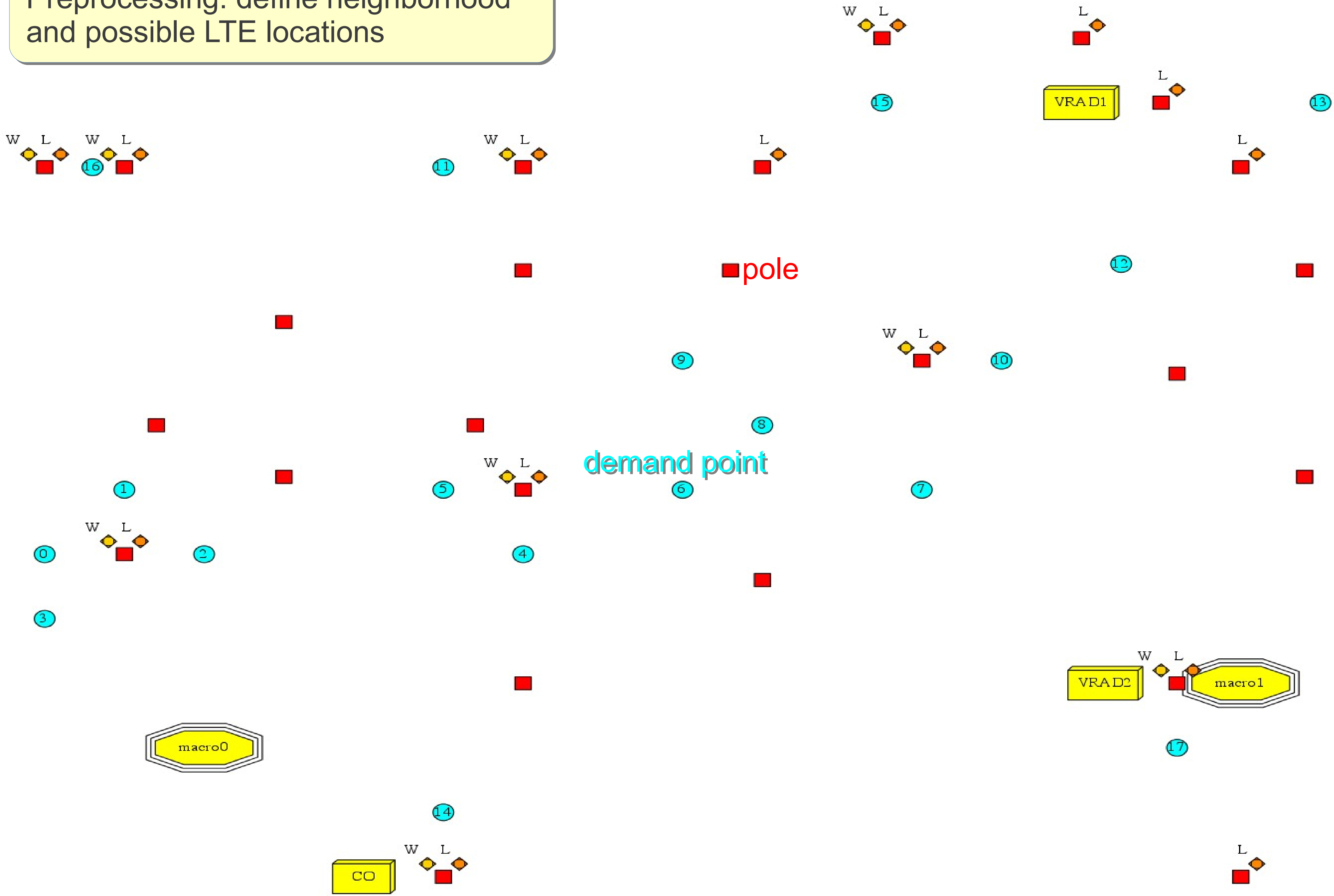
Decoder

- Define activation order & install LTE on poles
- Build backhaul graph
- Remove unused equipment
- Compute maximum flow from demand points to FAPs
- Remove unused equipment & poles
- Compute cost and revenue and return objective function value

Example: 25 poles, 2 macros, 2 RTs, 1 CO, 17 demand points



Preprocessing: define neighborhood and possible LTE locations



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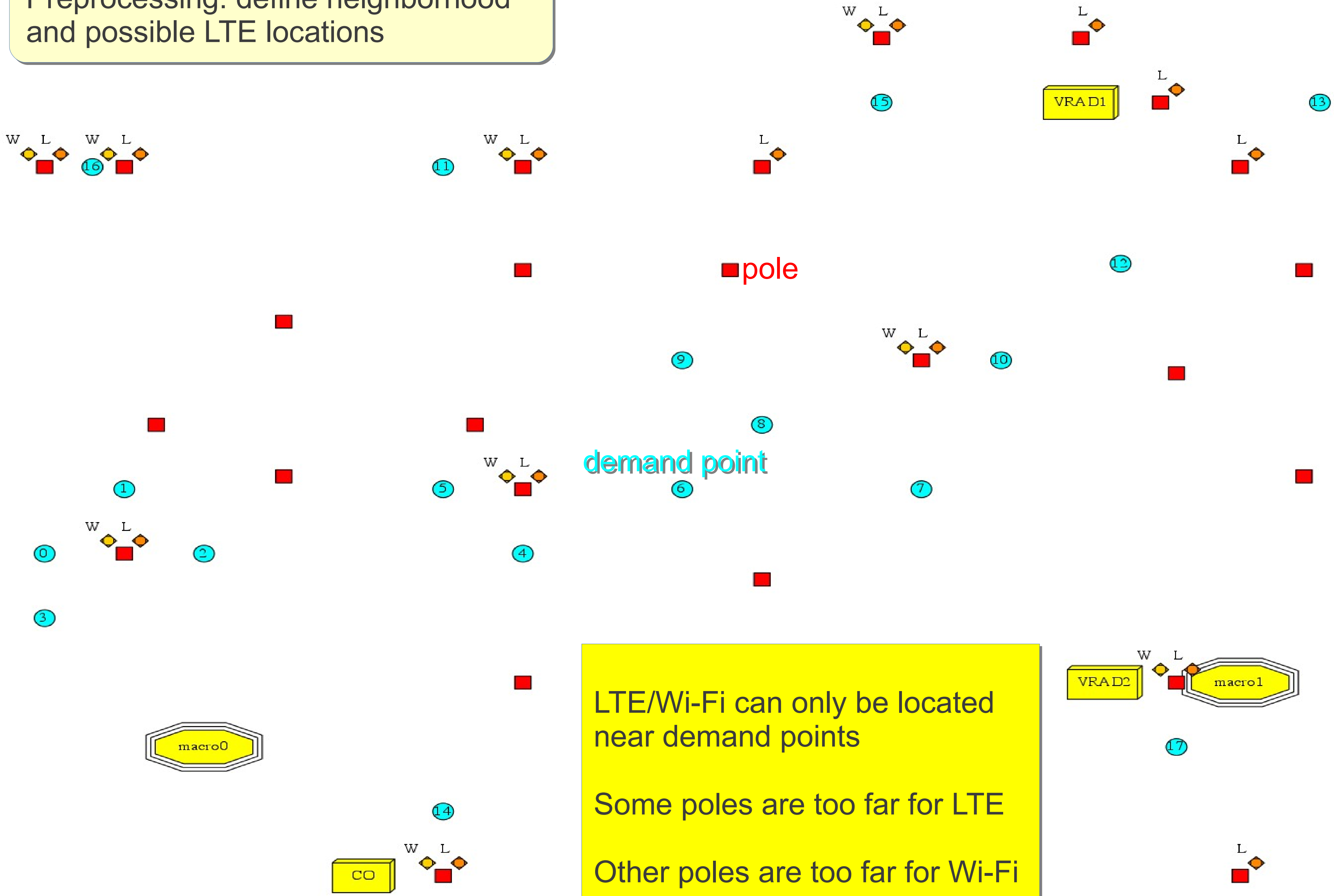


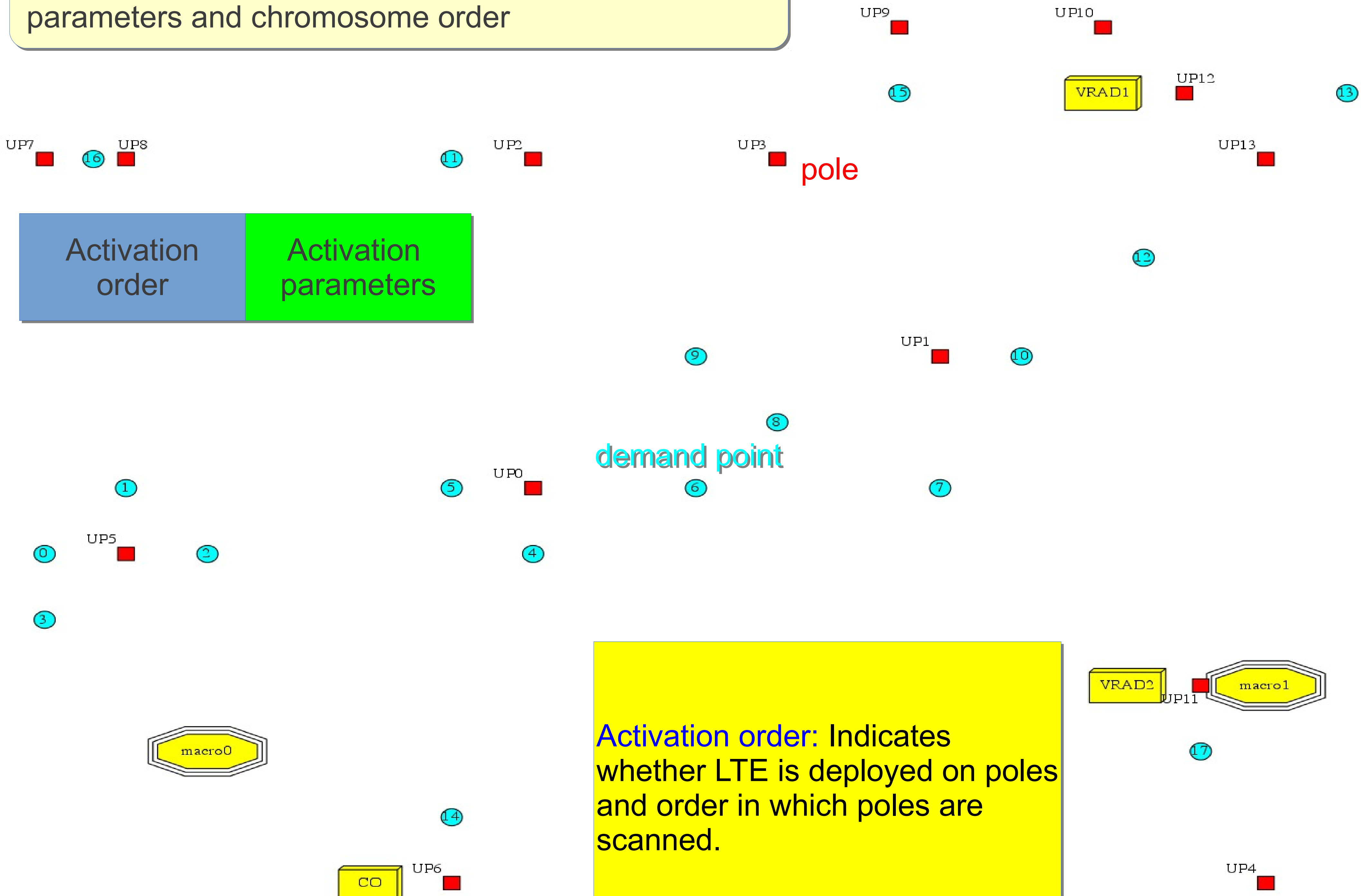
LTE/Wi-Fi can only be located near demand points

Some poles are too far for LTE

Other poles are too far for Wi-Fi

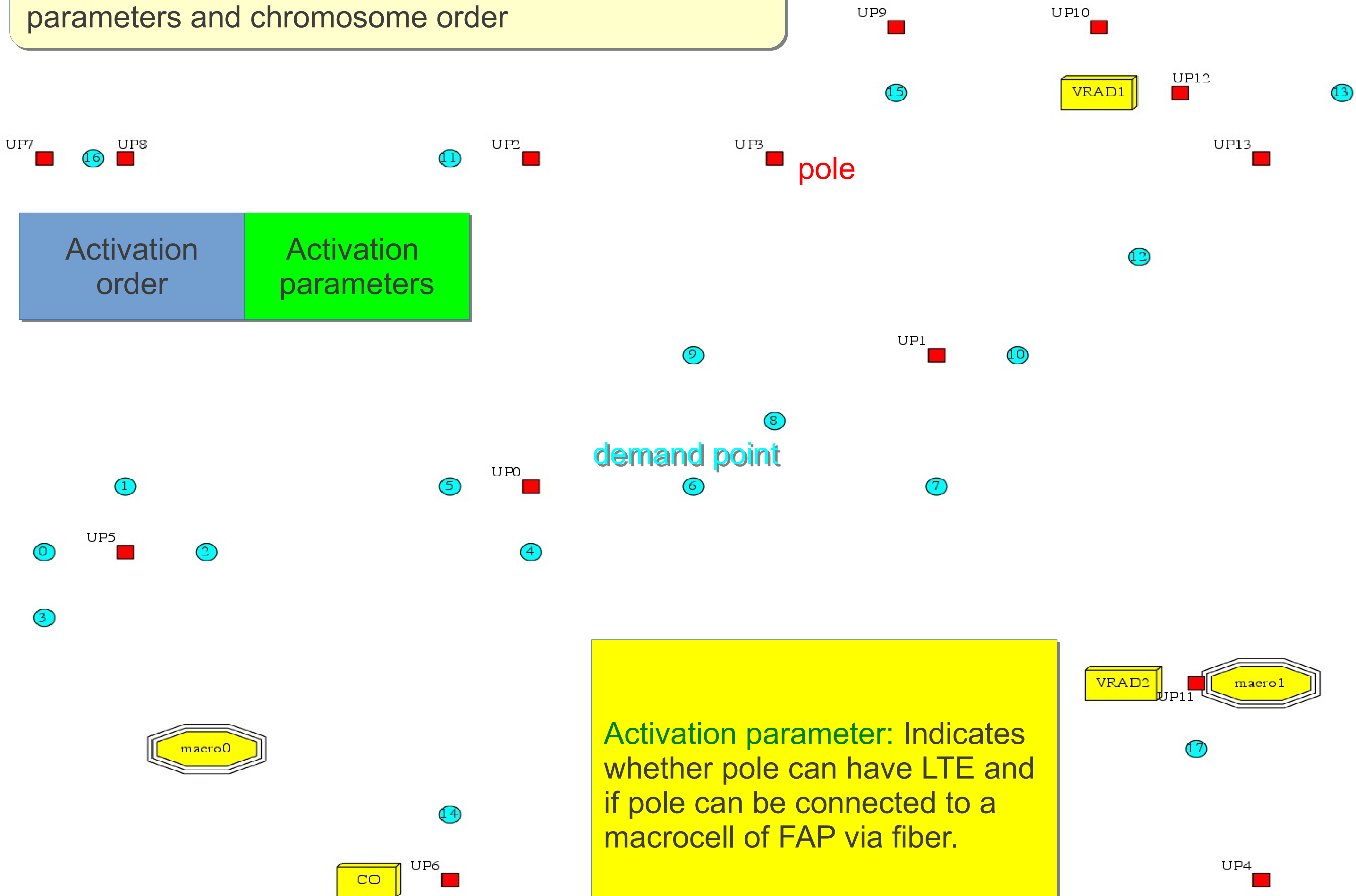
Activation: LTE deployment according to activation parameters and chromosome order

Activation order	Activation parameters



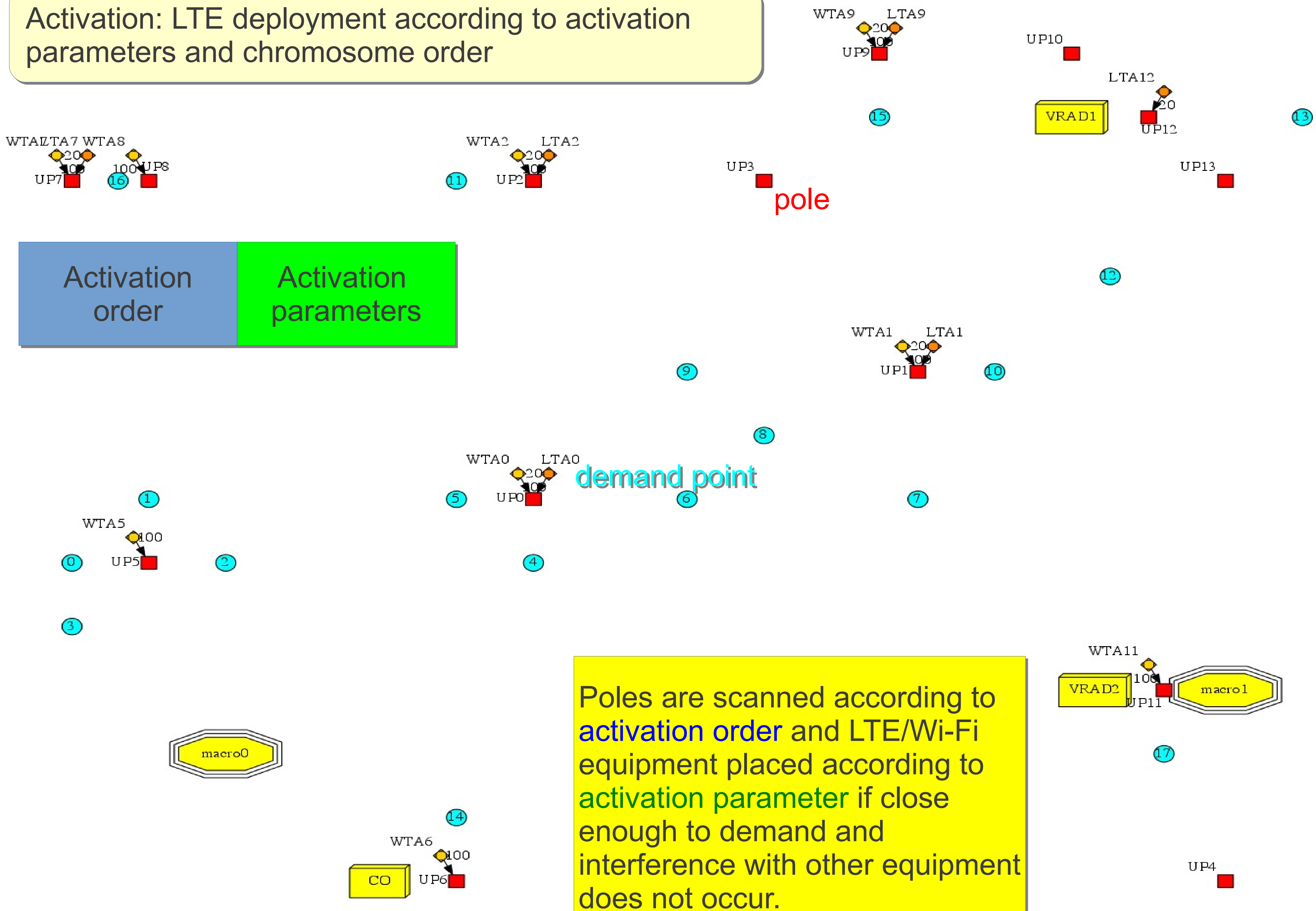
Activation order: Indicates whether LTE is deployed on poles and order in which poles are scanned.

Activation: LTE deployment according to activation parameters and chromosome order



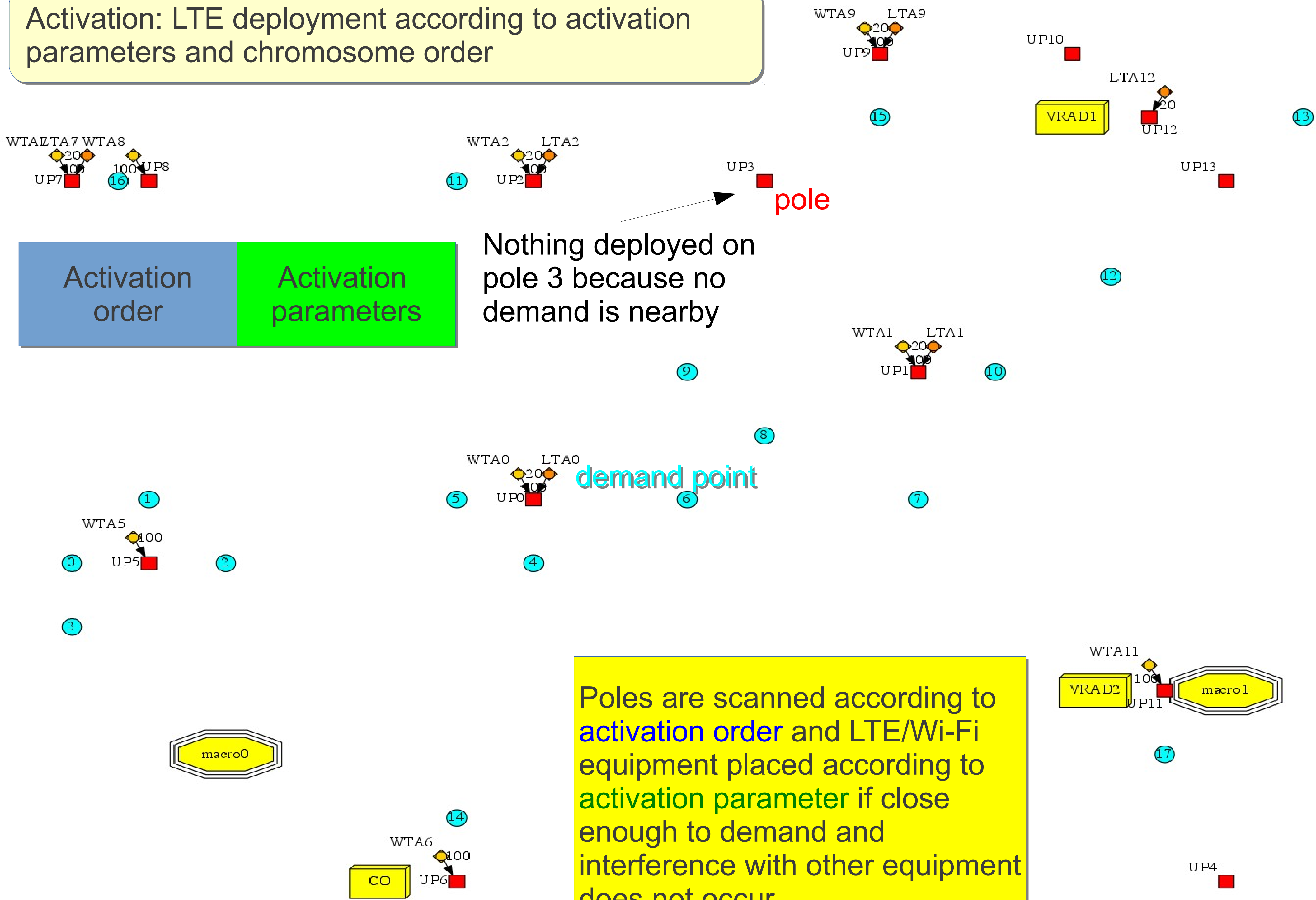
Activation parameter: Indicates whether pole can have LTE and if pole can be connected to a macrocell of FAP via fiber.

Activation: LTE deployment according to activation parameters and chromosome order



Poles are scanned according to activation order and LTE/Wi-Fi equipment placed according to activation parameter if close enough to demand and interference with other equipment does not occur.

Activation: LTE deployment according to activation parameters and chromosome order



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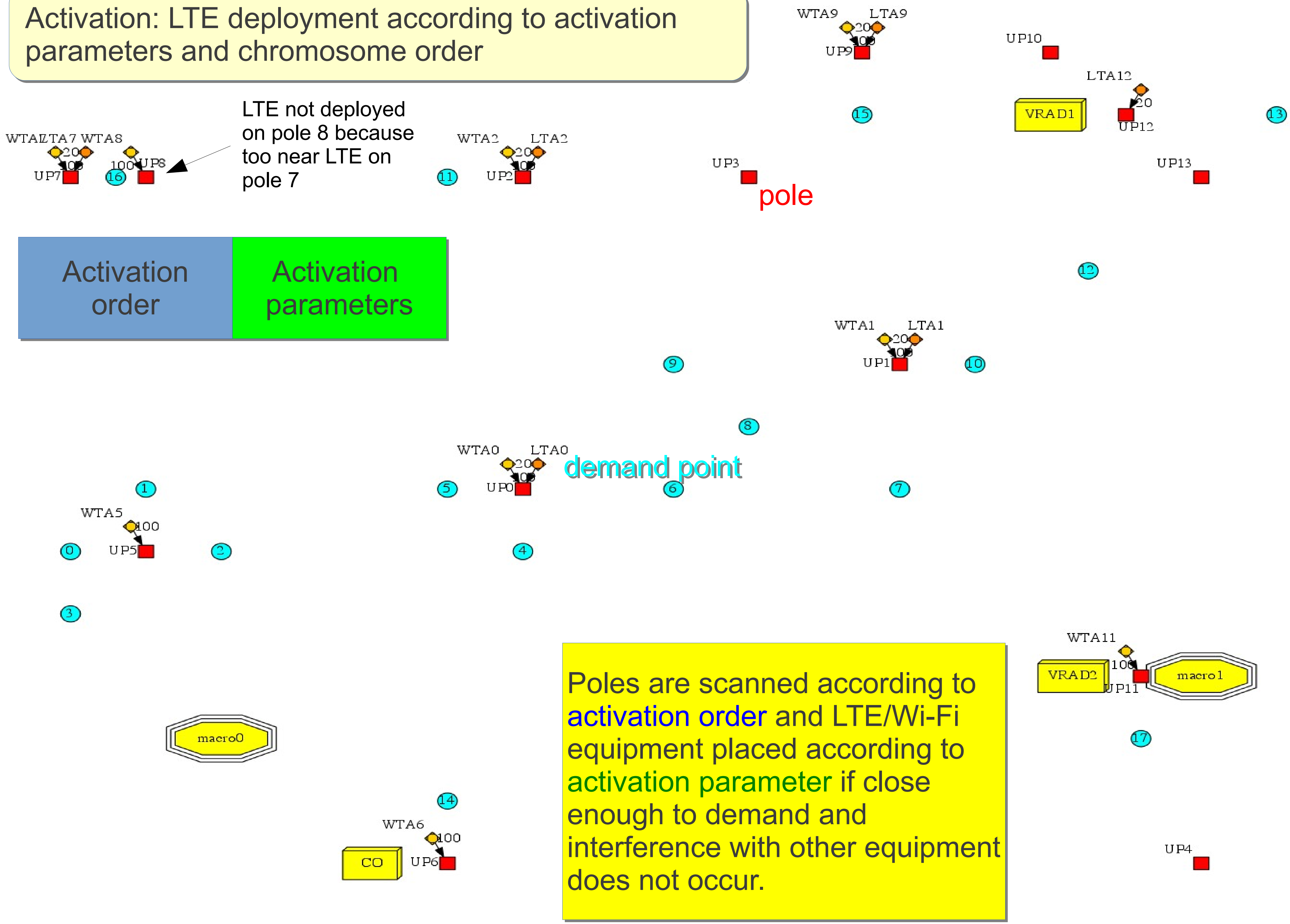

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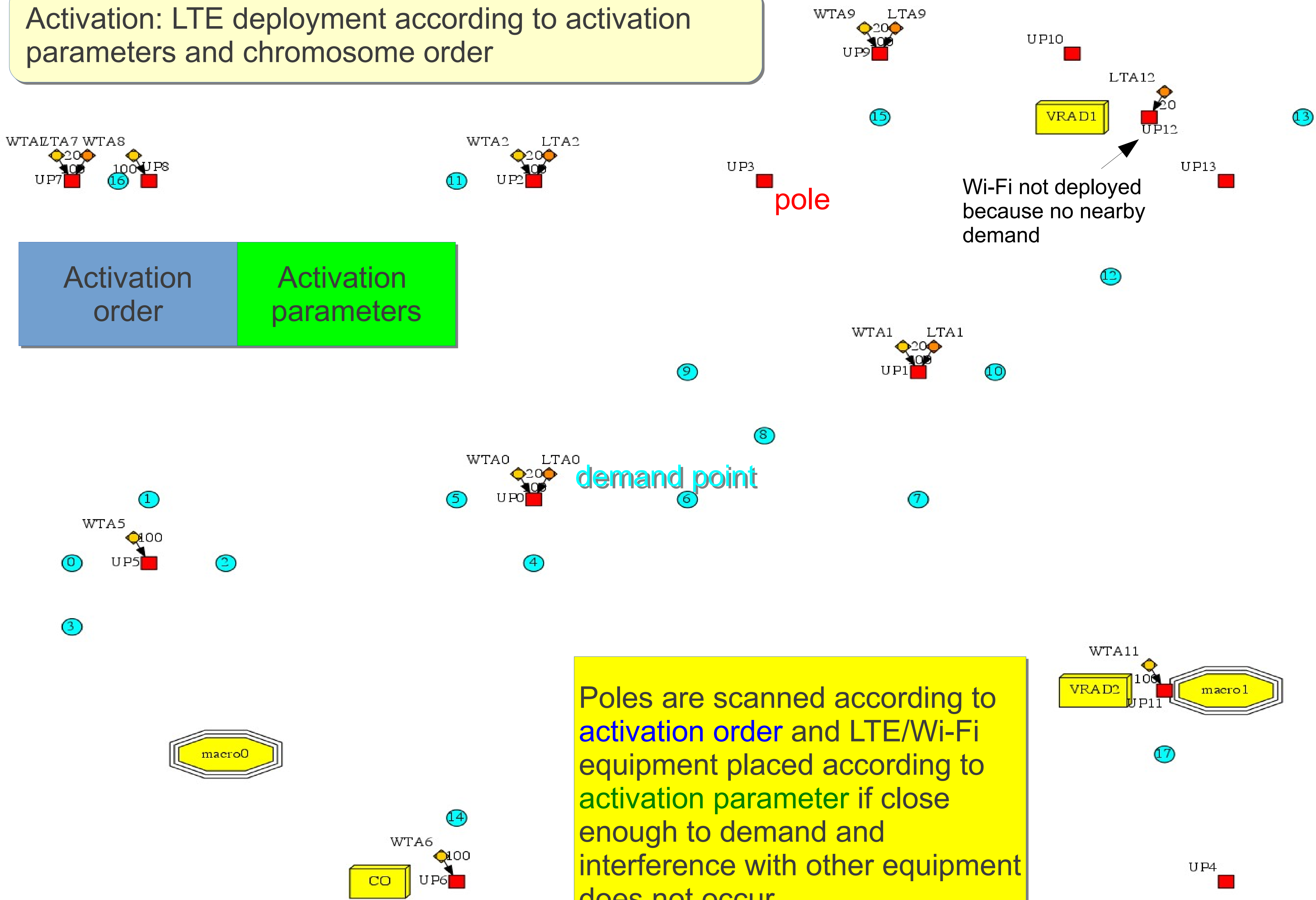
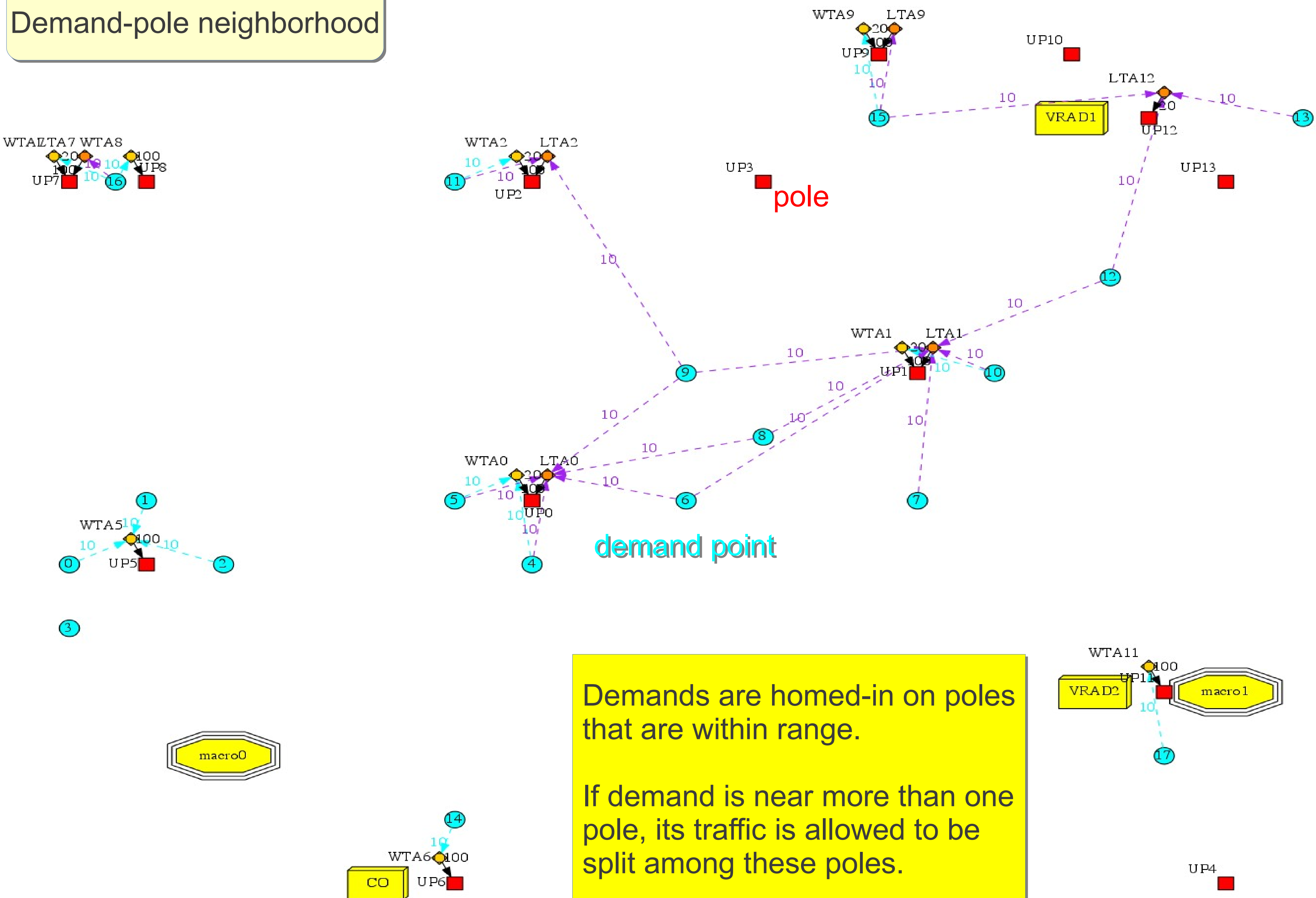


Poles are scanned according to activation order and LTE/Wi-Fi equipment placed according to activation parameter if close enough to demand and interference with other equipment does not occur.

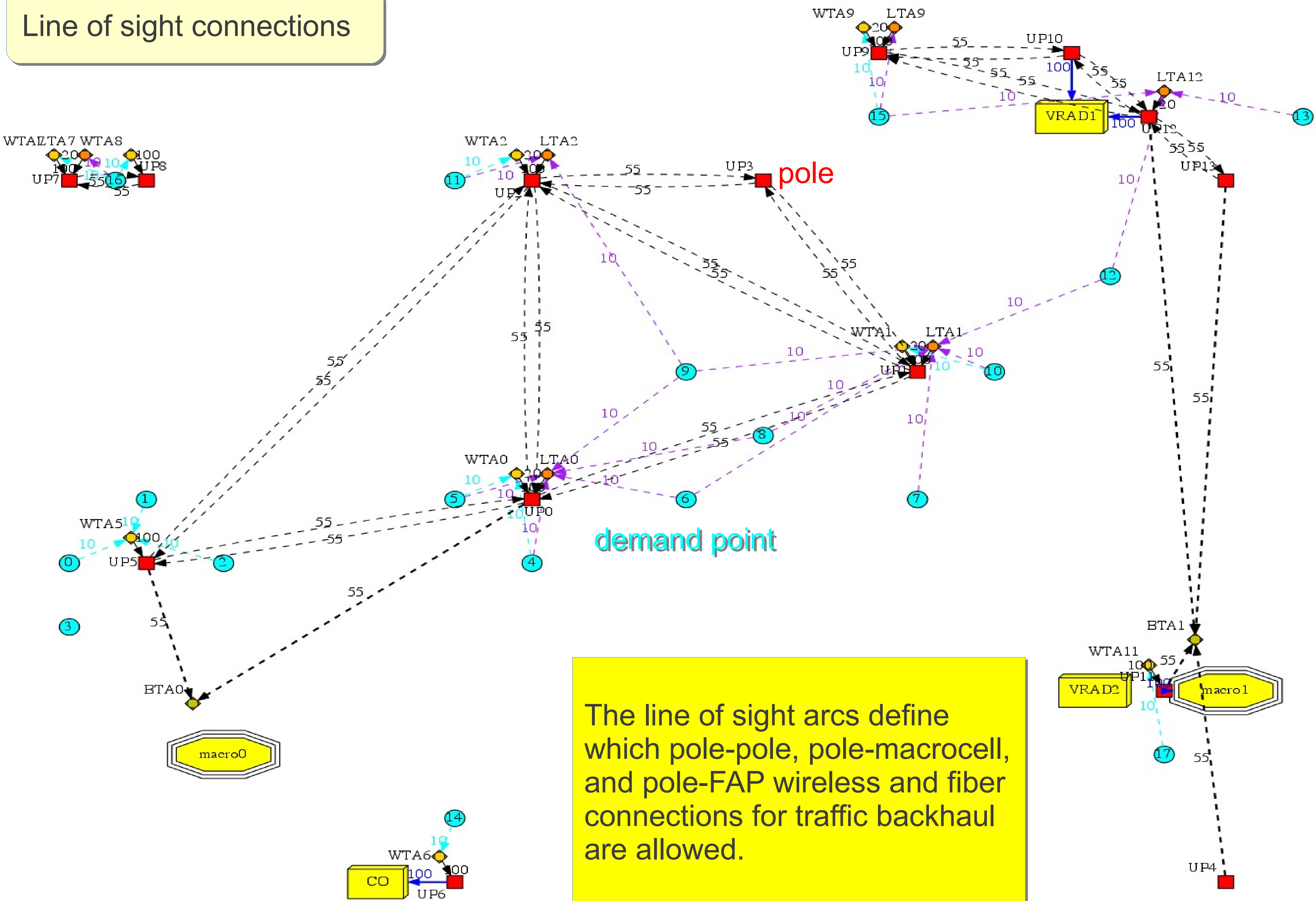
Demand-pole neighborhood



Demands are homed-in on poles that are within range.

If demand is near more than one pole, its traffic is allowed to be split among these poles.

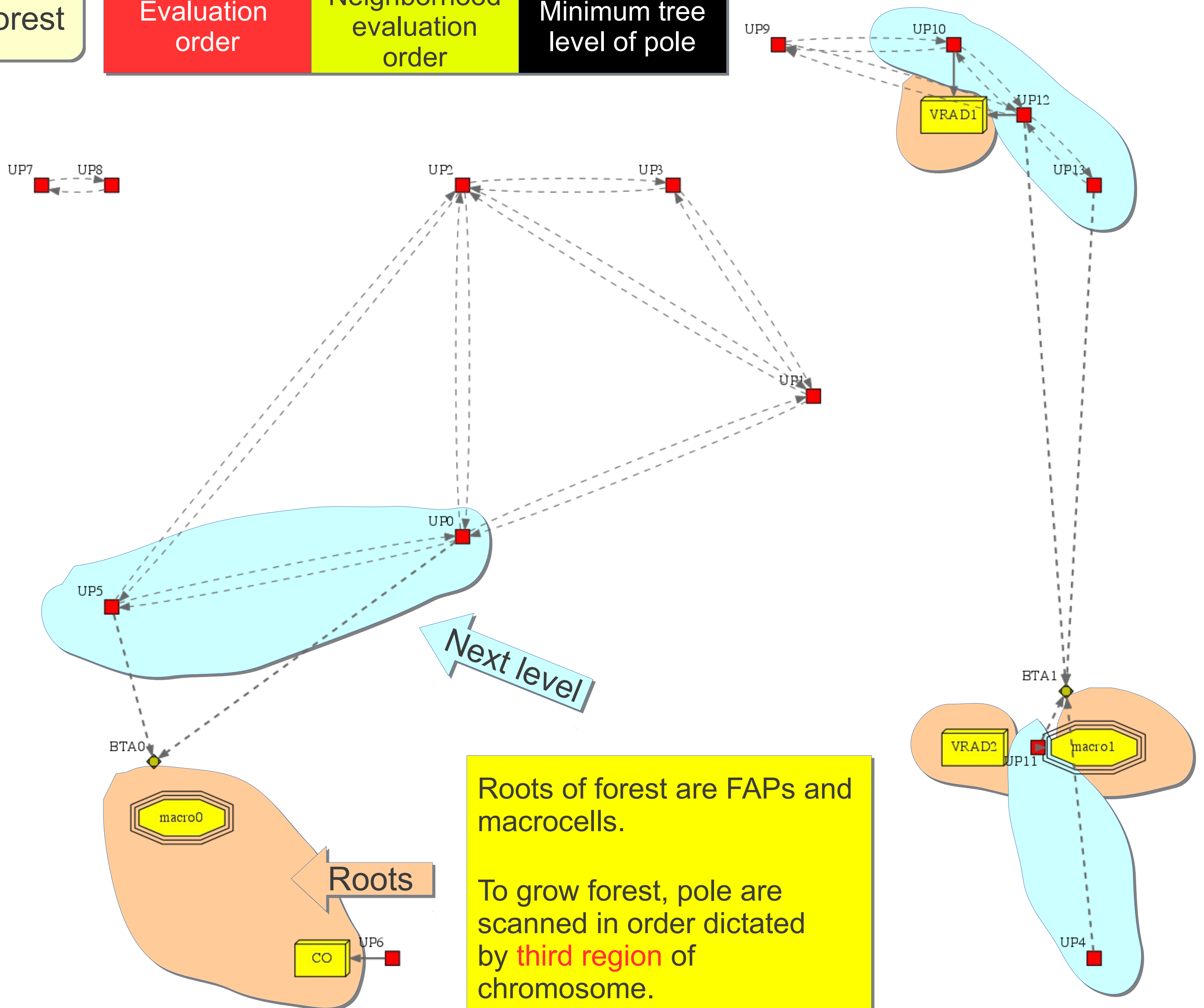
Line of sight connections



The line of sight arcs define which pole-pole, pole-macrocell, and pole-FAP wireless and fiber connections for traffic backhaul are allowed.

Growing forest

Evaluation order	Neighborhood evaluation order	Minimum tree level of pole

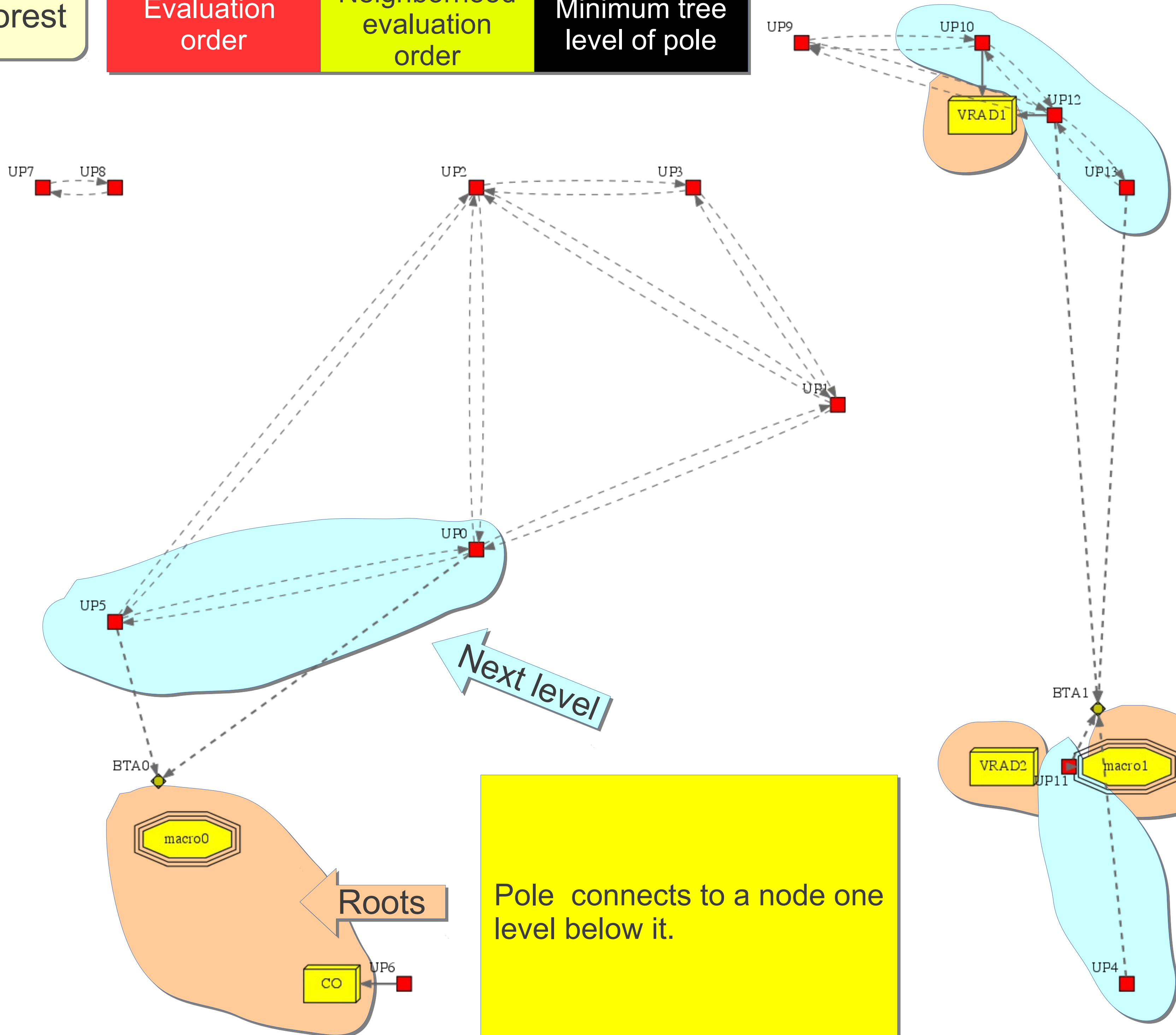


Roots of forest are FAPs and macrocells.

To grow forest, pole are scanned in order dictated by third region of chromosome.

Growing forest

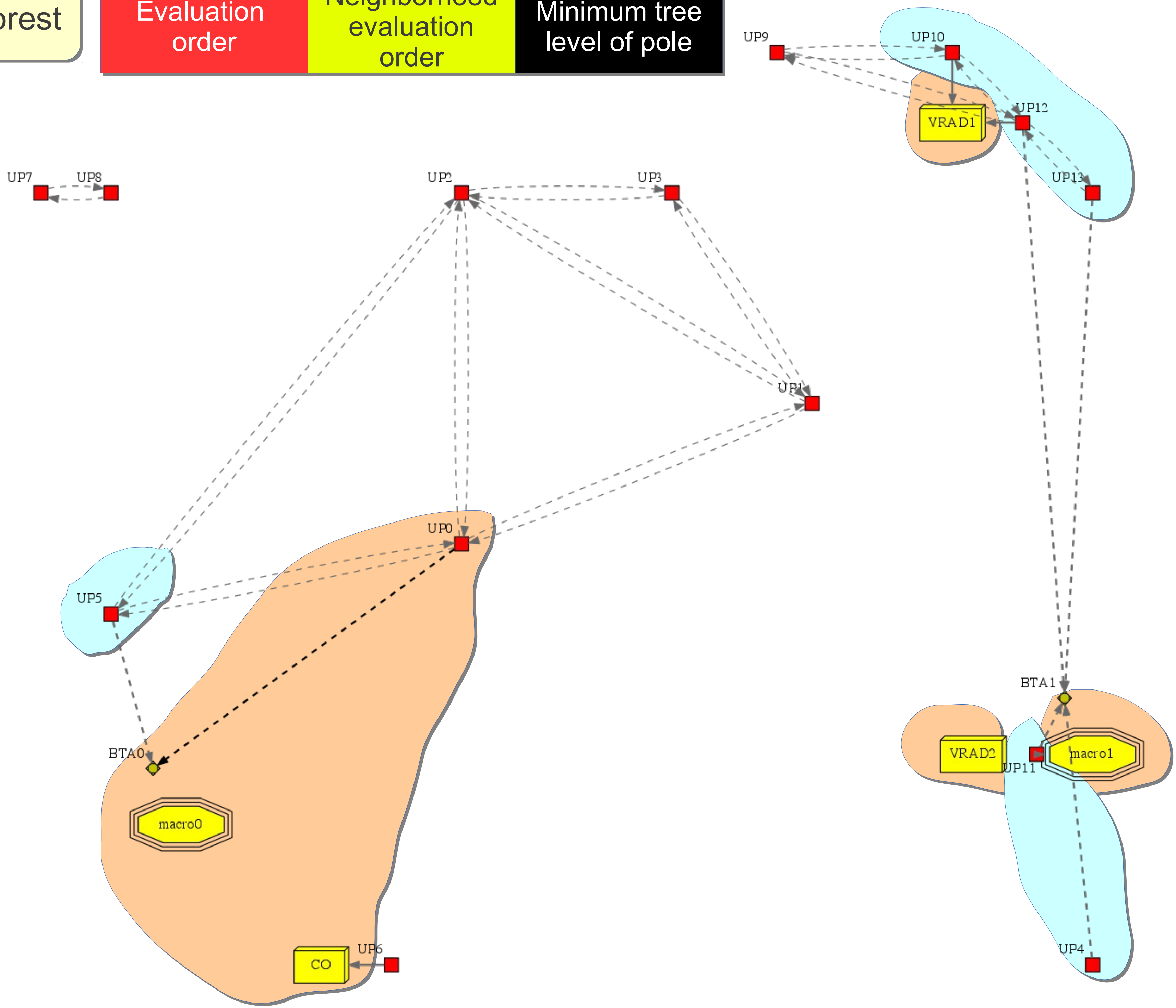
Evaluation order	Neighborhood evaluation order	Minimum tree level of pole



Pole connects to a node one level below it.

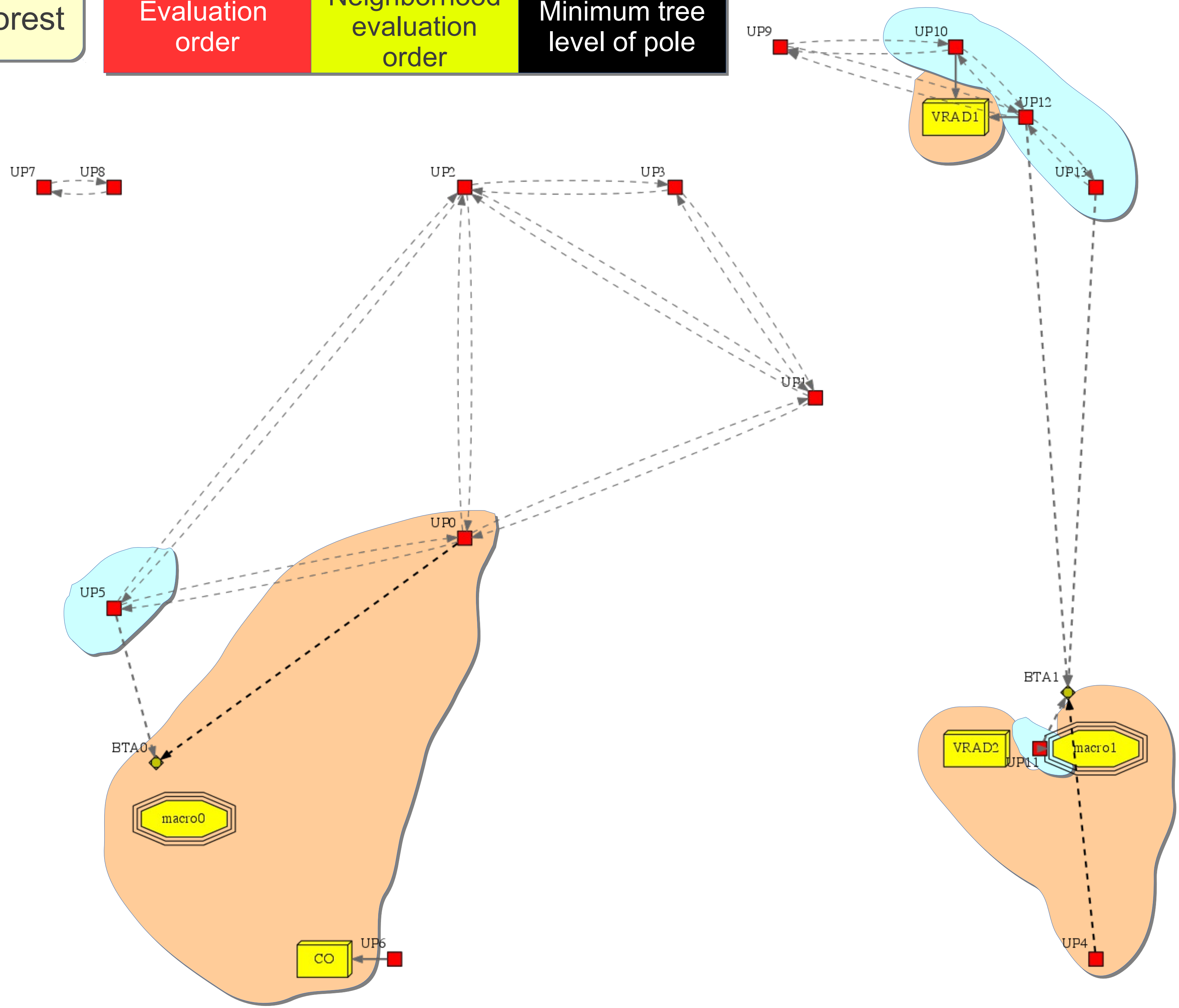
Growing forest

Evaluation order	Neighborhood evaluation order	Minimum tree level of pole



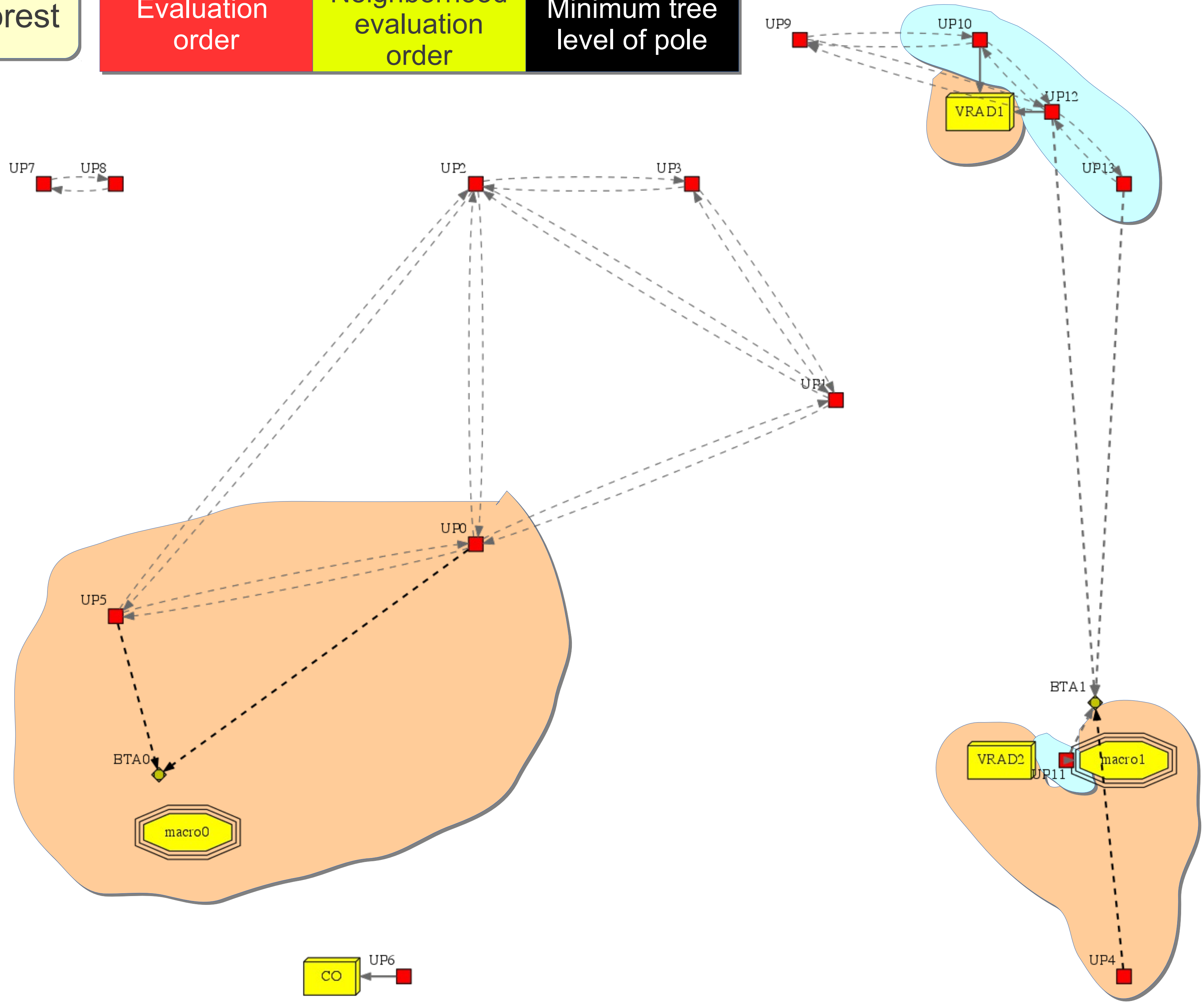
Growing forest

Evaluation order	Neighborhood evaluation order	Minimum tree level of pole



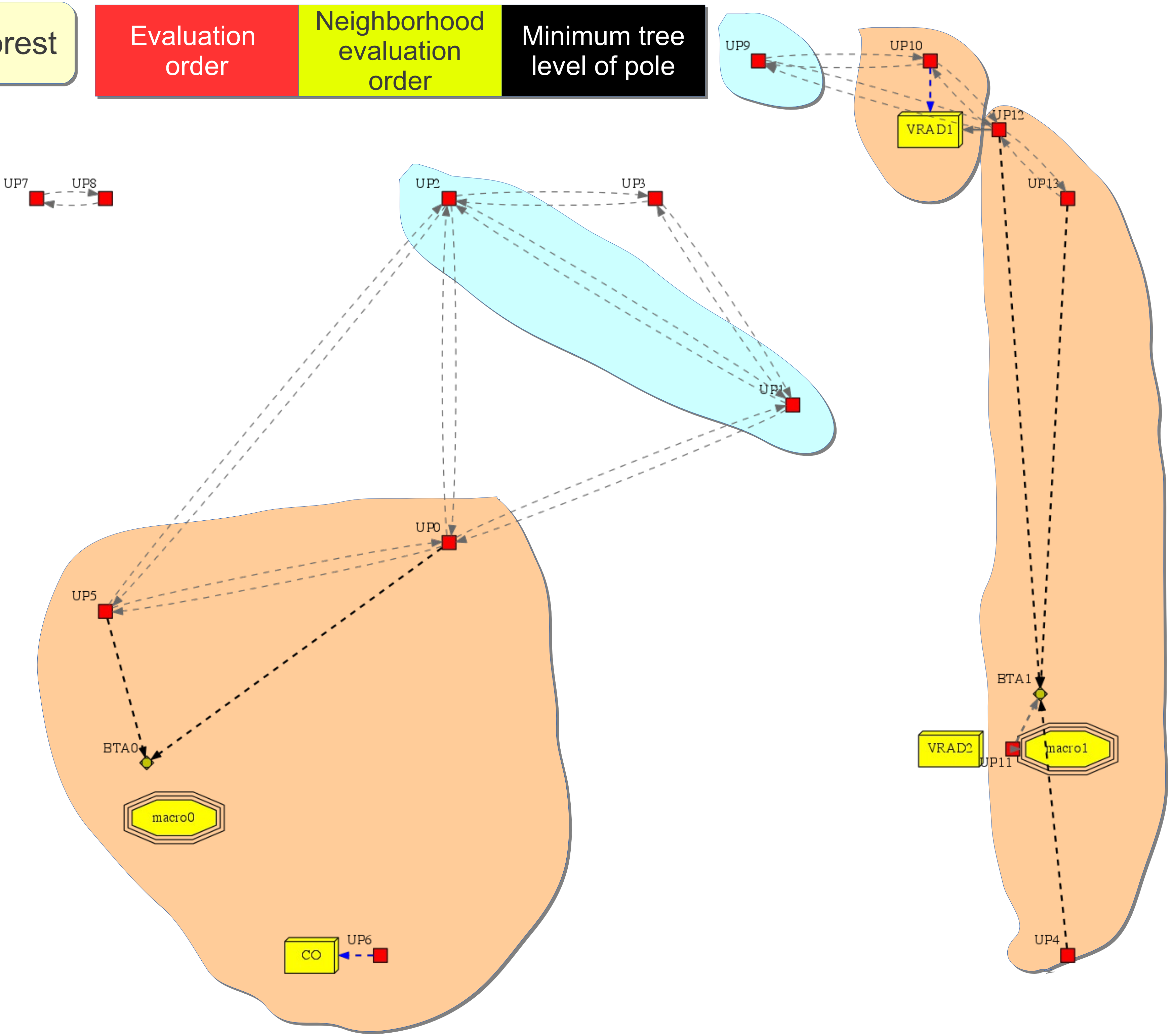
Growing forest

Evaluation order	Neighborhood evaluation order	Minimum tree level of pole



Growing forest

Evaluation order	Neighborhood evaluation order	Minimum tree level of pole

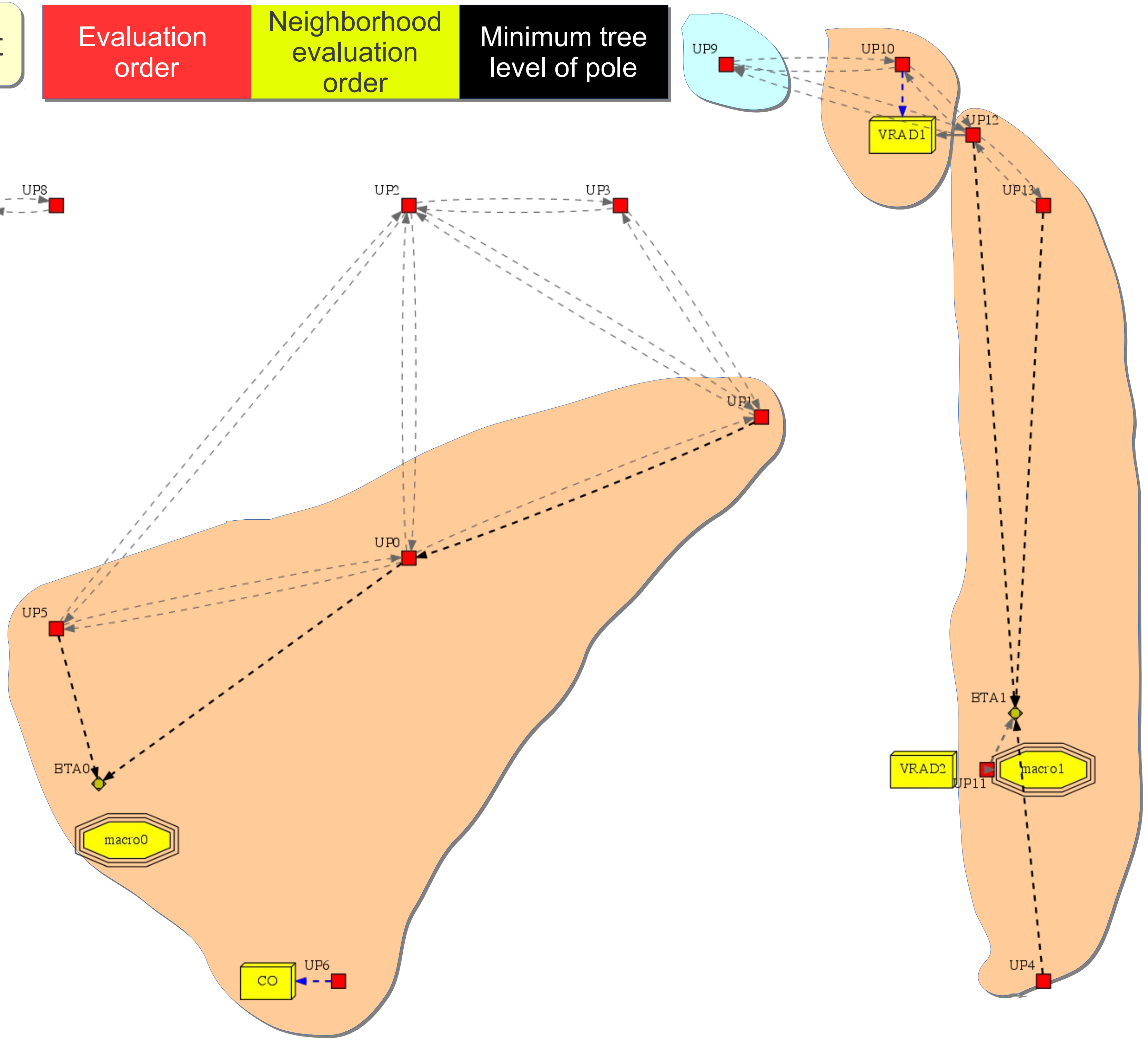


Growing forest

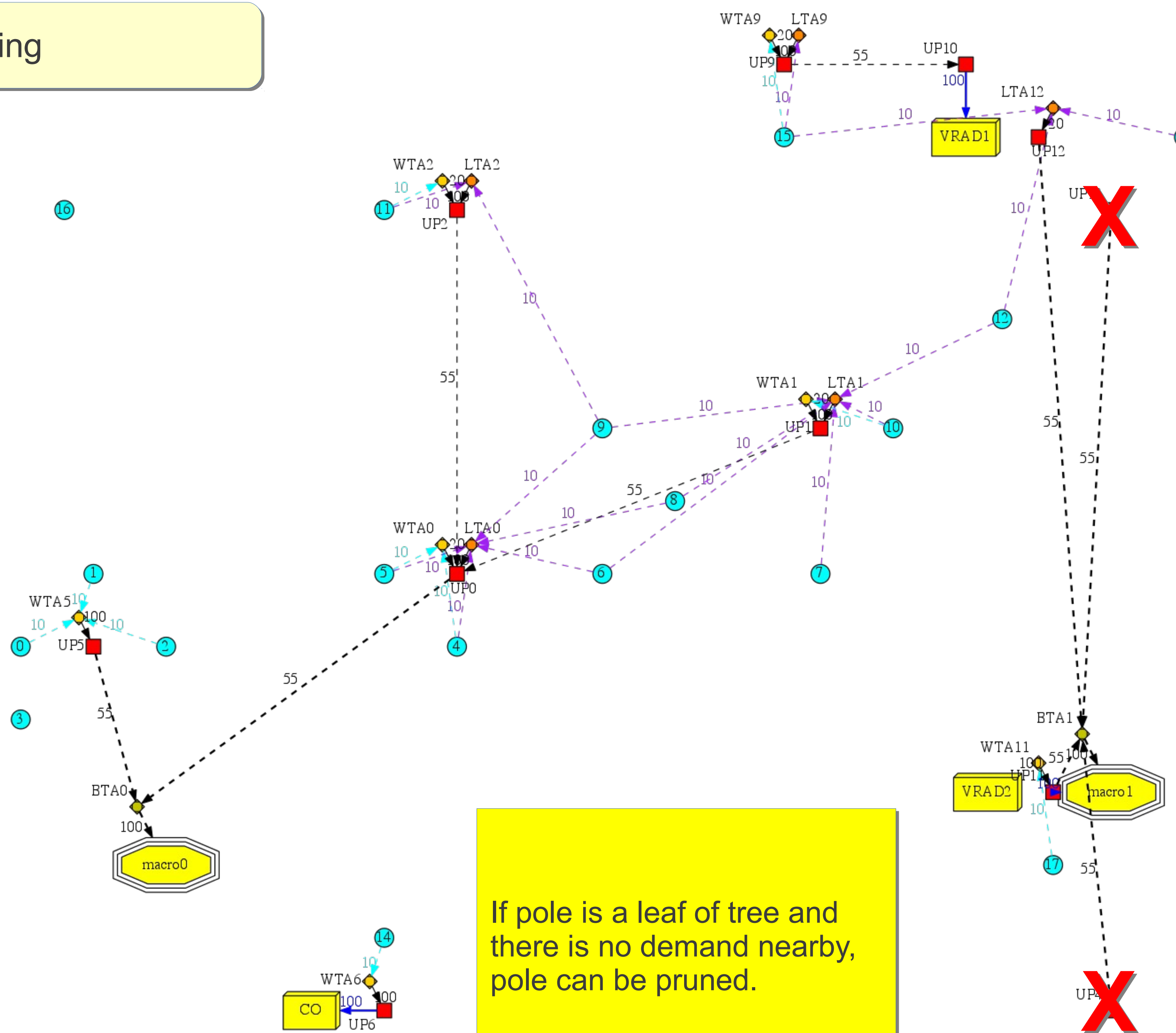
Evaluation order

Neighborhood evaluation order

Minimum tree level of pole

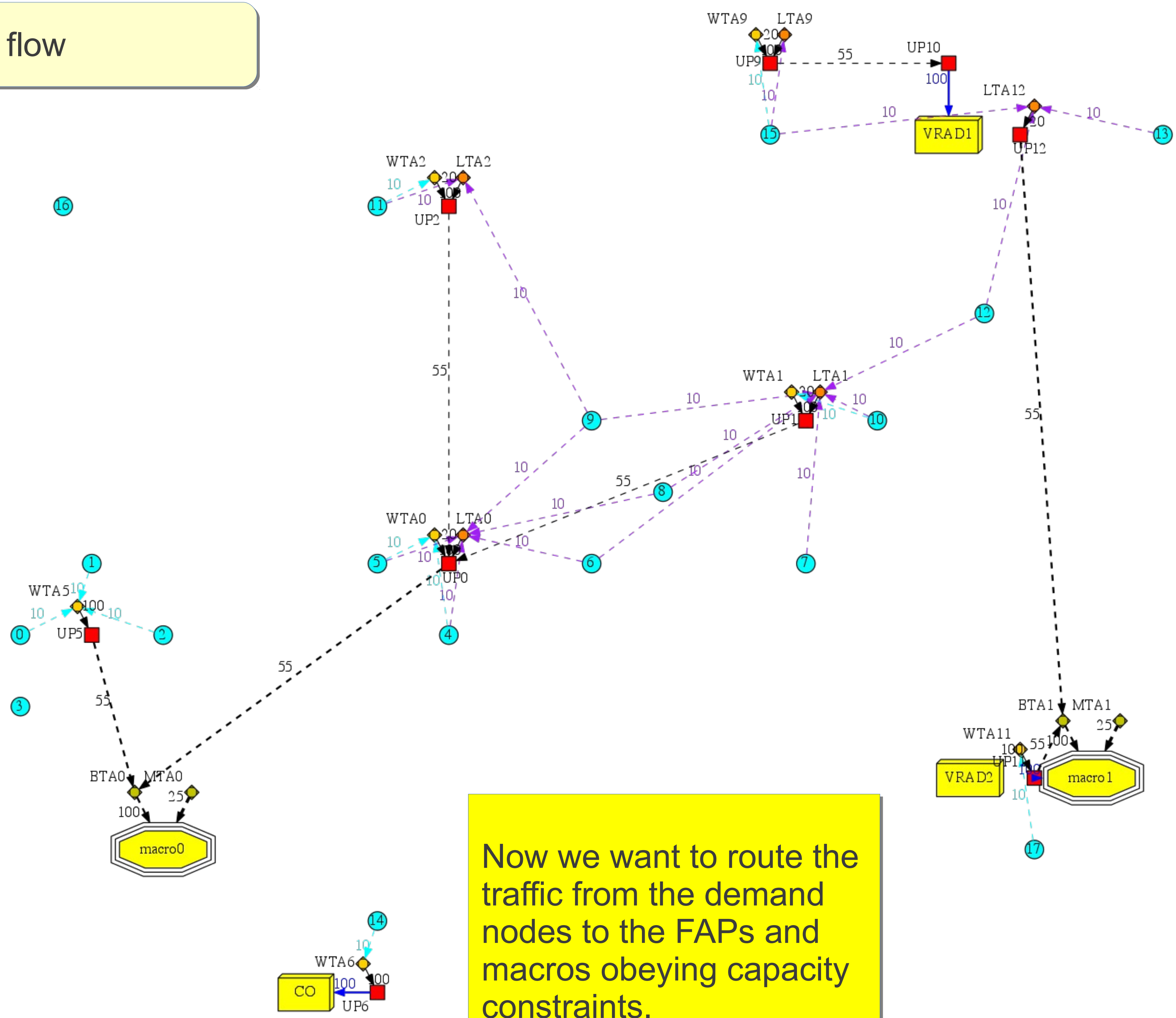


First Pruning



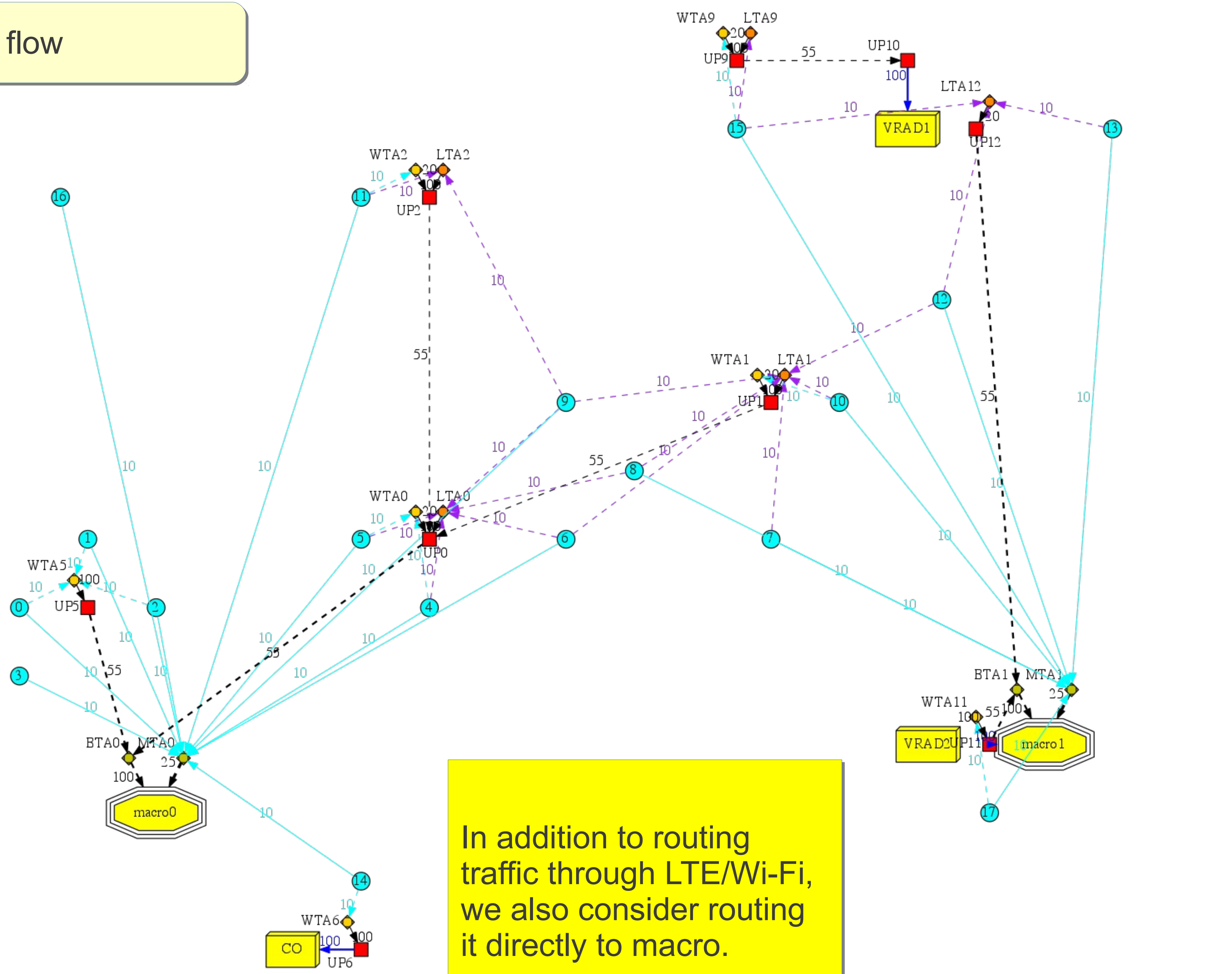
If pole is a leaf of tree and there is no demand nearby, pole can be pruned.

Maximum flow



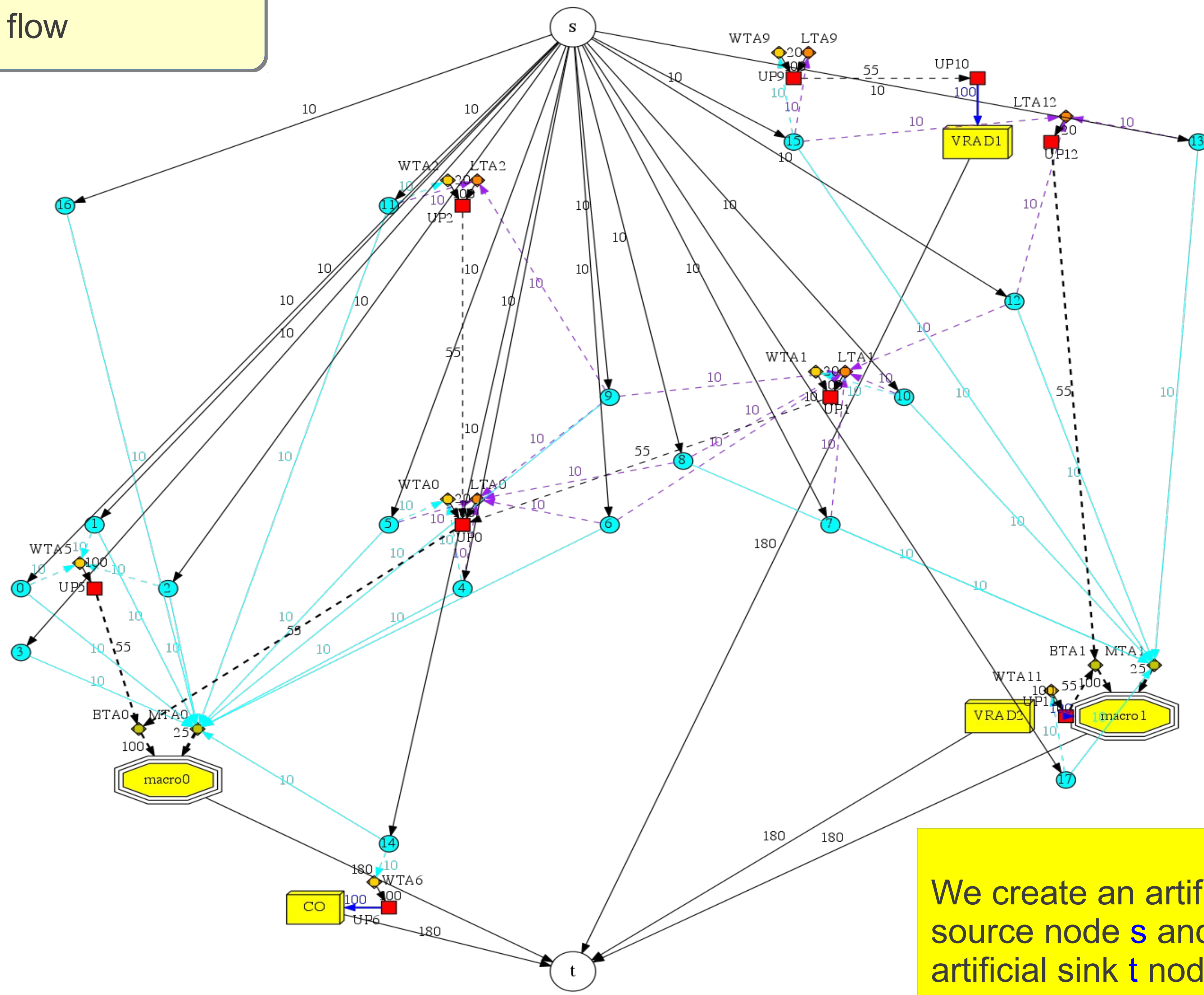
Now we want to route the traffic from the demand nodes to the FAPs and macros obeying capacity constraints.

Maximum flow



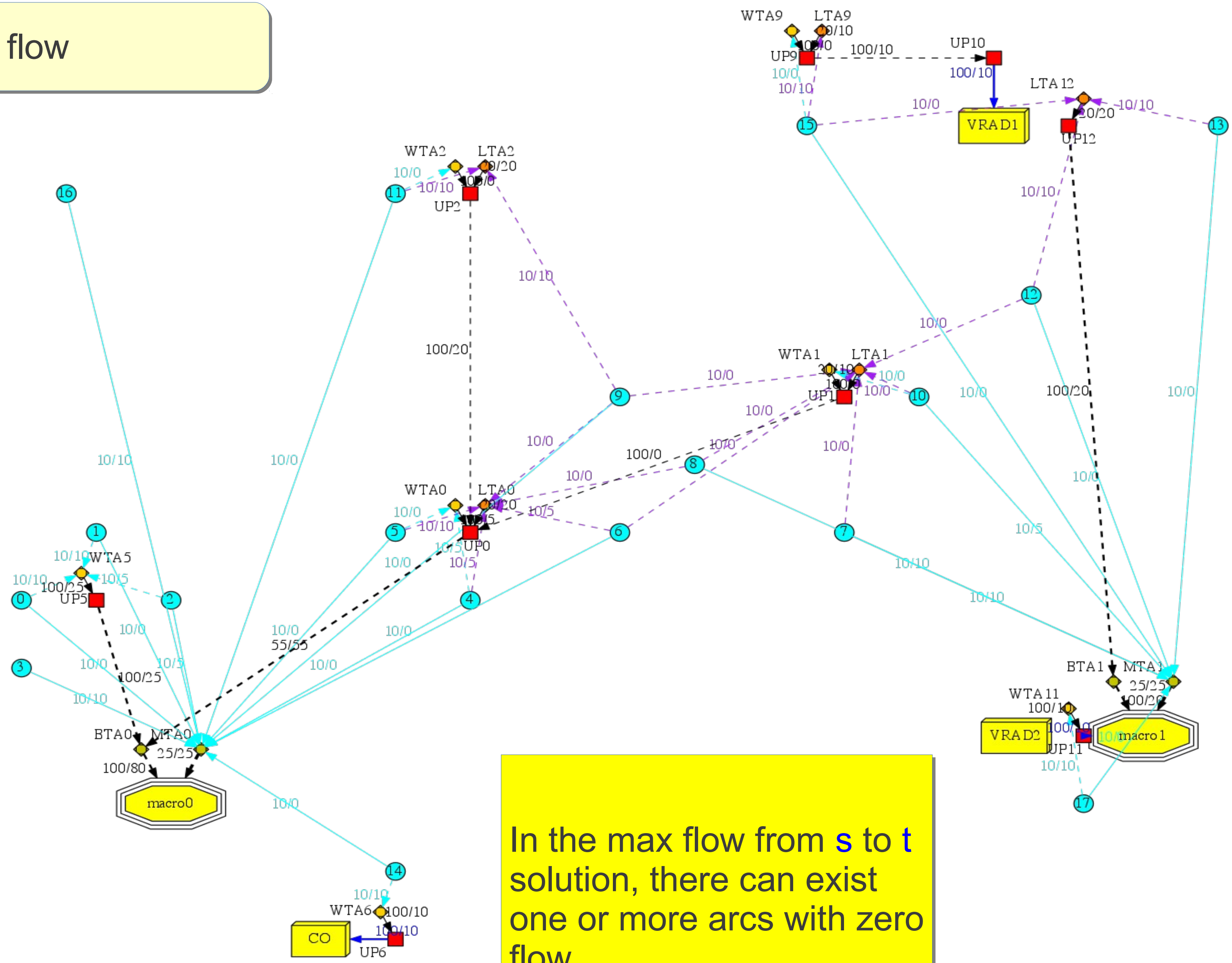
In addition to routing traffic through LTE/Wi-Fi, we also consider routing it directly to macro.

Maximum flow



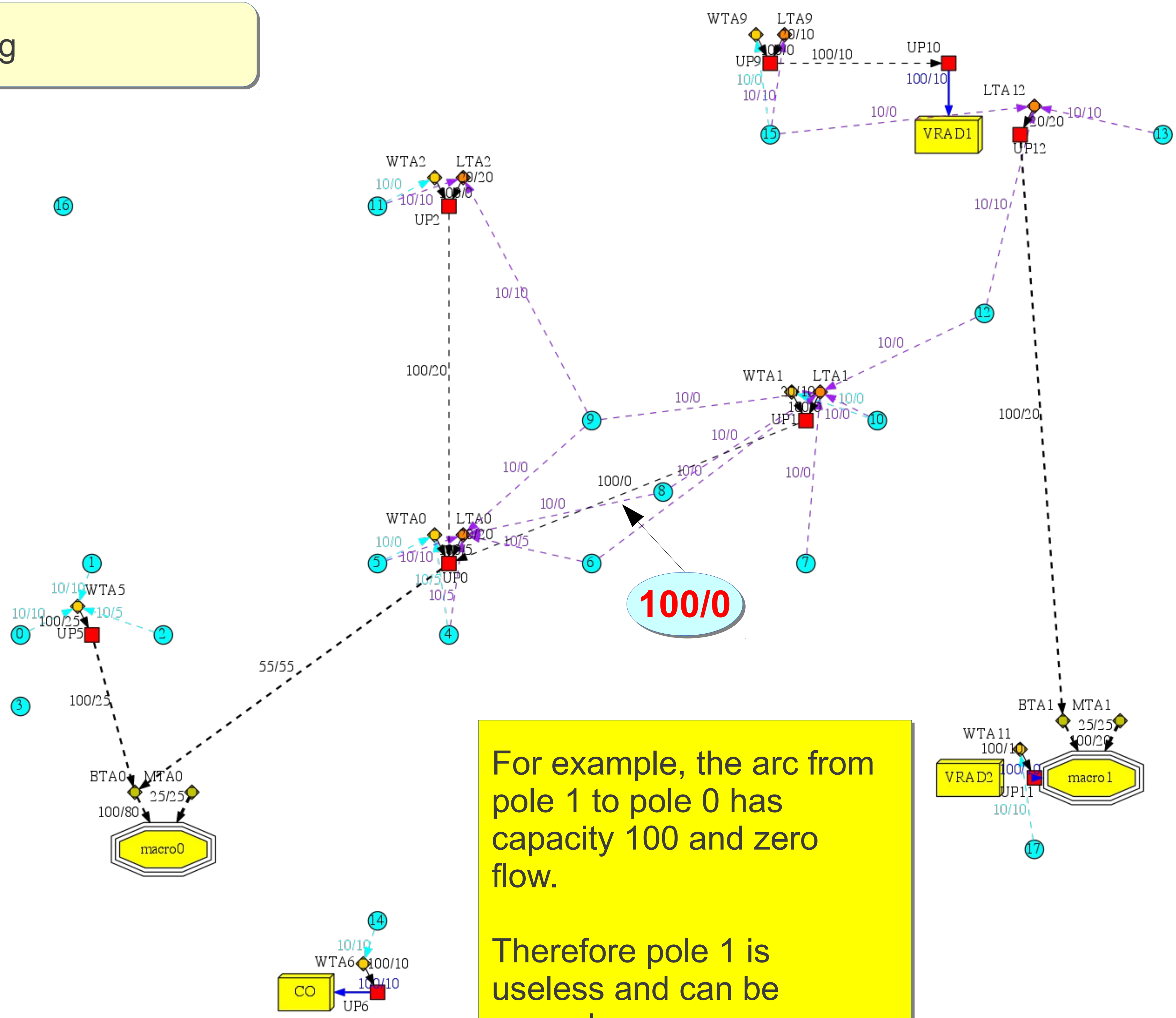
We create an artificial source node s and an artificial sink t node and compute the max flow from s to t.

Maximum flow



In the max flow from s to t solution, there can exist one or more arcs with zero flow.

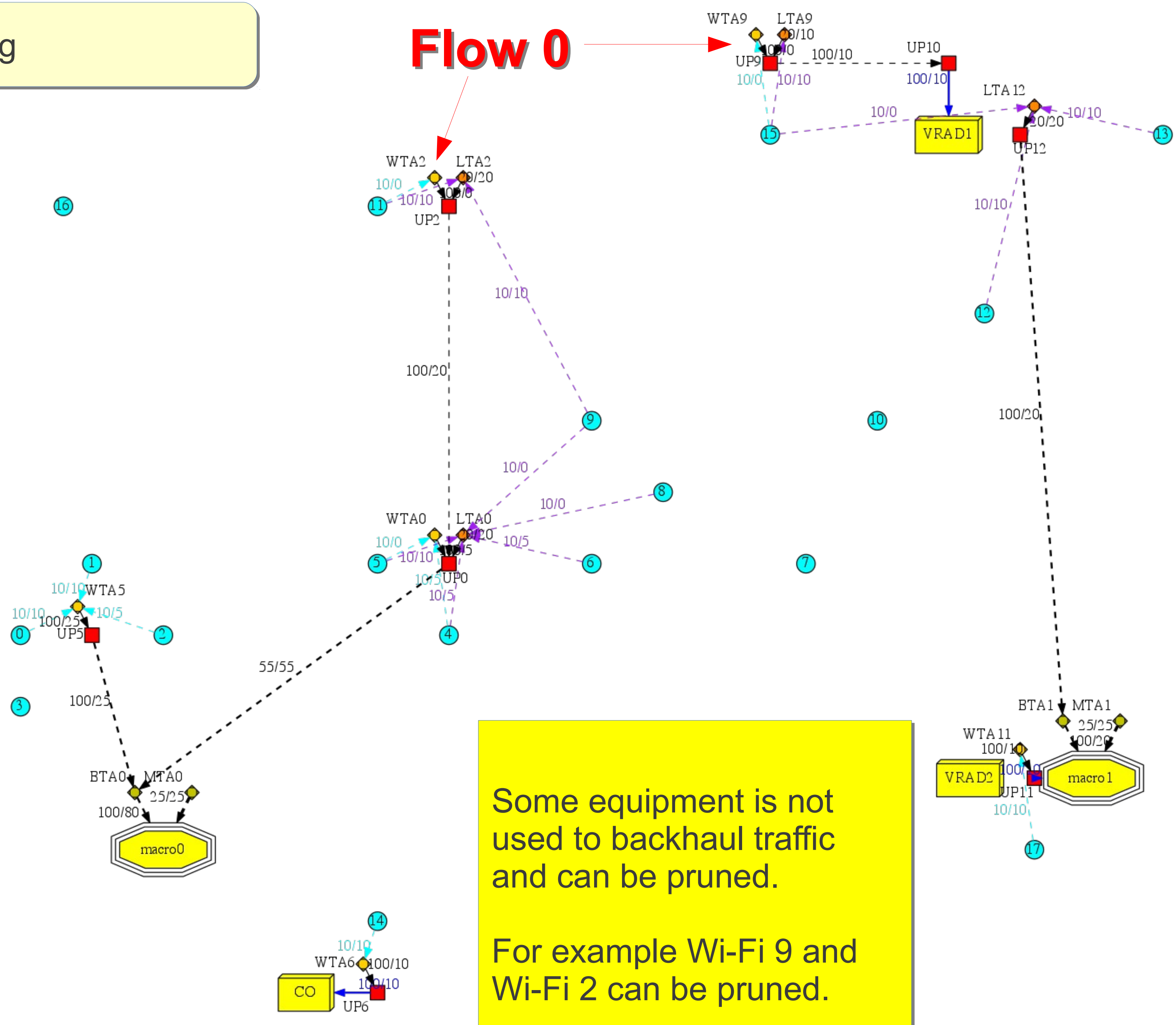
2nd Pruning



For example, the arc from pole 1 to pole 0 has capacity 100 and zero flow.

Therefore pole 1 is useless and can be pruned.

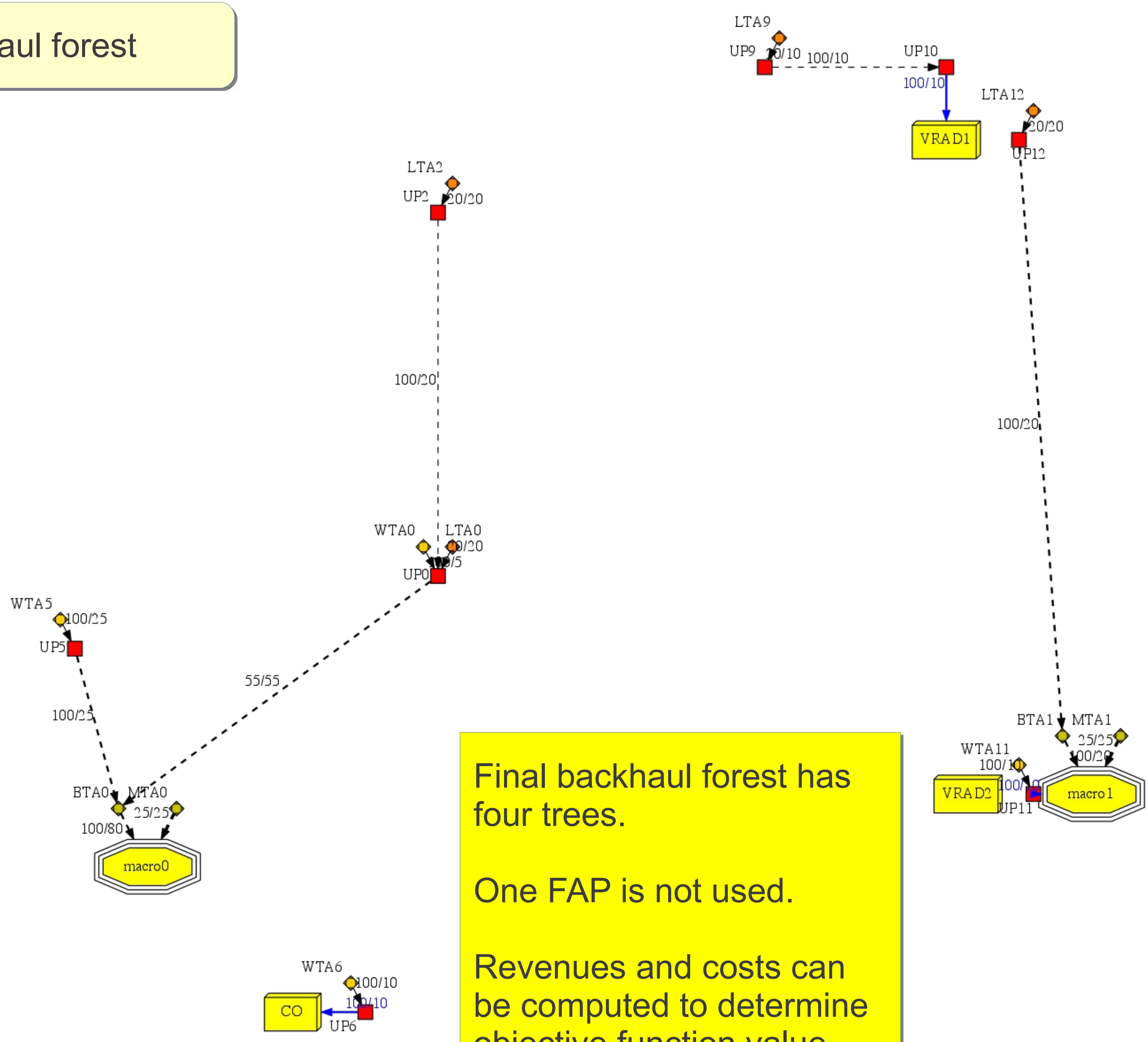
2nd Pruning



Some equipment is not used to backhaul traffic and can be pruned.

For example Wi-Fi 9 and Wi-Fi 2 can be pruned.

Final backhaul forest



Final backhaul forest has four trees.

One FAP is not used.

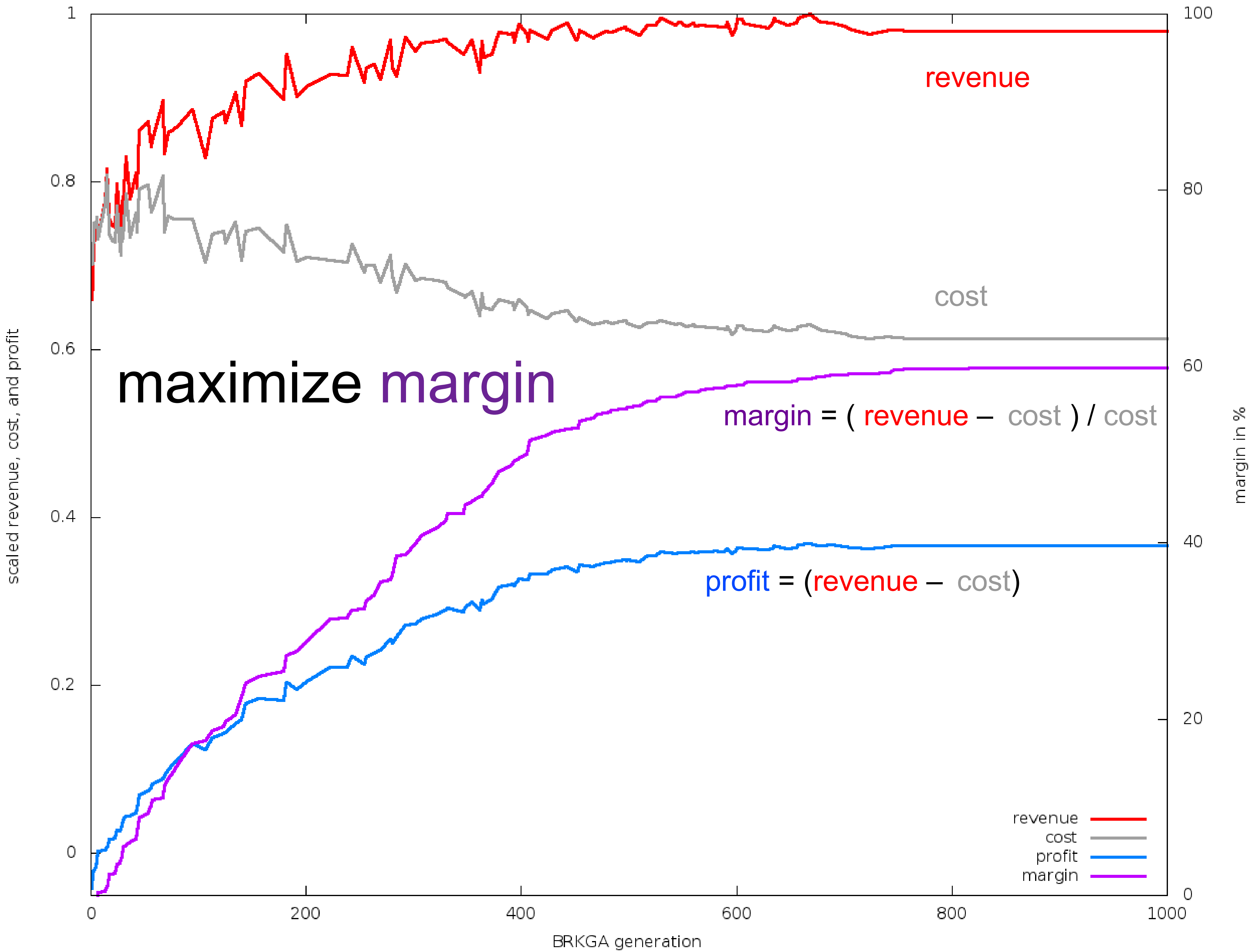
Revenues and costs can be computed to determine objective function value.

An example optimization

We ran the BRKGA on a "real" instance from a large Tier 1 Internet Service Provider

- Number of utility poles: 4395
- Number of demand points: 3236
- Number of fibered access points (FAP) excluding macrocells: 482
- Number of macrocells: 31

- Objective function: maximize margin = (revenue - cost)/cost



maximize margin
revenue
cost
margin = (revenue – cost) / cost
profit = (revenue – cost)
scaled revenue, cost, and profit
margin in %
BRKGA generation
1
0.8
0.6
0.4
0.2
0
100
80
60
40
20
0
0
200
400
600
800
1000
revenue
cost
profit
margin

Concluding remarks

- We described the prize collecting directed k-hop Steiner forest (PCk-HSF) problem
- We modeled a wireless backhaul network planning problem as a PCk-HSF problem with additional constraints
- We described a biased random-key genetic algorithm (BRKGA) for the wireless backhaul network planning problem focusing on the decoder
- We applied the BRKGA to a "real" instance of the wireless backhaul network planning problem

The End

These slides are available at http://mauricioresende.com