

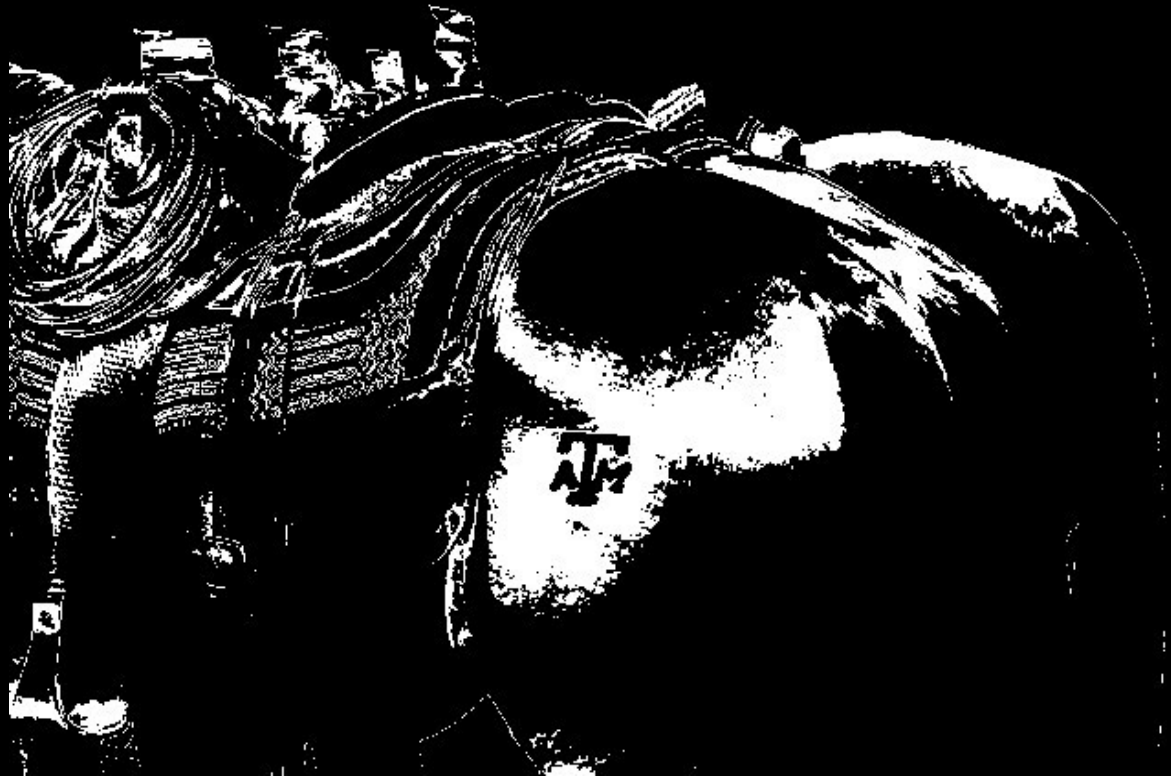
Packing with biased random-key genetic algorithms

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College Station, Texas ♣ March 28, 2014

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Summary

- Metaheuristics and basic concepts of genetic algorithms
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
 - Encoding / Decoding
 - Initial population
 - Evolutionary mechanisms
 - Problem independent / problem dependent components
 - Multi-start strategy
 - Specifying a BRKGA
 - Application programming interface (API) for BRKGA
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing

Metaheuristics

Metaheuristics are heuristics to devise heuristics.

Metaheuristics

Metaheuristics are high level procedures that coordinate simple heuristics, such as **local search**, to find solutions that are of better quality than those found by the simple heuristics alone.

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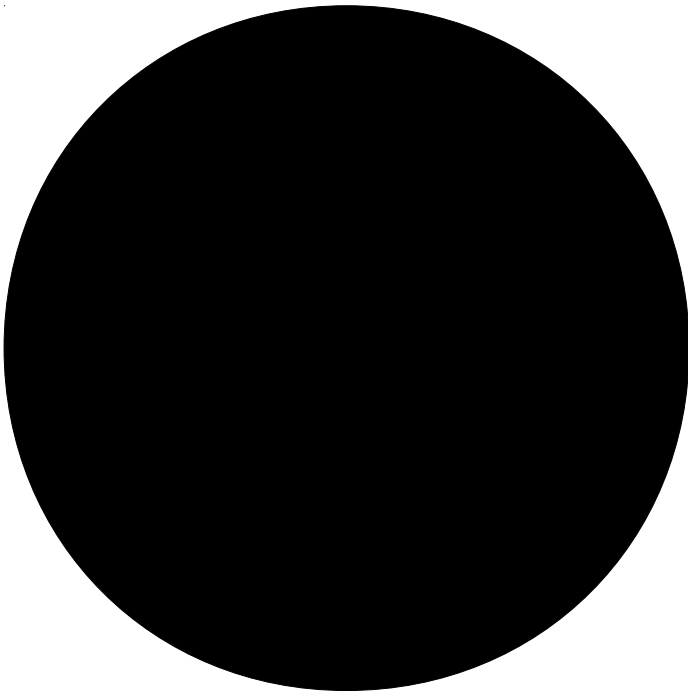
Examples: GRASP and C-GRASP, simulated annealing, genetic algorithms, tabu search, scatter search, ant colony optimization, variable neighborhood search, and **biased random-key genetic algorithms (BRKGA)**.

Genetic algorithms

Genetic algorithms

Holland (1975)

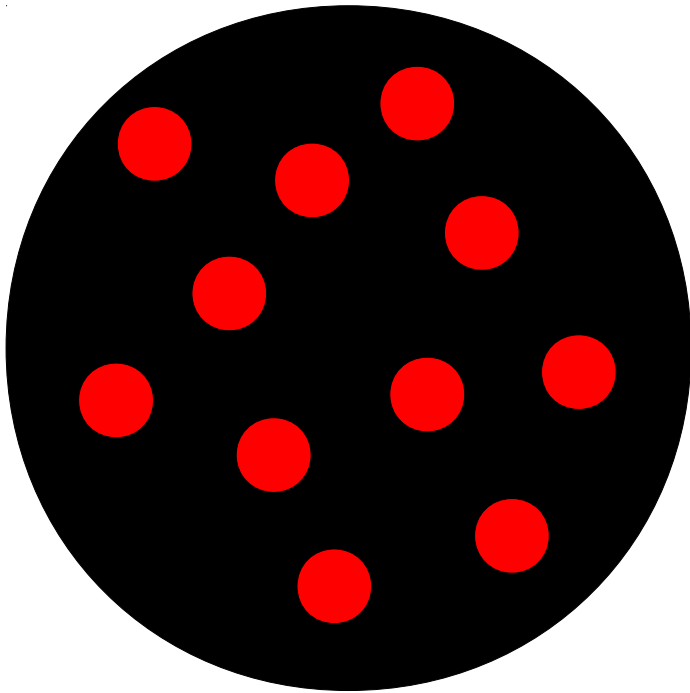
Adaptive methods that are used to solve search and optimization problems.



Individual: solution



Genetic algorithms

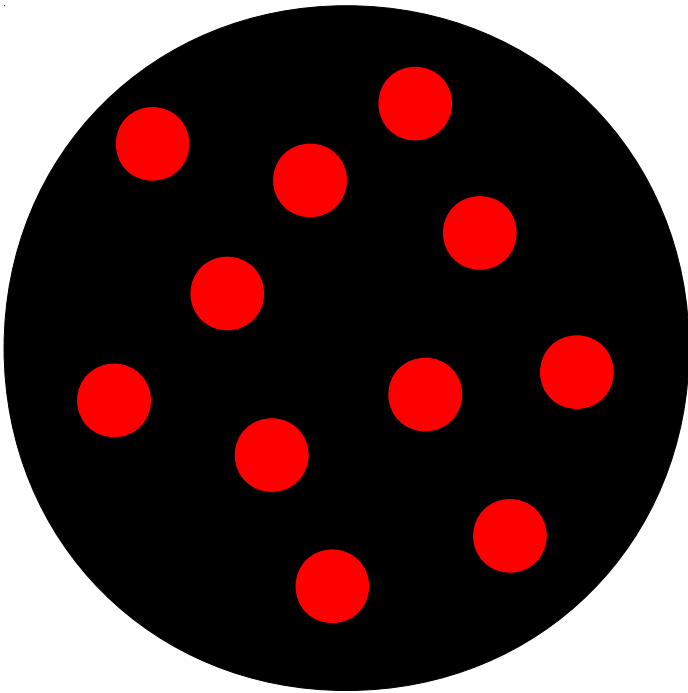


Individual: solution (chromosome = string of genes)

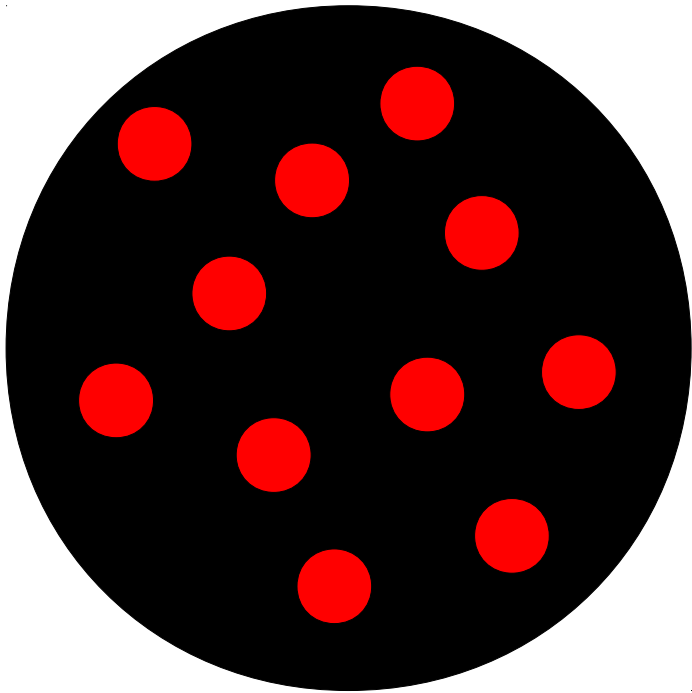
Population: set of fixed number of individuals

Genetic algorithms

Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.



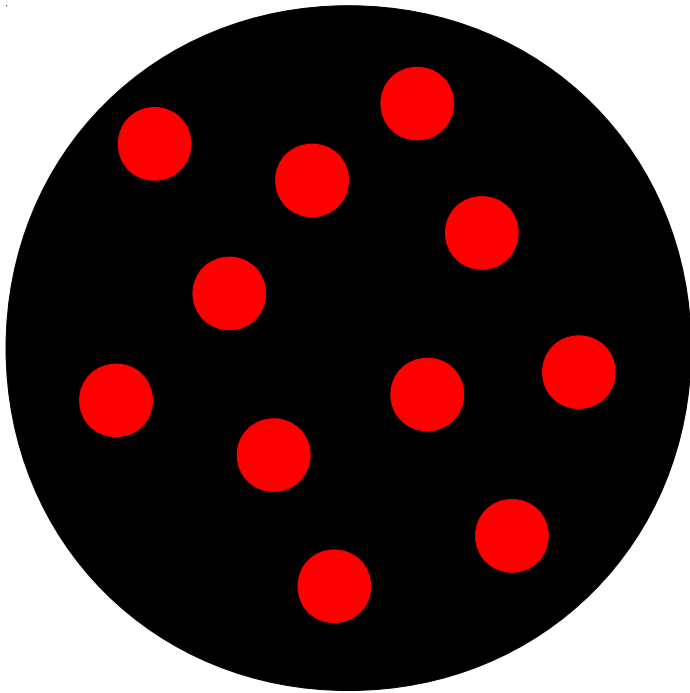
Genetic algorithms



Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

Genetic algorithms

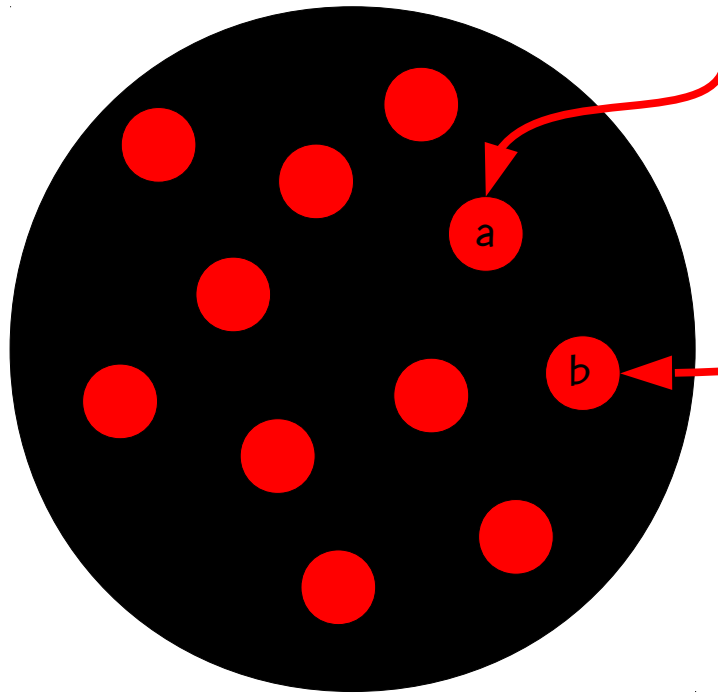


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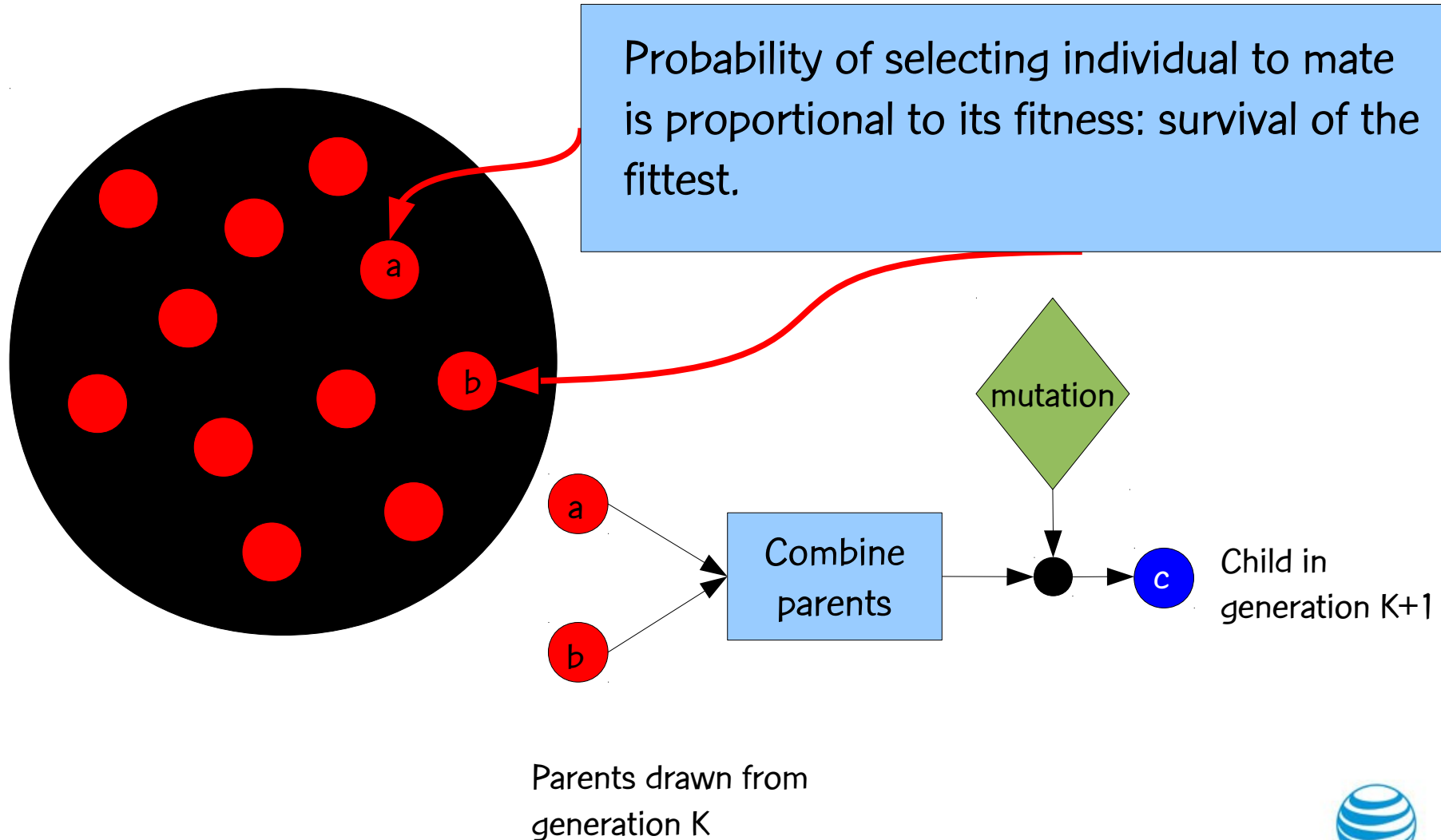
Individuals from one generation are combined to produce offspring that make up next generation.

Genetic algorithms

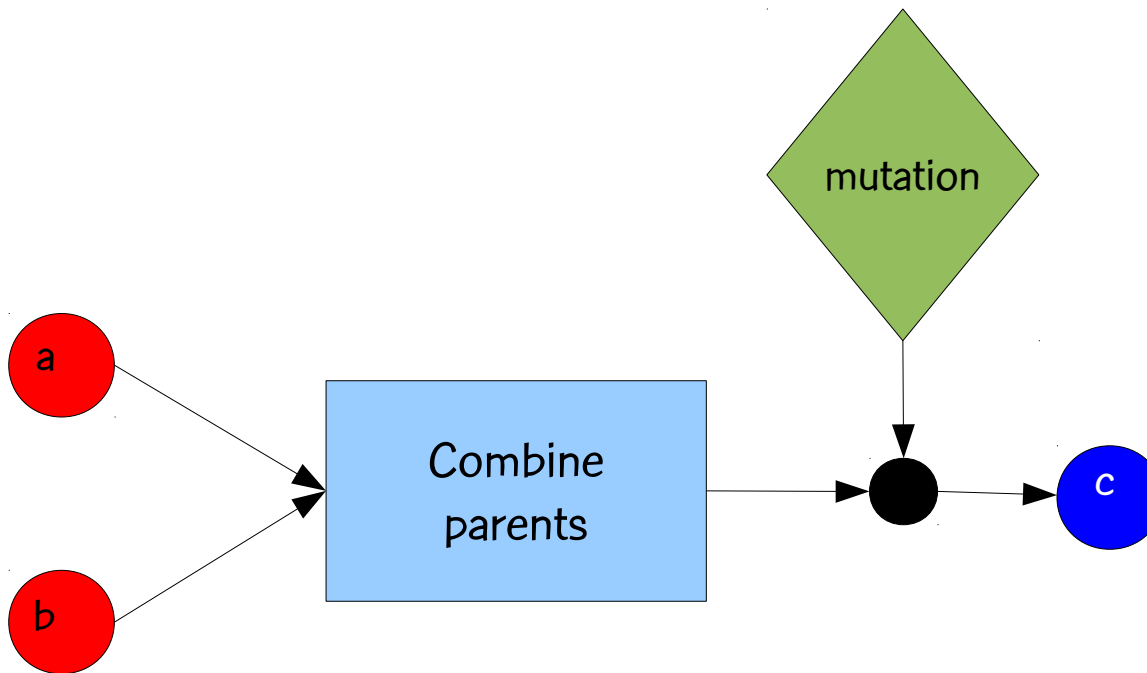


Probability of selecting individual to mate is proportional to its fitness: survival of the fittest.

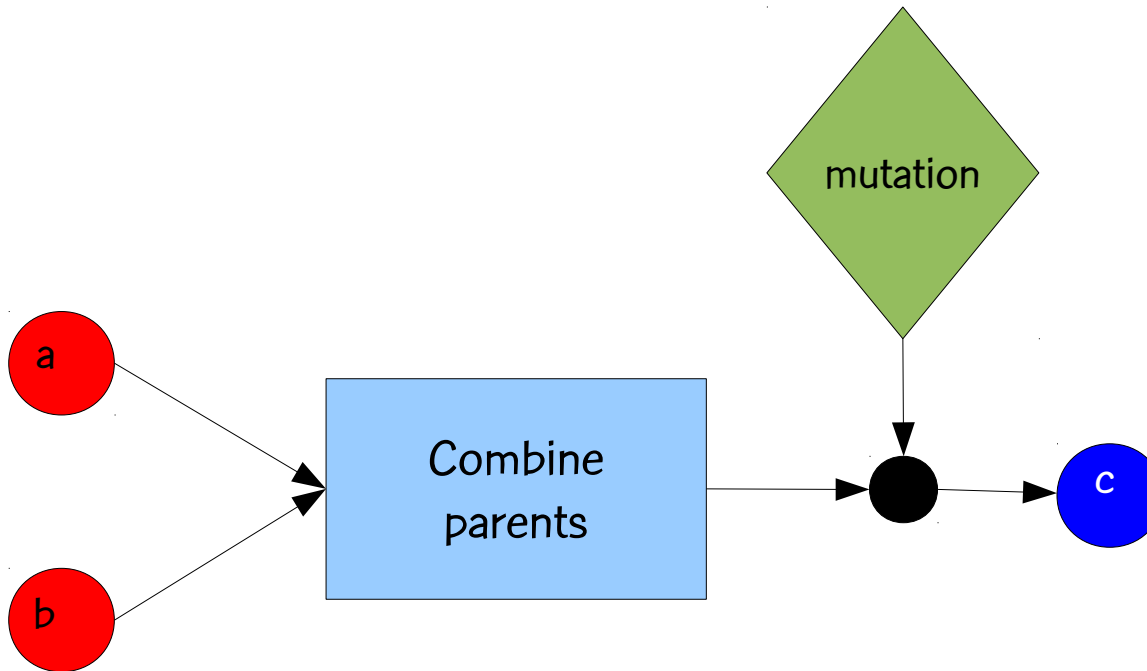
Genetic algorithms



Crossover and mutation



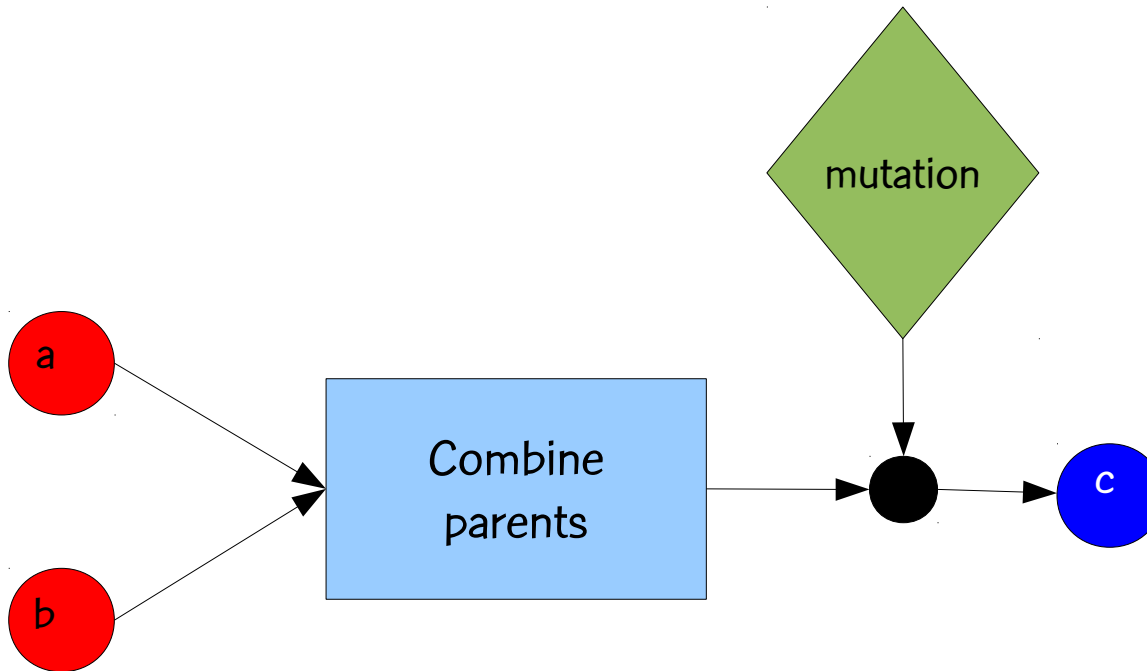
Crossover and mutation



Crossover: Combines parents ... passing along to offspring characteristics of each parent ...

Intensification of search

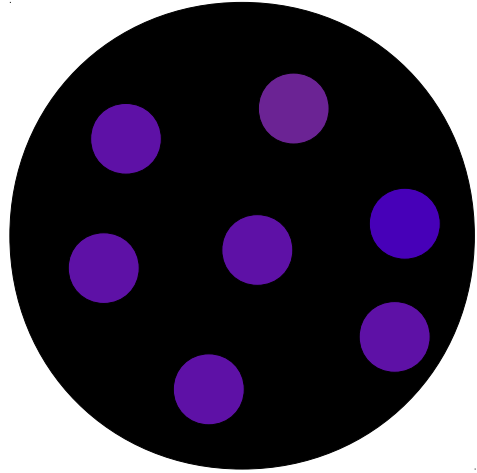
Crossover and mutation



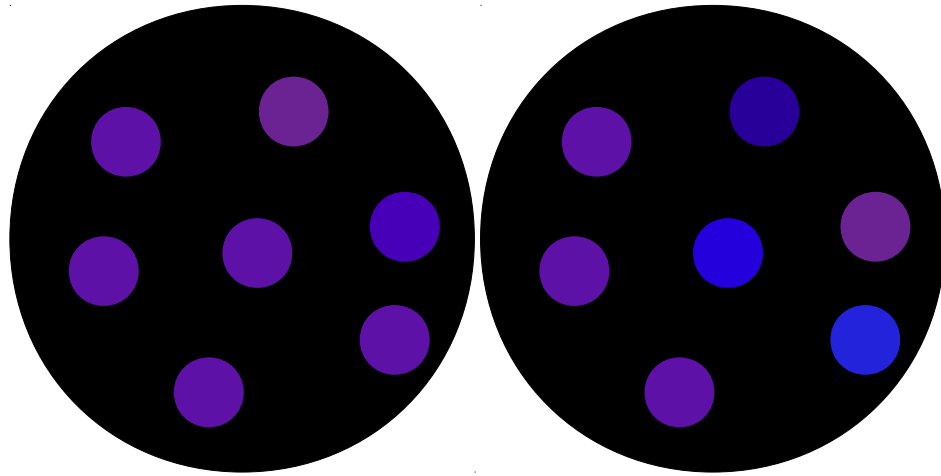
Mutation: Randomly changes chromosome of offspring ...
Driver of evolutionary process ...

Diversification of search

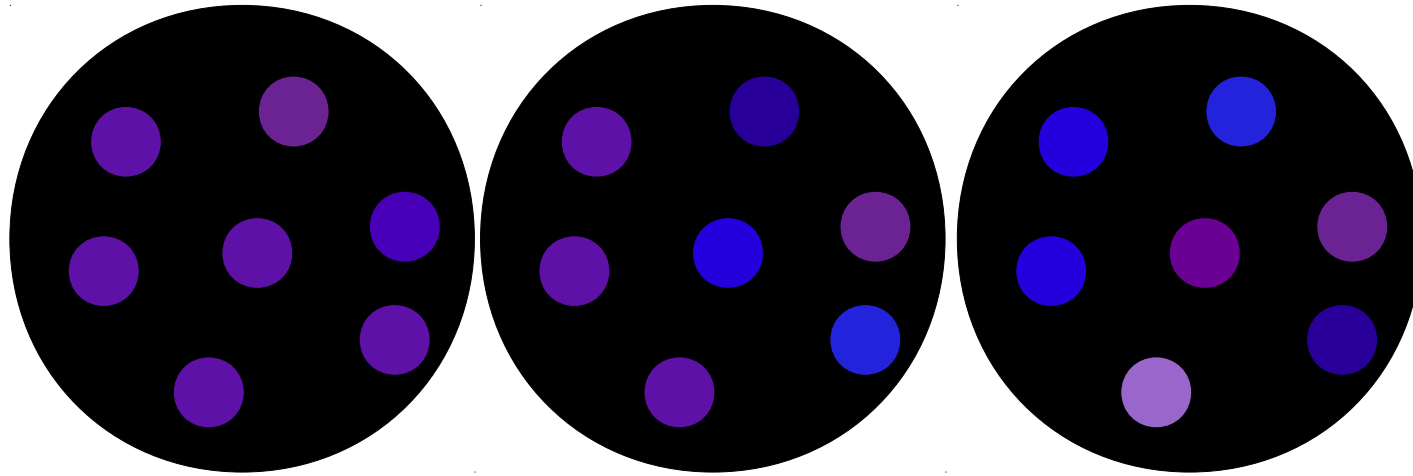
Evolution of solutions



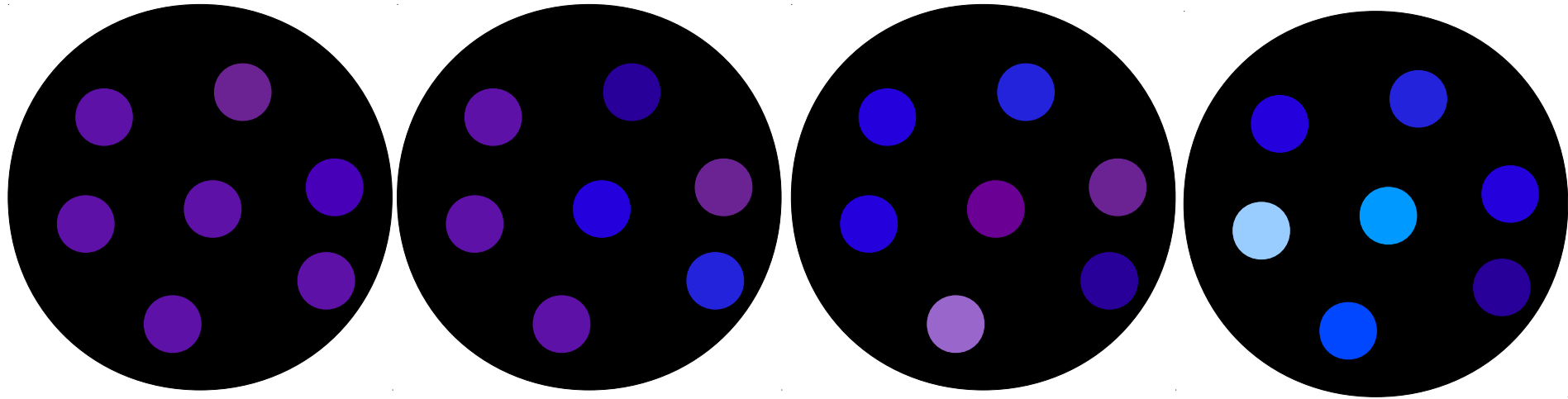
Evolution of solutions



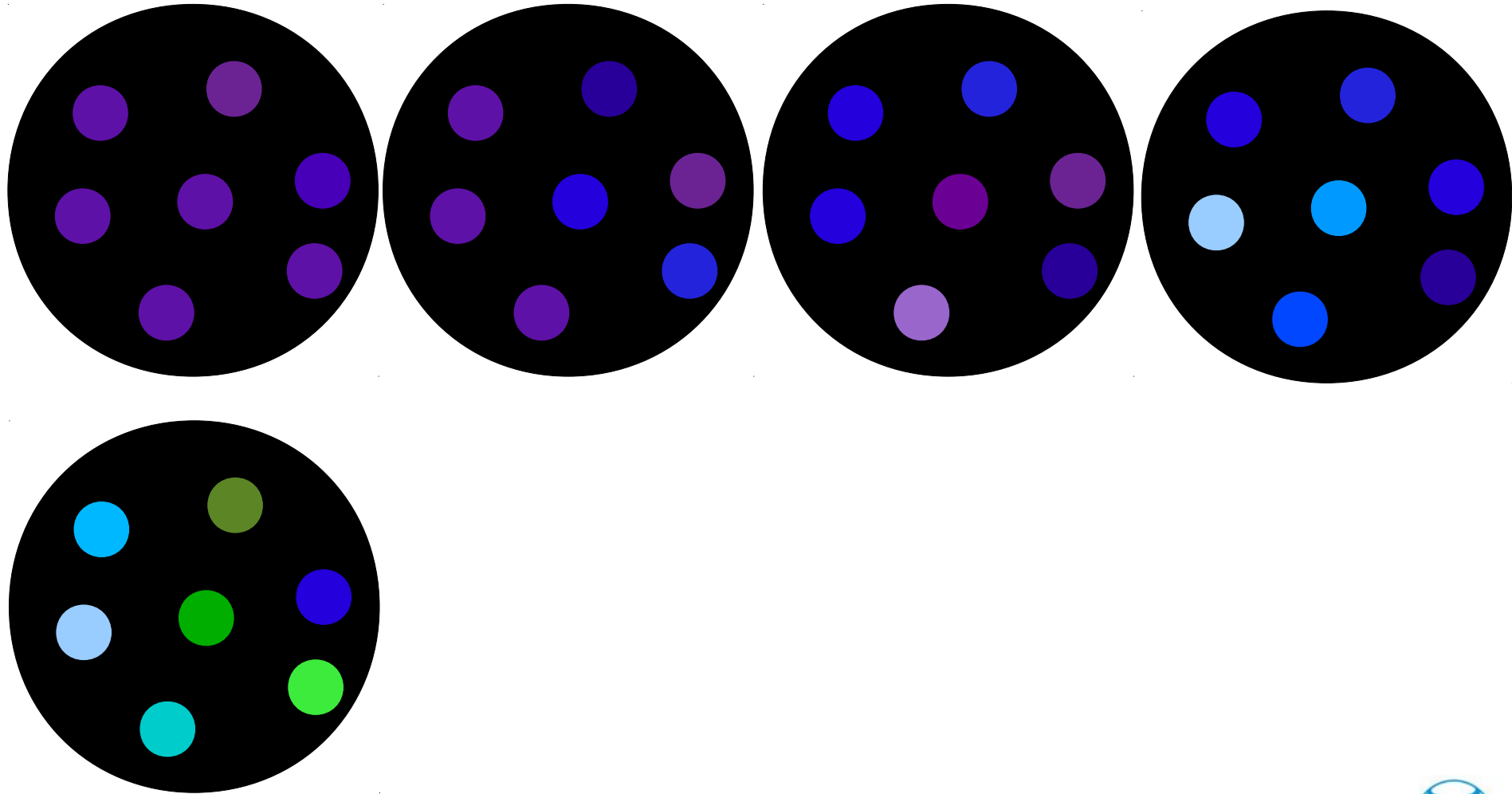
Evolution of solutions



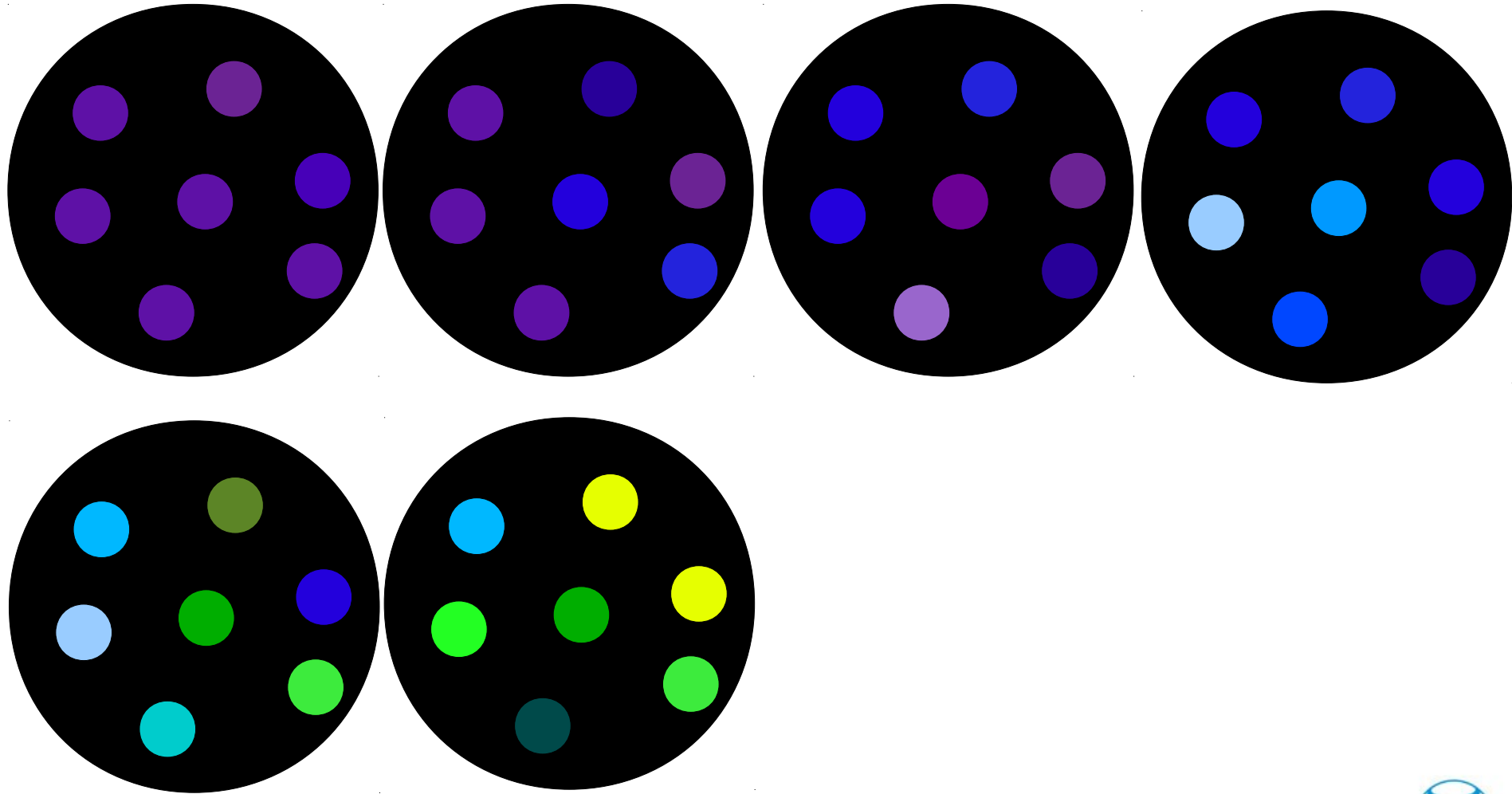
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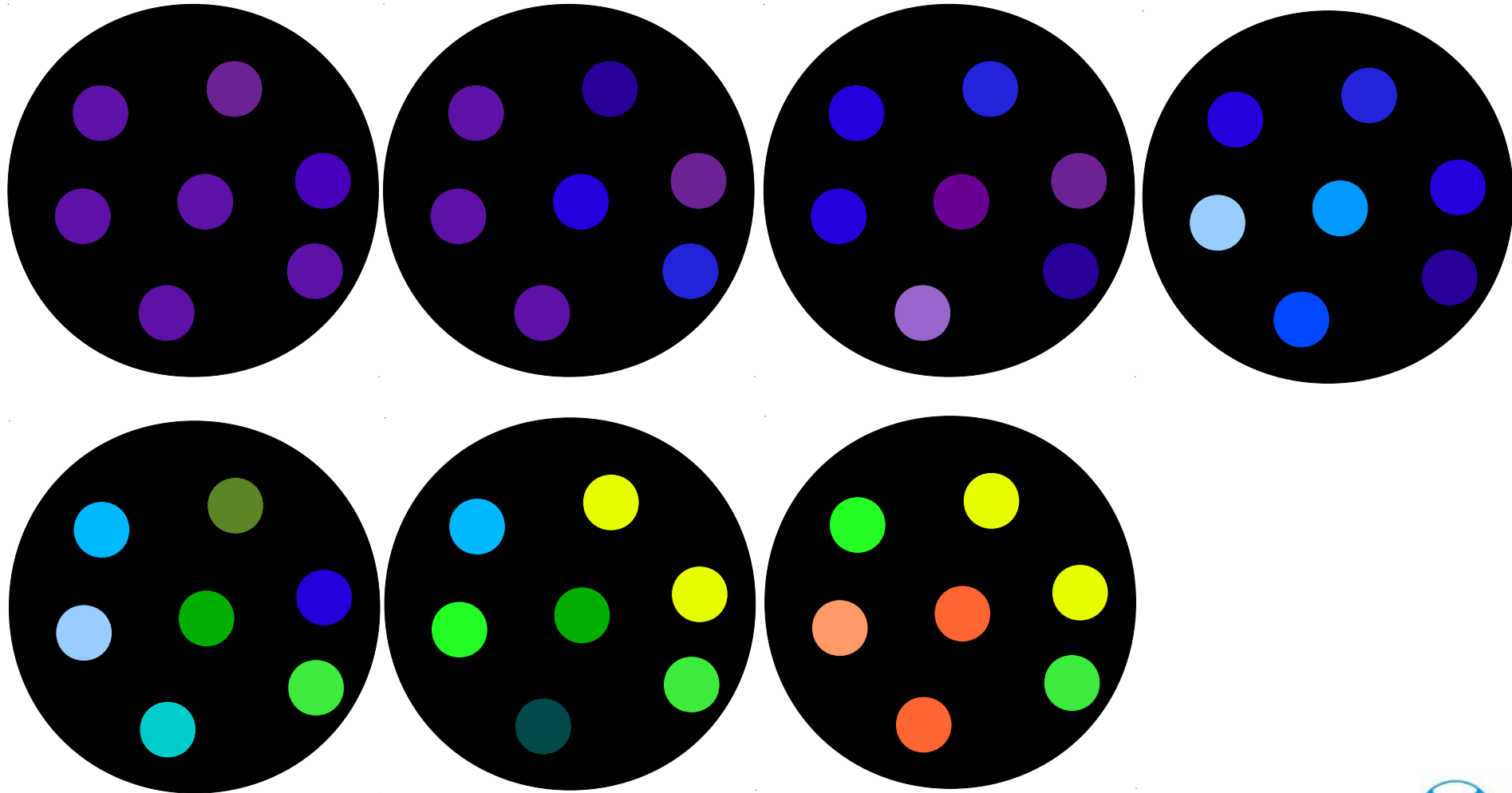
Evolution of solutions



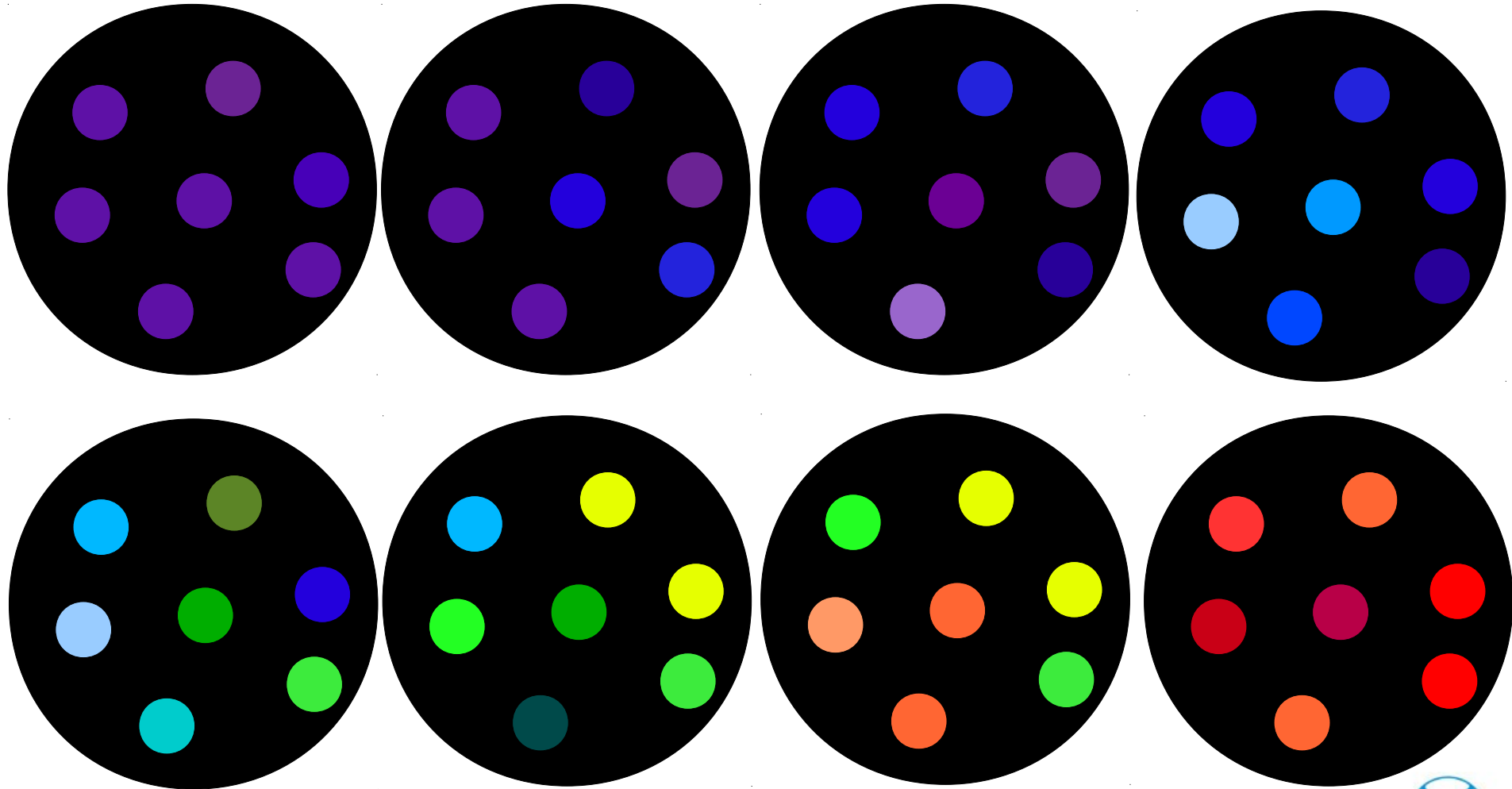
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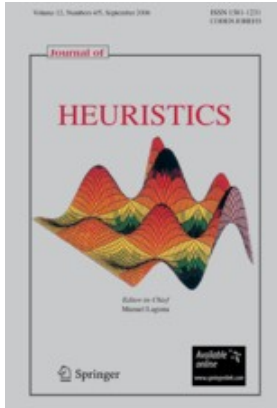
Evolution of solutions



Evolution of solutions



Reference



J.F. Gonçalves and M.G.C.R., “**Biased random-key genetic algorithms for combinatorial optimization,**” J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:

<http://www.research.att.com/~mgcr/doc/srkga.pdf>

Encoding solutions with random keys

Encoding with random keys

- A random key is a real random number in the continuous interval $[0,1)$.

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- A random key is a real random number in the continuous interval $[0,1)$.
- A vector X of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.

Encoding with random keys: Sequencing

Encoding

[1, 2, 3, 4, 5]

$X = [0.099, 0.216, 0.802, 0.368, 0.658]$

Encoding with random keys: Sequencing

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[1, 2, 3, 4, 5]

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Decode by sorting vector of random keys

[1, 2, 4, 5, 3]

$X = [0.099, 0.216, 0.368, 0.658, 0.802]$

Encoding with random keys: Sequencing

Therefore, the vector of random keys:

$$X = [0.099, 0.216, 0.802, 0.368, 0.658]$$

encodes the sequence: 1 – 2 – 4 – 5 – 3

Encoding with random keys: Subset selection (select 3 of 5 elements)

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Encoding with random keys: Subset selection (select 3 of 5 elements)

Therefore, the vector of random keys:

$X = [0.099, 0.216, 0.802, 0.368, 0.658]$

encodes the subset: $\{1, 2, 4\}$

Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

Encoding

[1, 2, 3, 4, 5 | 1, 2, 3, 4, 5]

$X = [0.099, 0.216, 0.802, 0.368, 0.658 \mid 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]$

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$X = [0.099, 0.216, 0.802, 0.368, 0.658 \mid 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]$

Decode by sorting the first 5 keys and assign as the weight the value

$W_i = \mathbf{floor} [10 X_{5+i}] + 1$ to the 3 elements with smallest keys X_i , for $i = 1, \dots, 5$.

Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

Therefore, the vector of random keys:

$X = [0.099, 0.216, 0.802, 0.368, 0.658 \mid 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]$

encodes the weight vector $W = (5, 6, -, 5, -)$

Genetic algorithms and random keys

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.

GAs and random keys

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- Individuals are strings of real-valued numbers (random keys) in the interval $[0,1)$.

$$S = (\begin{matrix} 0.25, & 0.19, & 0.67, & 0.05, & 0.89 \end{matrix})$$

$s(1) \quad s(2) \quad s(3) \quad s(4) \quad s(5)$

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Individuals are strings of real-valued numbers (random keys) in the interval $[0,1)$.
- Sorting random keys results in a sequencing order.

$$S = (\begin{matrix} 0.25 & 0.19 & 0.67 & 0.05 & 0.89 \end{matrix}) \\ \begin{matrix} s(1) & s(2) & s(3) & s(4) & s(5) \end{matrix}$$

$$S' = (\begin{matrix} 0.05 & 0.19 & 0.25 & 0.67 & 0.89 \end{matrix}) \\ \begin{matrix} s(4) & s(2) & s(1) & s(3) & s(5) \end{matrix}$$

Sequence: 4 – 2 – 1 – 3 – 5

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)

$a = (0.25, 0.19, 0.67, 0.05, 0.89)$

$b = (0.63, 0.90, 0.76, 0.93, 0.08)$

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)
- For each gene, flip a biased coin to choose which parent passes the allele (key, or value of gene) to the child.

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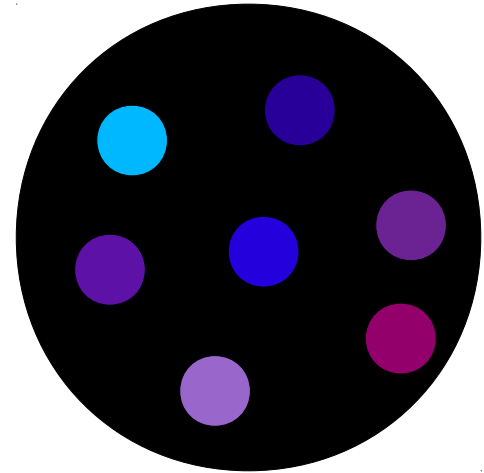
$b = (0.63, 0.90, 0.76, 0.93, 0.08)$

$c = (0.25, 0.90, 0.76, 0.05, 0.89)$

If every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

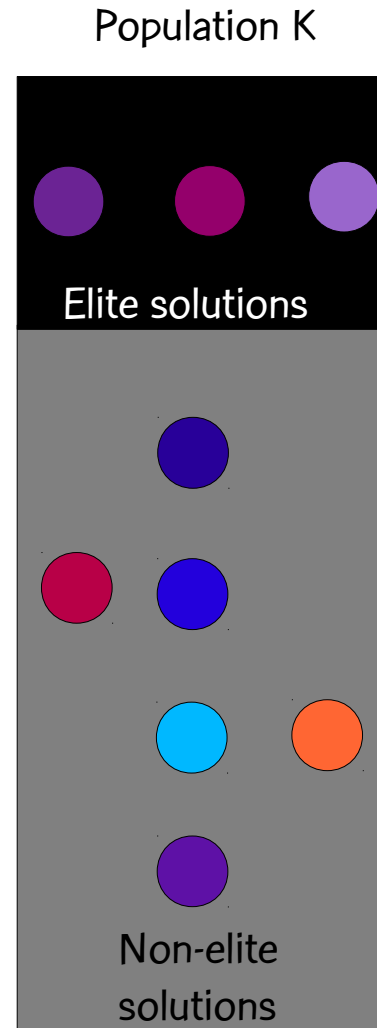
GAs and random keys

Initial population is made up of P random-key vectors, each with N keys, each having a value generated uniformly at random in the interval $[0,1)$.



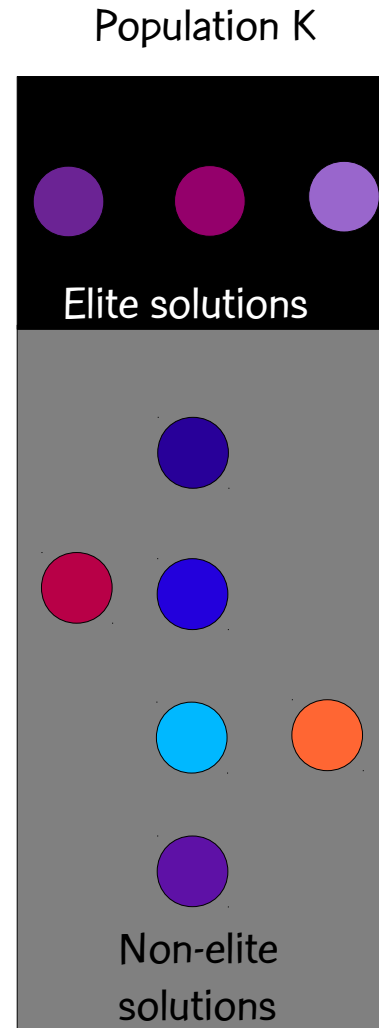
GAs and random keys

At the K-th generation,
compute the cost of each
solution ...



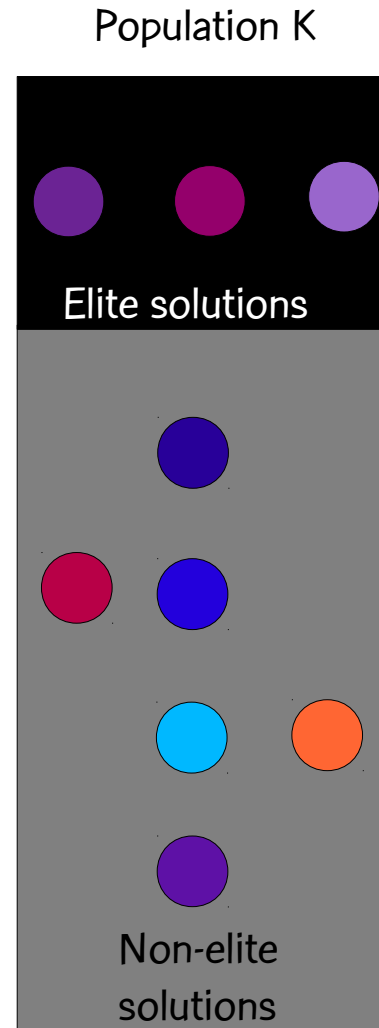
GAs and random keys

At the K -th generation,
compute the cost of each
solution and partition the
solutions into two sets:



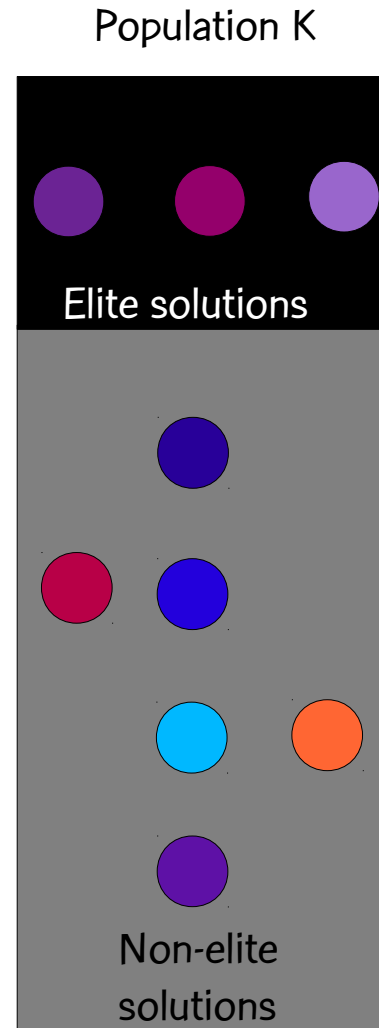
GAs and random keys

At the K-th generation,
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elite solutions and non-elite
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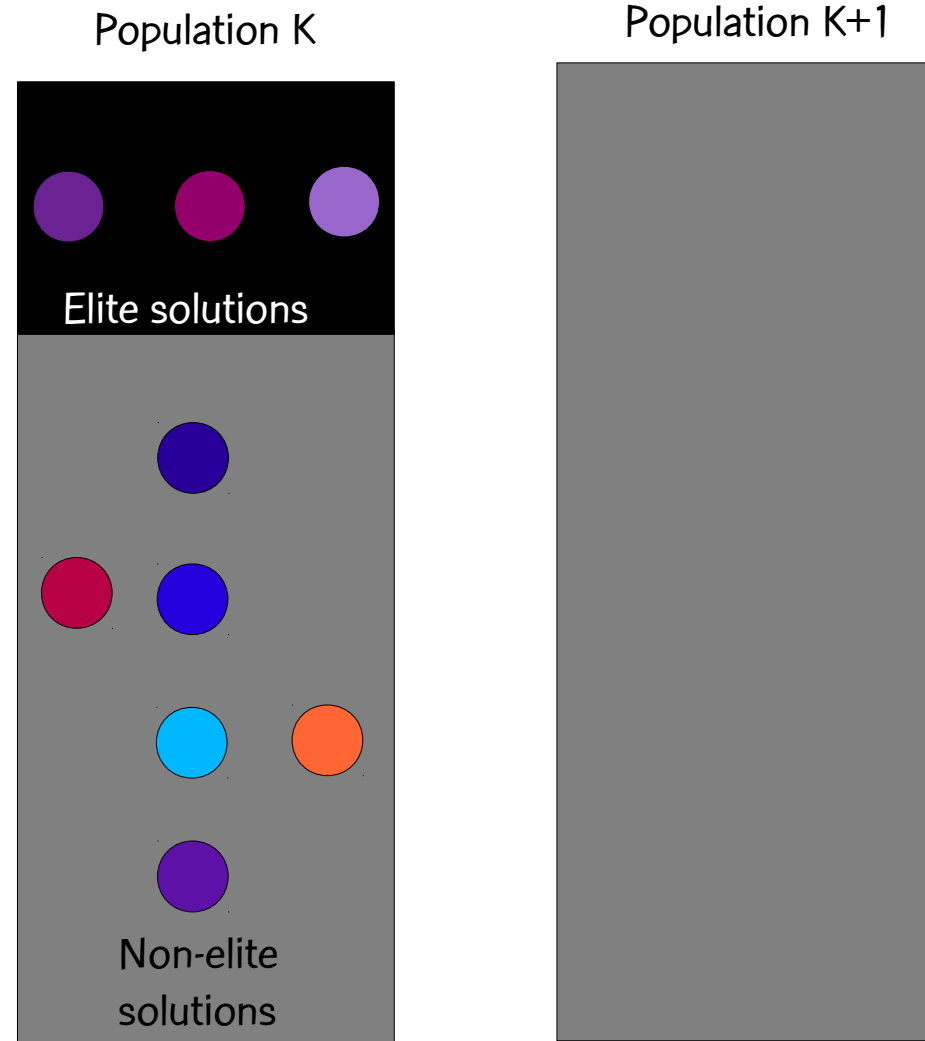
GAs and random keys

At the K -th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.



GAs and random keys

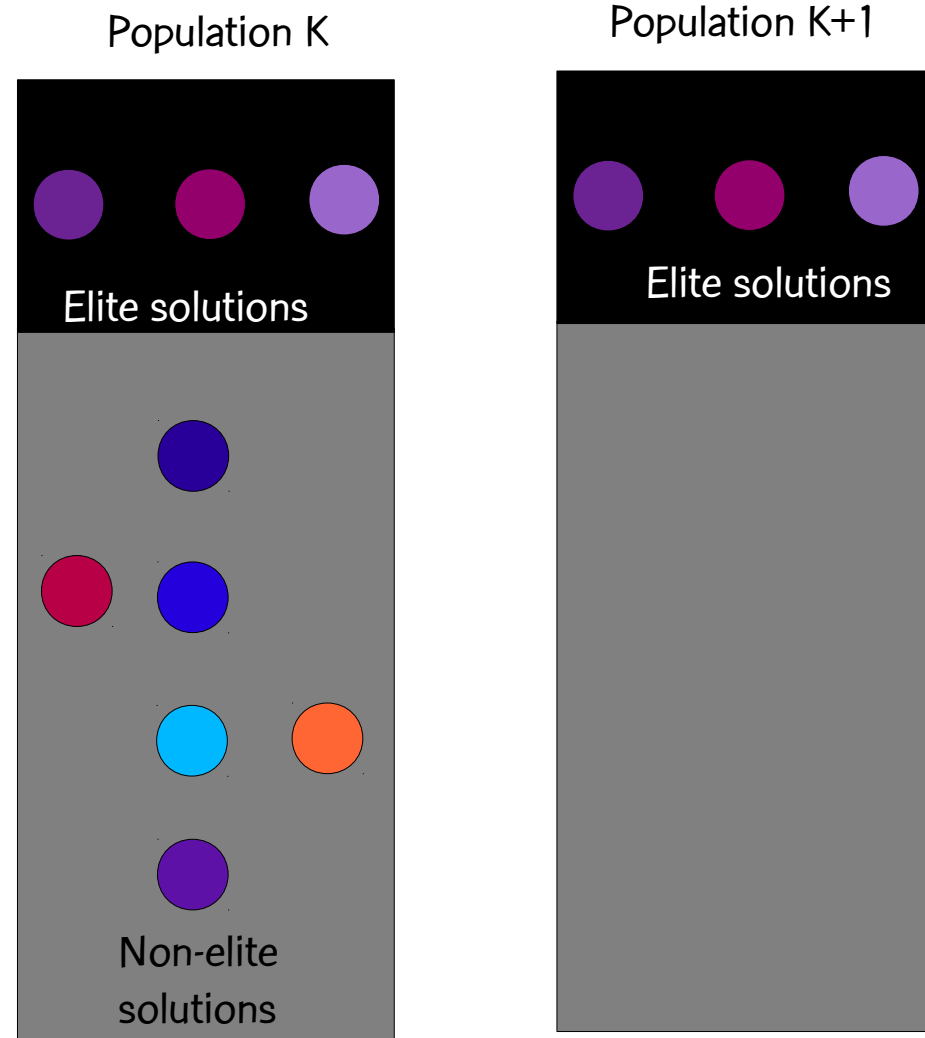
Evolutionary dynamics



GAs and random keys

Evolutionary dynamics

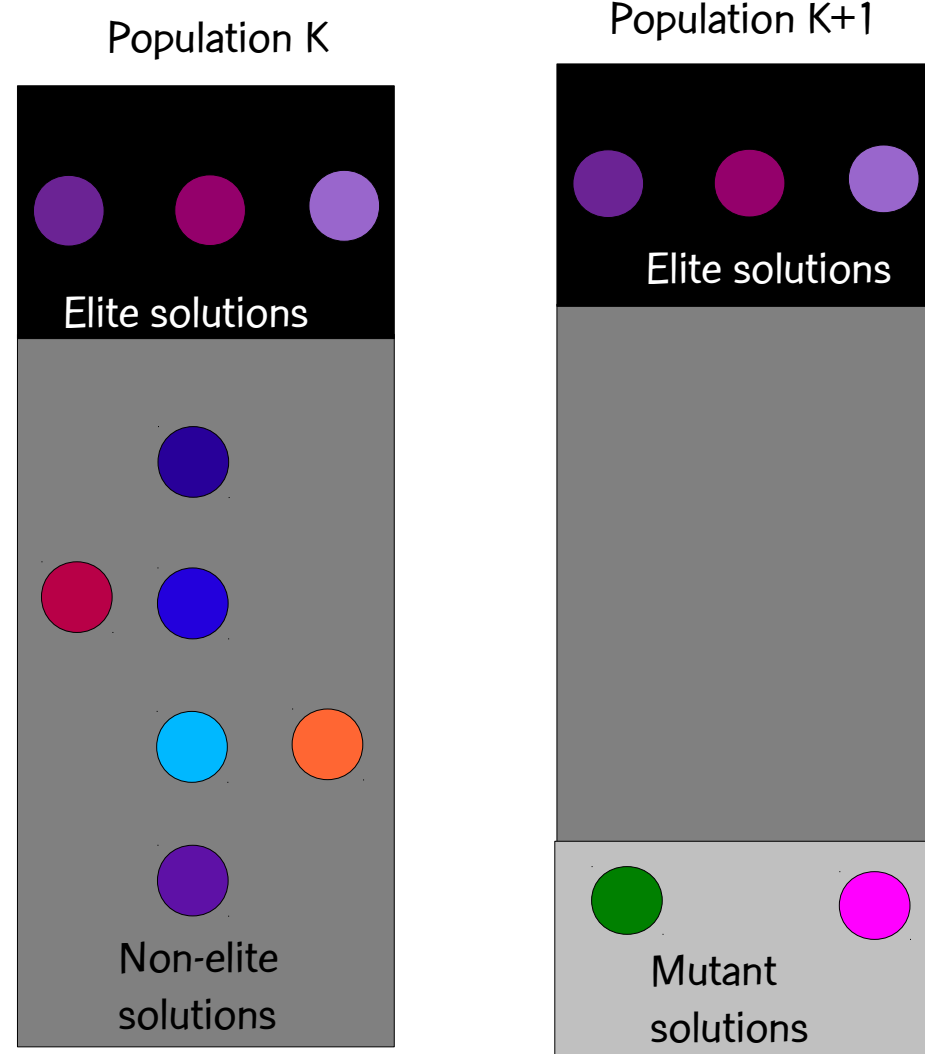
- Copy elite solutions from population K to population K+1



GAs and random keys

Evolutionary dynamics

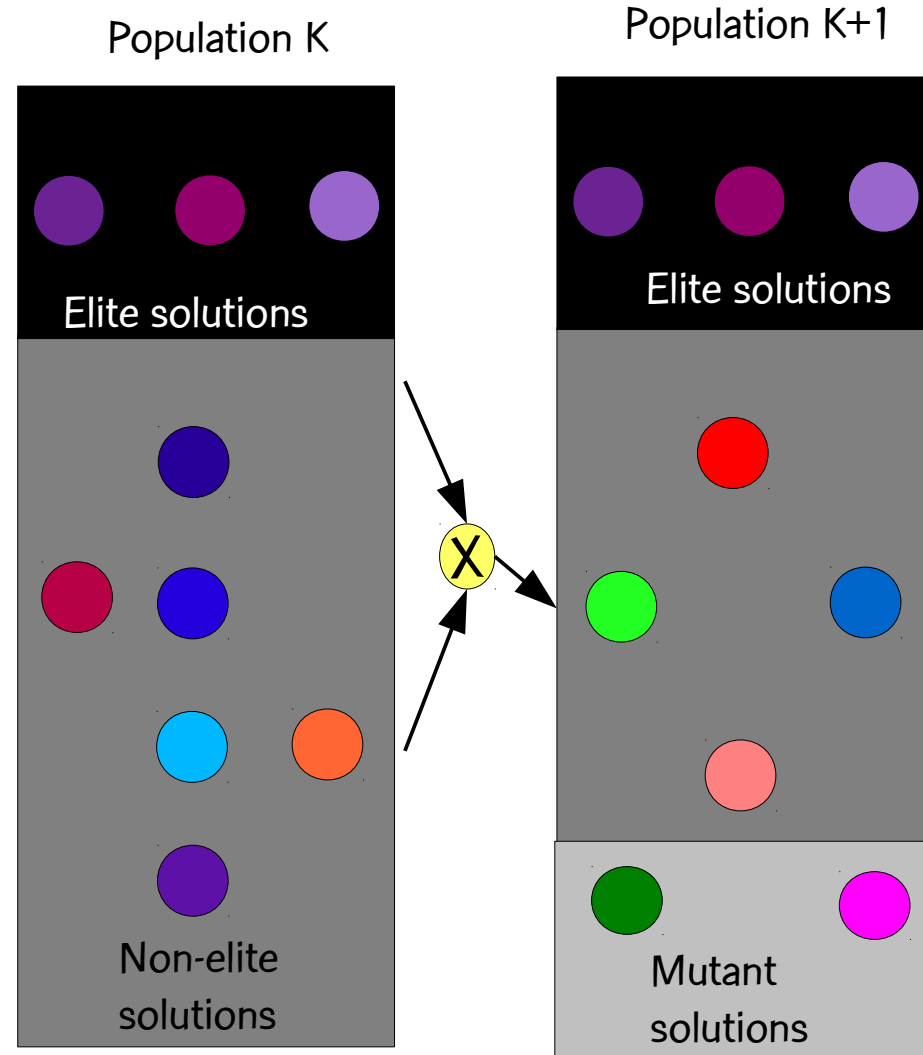
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1



GAs and random keys

Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population $< P$
 - **RANDOM-KEY GA:** Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.



Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).

Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.

How RKGA & BRKGA differ

RKGA

both parents chosen at
random from entire
population

BRKGA

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

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RKGA

both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

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BRKGA

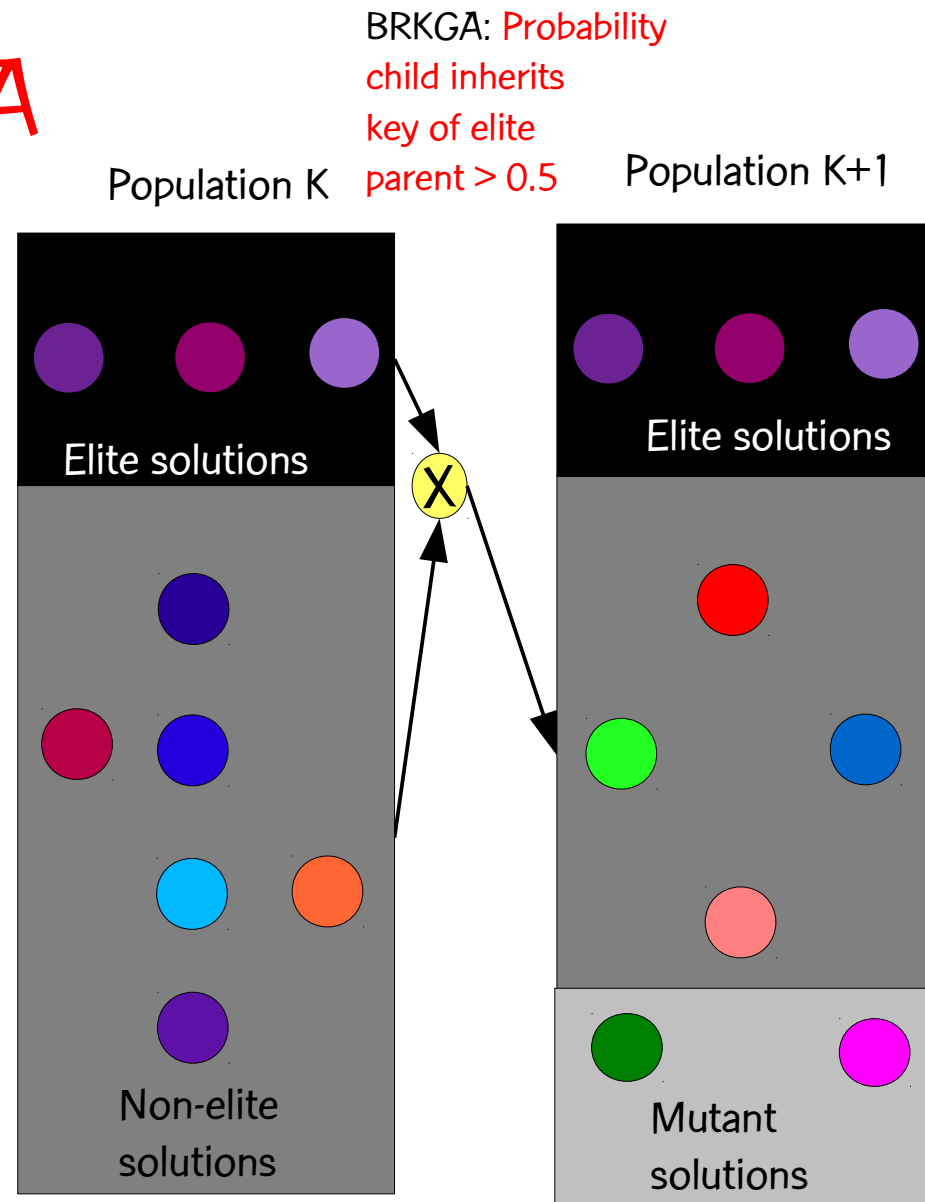
both parents chosen at random but one parent chosen from population of elite solutions

best fit parent is parent A in parametrized uniform crossover

Biased random key GA

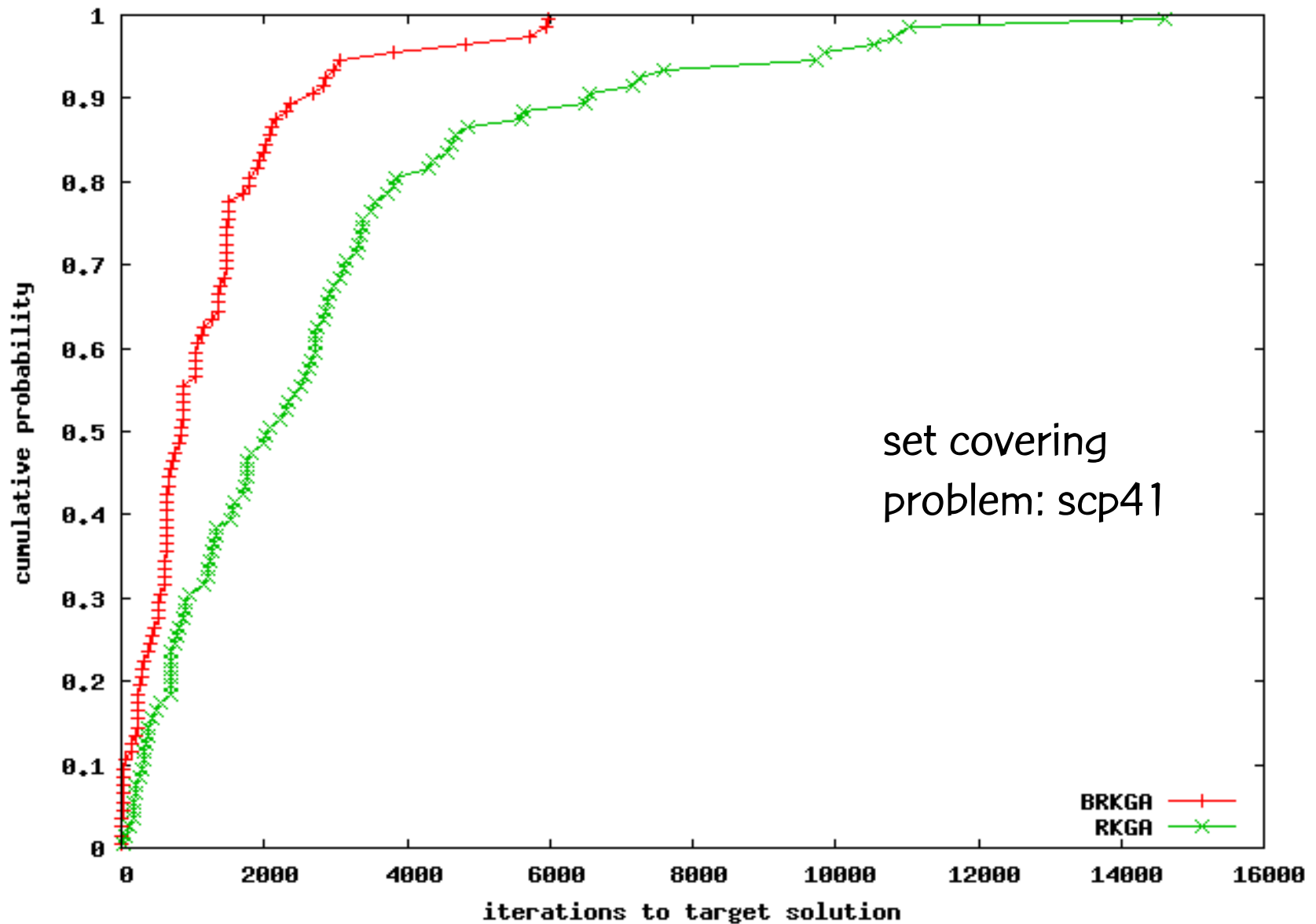
Evolutionary dynamics

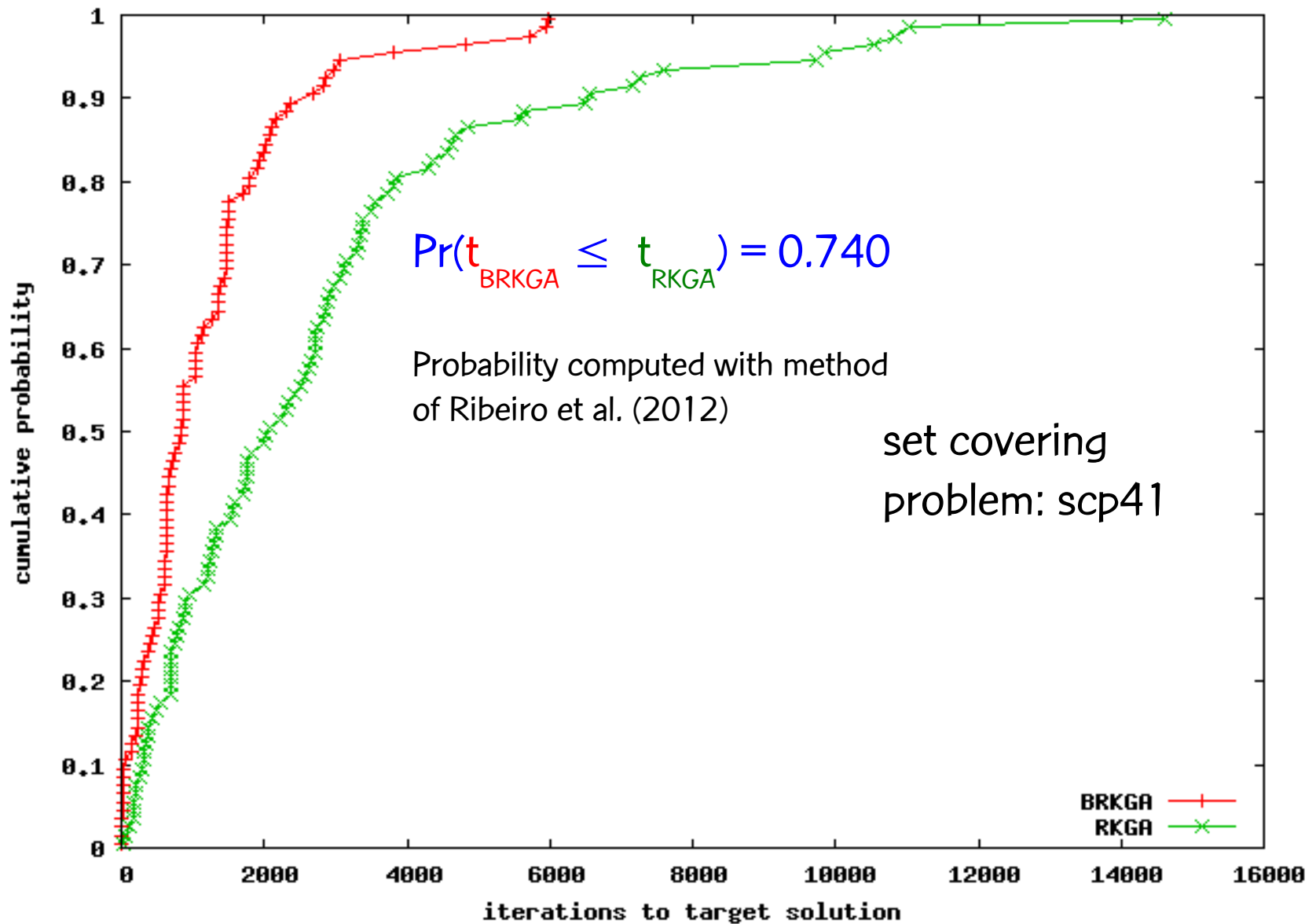
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population $< P$
 - **RANDOM-KEY GA:** Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.
 - **BIASED RANDOM-KEY GA:** Mate elite solution with other solution of population K to produce child in population K+1. Mates are chosen at random.

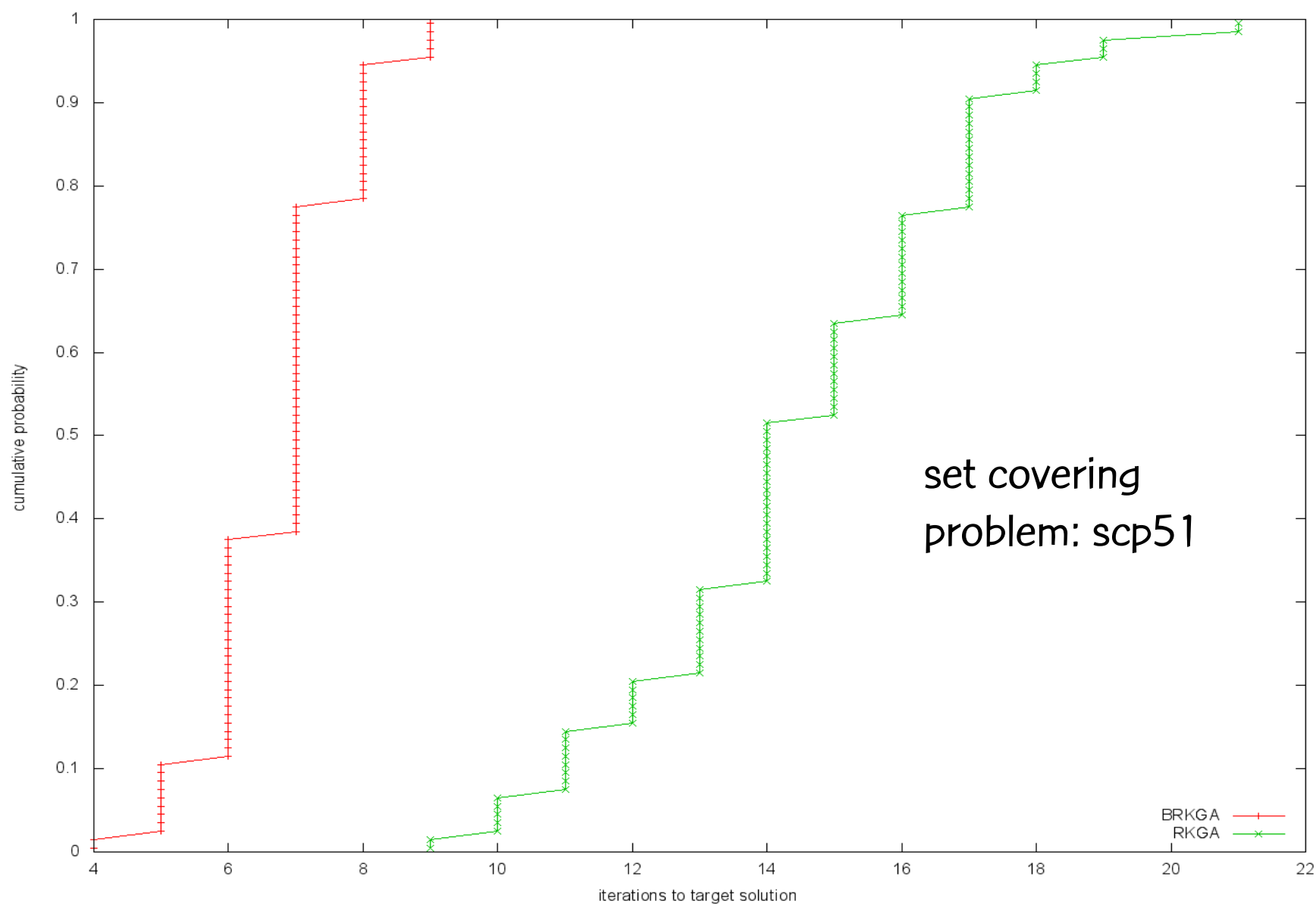


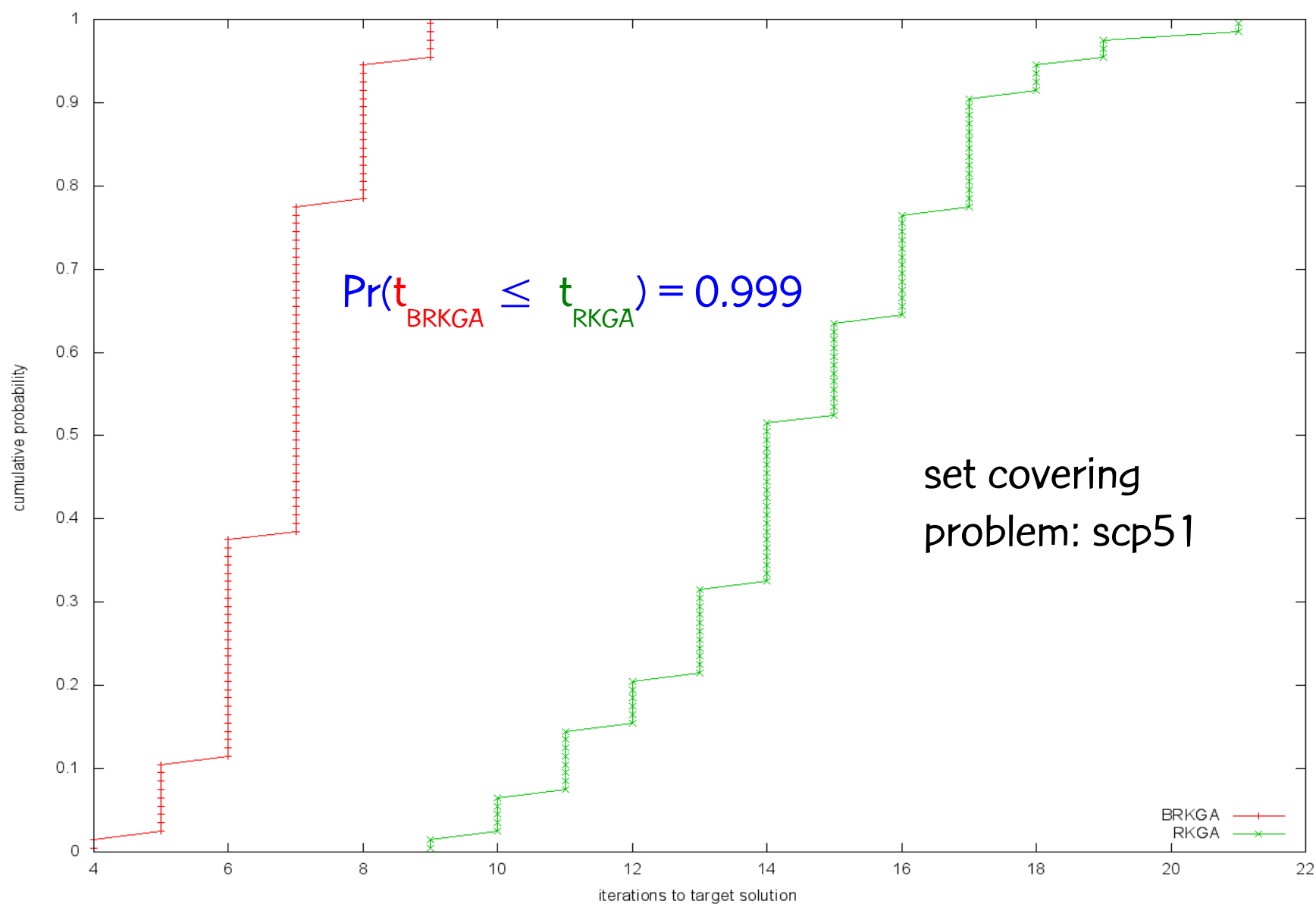
Paper comparing BRKGA and Bean's Method

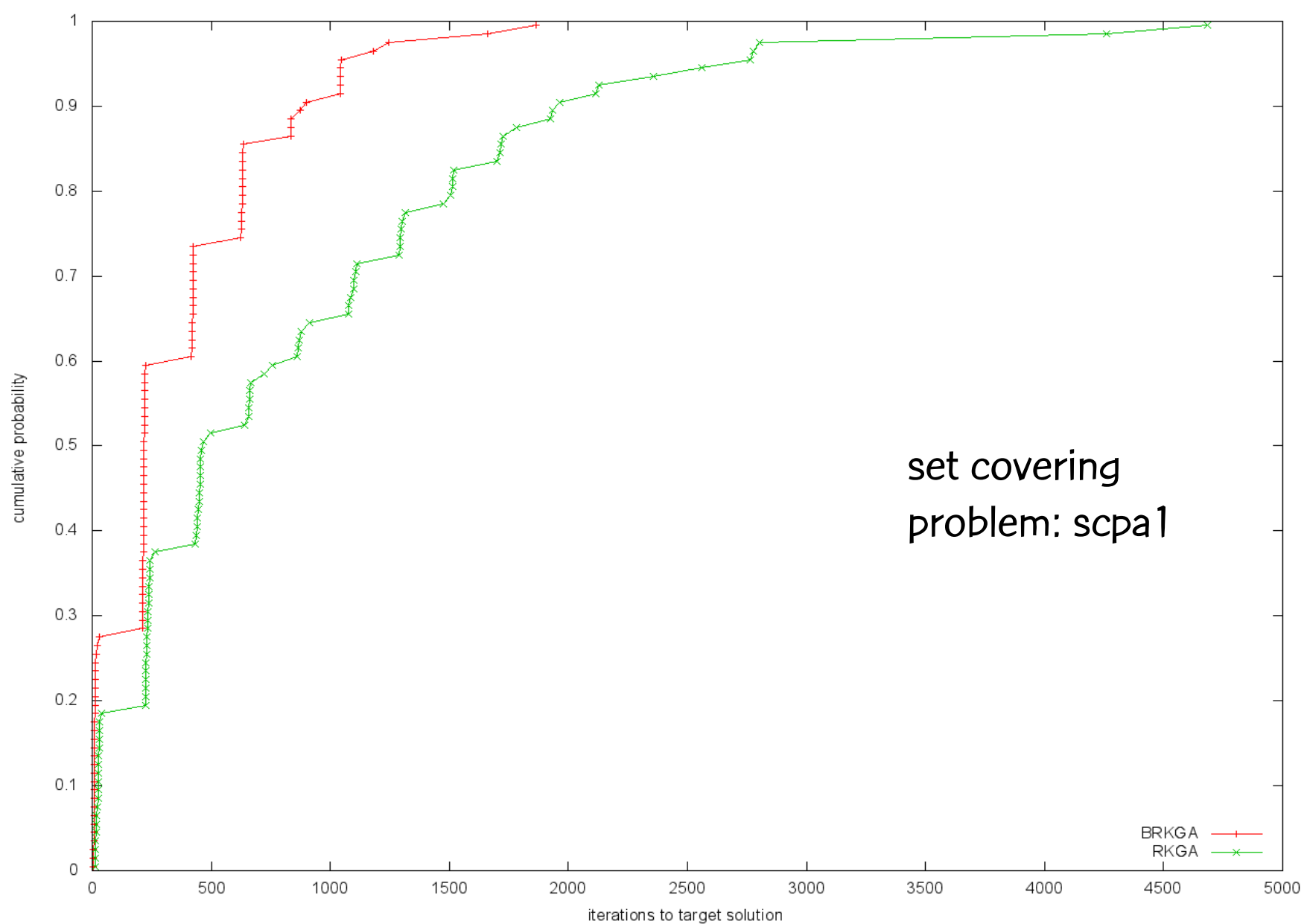
Gonçalves, R., and Toso, “Biased and unbiased random-key genetic algorithms: An experimental analysis”, Proceedings of the 10th Metaheuristics International Conference, Singapore, August 2013.

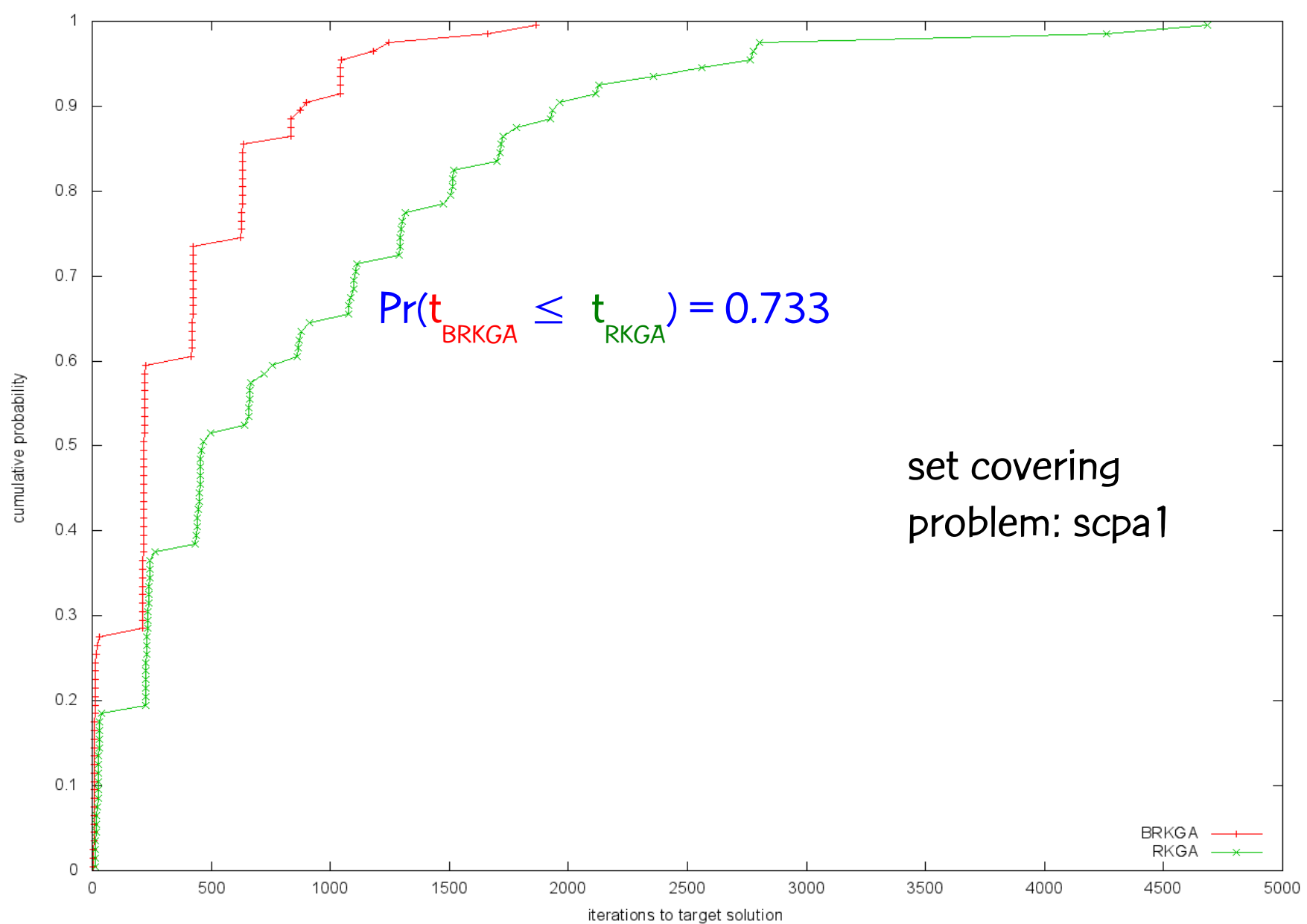


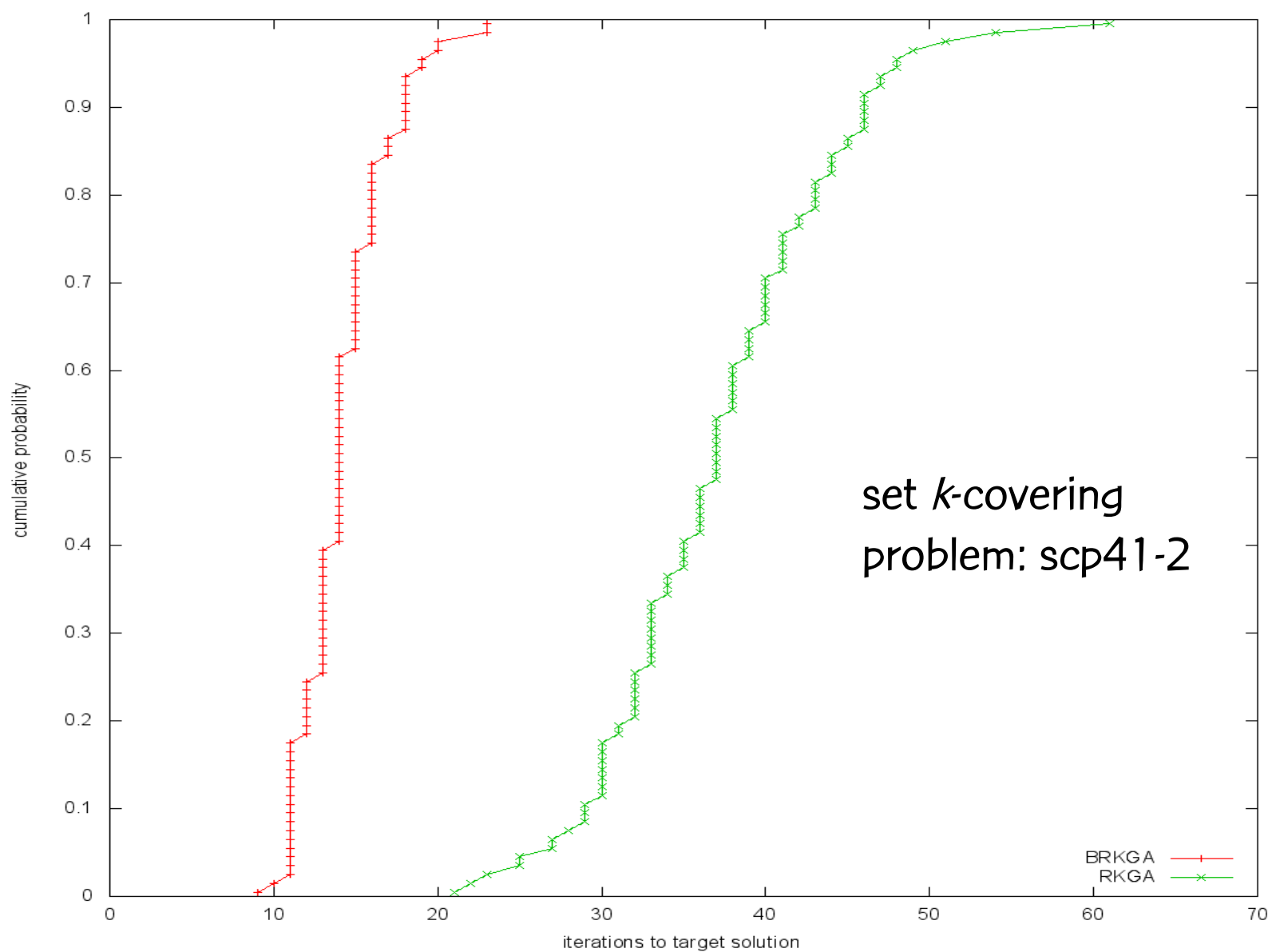


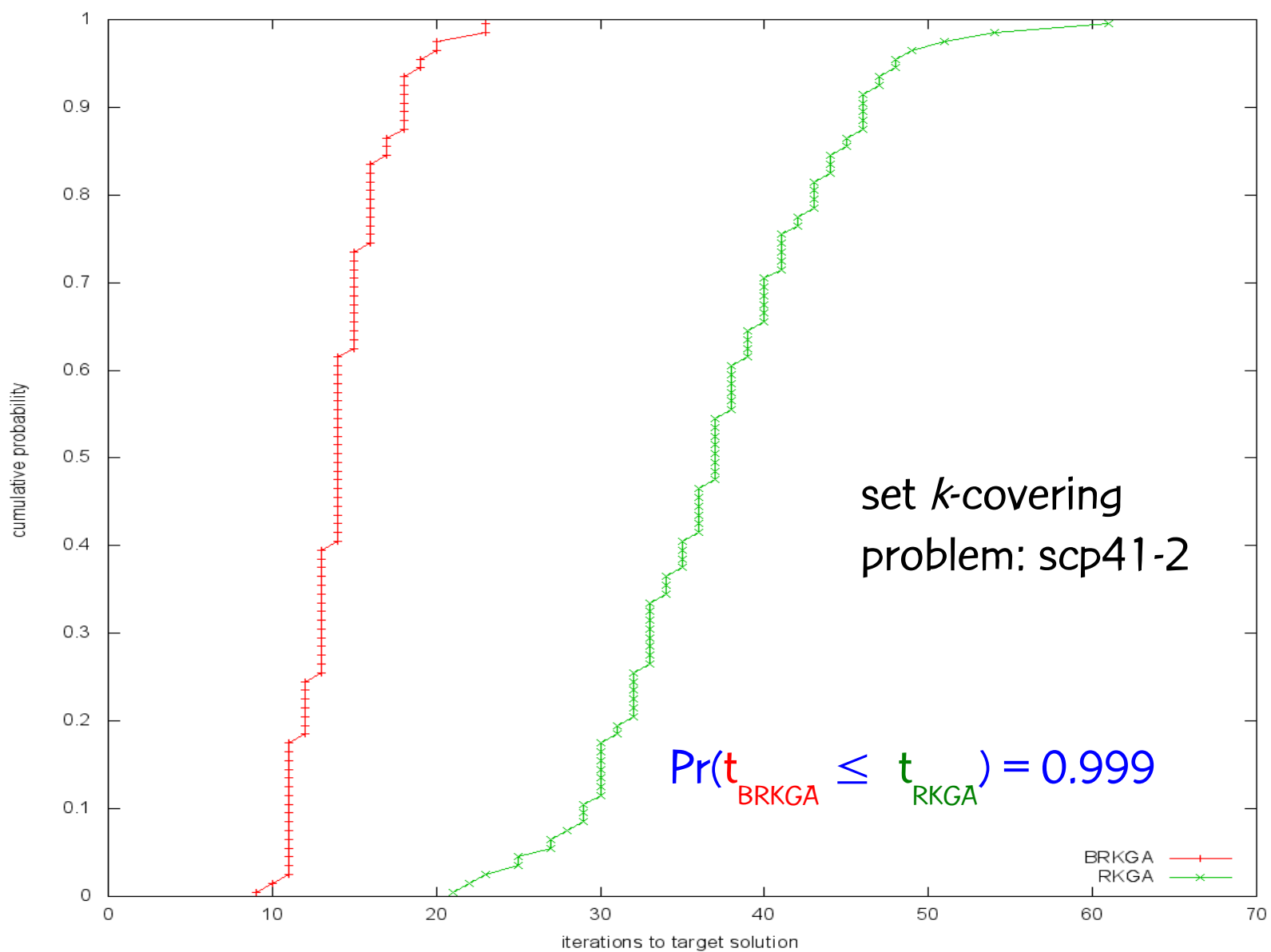


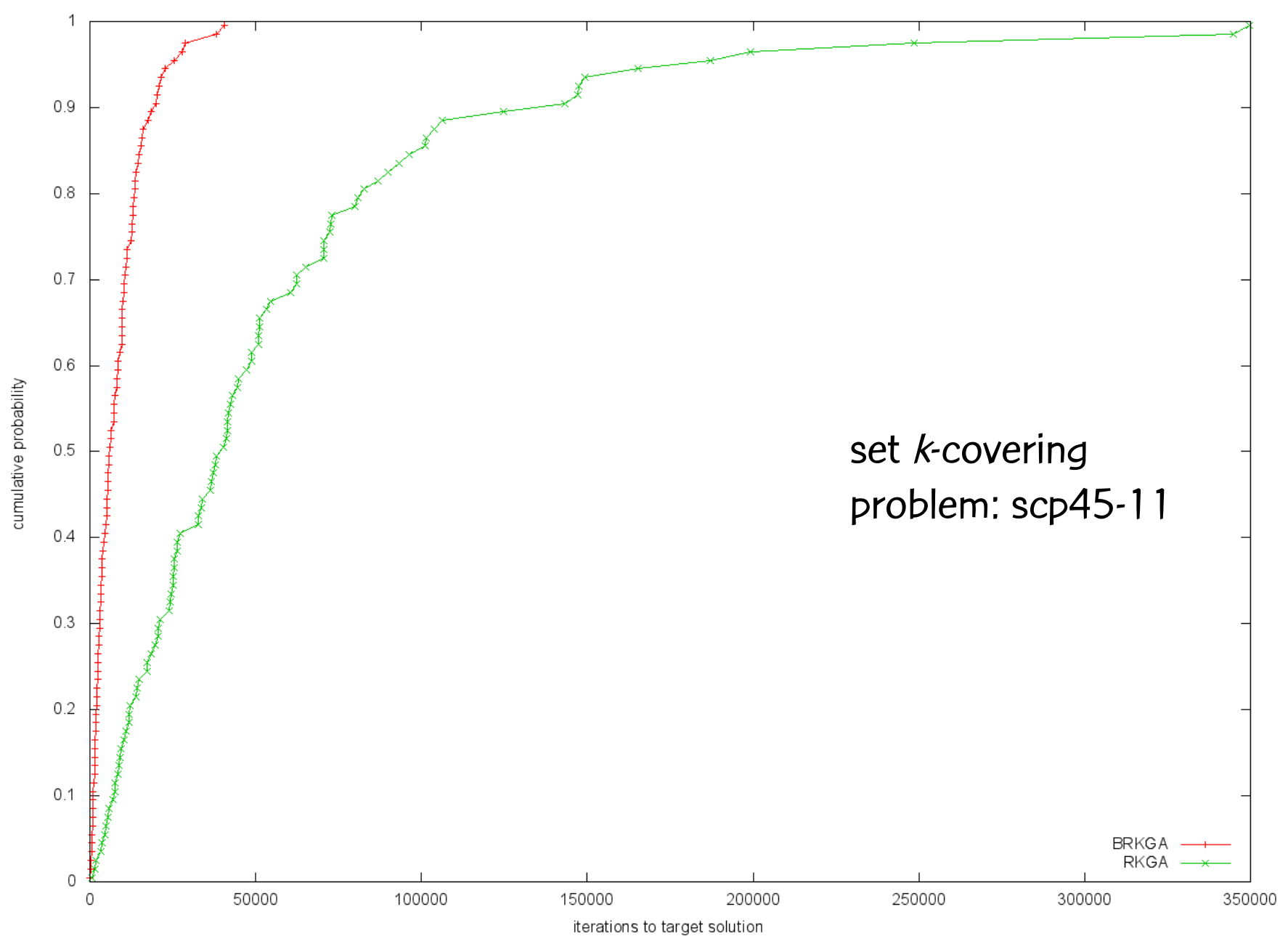


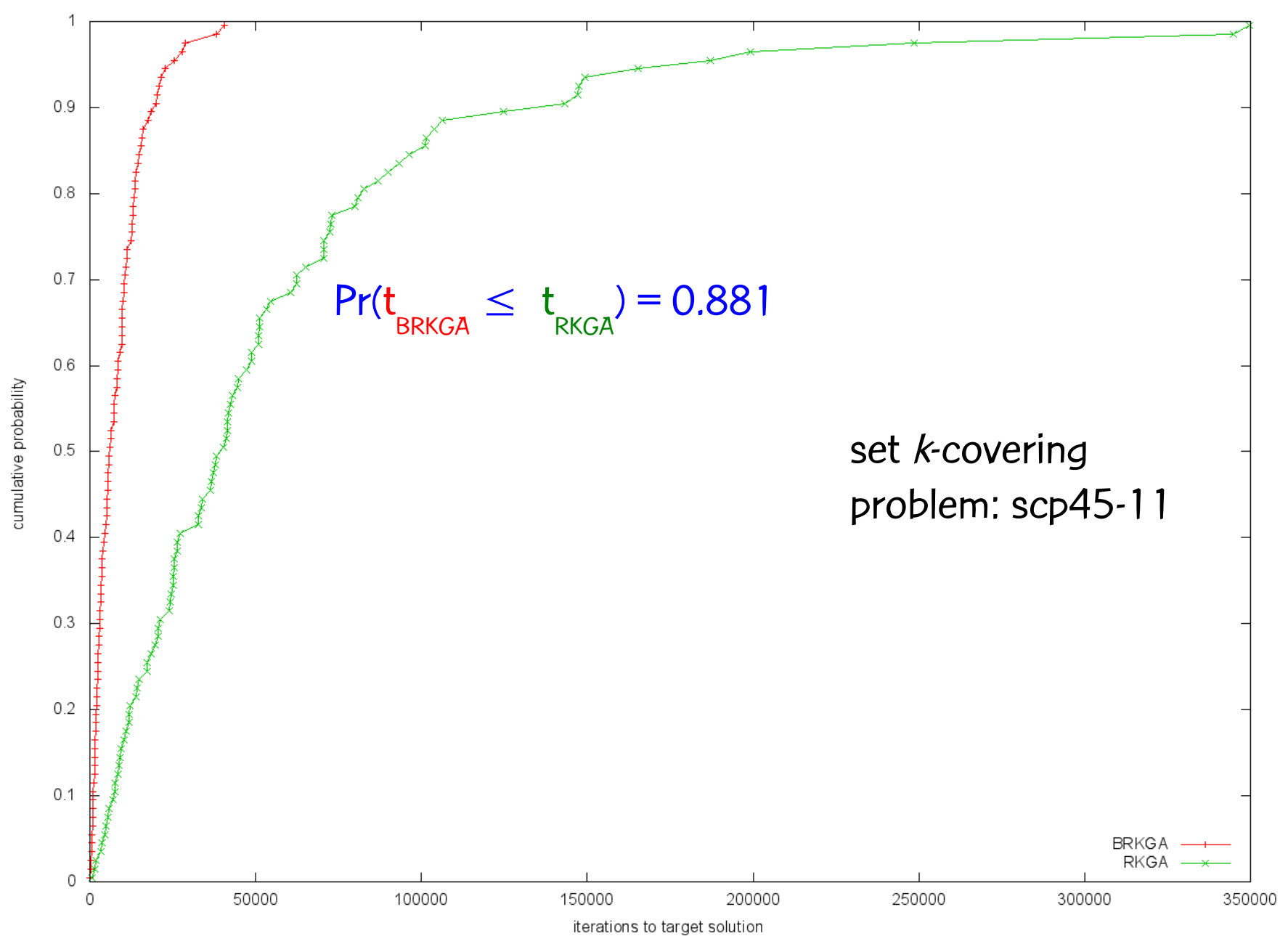


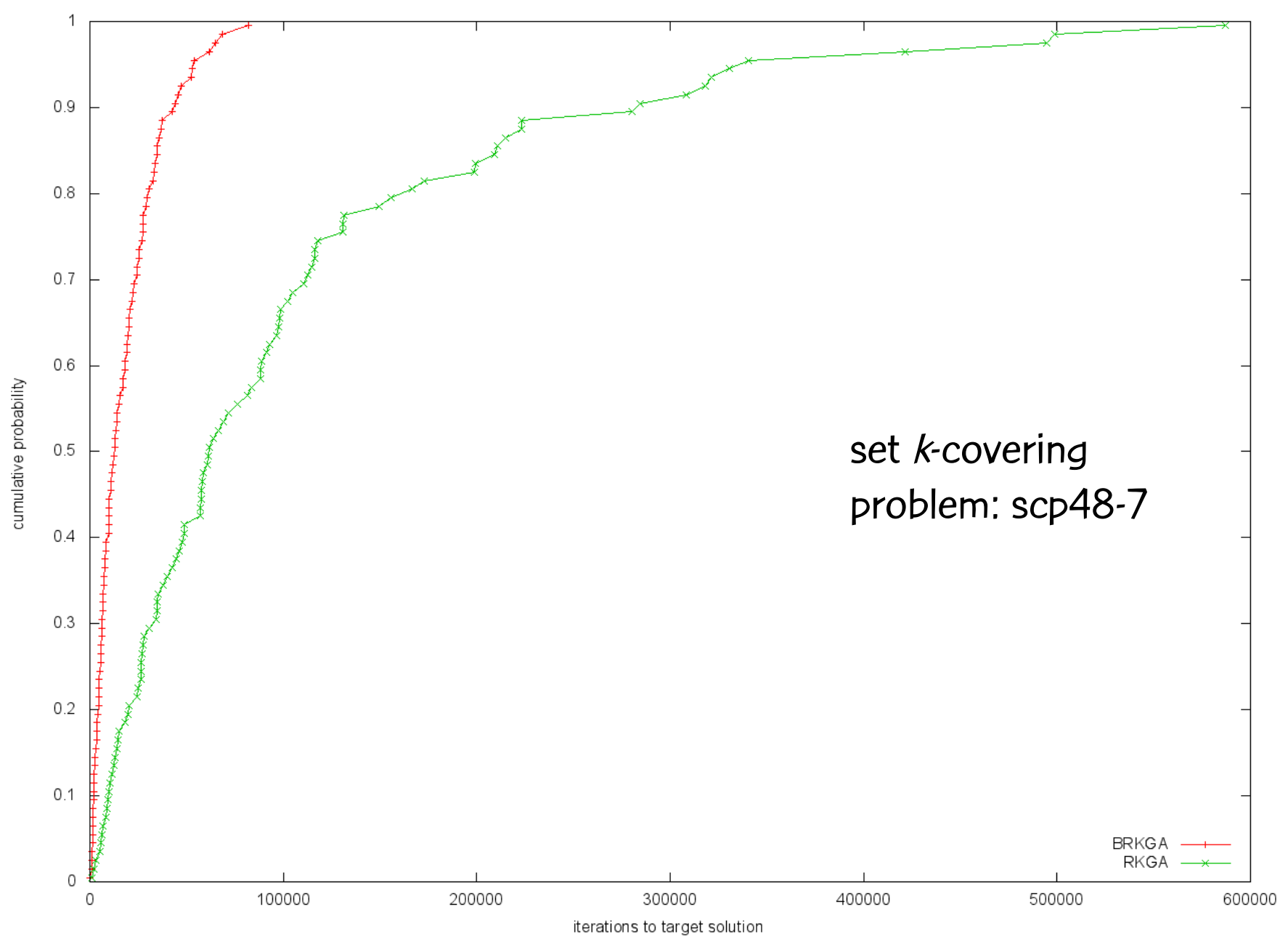


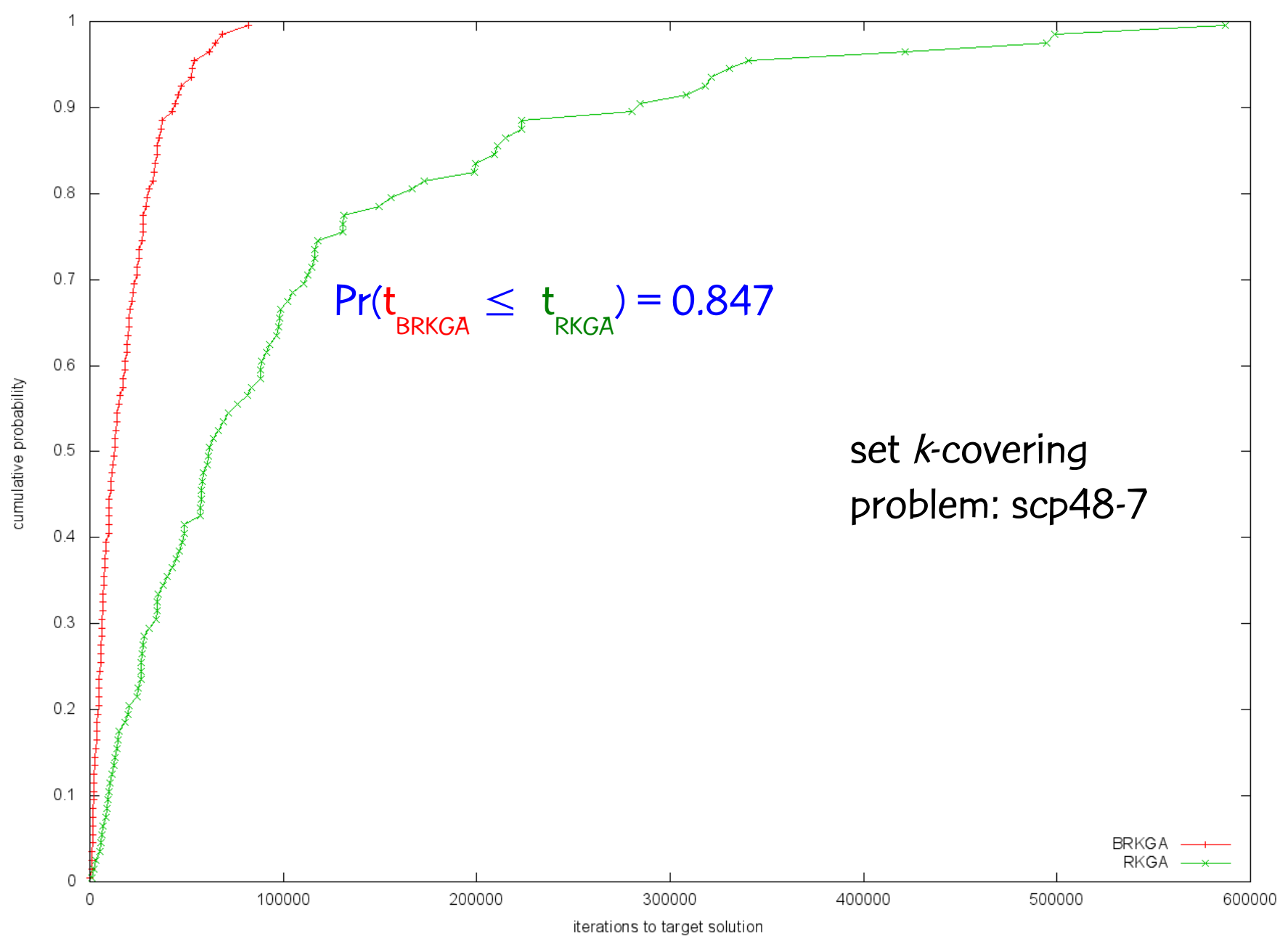












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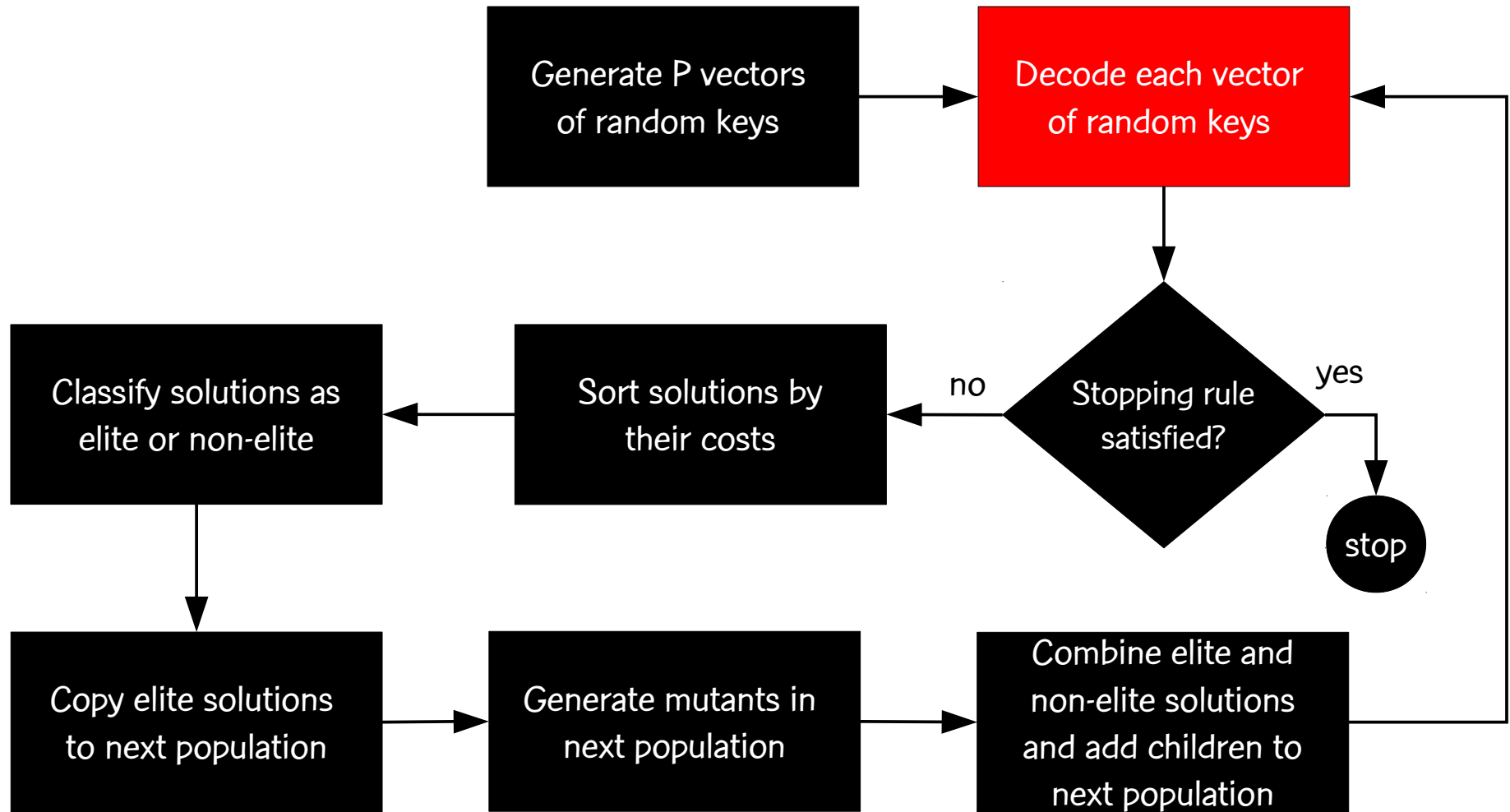
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- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent > 0.5 Not so in the RKGA of Bean.

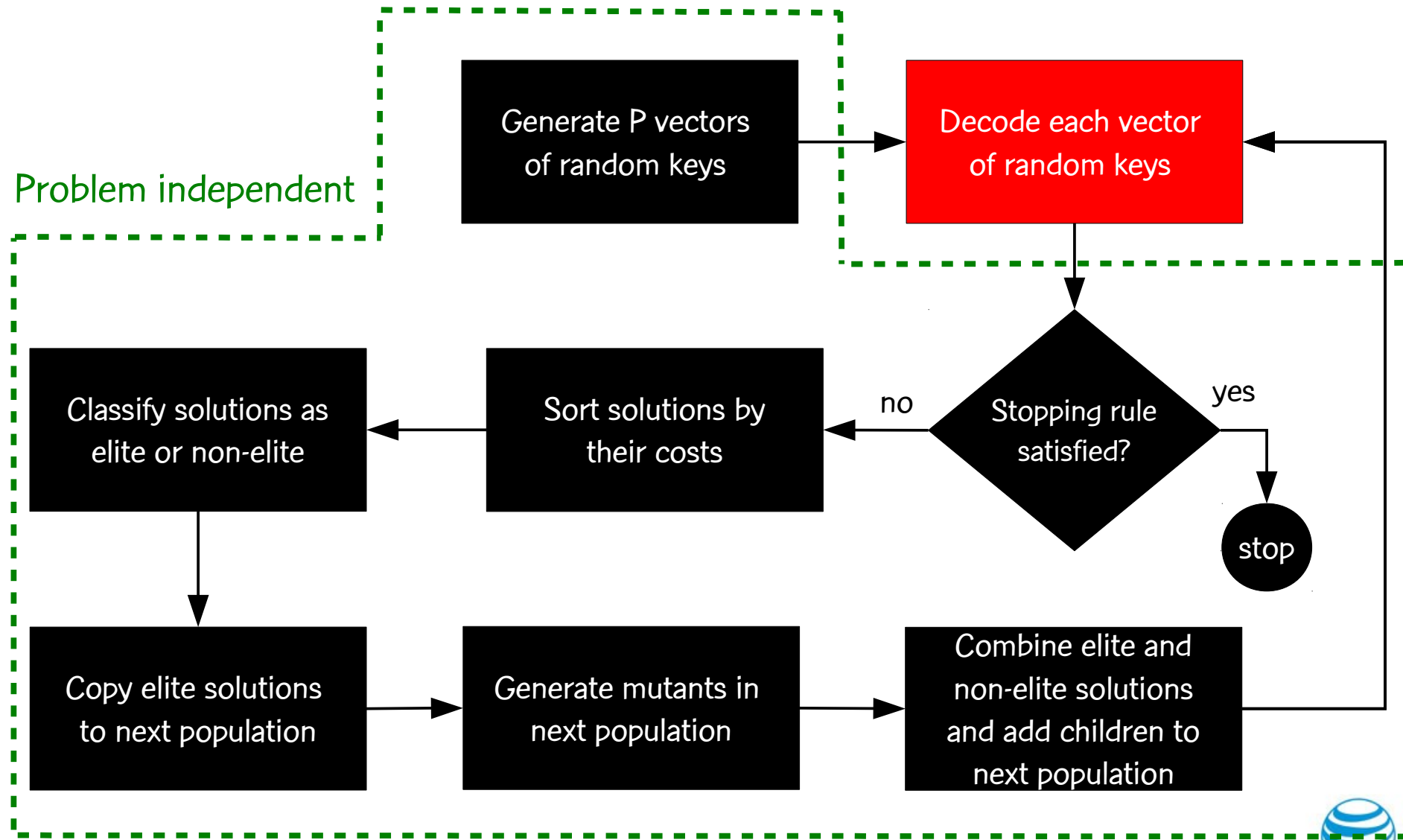
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- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs ... help escape local optima)

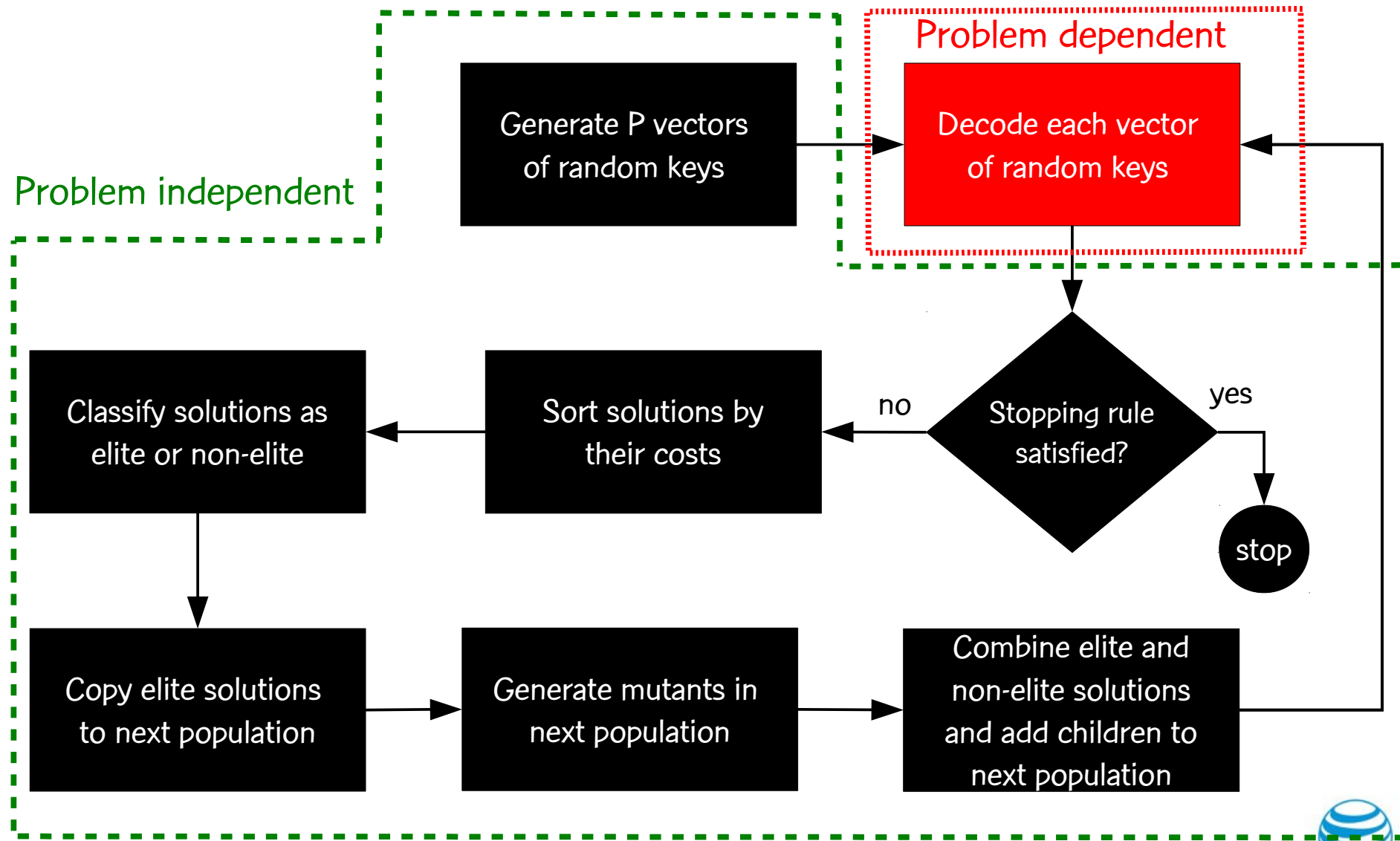
Framework for biased random-key genetic algorithms



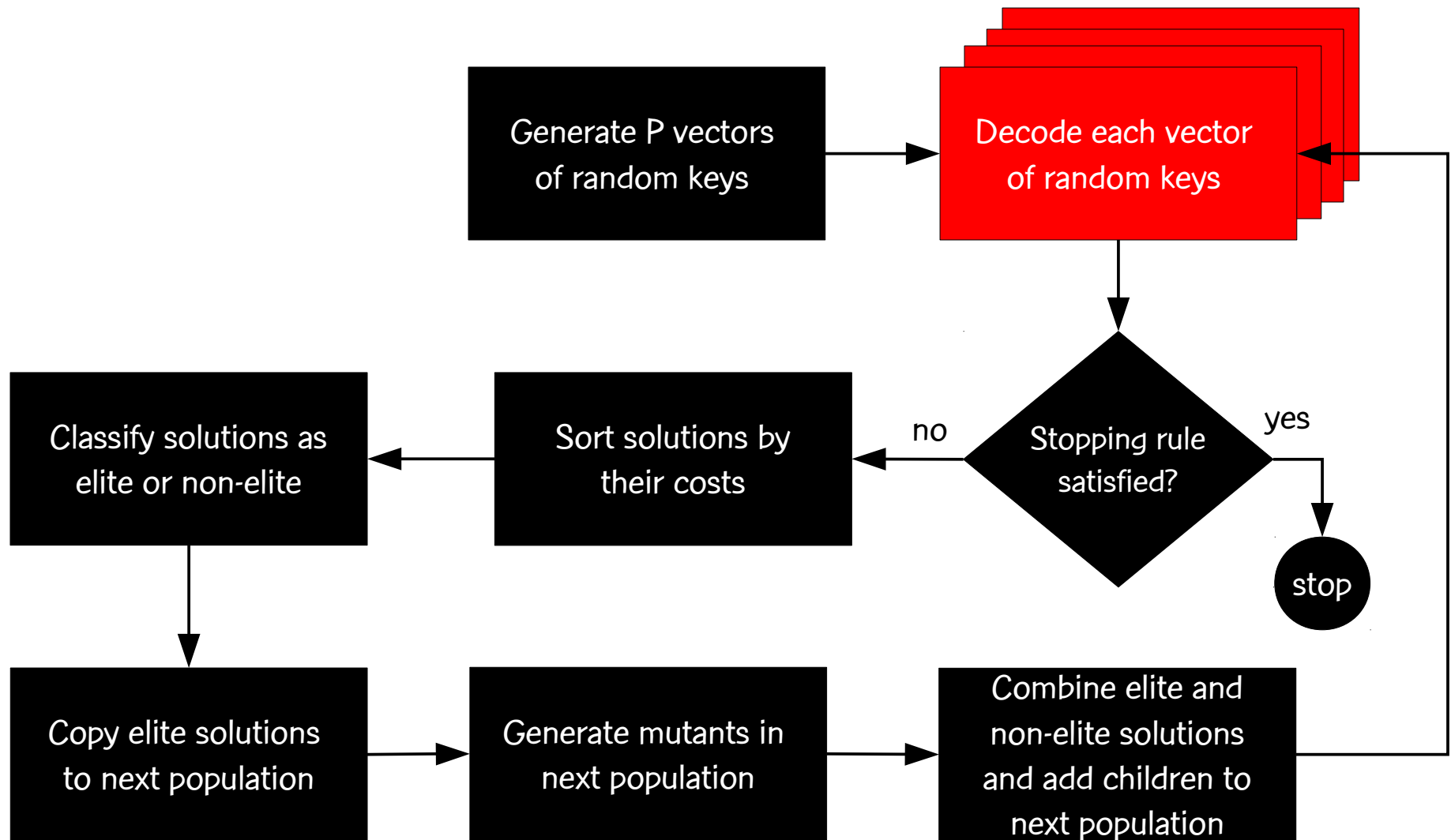
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Framework for biased random-key genetic algorithms



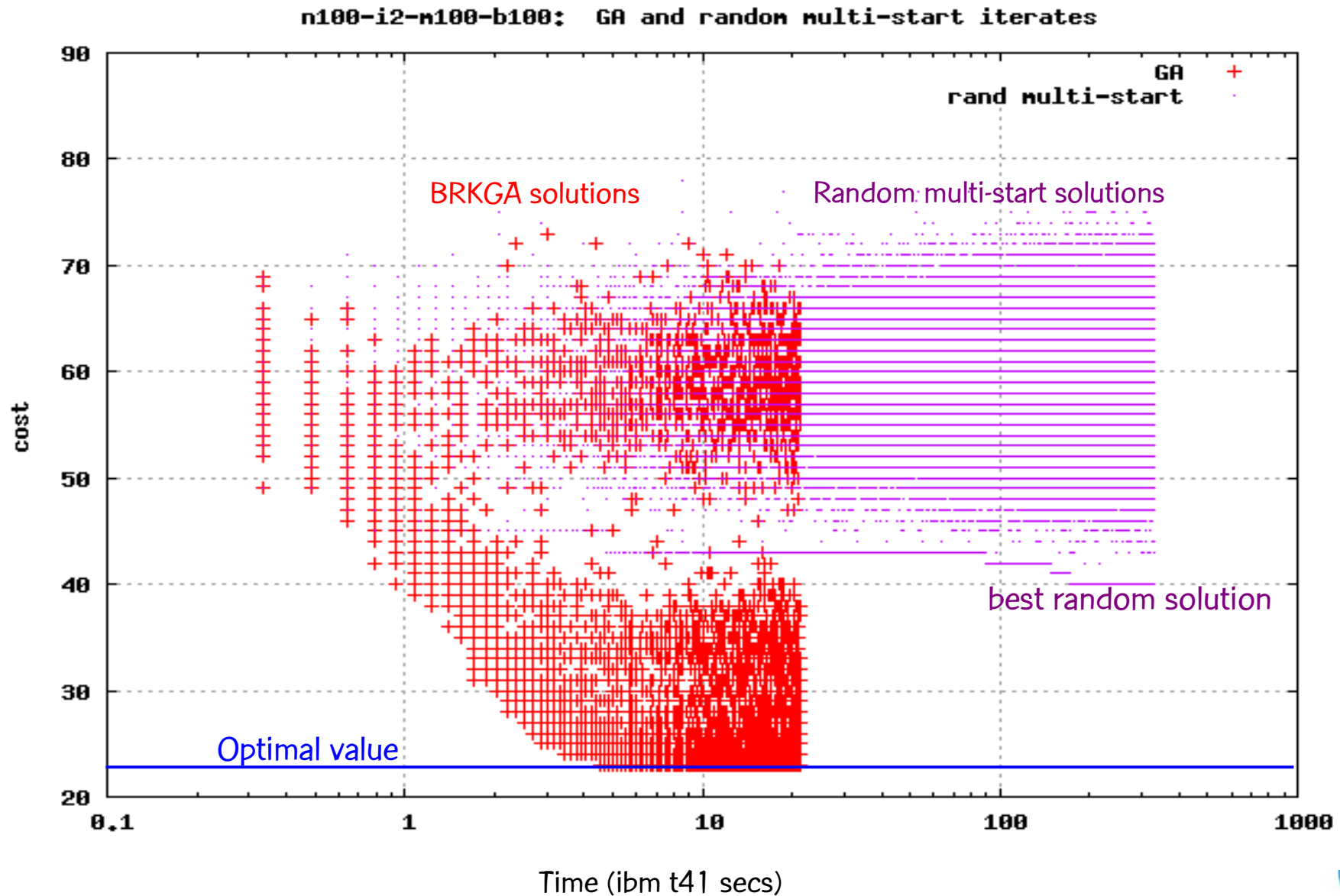
Decoding of random key vectors can be done in parallel



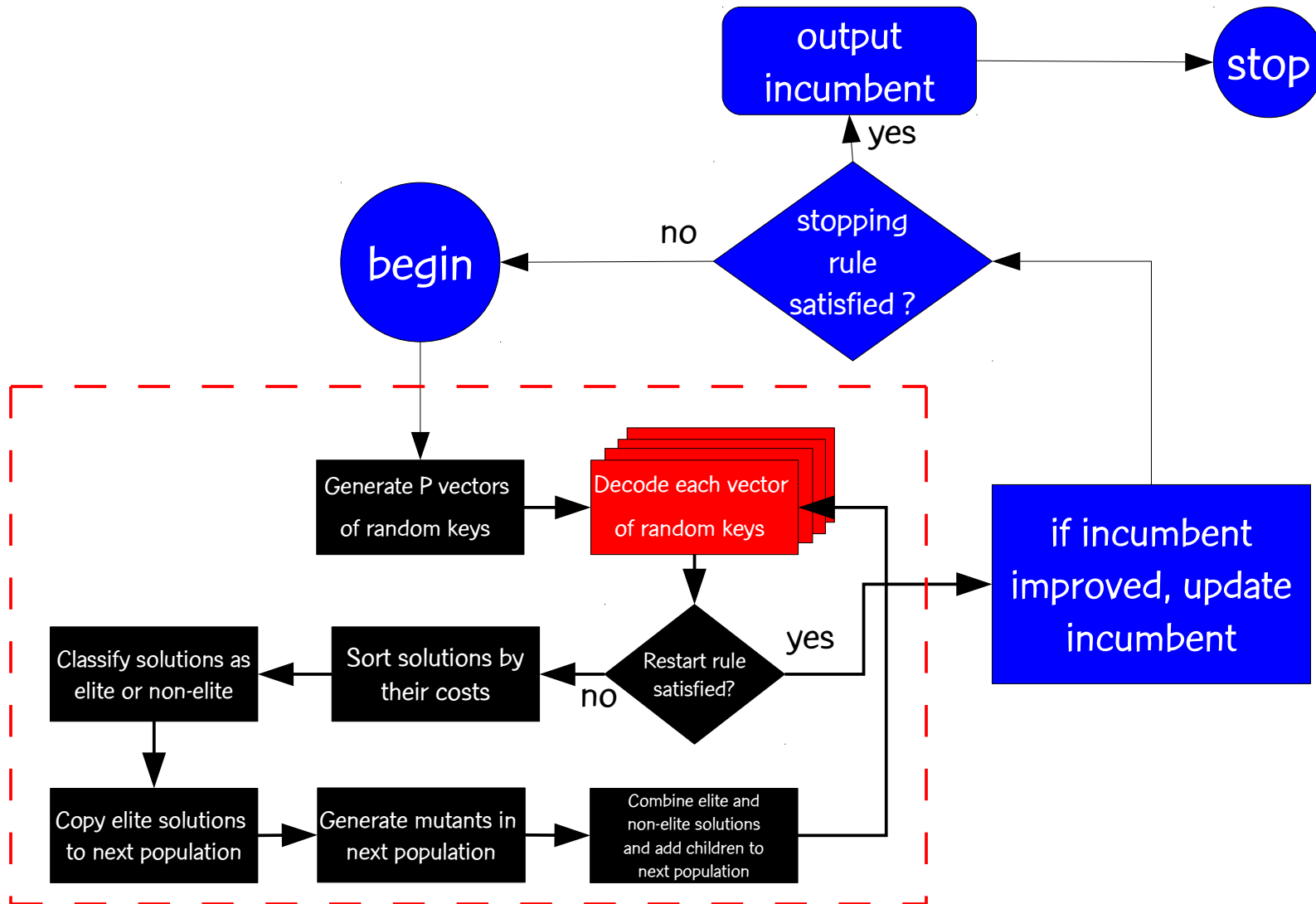
Is a BRKGA any different from applying the decoder to random keys?

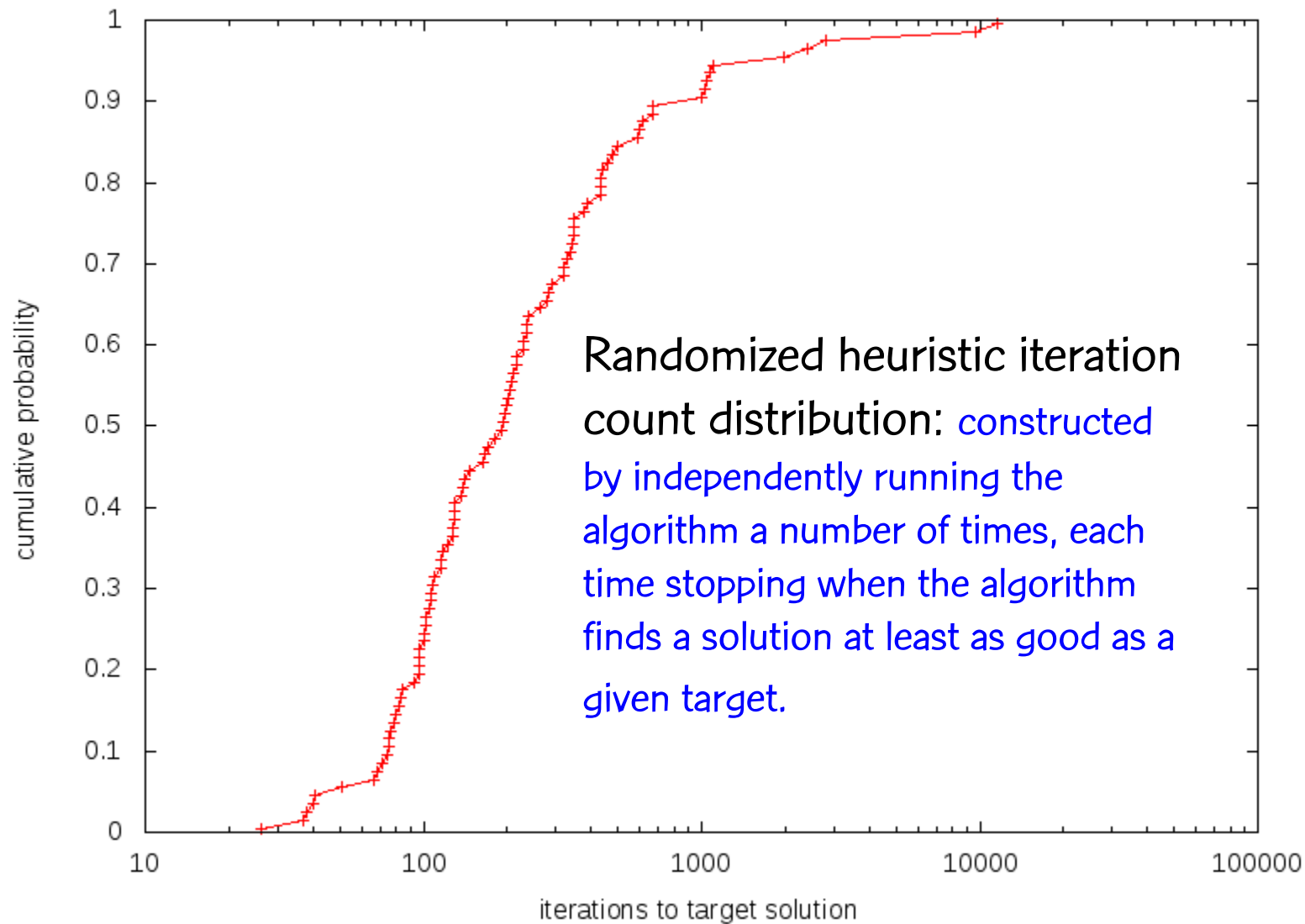
- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to $P-1$
- Each iteration, best solution is maintained in elite set and $P-1$ random key vectors are generated as mutants ... no mating is done since population already has P individuals

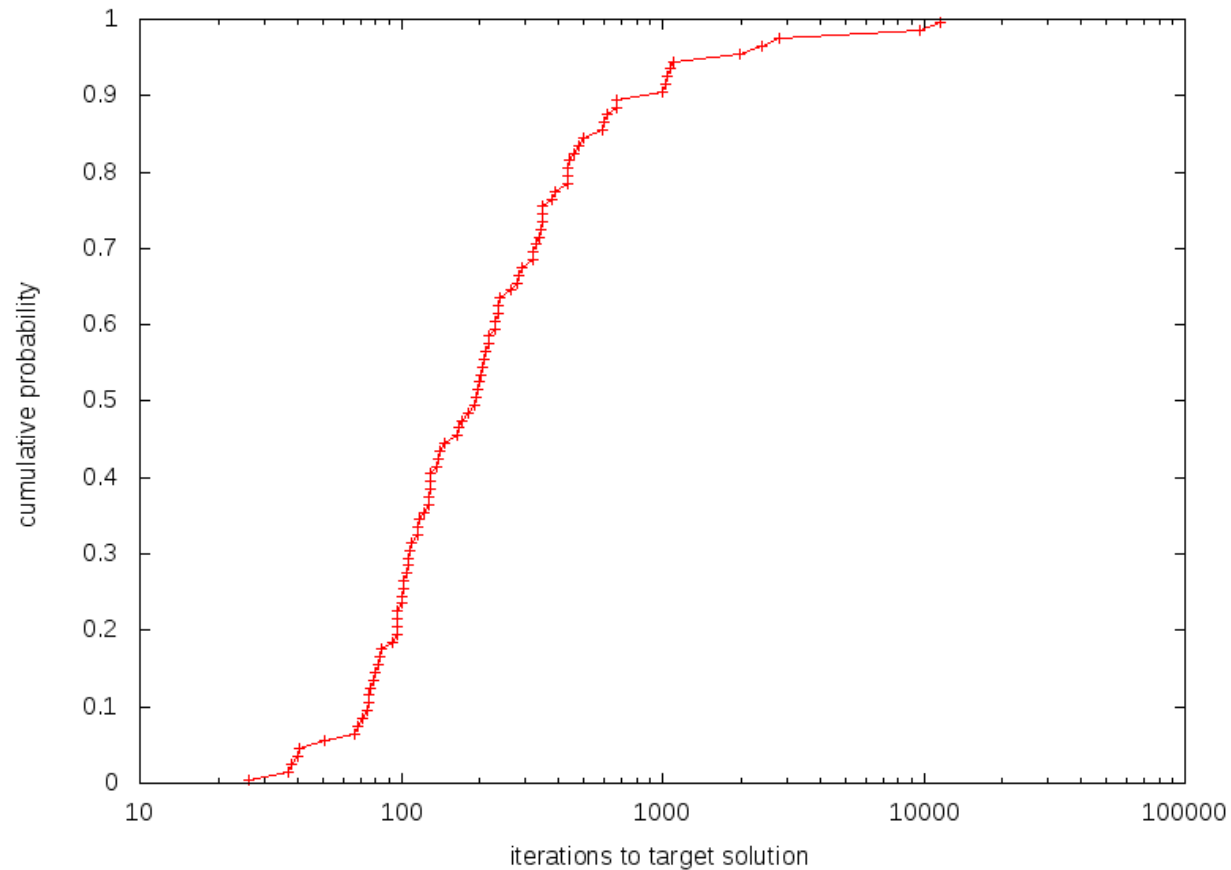
solution



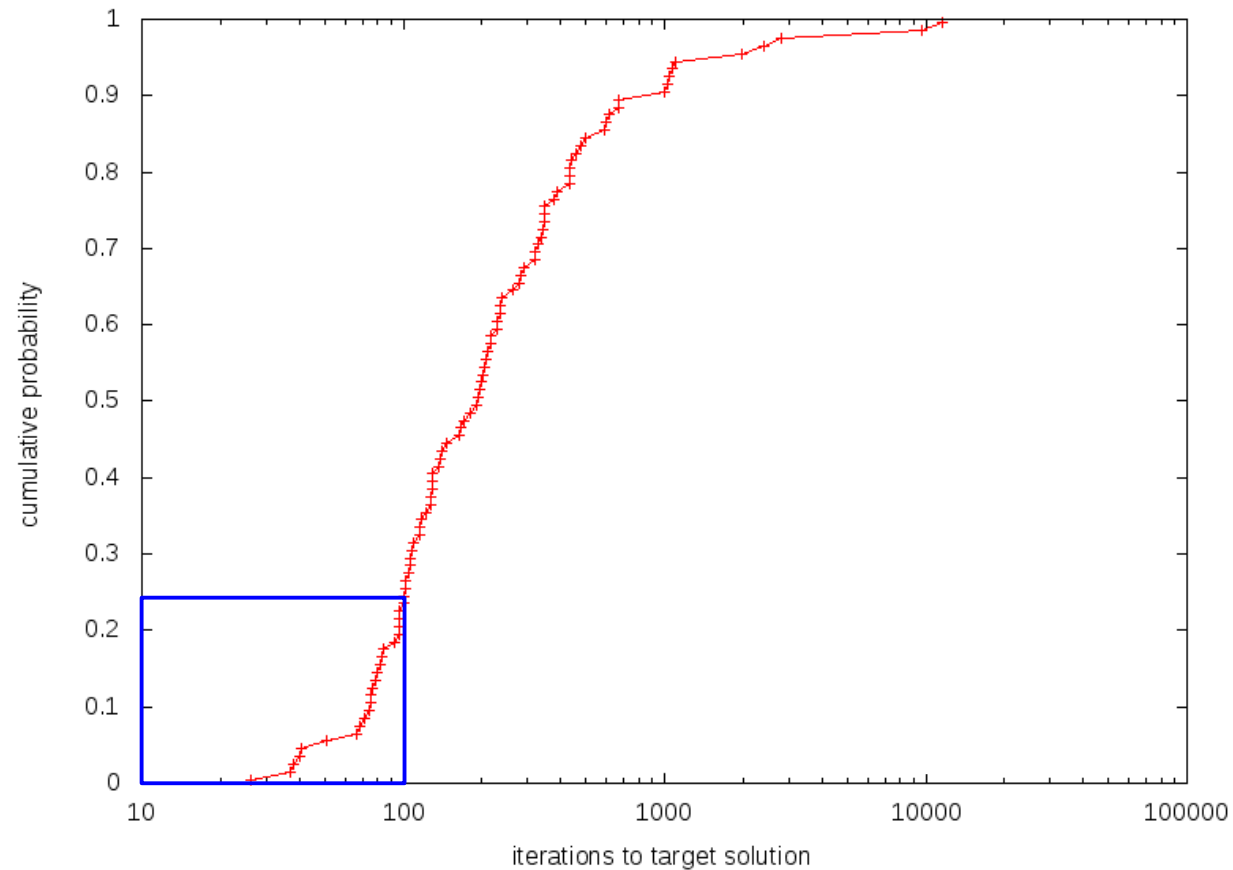
BRKGA in multi-start strategy



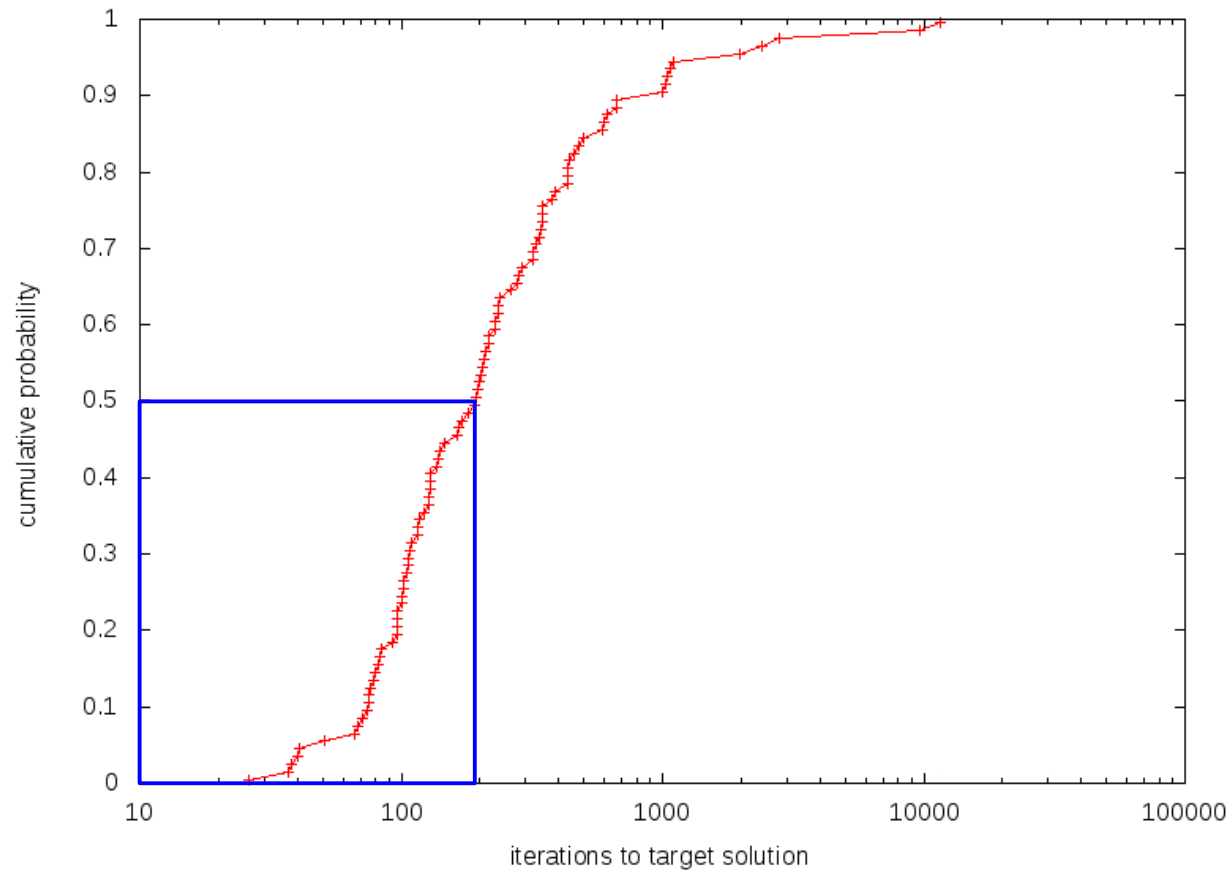




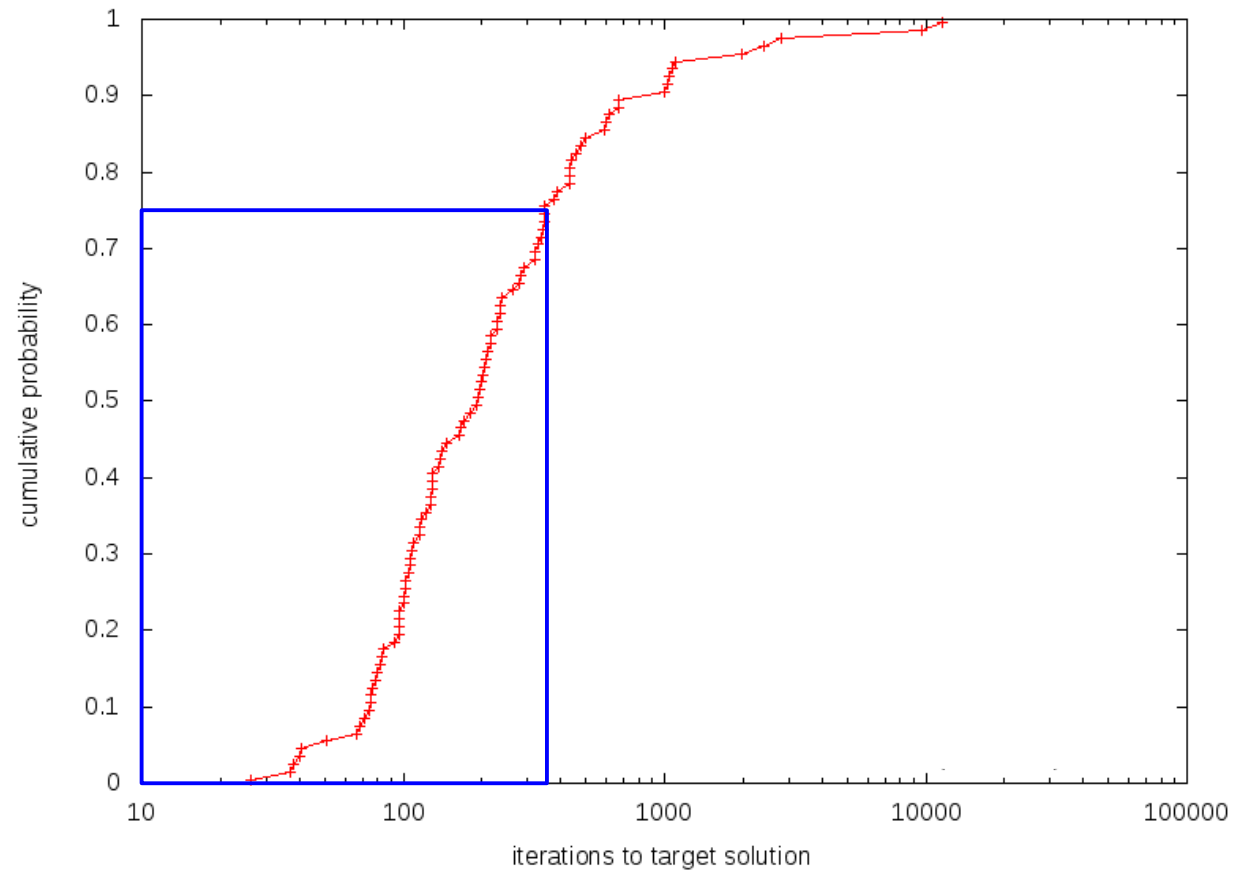
In most of the independent runs, the algorithm finds the target solution in relatively few iterations:



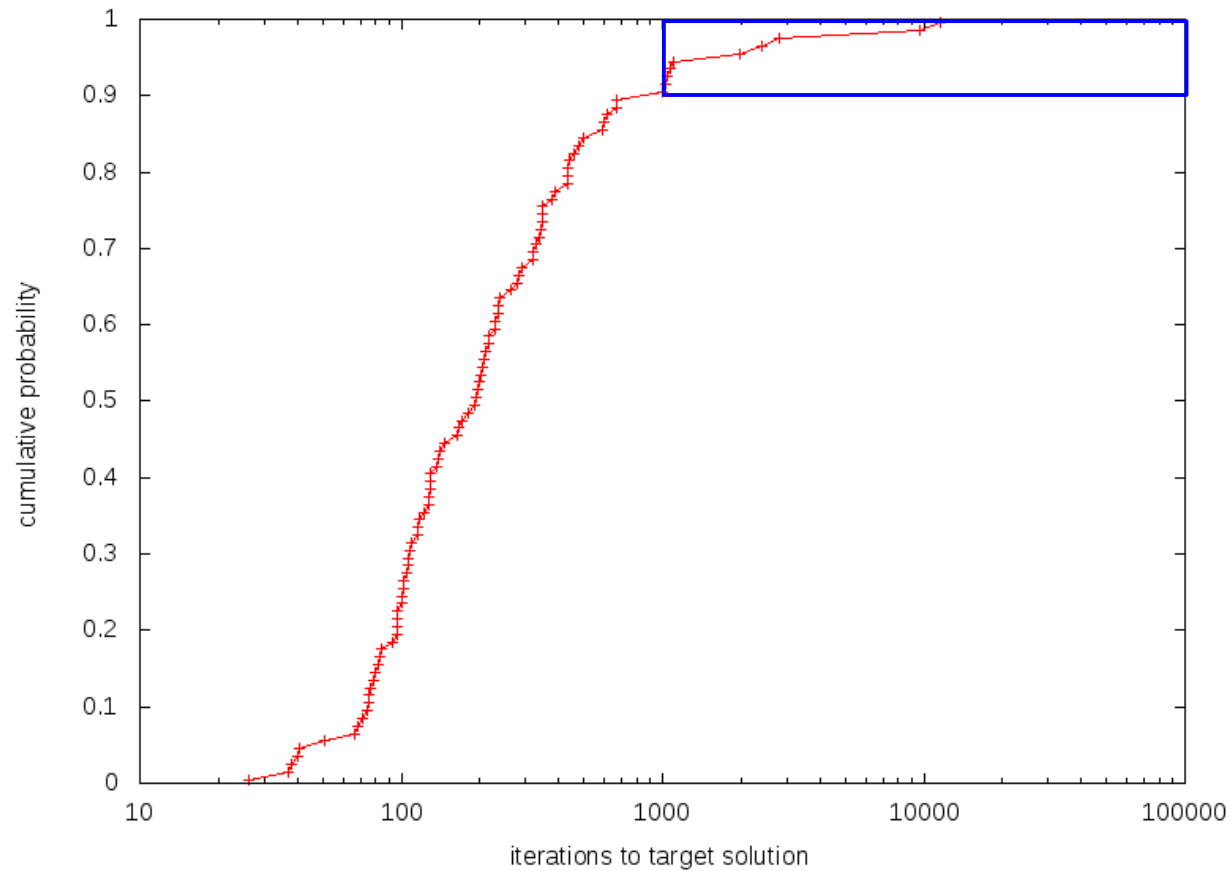
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 25% of the runs take fewer than 101 iterations



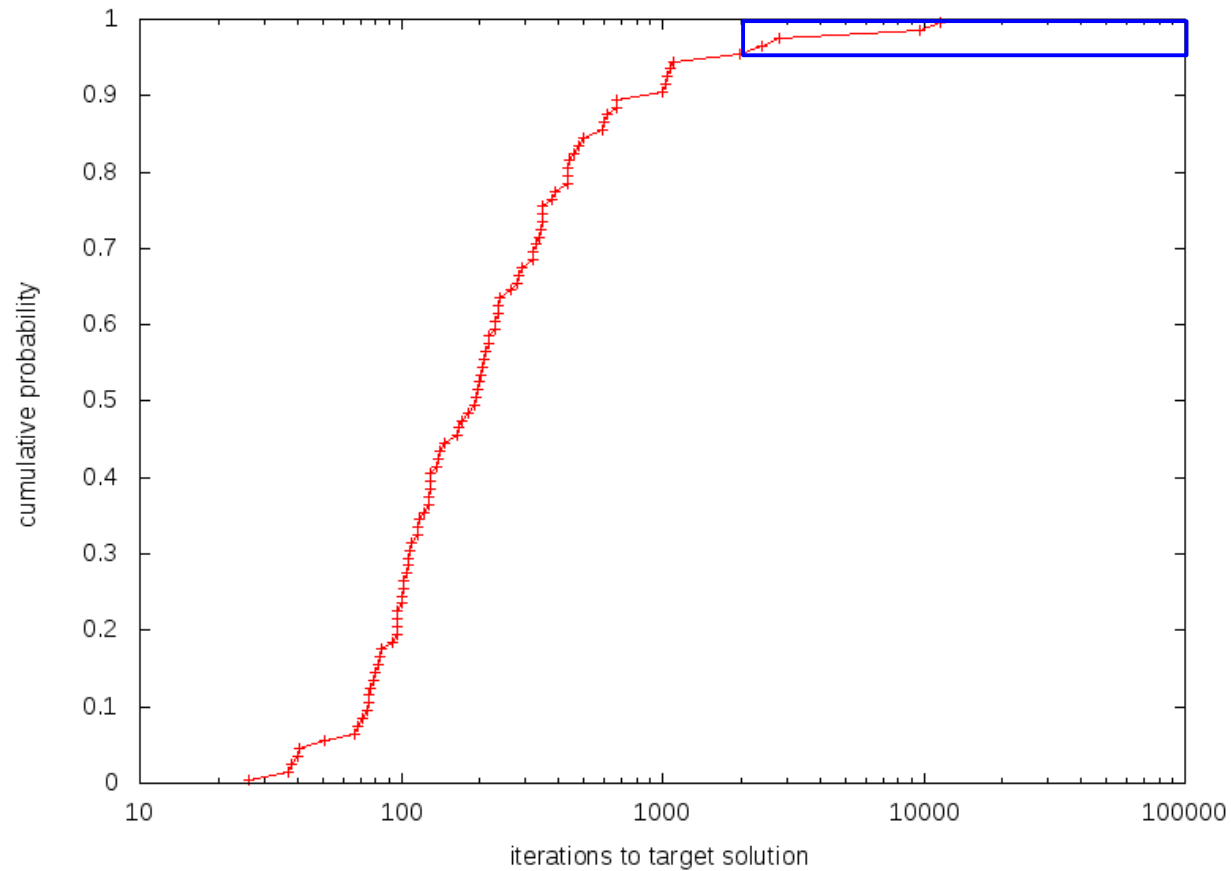
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 50% of the runs take fewer than 192 iterations



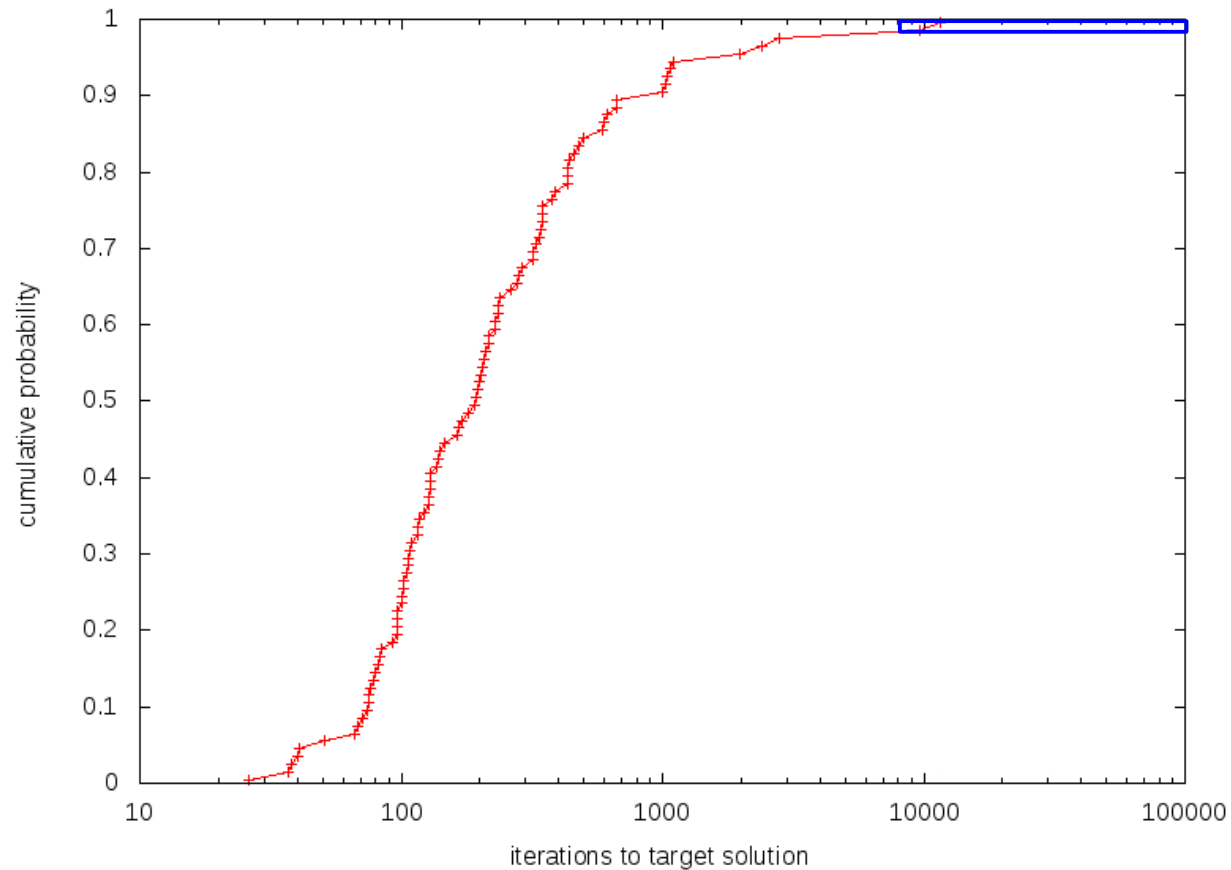
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 75% of the runs take fewer than 345 iterations



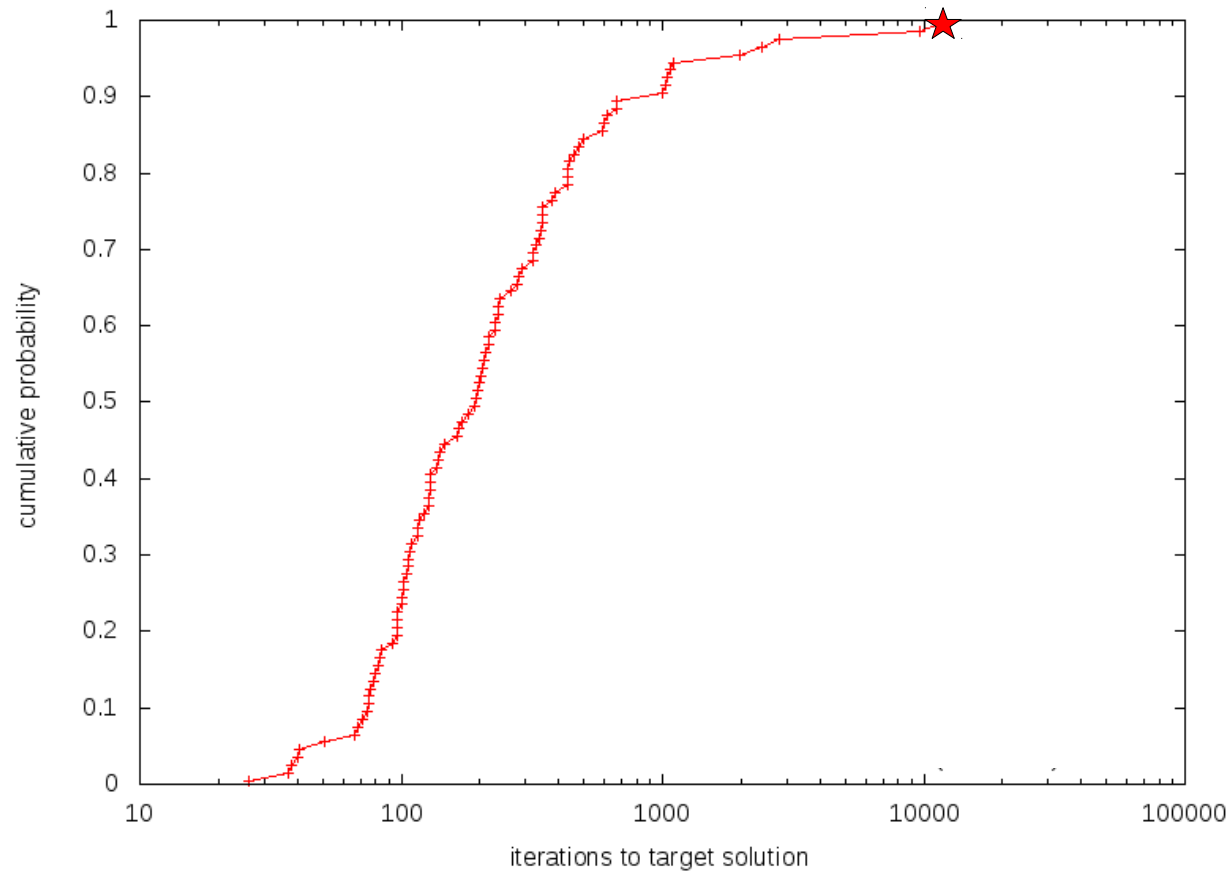
However, some runs take much longer: 10% of the runs take over 1000 iterations



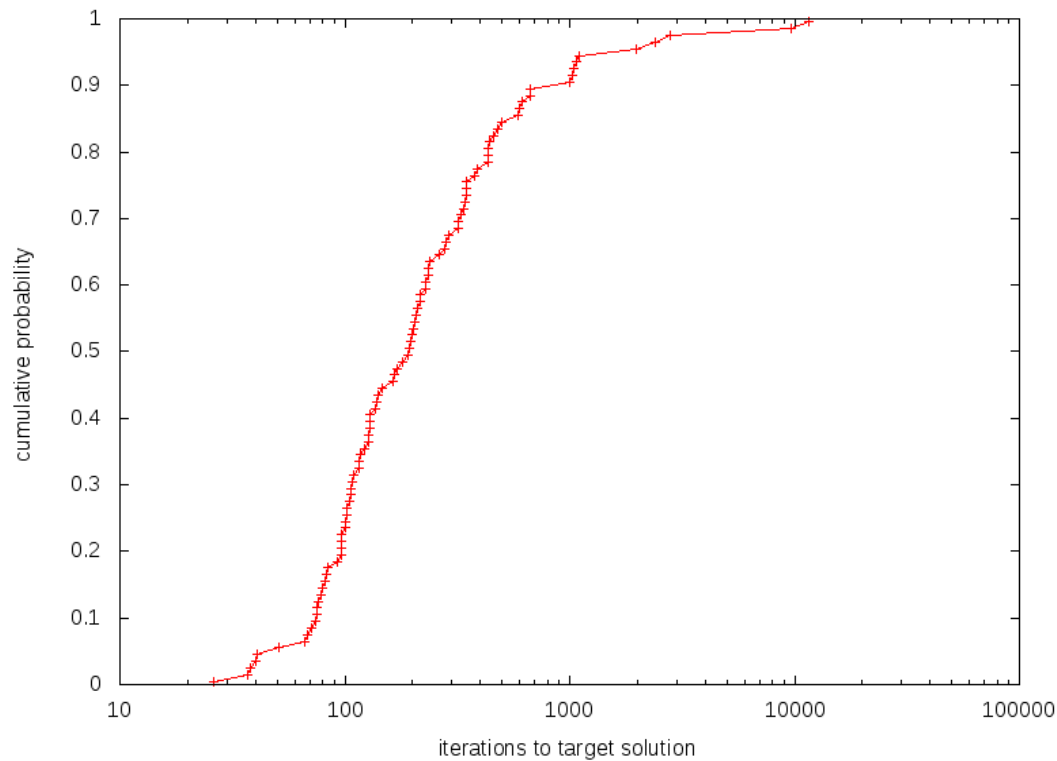
However, some runs take much longer: 5% of the runs take over 2000 iterations



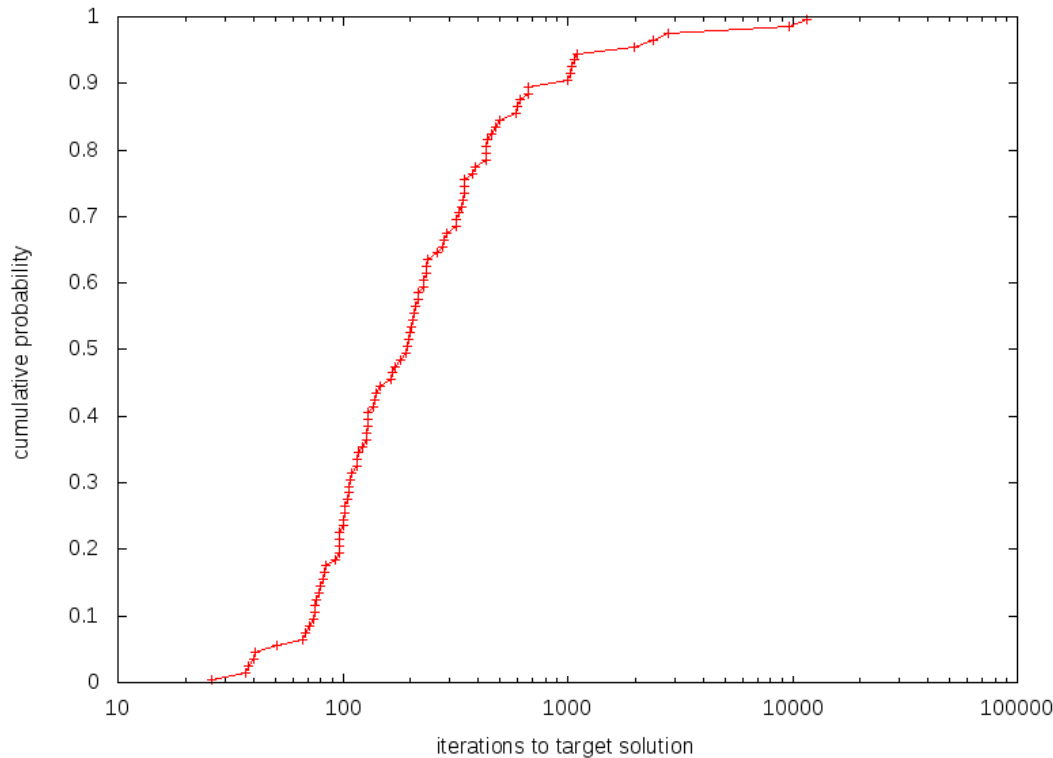
However, some runs take much longer: 2% of the runs take over 9715 iterations



However, some runs take much longer: the longest run took 11607 iterations



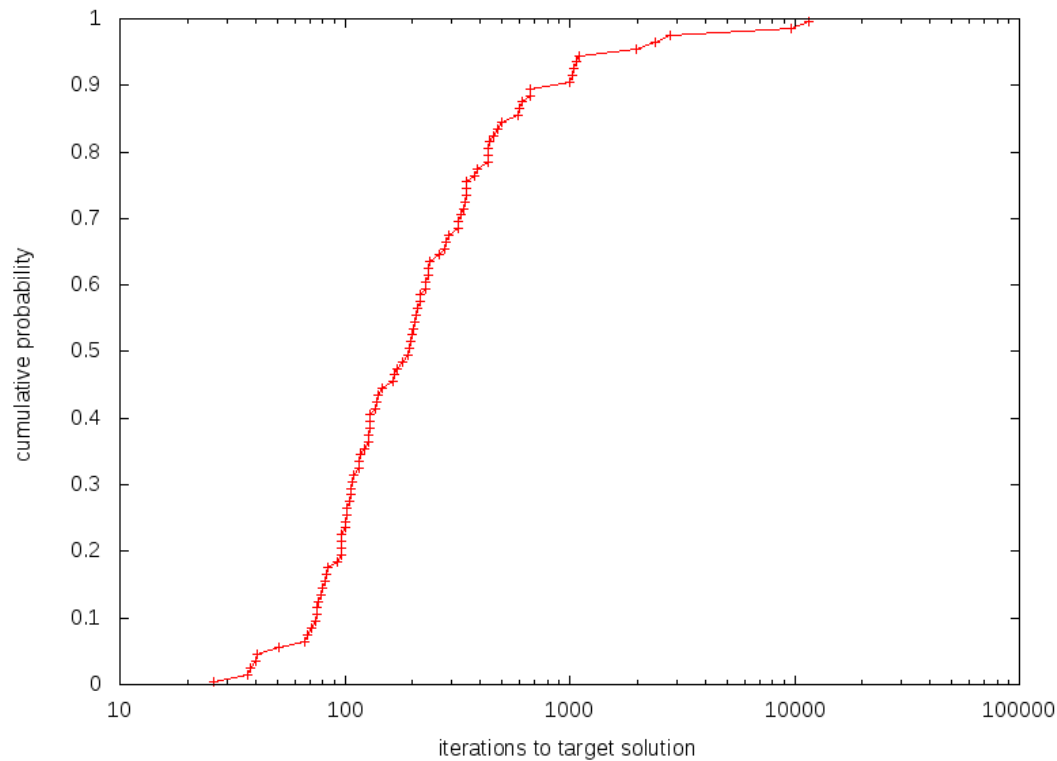
Probability that algorithm will take
over 345 iterations: $25\% = 1/4$



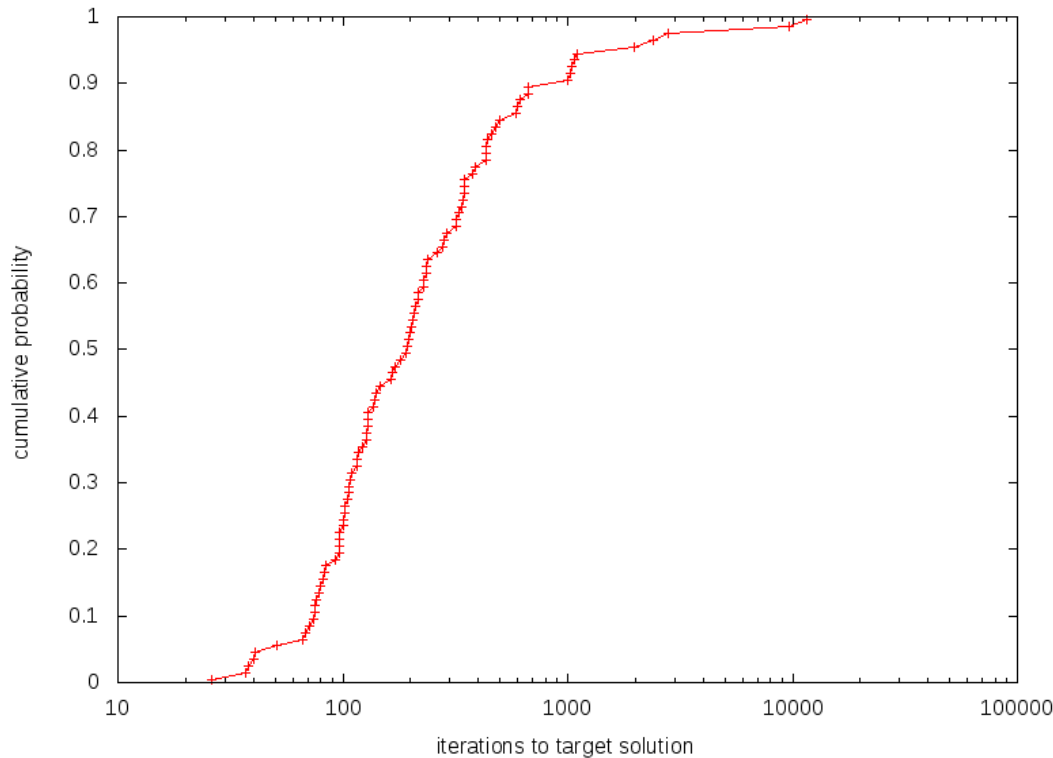
Probability that algorithm will take over 345 iterations: $25\% = 1/4$

By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: $25\% = 1/4$

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 \times probability of taking over 690 iterations given it took over 345 = $1/4 \times 1/4 = 1/4^2$

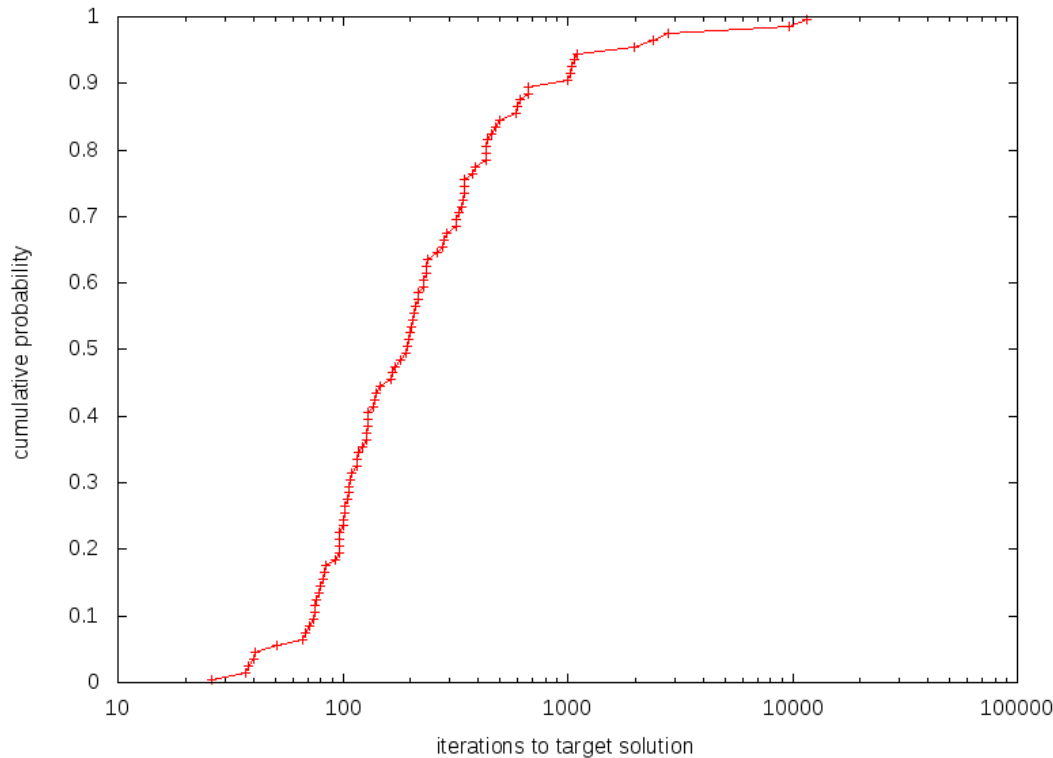


Probability that algorithm will still be
running after K periods of 345
iterations: $1/4^K$



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For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$



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For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$

This is much less than the 5% probability that the algorithm without restart will take over 2000 iterations.

Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S = \{\tau_1, \tau_2, \tau_3, \dots\}$ which define epochs $\tau_1, \tau_1 + \tau_2, \tau_1 + \tau_2 + \tau_3, \dots$ when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $\tau_1 = \tau_2 = \tau_3 = \dots = \tau^*$, where τ^* is a constant.

Restart strategy for BRKGA

- Recall the restart strategy of Luby et al. where equal time intervals $\tau_1 = \tau_2 = \tau_3 = \dots = \tau^*$ pass between restarts.
- Strategy requires τ^* as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
 - choosing τ^* too small: restart variant may take long to converge
 - choosing τ^* too big: restart variant may become like no-restart variant

Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy $\text{restart}(K)$

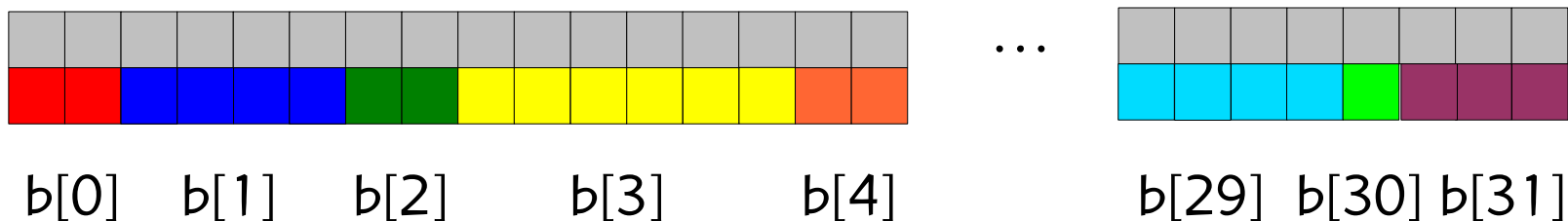
Example of restart strategy for BRKGA: Load balancing

Given an unordered sequence of 1024 integers $p[0], p[1], \dots, p[1023]$



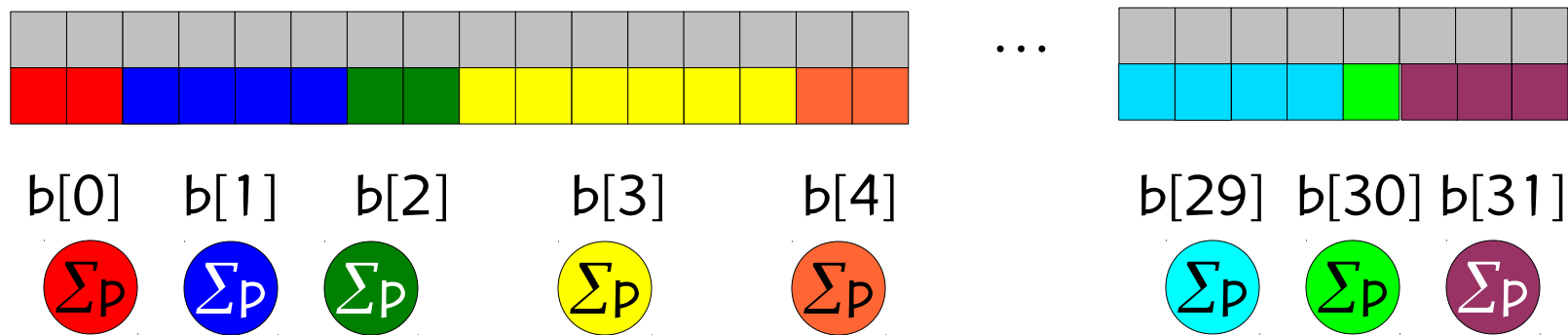
Example of restart strategy for BRKGA: Load balancing

Place consecutive numbers in 32 buckets $b[0]$, $b[1]$, ..., $b[31]$



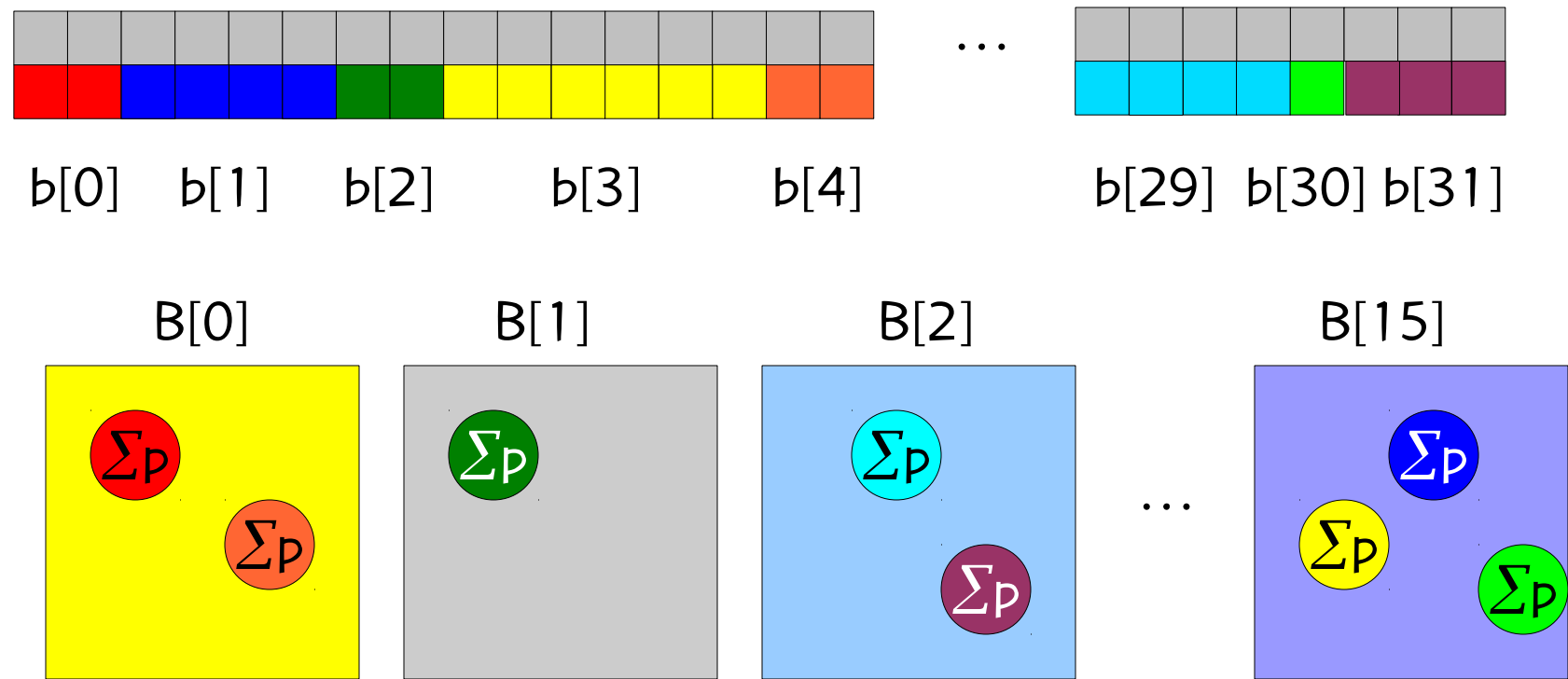
Example of restart strategy for BRKGA: Load balancing

Add the numbers in each bucket $b[0]$, $b[1]$, ..., $b[31]$



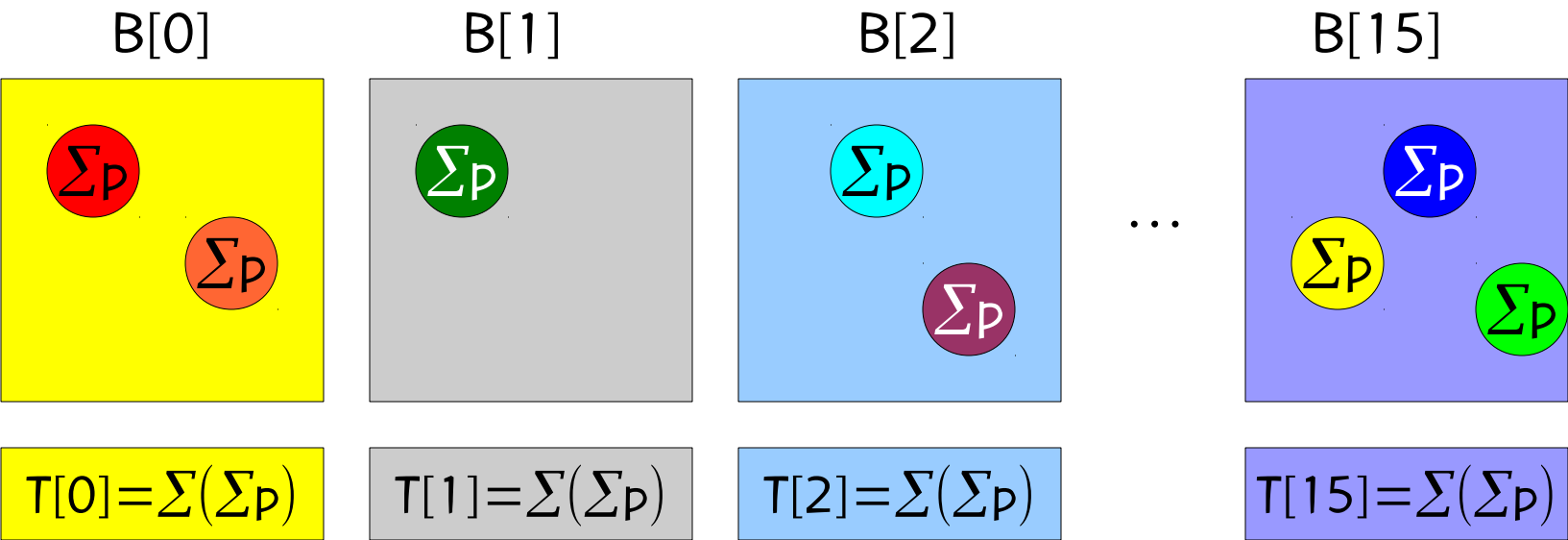
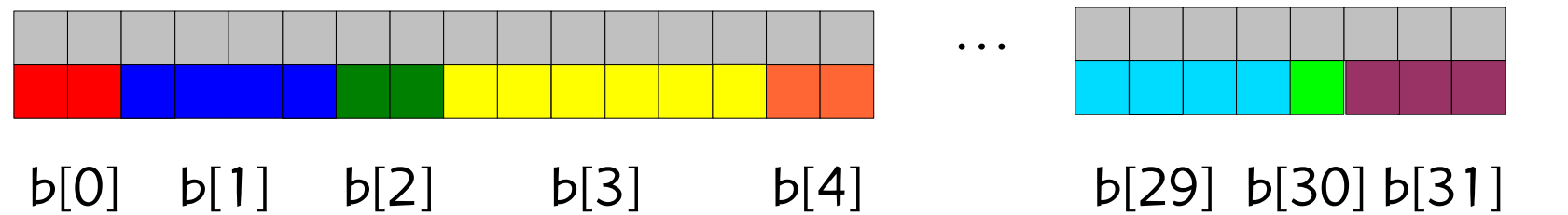
Example of restart strategy for BRKGA: Load balancing

Place the buckets in 16 bins $B[0], B[1], \dots, B[15]$



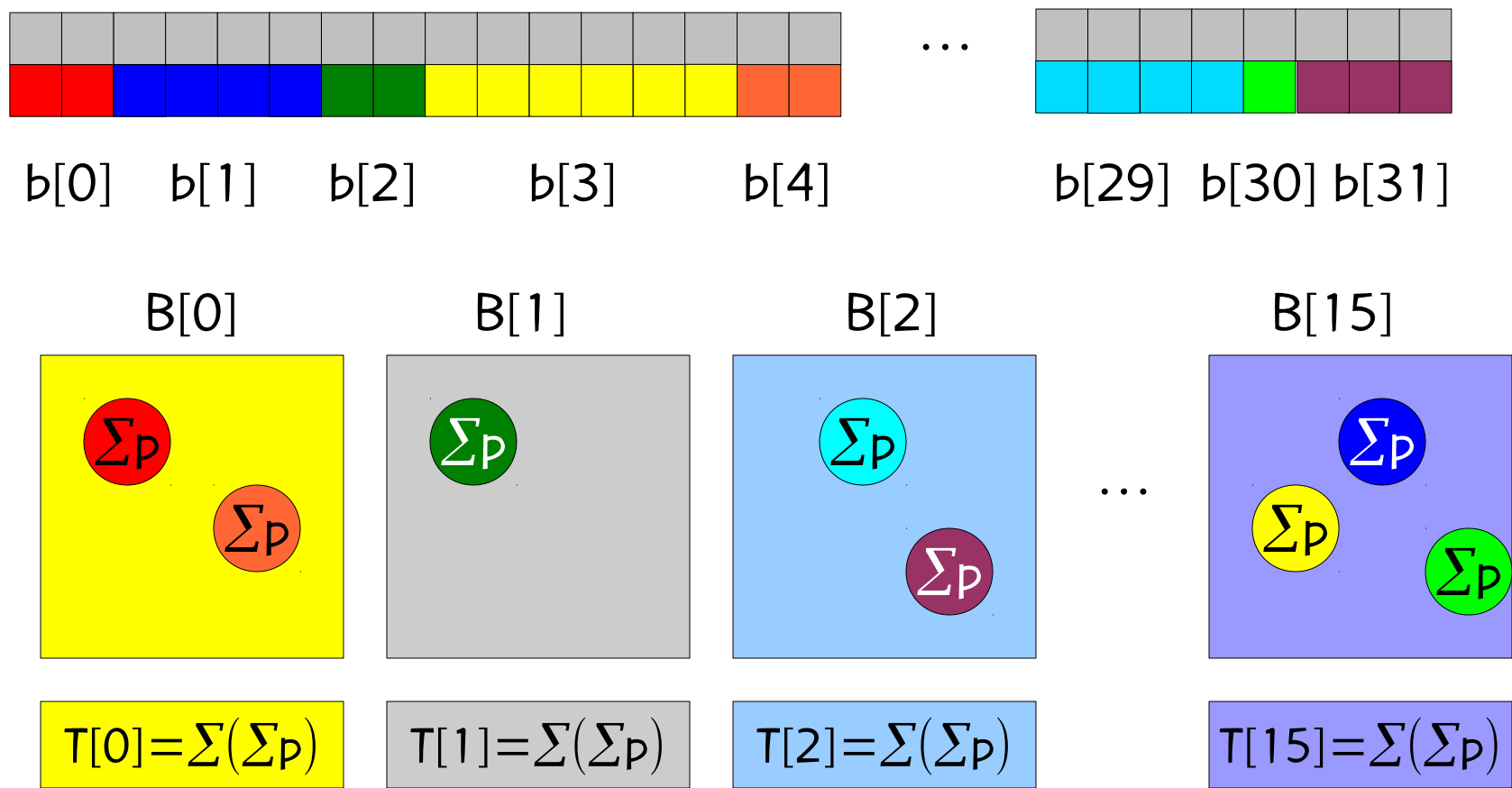
Example of restart strategy for BRKGA: Load balancing

Add up the numbers in each bin $B[0], B[1], \dots, B[15]$

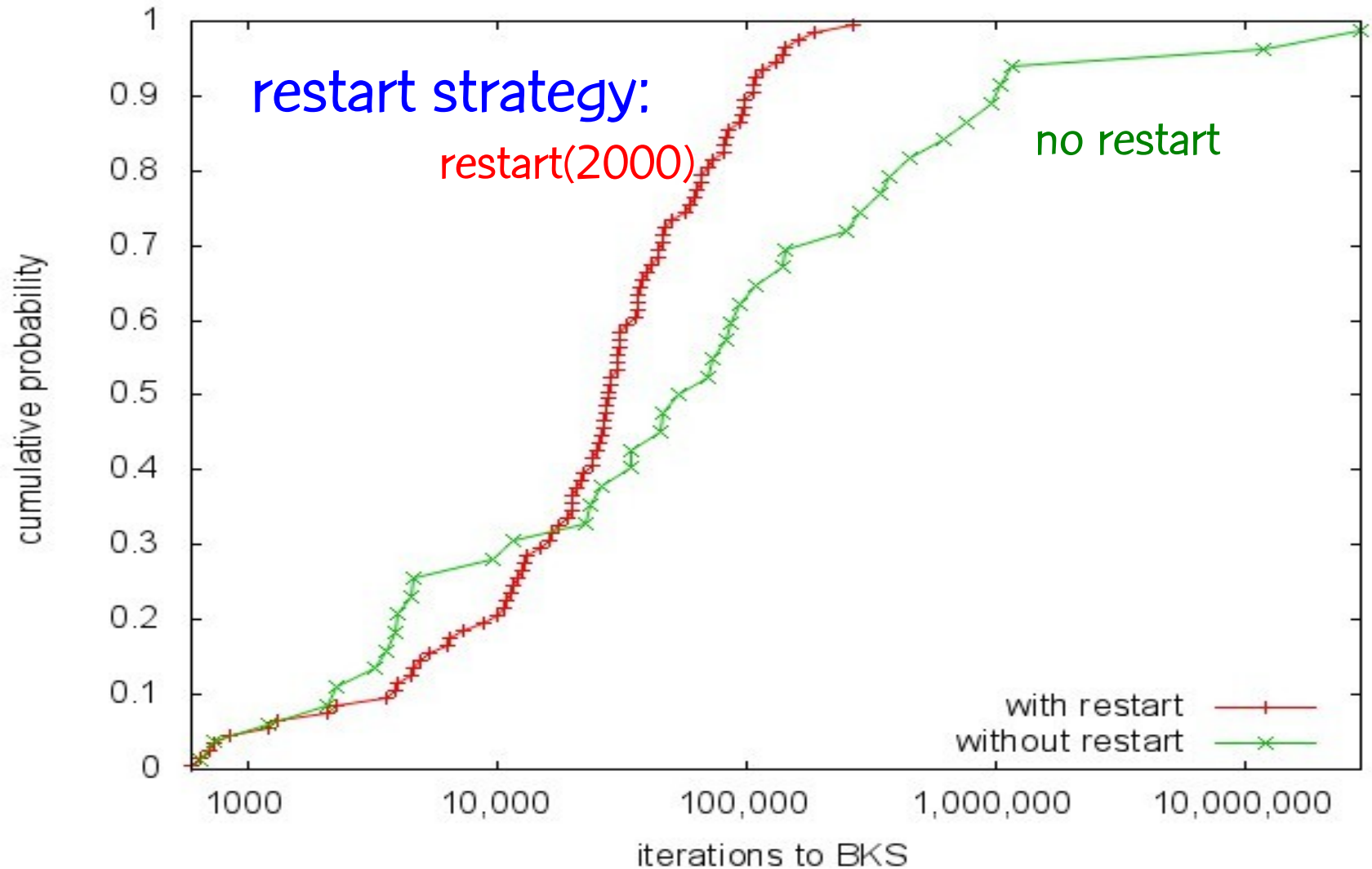


Example of restart strategy for BRKGA: Load balancing

OBJECTIVE: Minimize { Maximum (T[0], T[1], ..., T[15]) }



Example of restart strategy for BRKGA: Load balancing



Specifying a BRKGA

Specifying a biased random-key GA

- Encoding is always done the same way, i.e. with a vector of N random-keys (parameter N must be specified)

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- Parameters

Specifying a biased random-key GA

Parameters:

- Size of population
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

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- Size of population: a function of N , say N or $2N$
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Specifying a biased random-key GA

Parameters:

- Size of population: a function of N , say N or $2N$
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- Stopping criterion: e.g. time, # generations, solution quality, # generations without improvement

brkgaAPI: A C++ API for BRKGA

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.

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- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.
- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
 - population management
 - evolutionary dynamics

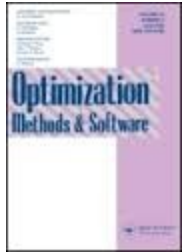
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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
 - population management
 - evolutionary dynamics
- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.

brkgaAPI: A C++ API for BRKGA



Paper: Rodrigo F. Toso and M.G.C.R.,

“A C++ Application Programming Interface
for Biased Random-Key Genetic Algorithms,”

Optimization Methods & Software, published online 13 March 2014.

Software: <http://www.research.att.com/~mgcr/src/brkgaAPI>

An example BRKGA: Packing weighted rectangles

Reference



J.F. Gonçalves and M.G.C.R., “A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal packing problem,” *Journal of Combinatorial Optimization*, vol. 22, pp. 180-201, 2011.

Tech report:

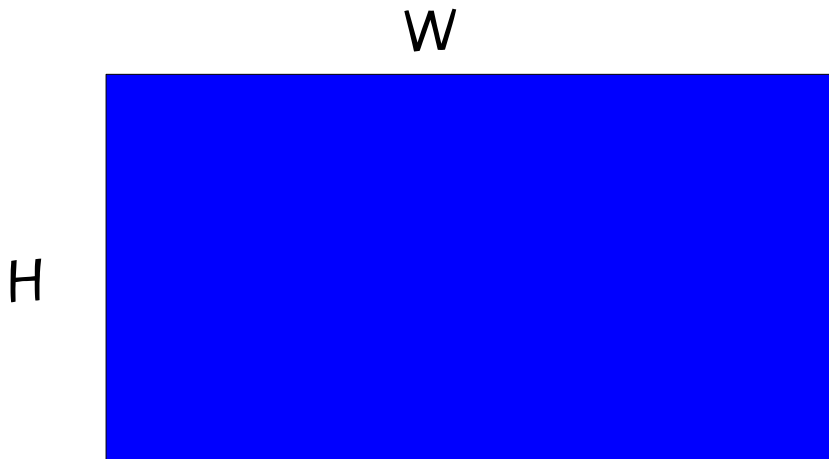
<http://www.research.att.com/~mgcr/doc/pack2d.pdf>

Constrained orthogonal packing

- Given a large planar stock rectangle (W , H) of width W and height H ;

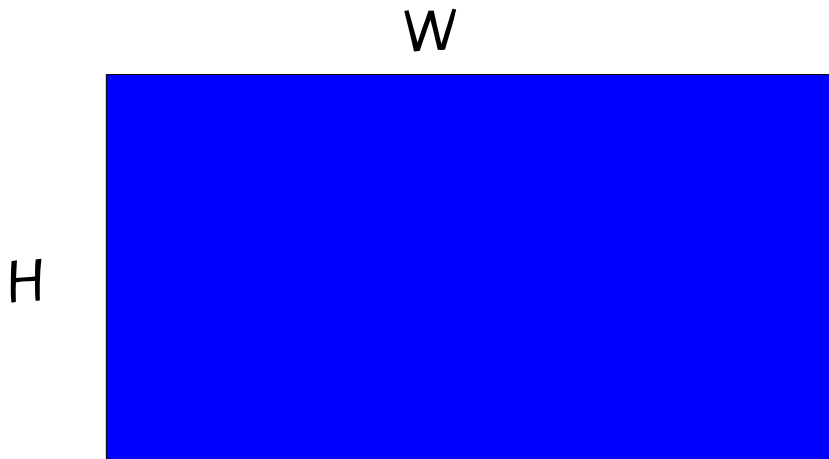
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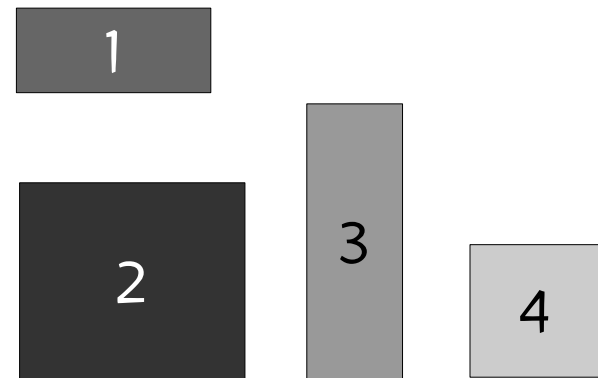
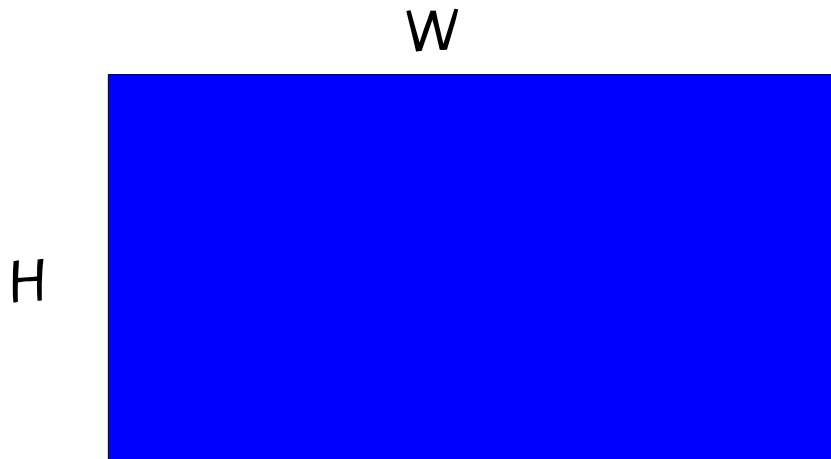
Constrained orthogonal packing

- Given a large planar stock rectangle (W , H) of width W and height H ;
- Given N smaller rectangle types ($w[i]$, $h[i]$), $i = 1, \dots, N$, each of width $w[i]$, height $h[i]$, and value $v[i]$;



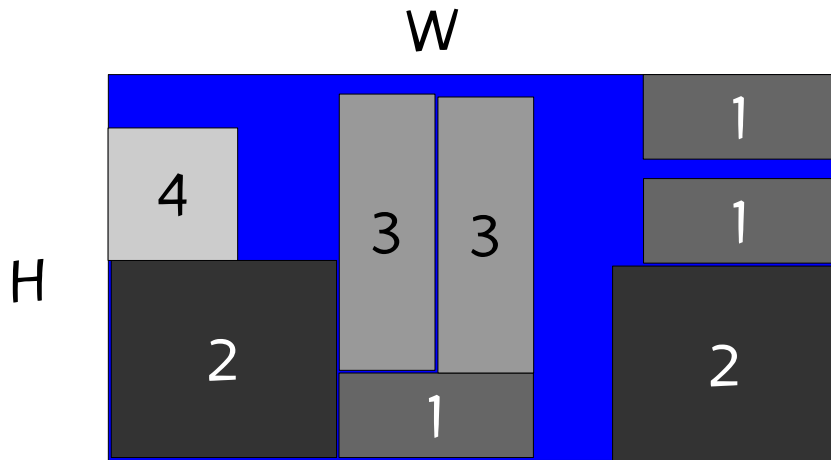
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Constrained orthogonal packing

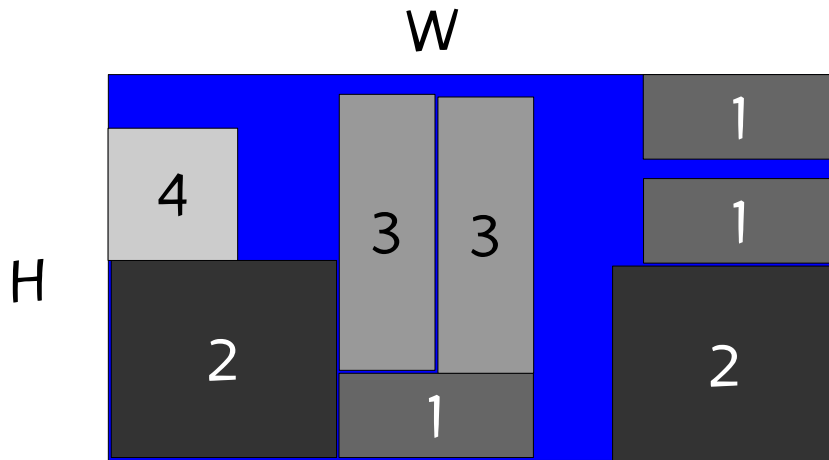
- $r[i]$ rectangles of type $i = 1, \dots, N$ are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;



Constrained orthogonal packing

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- For $i = 1, \dots, N$, we require that:

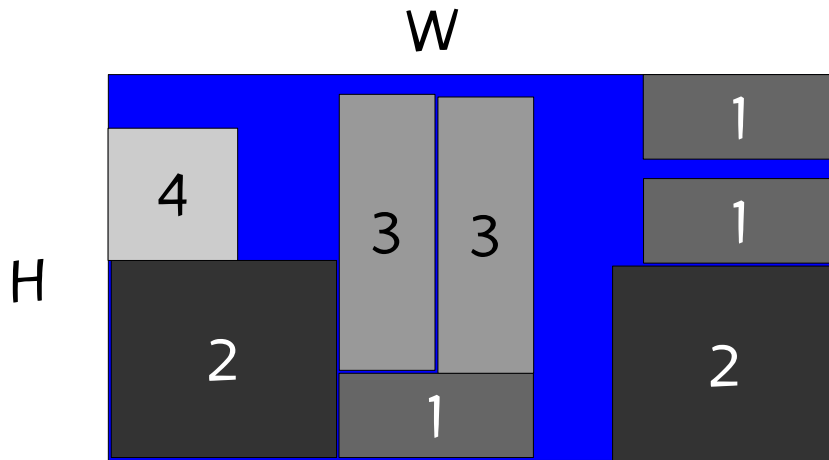
$$0 \leq P[i] \leq r[i] \leq Q[i]$$



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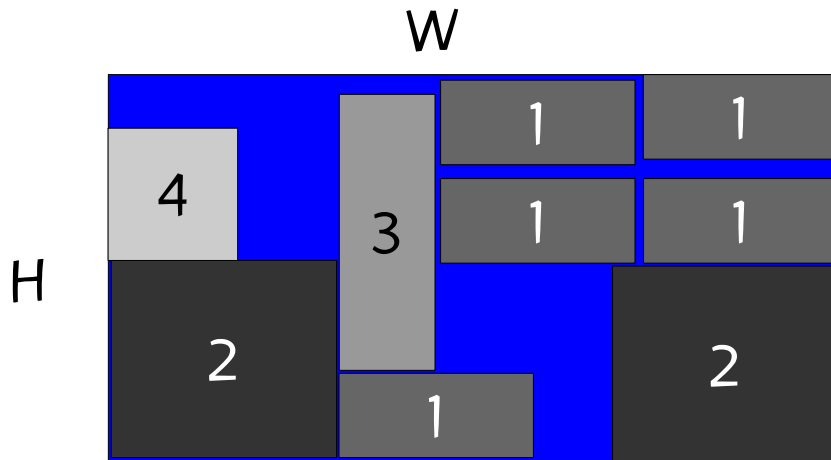


Suppose $5 \leq r[1] \leq 12$

Constrained orthogonal packing

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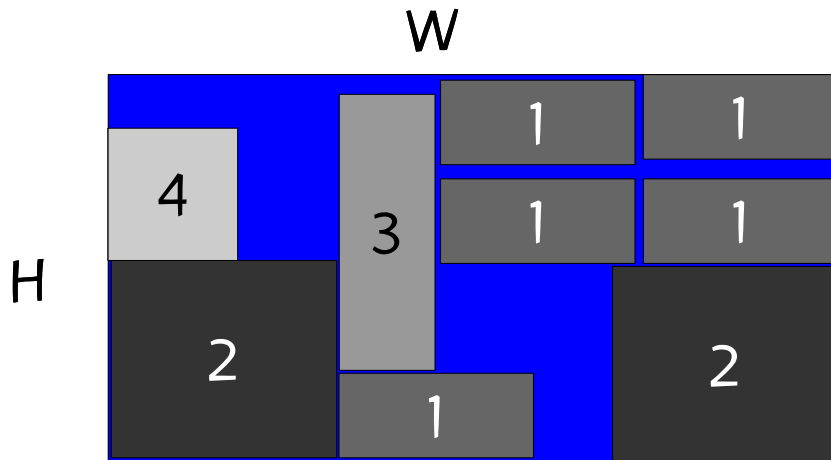


Suppose $5 \leq r[1] \leq 12$

Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

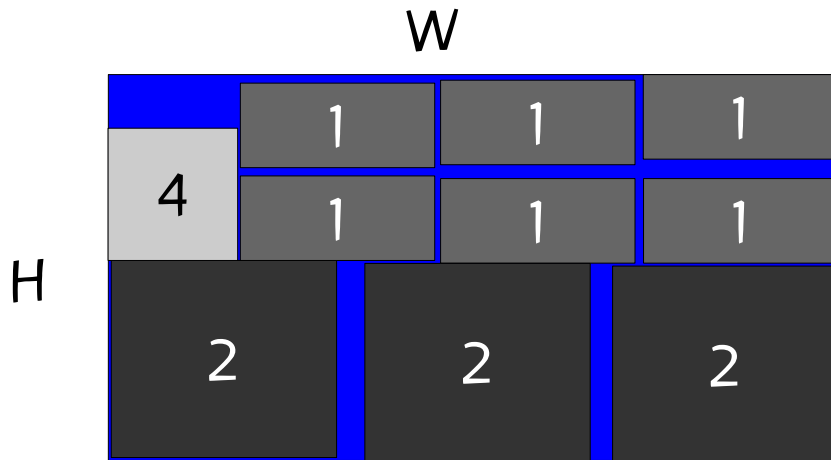
$$v[1] r[1] + v[2] r[2] + \dots + v[N] r[N]$$



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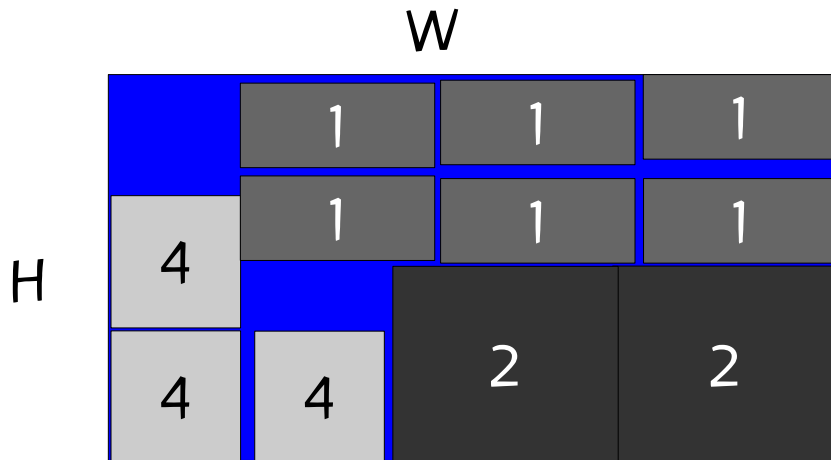
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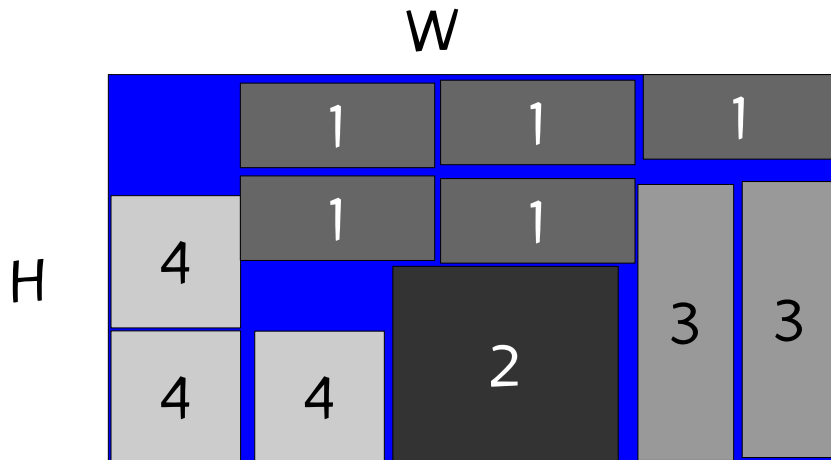
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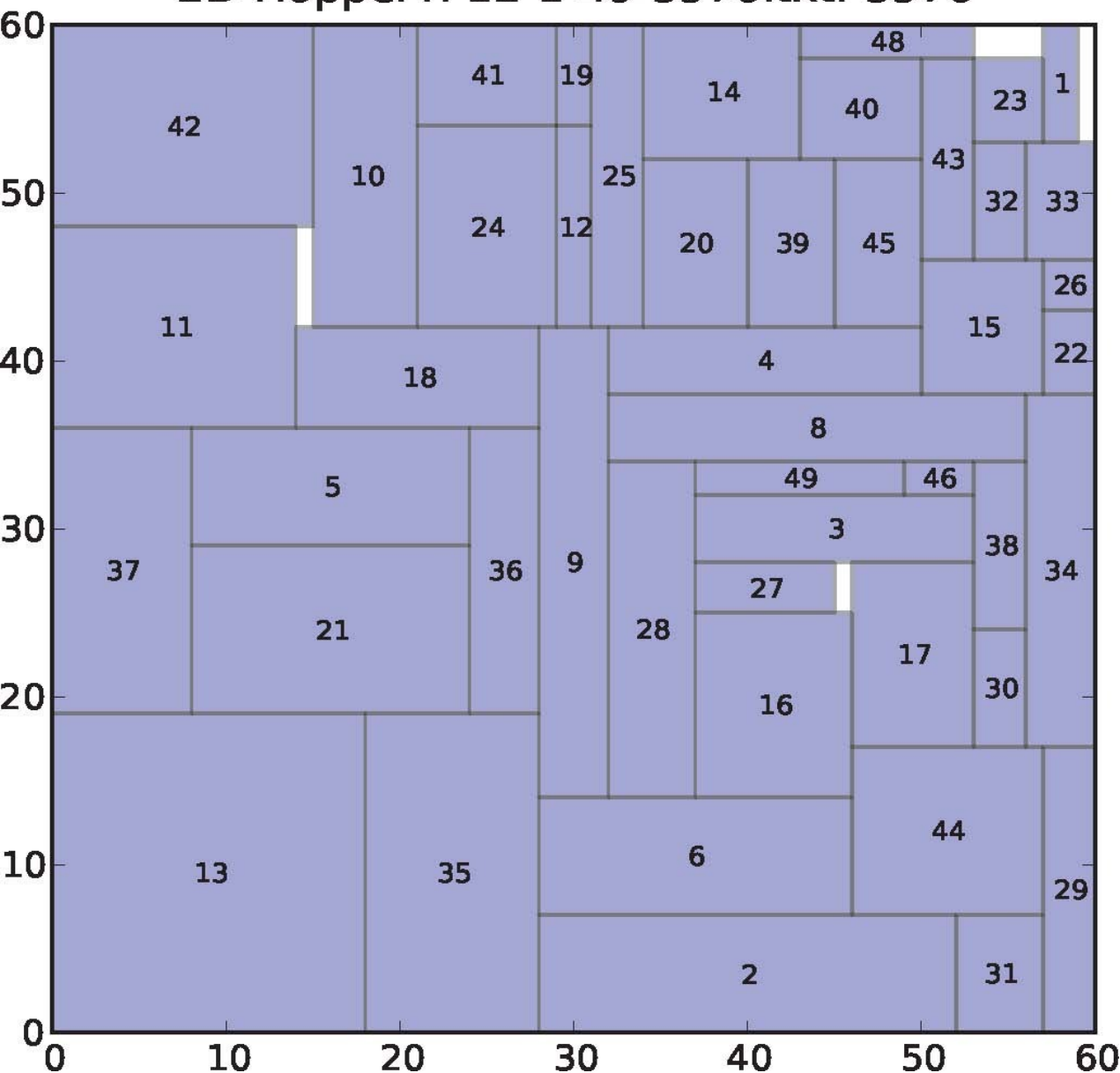
Applications

Problem arises in several production processes, e.g.

- Textile
- Glass
- Wood
- Paper

where rectangular figures are cut from large rectangular sheets of materials.

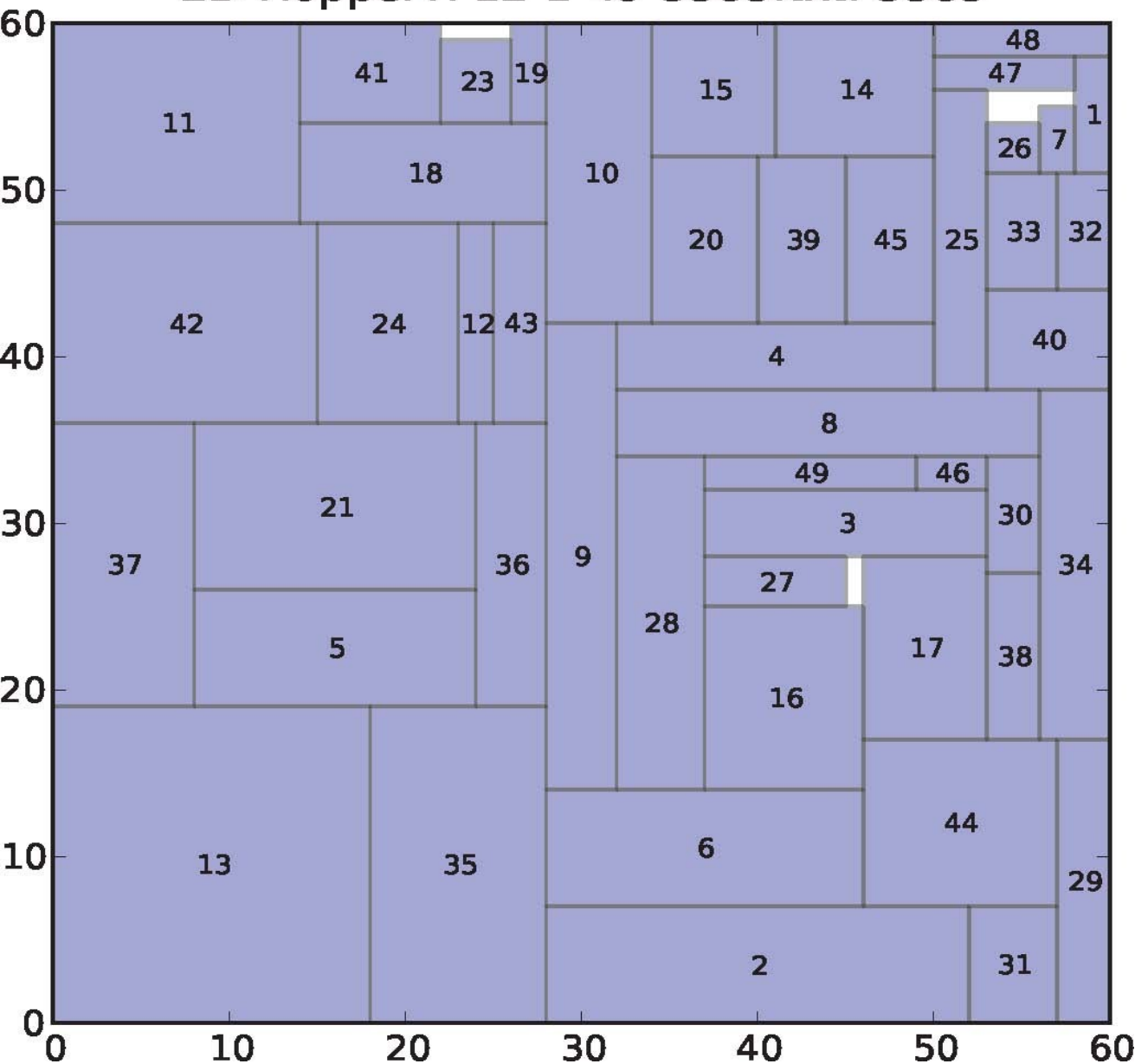
2D-HopperTP12-1-49-3576.txt: 3576



Hopper & Turton, 2001
 Instance 4-1 60 x 60
 Value: 3576

Previous best: 3580 by a
 Tabu Search heuristic
 (Alvarez-Valdes et al., 2007)

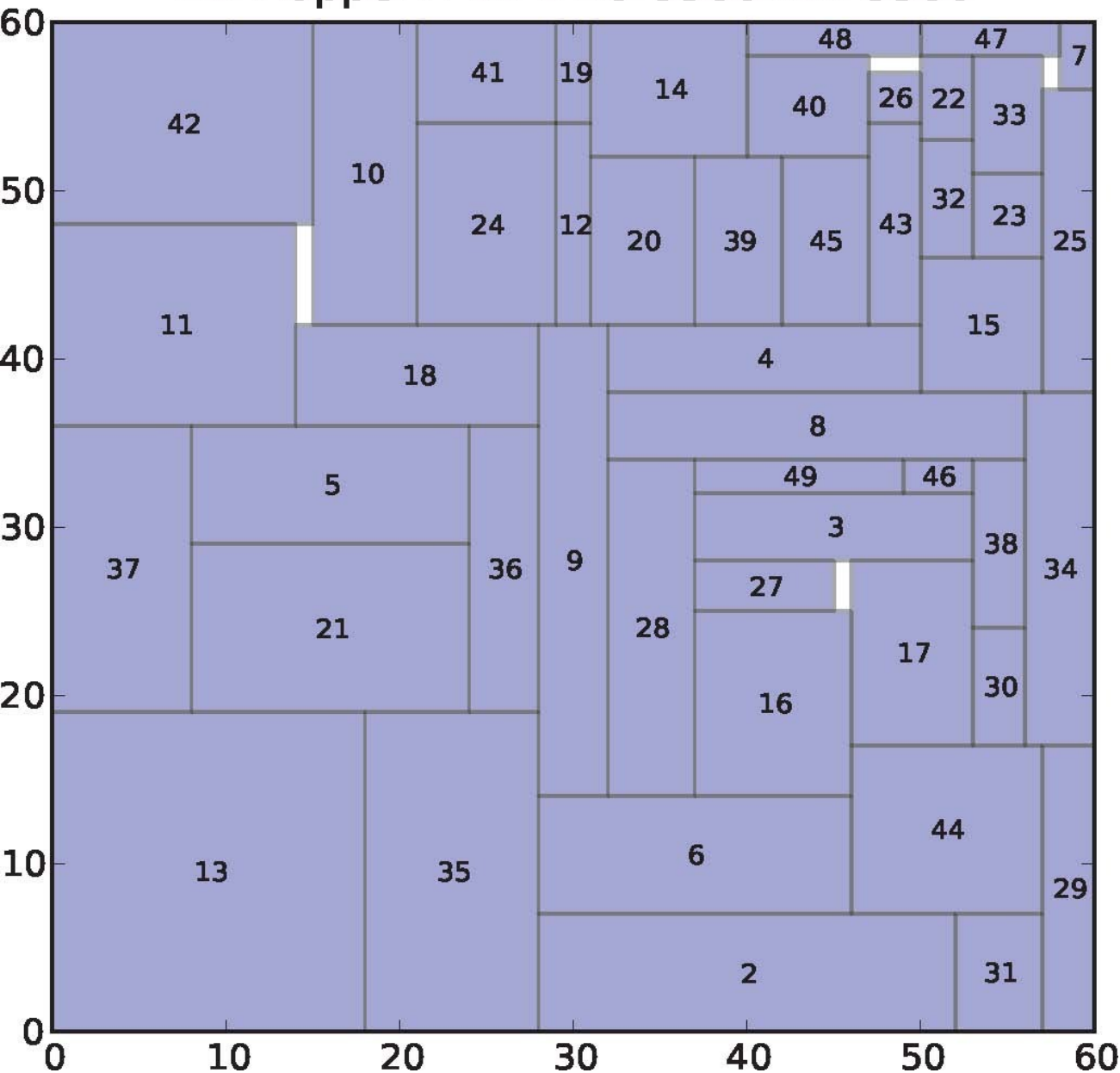
2D-HopperTP12-1-49-3585.txt: 3585



Hopper & Turton, 2001
 Instance 4-2 60 x 60
 Value: 3585

Previous best: 3580 by a
 Tabu Search heuristic
 (Alvarez-Valdes et al., 2007)

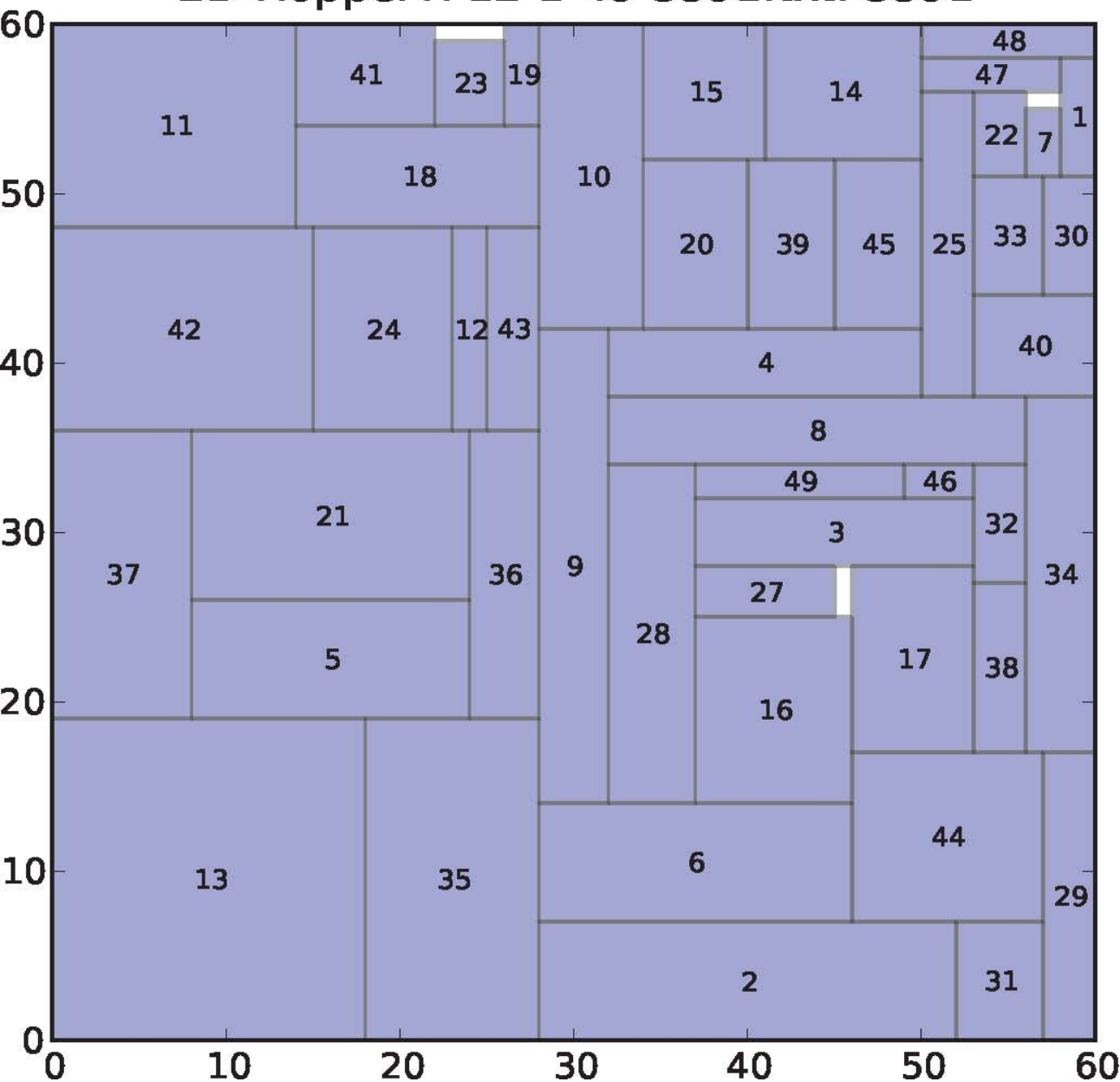
2D-HopperTP12-1-49-3586.txt: 3586



Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3586

Previous best: 3580 by a
Tabu Search heuristic
(Alvarez-Valdes et al., 2007)

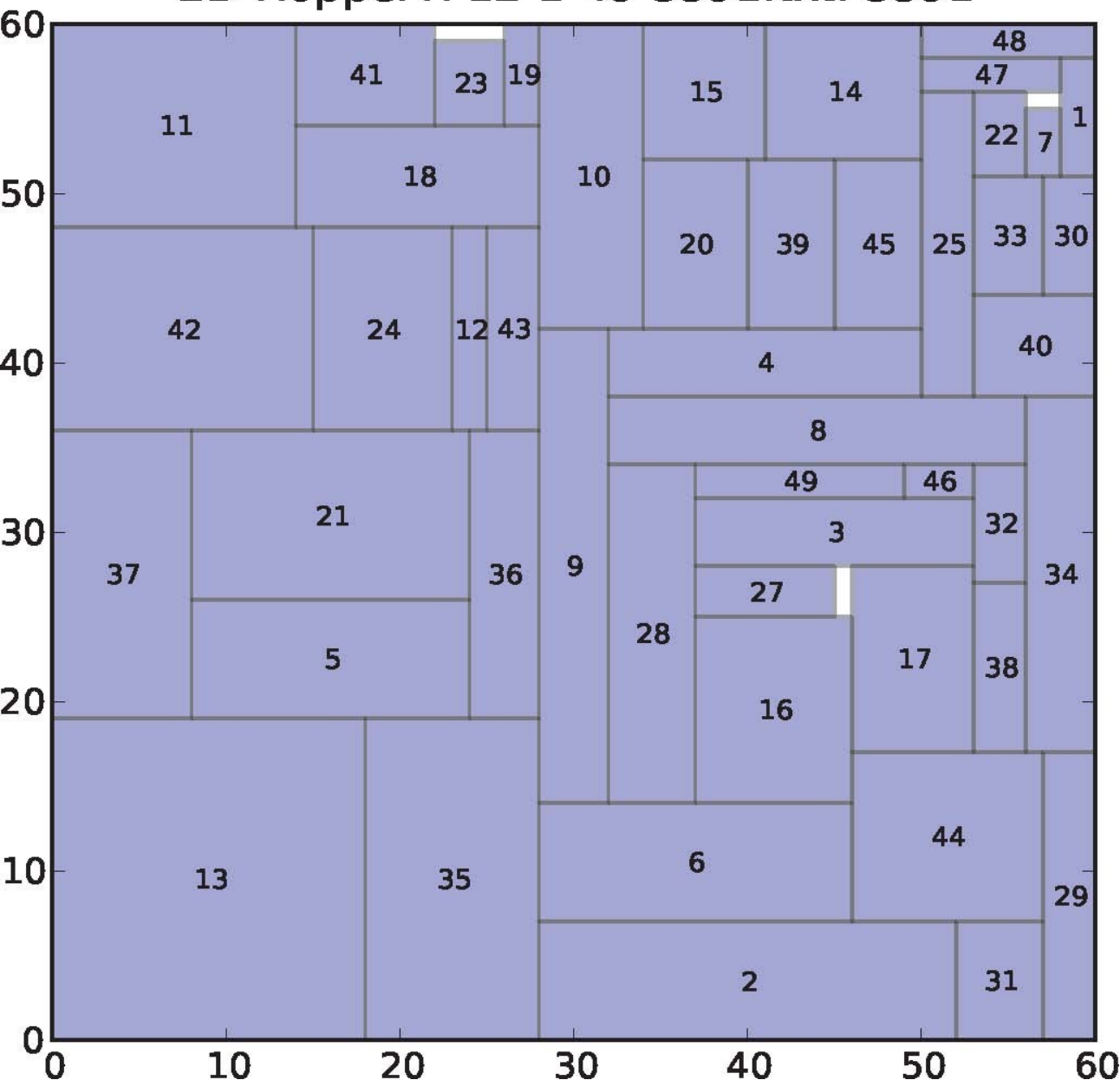
2D-HopperTP12-1-49-3591.txt: 3591



Hopper & Turton, 2001
 Instance 4-2 60 x 60
 Value: 3591

Previous best: 3580 by a
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2D-HopperTP12-1-49-3591.txt: 3591



Hopper & Turton, 2001

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New best known solution!

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BRKGA for constrained 2-dim orthogonal packing

Encoding

- Solutions are encoded as vectors X of
 $2N' = 2 \{ Q[1] + Q[2] + \dots + Q[N] \}$
random keys, where $Q[i]$ is the maximum number
of rectangles of type i (for $i = 1, \dots, N$) that can be
packed.
- $X = (X[1], \dots, X[N'], \quad X[N'+1], \dots, X[2N'])$

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Decoding

- Simple heuristic to pack rectangles:
 - Make $Q[i]$ copies of rectangle i , for $i = 1, \dots, N$.
 - Order the $N' = Q[1] + Q[2] + \dots + Q[N]$ rectangles in some way.
 - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: **bottom-left (BL)** or **left-bottom (LB)**. If **rectangle cannot be positioned, discard it** and go on to the next rectangle in the order.

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 - Make $Q[i]$ copies of rectangle i , for $i = 1, \dots, N$.
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Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
- Let $(x[i], y[i])$ be the coordinates of the bottom left corner of the i -th ERS.

Decoding

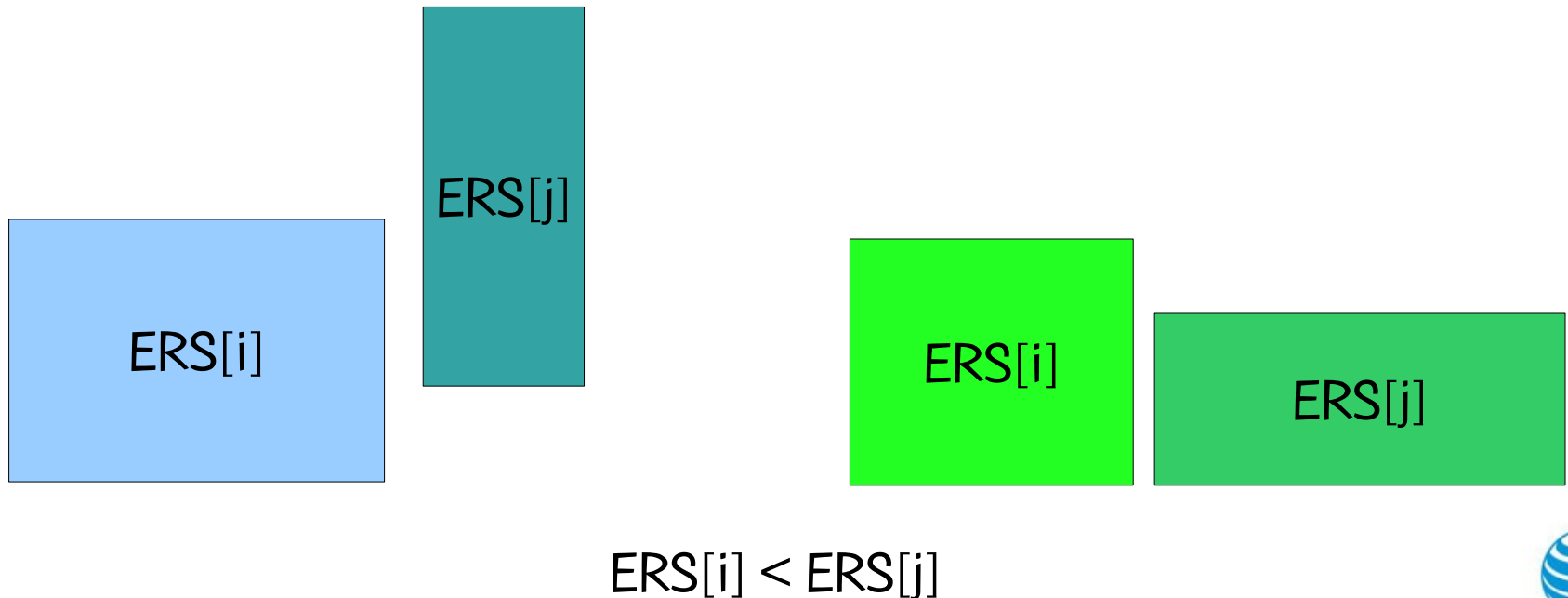
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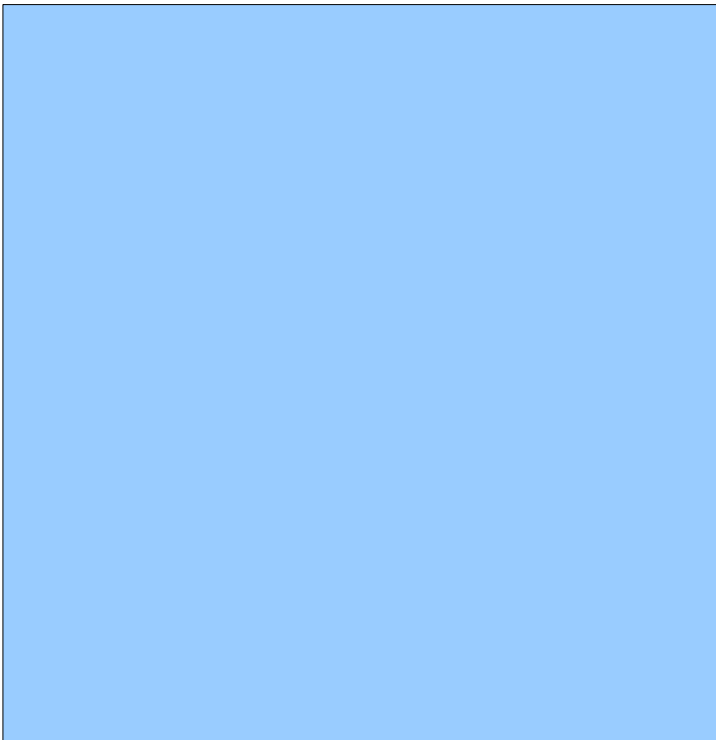
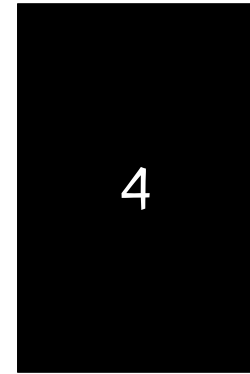
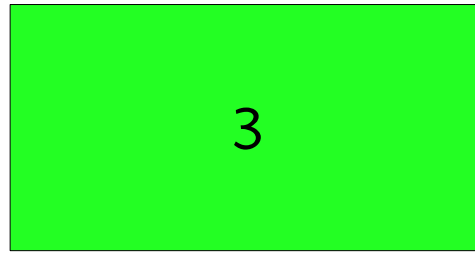
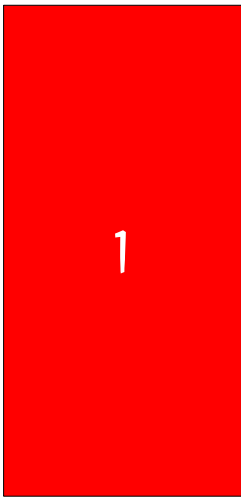


$(x[i], y[i])$

Decoding

- If BL is used, ERSs are ordered such that $ERS[i] < ERS[j]$ if $y[i] < y[j]$ or $y[i] = y[j]$ and $x[i] < x[j]$.

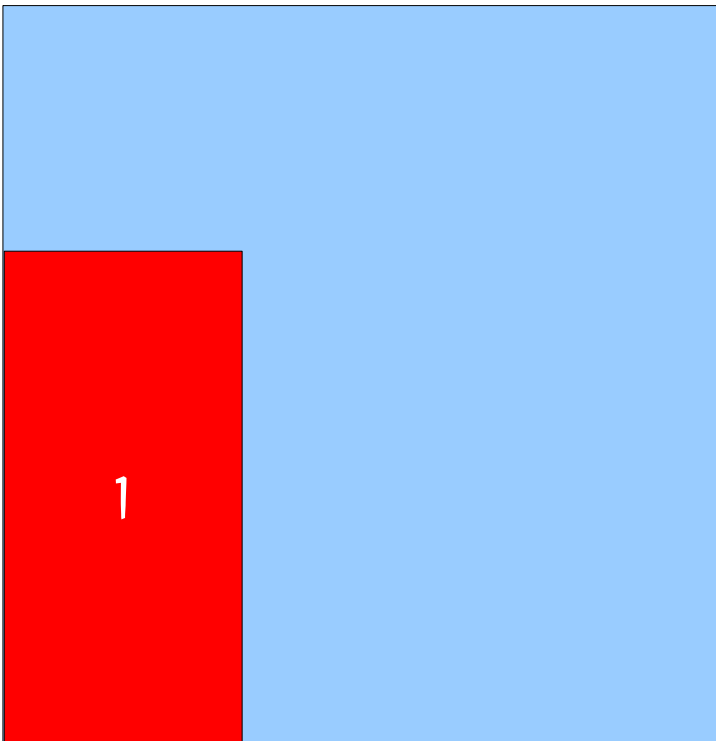
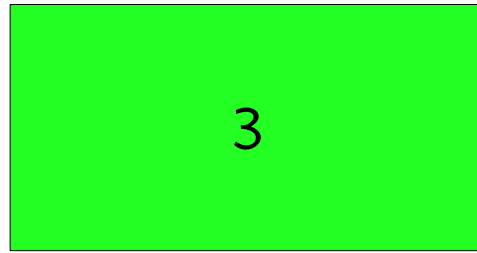




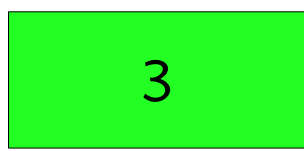
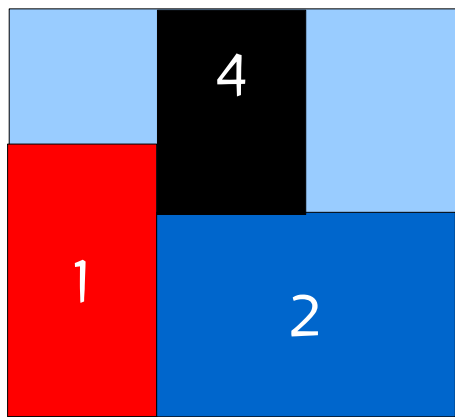
BL can run into problems even on small instances (Liu & Teng, 1999).

Consider this instance with 4 rectangles.

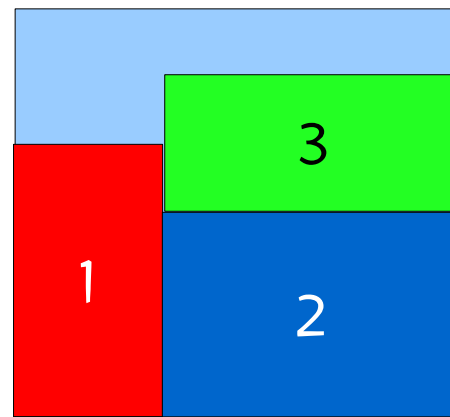
BL cannot find the optimal solution for any RTPS.



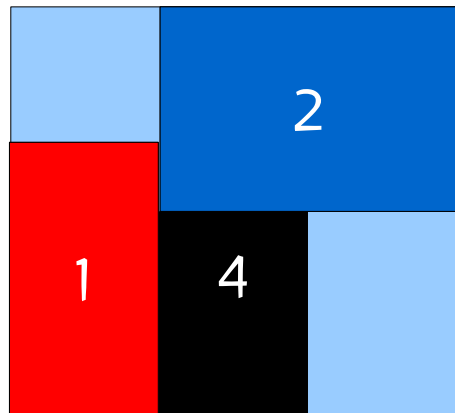
We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



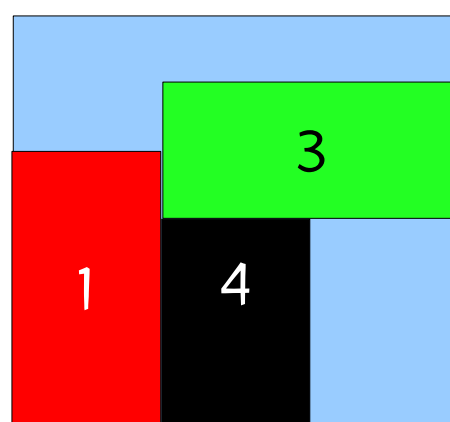
RTPS: 1-2-4-3



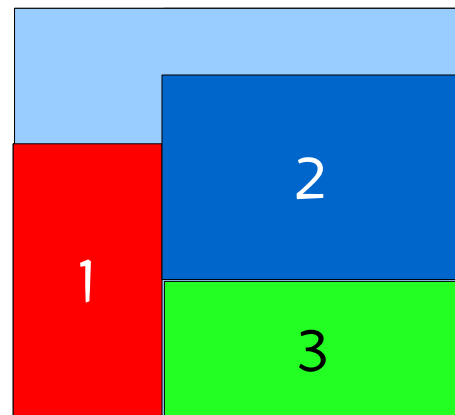
RTPS: 1-2-3-4



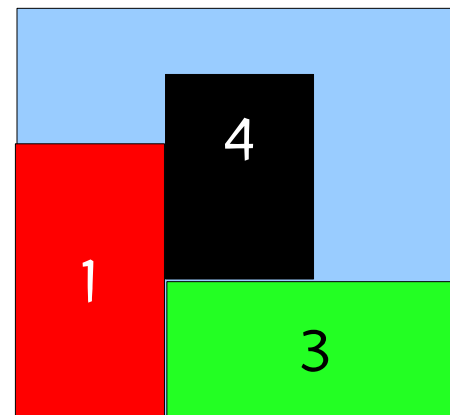
RTPS: 1-4-2-3



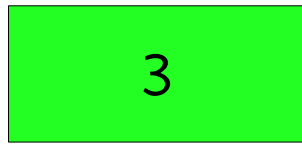
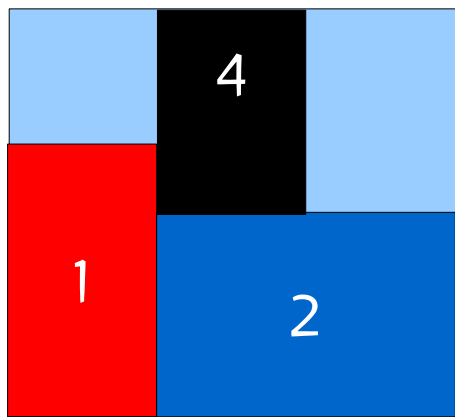
RTPS: 1-4-3-2



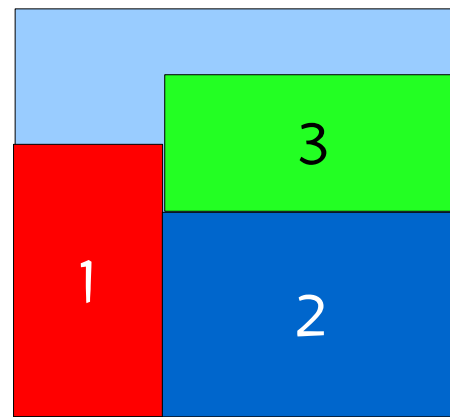
RTPS: 1-3-2-4



RTPS: 1-3-4-2

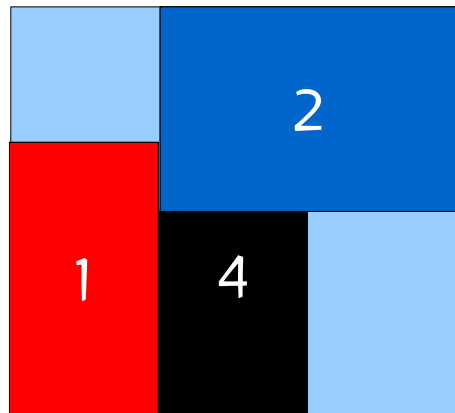


RTPS: 1-2-4-3

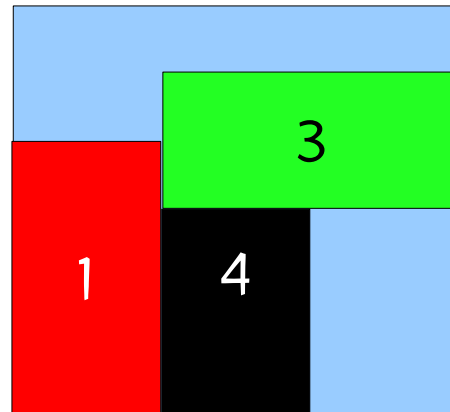


RTPS: 1-2-3-4

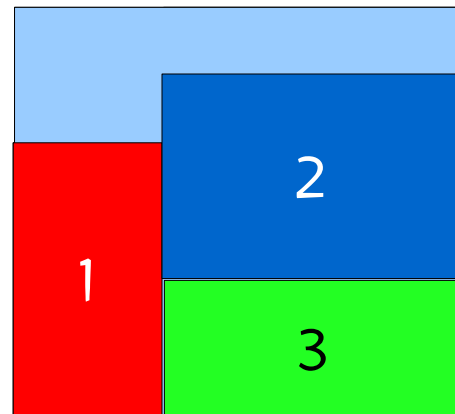
Similar infeasibilities are observed if 2, 3, or 4 is the first rectangle in the RTPS.



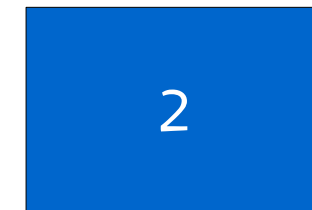
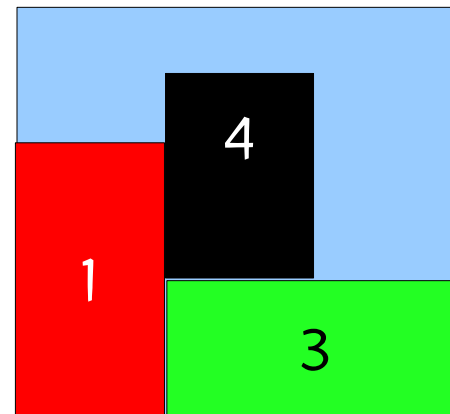
RTPS: 1-4-2-3



RTPS: 1-4-3-2



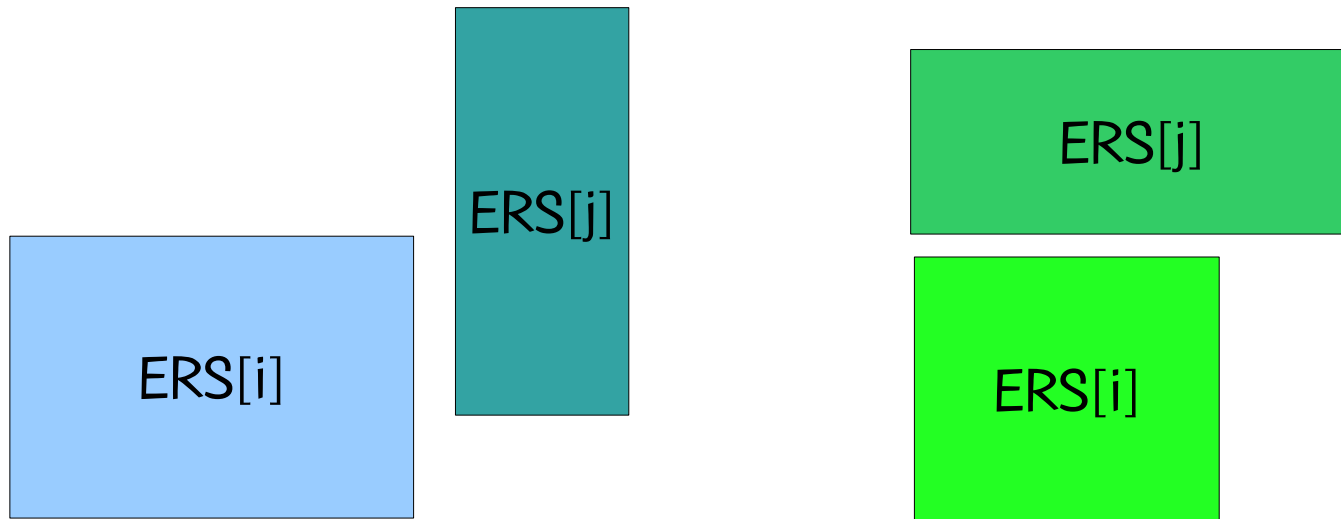
RTPS: 1-3-2-4



RTPS: 1-3-4-2

Decoding

- If LB is used, ERSs are ordered such that $ERS[i] < ERS[j]$ if $x[i] < x[j]$ or $x[i] = x[j]$ and $y[i] < y[j]$.



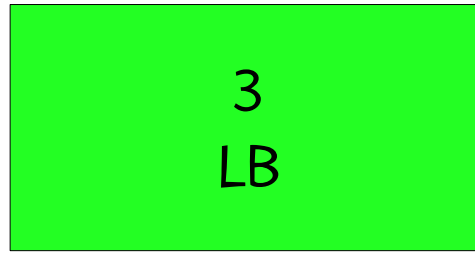
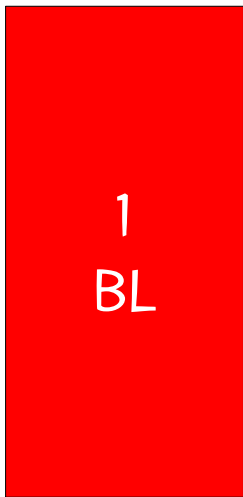
$ERS[i] < ERS[j]$

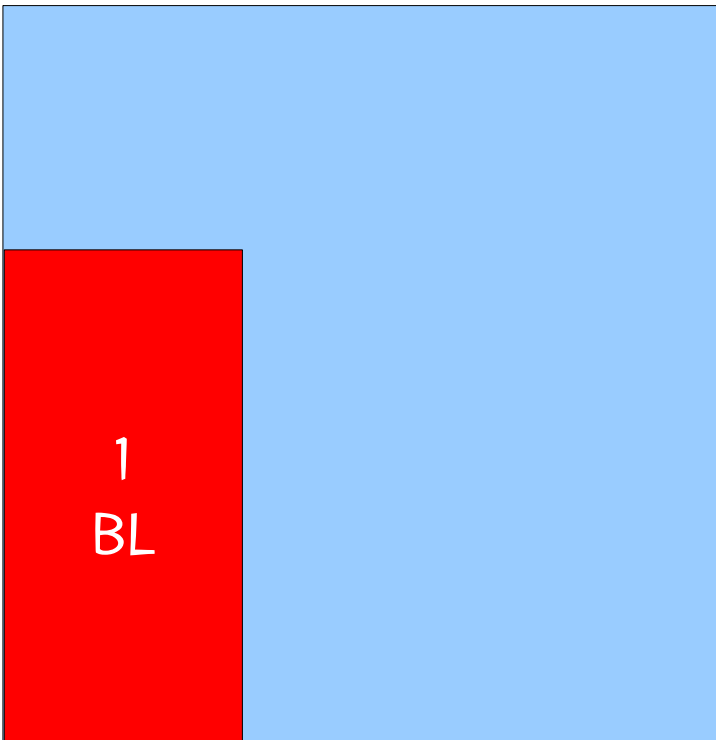
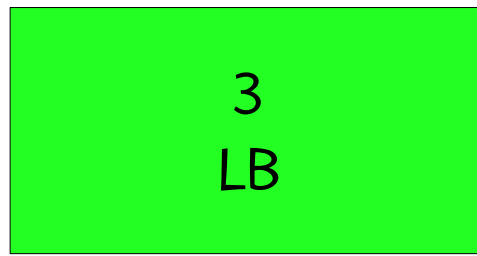
1
BL

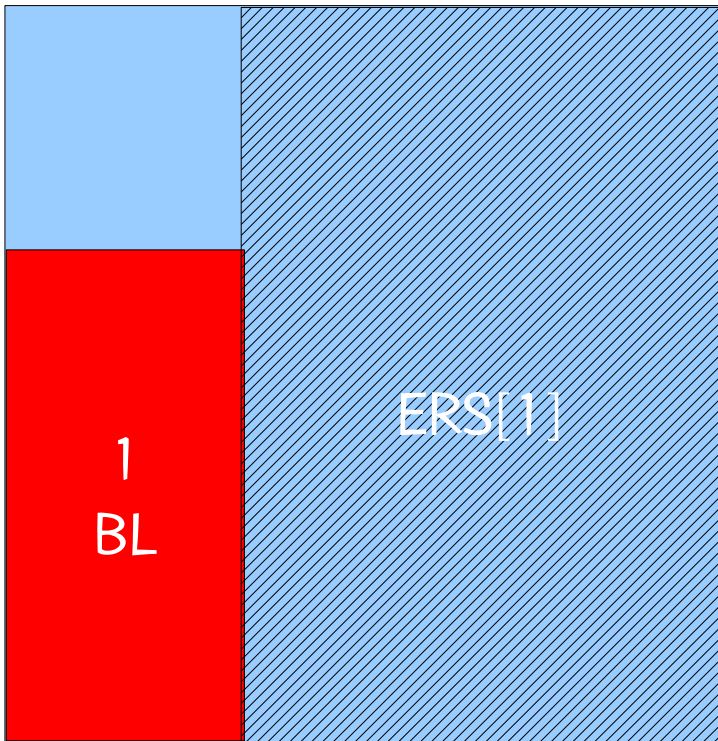
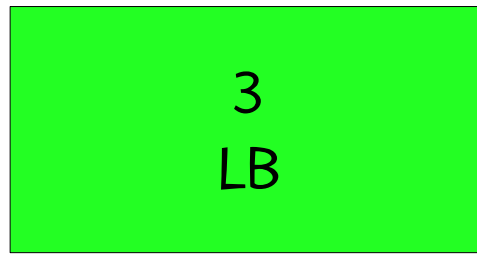
2
BL

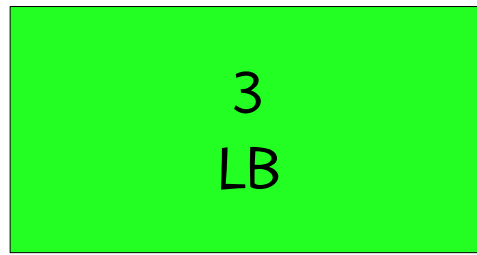
3
LB

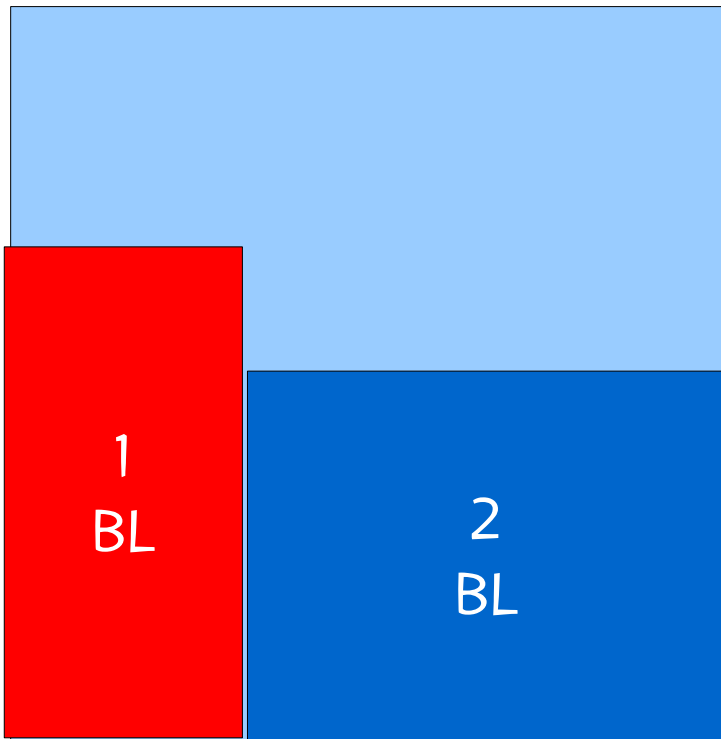
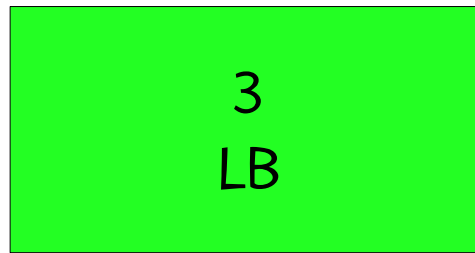
4
BL

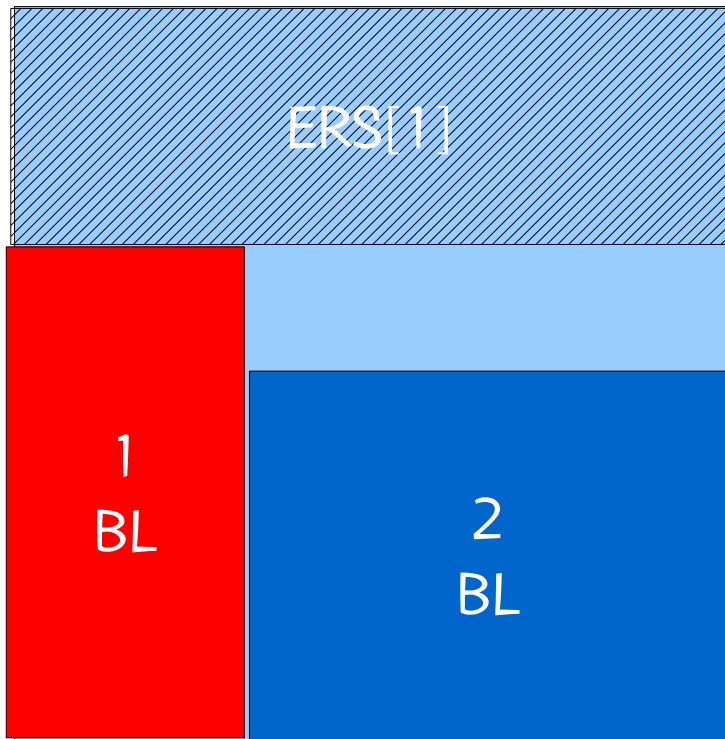
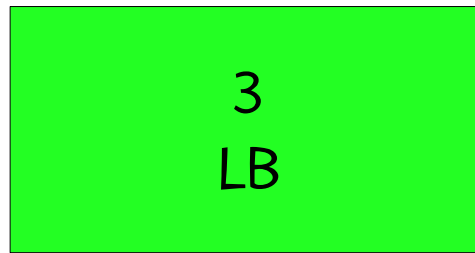


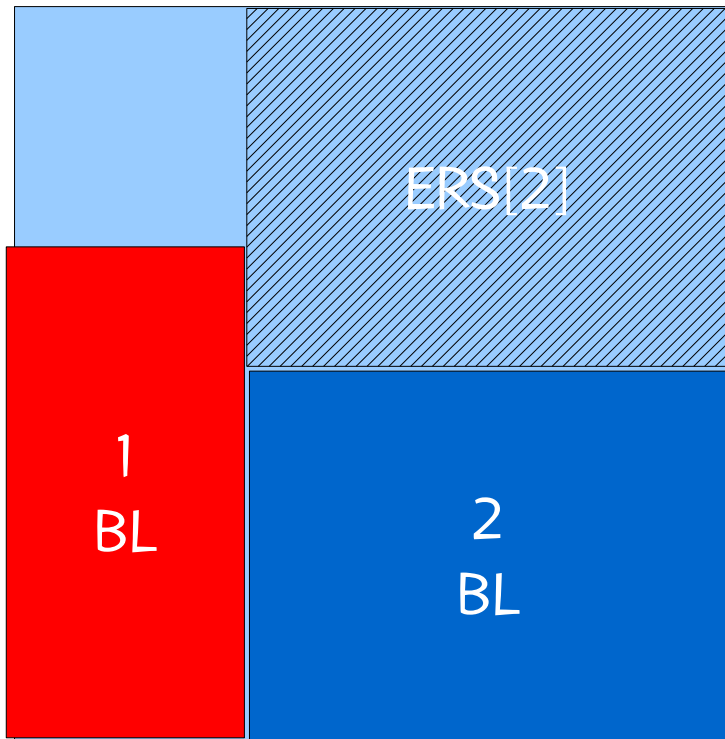
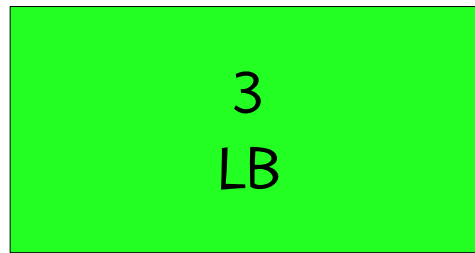


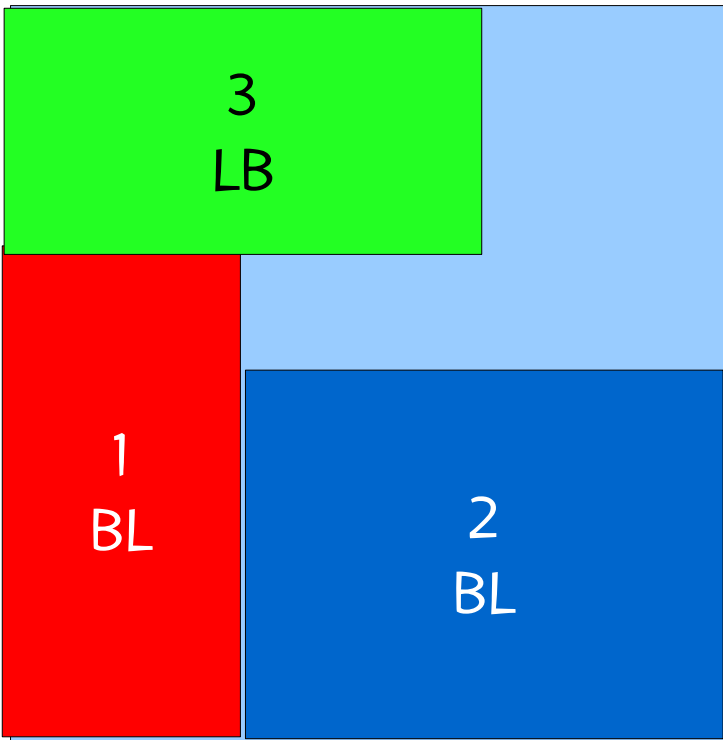








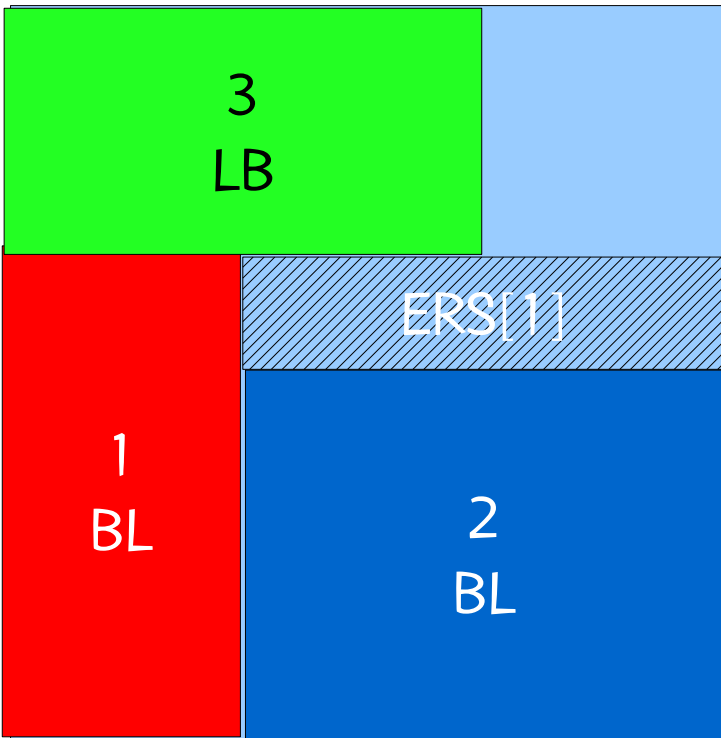




Texas A&M, College Station, Texas ♣ Mar. 28, 2014

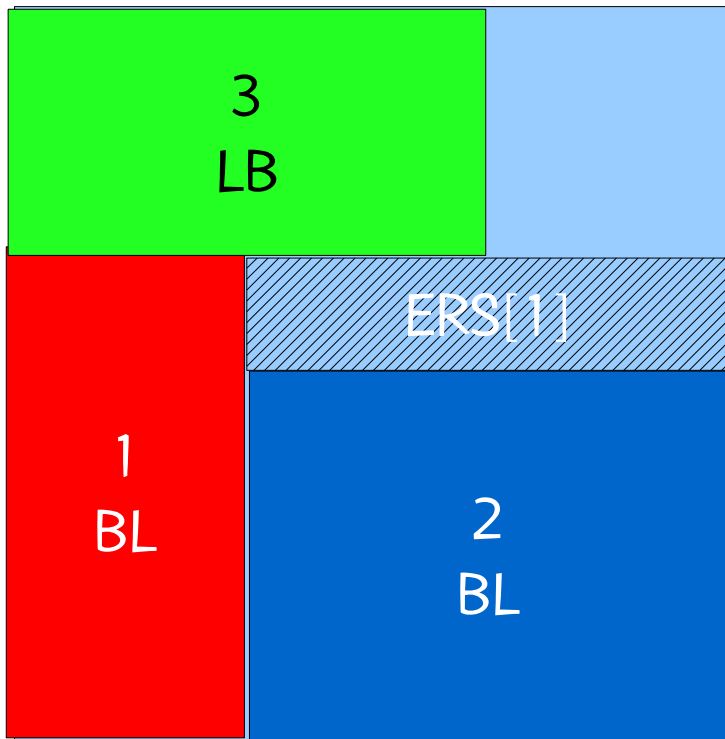
Packing with a BRKGA

4
BL



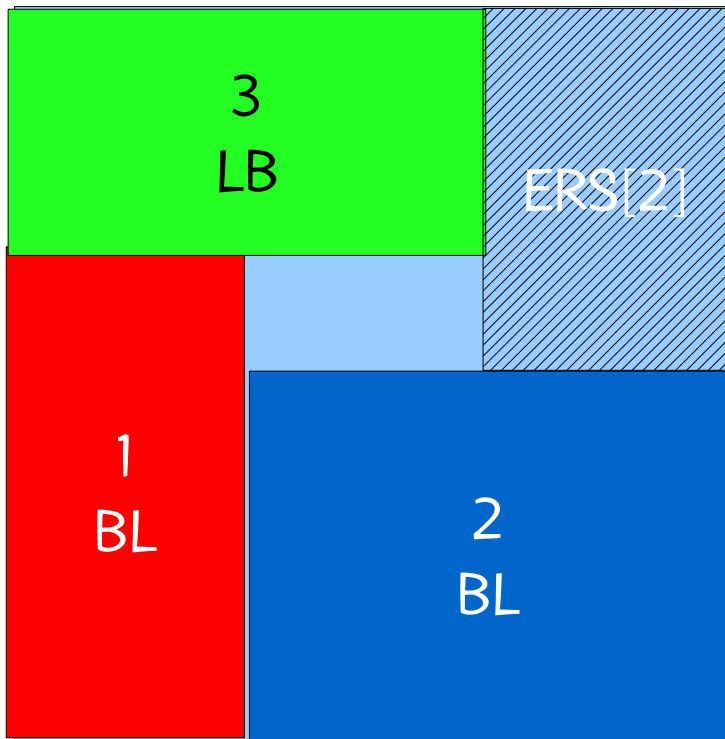


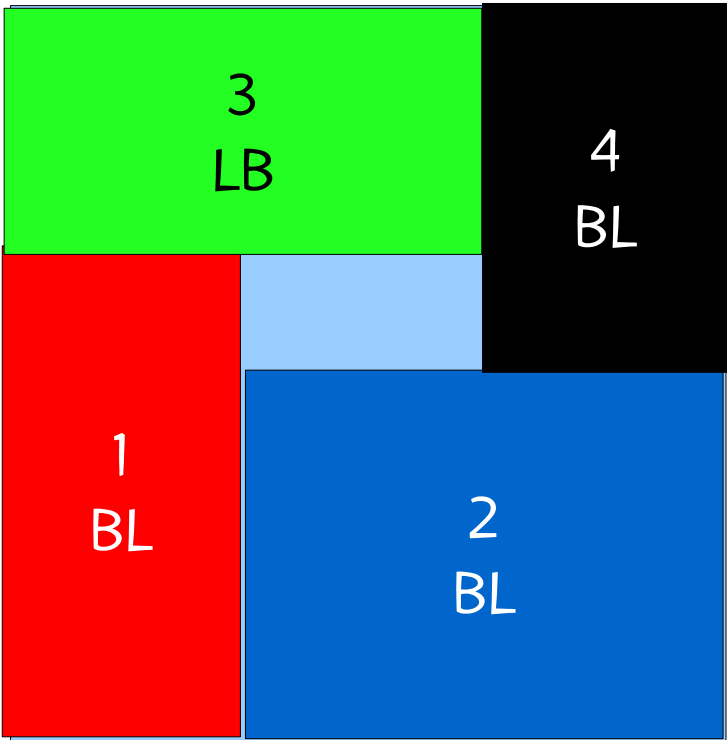
4 does not fit
in ERS[1].





4 does fit
in ERS[2].





Optimal solution!

Experimental results

Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:

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Design

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 - **GRASP**: greedy randomized adaptive search procedure of Alvarez-Valdes et al. (2005)
 - **TABU**: tabu search of Alvarez-Valdes et al. (2007)

Number of best solutions / total instances

Problem	PH	GA	GRASP	TABU	BRKGA BL-LB-L-4NR
From literature (optimal)	13/21	21/21	18/21	21/21	21/21
Large random*	0/21	0/21	5/21	8/21	20/21
Zero-waste			5/31	17/31	30/31
Doubly constrained	11/21		12/21	17/21	19/21

* For large random: number of best average solutions / total instance classes

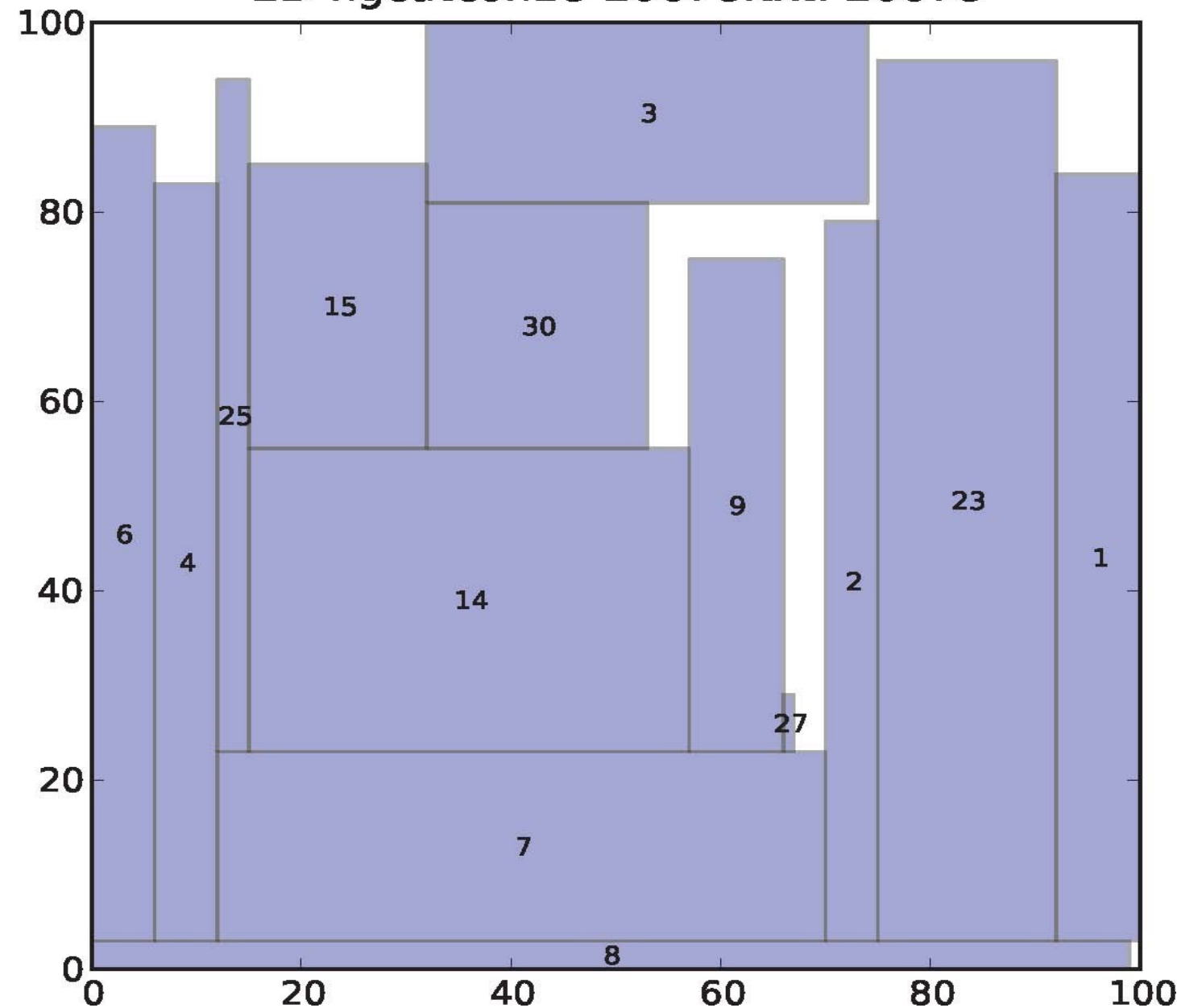
Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

Problem	Min solution time (secs)	Avg solution time (secs)	Max solution time (secs)
From literature (optimal)	0.00	0.05	0.55
Large random	1.78	23.85	72.70
Zero-waste	0.01	82.21	808.03
Doubly constrained	0.00	1.16	16.87

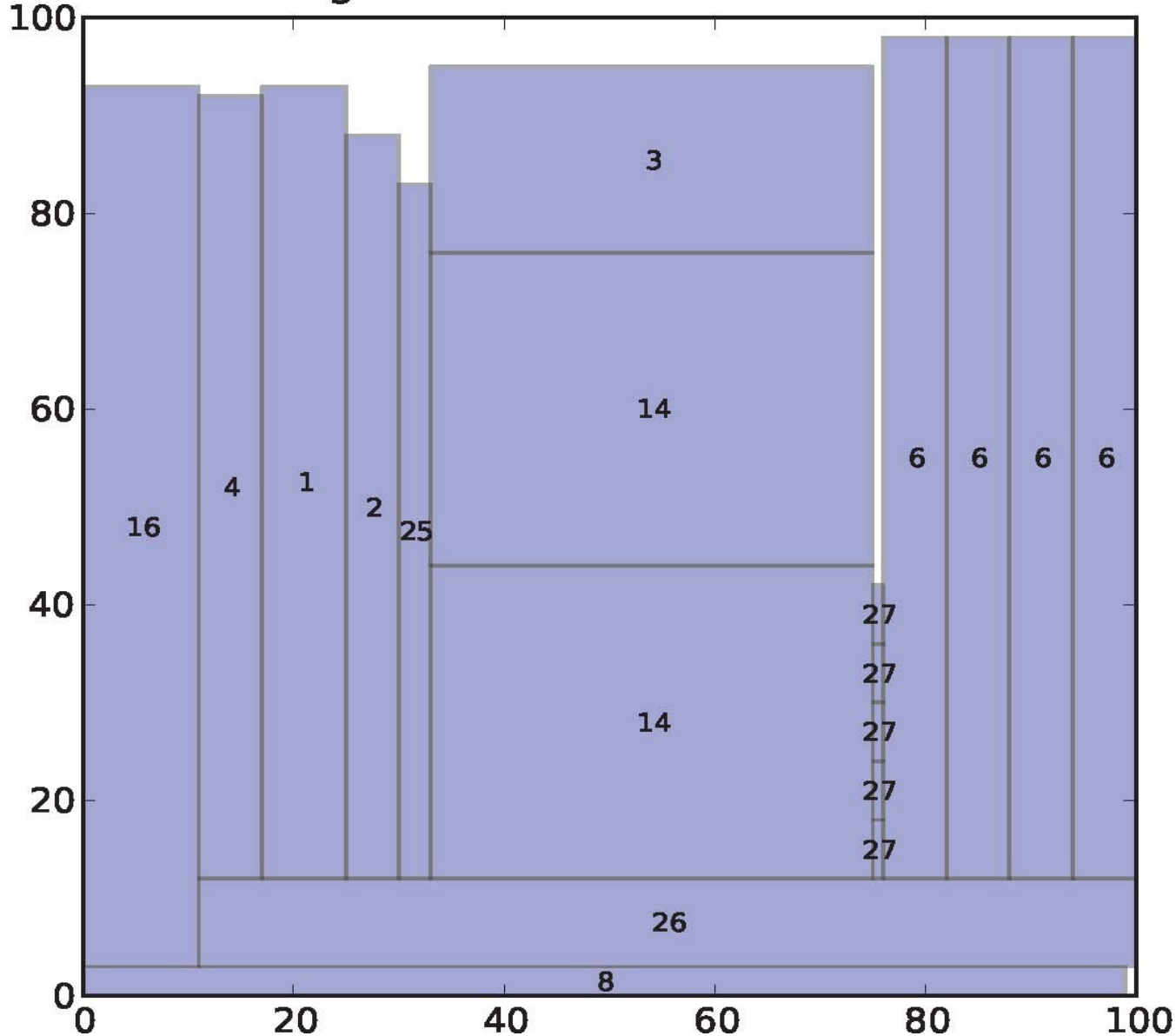
2D-ngcutcon18-20678.txt: 20678

New BKS
for a 100 x100
doubly
constrained
instance of
Fekete &
Schepers (1997)
of value **20678**.
Previous best
was **19657** by
tabu search of
Alvarez-Valdes et
al., (2007).

30 types
30 rectangles



2D-ngcutcon21-22140-1.txt: 22140



New BKS for a 100 x 100 doubly constrained instance Fekete & Schepers (1997) of value **22140**.

Previous BKS was **22011** by tabu search of Alvarez-Valdes et al. (2007).

29 types
97 rectangles

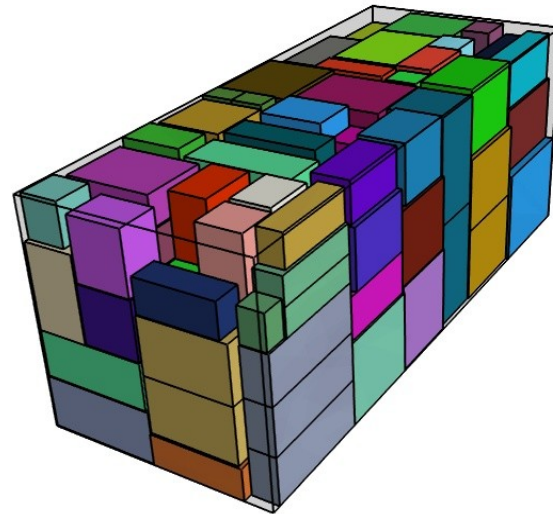
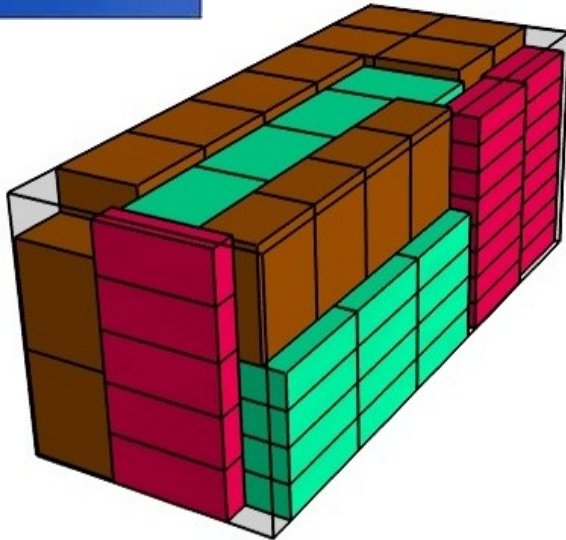
Some remarks



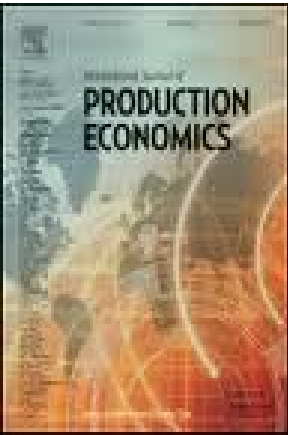
We have extended this to 3D packing:

J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers & Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: <http://www.research.att.com/~mgcr/doc/brkga-pack3d.pdf>



3D bin packing



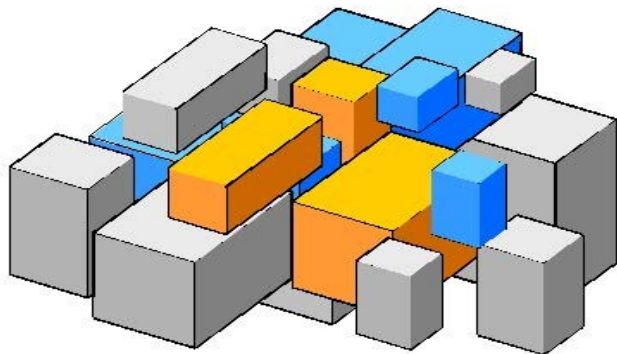
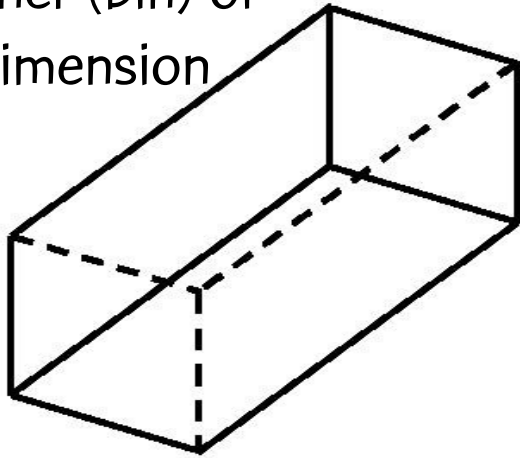
J.F. Gonçalves and M.G.C.R., “A biased
random-key genetic algorithm for 2D
and 3D bin packing problems,”

International J. of Production Economics, vol. 15, pp.
500–510, 2013.

<http://www.research.att.com/~mgcr/doc/brkga-binpacking.pdf>

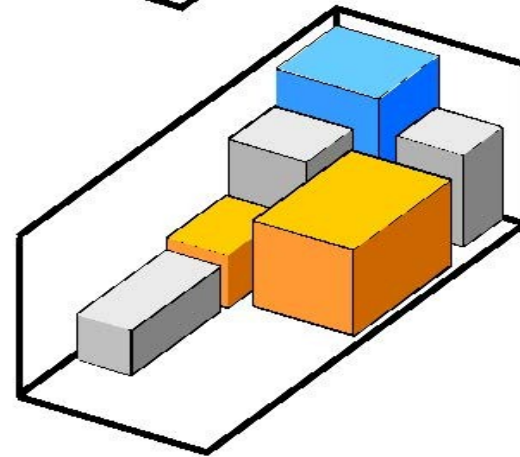
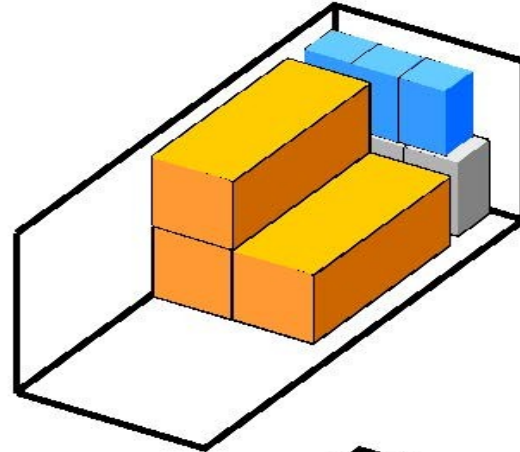
3D bin packing problem

Container (bin) of
fixed dimension



Boxes of different dimensions

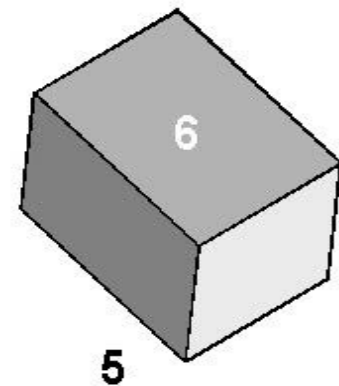
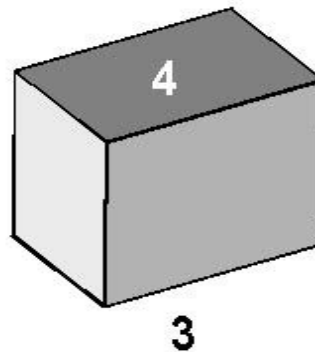
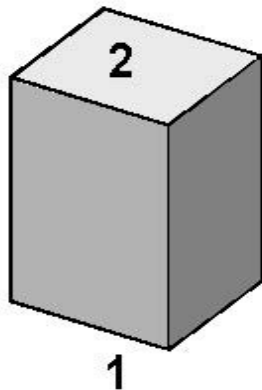
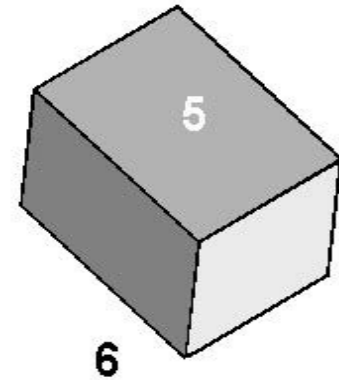
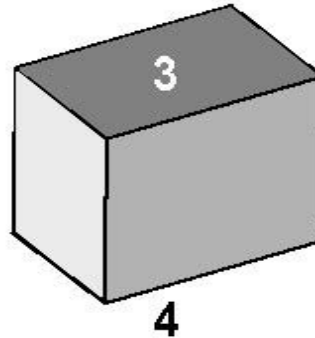
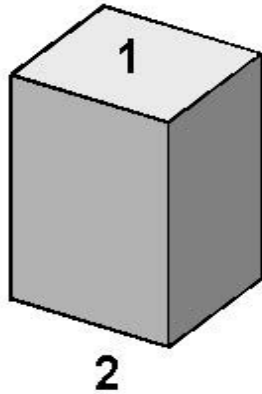
Minimize number of containers
(bins) needed to pack all boxes



3D bin packing constraints

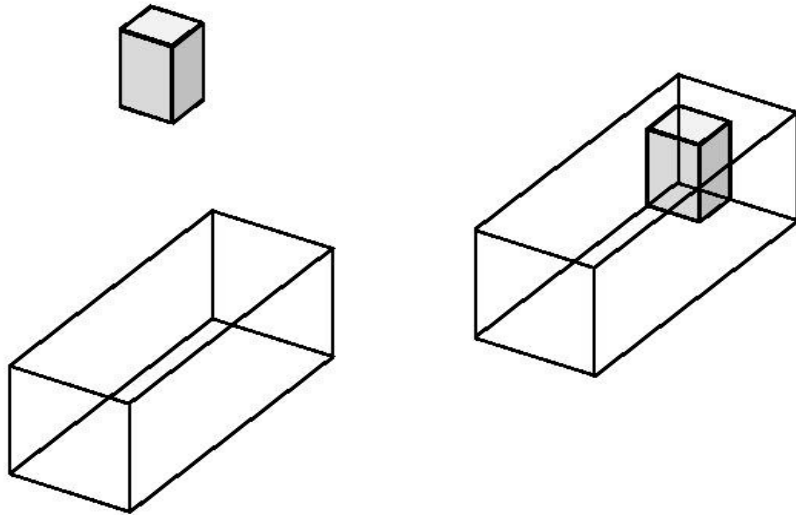
- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)

Six possible orientations for each box

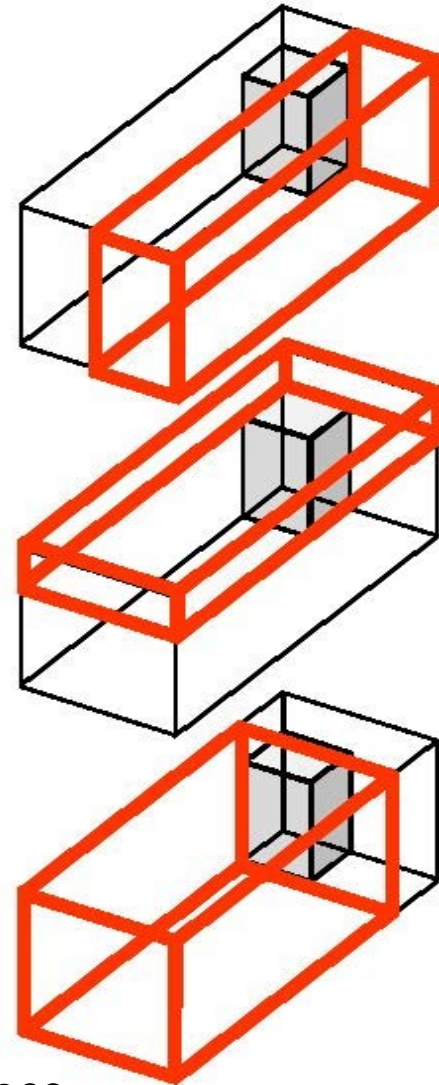


Difference process - DP

(Lai & Chan, 1997)



When box is placed in container ...
use DP to keep track of maximal free spaces



Encoding

Solutions are encoded as vectors of $3n$ random keys, where n is the number of boxes to be packed.

$$X = (\underbrace{x_1, x_2, \dots, x_n}_{\text{Box packing sequence}}, \underbrace{x_{n+1}, x_{n+2}, \dots, x_{2n}}_{\text{Placement heuristic}}, \underbrace{x_{2n+1}, x_{2n+2}, \dots, x_{3n}}_{\text{Box orientation}})$$

Decoding

- 1) Sort first n keys of X to produce sequence boxes will be packed;
- 2) Use second n keys of X to determine which placement heuristic to use (back-bottom-left or back-left-bottom):
 - if $x_{n+i} < 1/2$ then use back-bottom-left to pack i -th box
 - if $x_{n+i} \geq 1/2$ then use back-left-bottom to pack i -th box
- 3) Use third n keys of X to determine which of six orientations to use when packing box:
 - $x_{2n+i} \in [0, 1/6)$: orientation 1;
 - $x_{2n+i} \in [1/6, 2/6)$: orientation 2; ...
 - $x_{2n+i} \in [5/6, 1]$: orientation 6.

Decoding

For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation

Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness: aNB
- $aNB = NB + (\text{LeastLoad} / \text{BinVolume})$, where
 - × LeastLoad is load on least loaded bin
 - × BinVolume is volume of bin: $H \times W \times L$

Experiment

- Parameters:
 - population size: $p = 30n$
 - size of elite partition: $p_e = .10p$
 - number of of mutans: $p_m = .15p$
 - crossover probability: 0.7
 - stopping criterion: 300 generations

Experiment

- Instances:
 - 320 instances of Martello et al. (2000)
 - generator is available at <http://www.diku.dk/~pisinger/codes/html>
 - 8 classes
 - 40 instances per class
 - 10 instances for each value of $n \in \{50, 100, 150, 200\}$

Experiment

- We compare BRKGA with:
 - TS3, the tabu search of Lodi et al. (2002)
 - GLS, the guided local search of Faroe et al. (2003)
 - TS2PACK, the tabu search of Crainic et al. (2009)
 - GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)

Class 1 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.5	13.4	13.4	13.4	13.4	13.4
100	25.1	26.6	26.6	26.6	26.7	26.7
150	34.7	36.4	36.4	36.7	37.0	37.0
200	48.4	50.9	50.9	51.2	51.1	51.2
Sum:	120.7	127.3	127.3	127.9	128.2	128.3

Class 2 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.7	13.8	13.8	13.8	Did not run	
100	24.1	25.6	25.7	25.7		
150	35.1	36.7	36.9	37.2		
200	47.5	49.4	49.4	50.1		
Sum:	119.4	125.5	125.8	126.8		

Class 3 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.3	13.3	13.3	13.3	Did not run	
100	24.7	25.9	26.0	26.0		
150	36.0	37.5	37.6	37.7		
200	47.8	49.8	50.0	50.5		
Sum:	120.8	126.5	126.9	127.5		

Class 4 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	28.7	29.4	29.4	29.4	29.4	29.4
100	57.6	59.0	59.0	59.0	58.9	59.0
150	85.2	86.8	86.8	86.8	86.8	86.8
200	116.3	118.8	118.8	118.8	118.8	119.0
Sum:	287.8	294.0	294.0	294.0	293.9	294.2

Class 5 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	7.3	8.3	8.3	8.4	8.3	8.3
100	12.9	15.0	15.0	15.0	15.2	15.1
150	17.4	20.0	20.1	20.4	20.1	20.2
200	24.4	27.1	27.1	27.6	27.4	27.2
Sum:	62.0	70.4	70.5	71.4	71.0	70.8

Class 6 - Bin size: 10 x 10 x 10

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	8.7	9.8	9.8	9.9	9.8	9.8
100	17.5	18.8	19.0	19.1	19.1	19.1
150	26.9	29.2	29.2	29.4	29.2	29.4
200	35.0	37.2	37.4	37.7	37.7	37.7
Sum:	88.1	95.0	95.4	96.1	95.8	96.0

Class 7 - Bin size: 40 x 40 x 40

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	6.3	7.4	7.4	7.5	7.4	7.4
100	10.9	12.2	12.5	12.5	12.3	12.3
150	13.7	15.2	16.0	16.1	15.8	15.8
200	21.0	23.4	23.5	23.9	23.5	23.5
Sum:	51.9	58.2	59.4	60.0	59.0	59.0

Class 8 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	0.8	9.2	9.2	9.3	9.2	9.2
100	17.5	18.9	18.9	18.9	18.8	18.9
150	21.3	23.5	24.1	24.1	23.9	23.9
200	26.7	29.3	29.8	30.3	30.0	29.9
Sum:	66.3	80.9	82.0	82.6	81.9	81.9

Summary

Class	Bin size	BRKGA	GRASP	TS3	TS2PACK	GLS
1	100 ³	127.3	127.3	127.9	128.2	128.3
2	100 ³	125.5	125.8	126.8		
3	100 ³	126.5	126.9	127.5		
4	100 ³	294.0	294.0	294.0	293.9	294.2
5	100 ³	70.4	70.5	71.4	71.0	70.8
6	10 ³	95.0	95.4	96.1	95.8	96.0
7	40 ³	58.2	59.4	60.0	59.0	59.0
8	100 ³	80.9	82.0	82.6	81.9	81.9
Sum(rows 1, 4-8):		725.8	728.6	732.0	729.8	730.2
Sum(rows 1-8):		977.8	981.3	986.3		

Concluding remarks

- Reviewed BRKGA framework
- Applied framework to
 - 2D/3D packing to maximize value packed
 - 2D/3D bin packing to minimize number of bins
- All decoders were simple heuristics
- BRKGA “learned” how to “operate” the heuristics
- In all cases, several new best known solutions were produced

Thanks!

These slides and all of the papers cited in this lecture can be downloaded from my homepage:

<http://www.research.att.com/~mgcr>