

# Packing with biased random-key genetic algorithms

Mauricio G. C. Resende  
AT&T Labs Research  
Middletown, New Jersey

[mgcr@research.att.com](mailto:mgcr@research.att.com)

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# Summary

## **Part 1: Packing with BRKGA (~35 minutes)**

- Combinatorial optimization
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing

## **Part 2: My vision for the Department of Industrial & Systems Engineering at the University of Florida (~20 minutes)**

# Combinatorial optimization

Optimization problems are commonly classified into two groups:

- 1) Continuous optimization: Those with continuous variables, that in principle may take any real value.
- 2) Combinatorial optimization: Those represented by discrete variables, that may take only a finite or countably infinite set of values.

# Combinatorial optimization

Combinatorial optimization problems reduce to the search for a best solution (or object) in a finite (or countably infinite) set.

The solution set may typically be formed by binary or integer variables, permutations, paths, trees, cycles, or graphs.

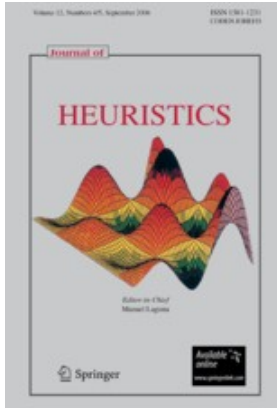
Applications abound: e.g. scheduling, routing, design, ...

# Genetic algorithms (GA)

Genetic algorithms are stochastic search methods for combinatorial optimization that apply Darwin's principle of survival of the fittest to evolve a population of solutions towards the optimal solution.

In this talk we describe a special class of genetic algorithm, the Biased Random-Key GA (BRKGA) and apply it to solve hard 2D and 3D packing problems.

# Reference



J.F. Gonçalves and M.G.C.R., “**Biased random-key genetic algorithms for combinatorial optimization,**” J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:

<http://www.research.att.com/~mgcr/doc/srkga.pdf>

# Random-key genetic algorithms

Random key – random number in  $[0,1)$



0.2384

# Random-key genetic algorithms

Random key – random number in  $[0,1)$



0.8394



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0.9234

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0.0143

# Random-key genetic algorithms

Solutions can be encoded by a vector of random keys

0.4756

0.9903

0.0012

0.7675

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0.5887

0.3929

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0.1029

0.3428

0.3233

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Decoder takes vector of random keys as input and outputs solution to optimization problem

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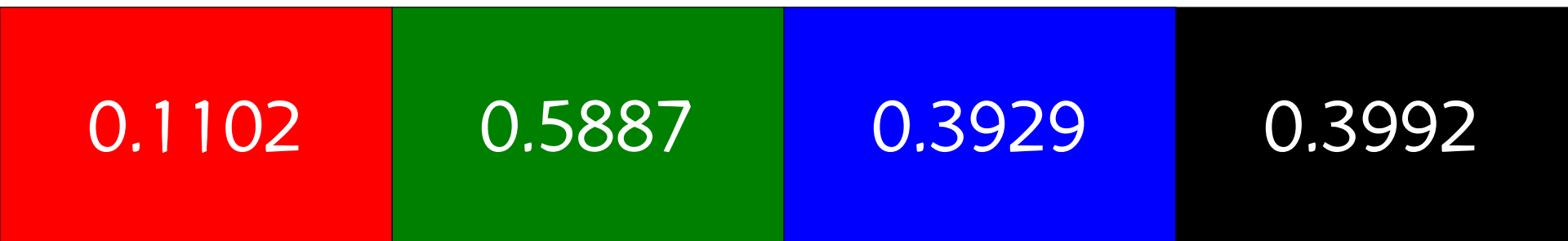
0.5887

0.3929

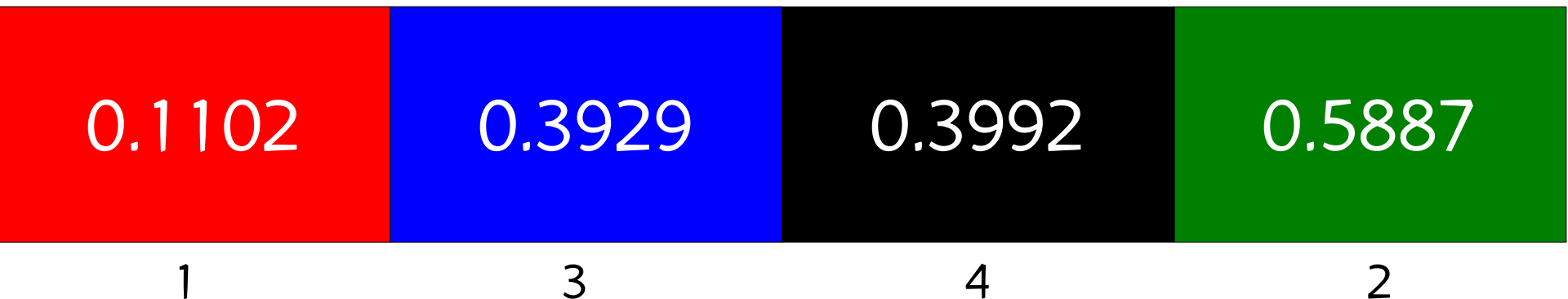
0.3992

# Random-key genetic algorithms

Decoder takes vector of random keys as input and outputs solution to optimization problem



Sorting, for example, ... gives us a permutation

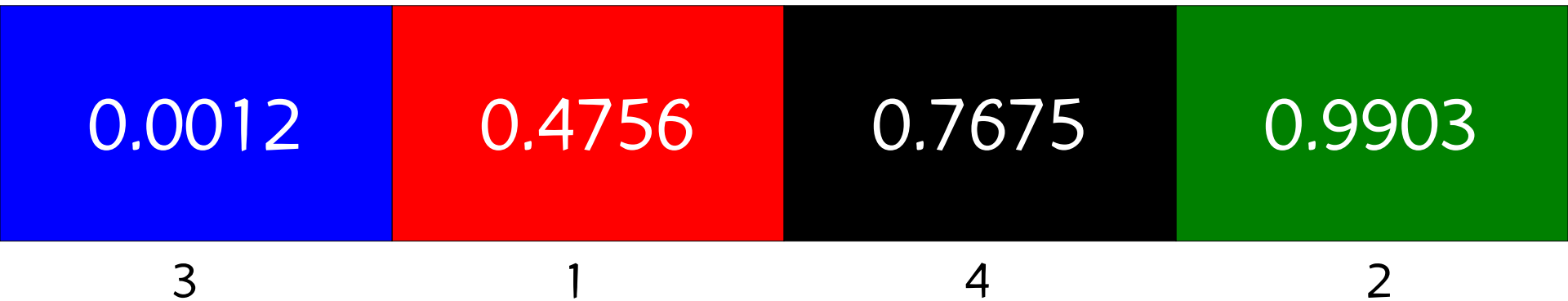




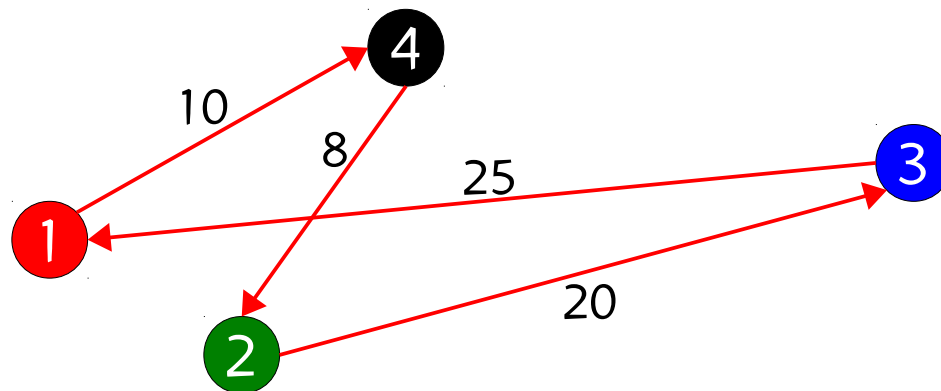
# Random-key genetic algorithms

Bean (1994) – proposed a GA based on random keys for problems whose solutions can be encoded as permutations (e.g. sequencing, assignment, TSP)

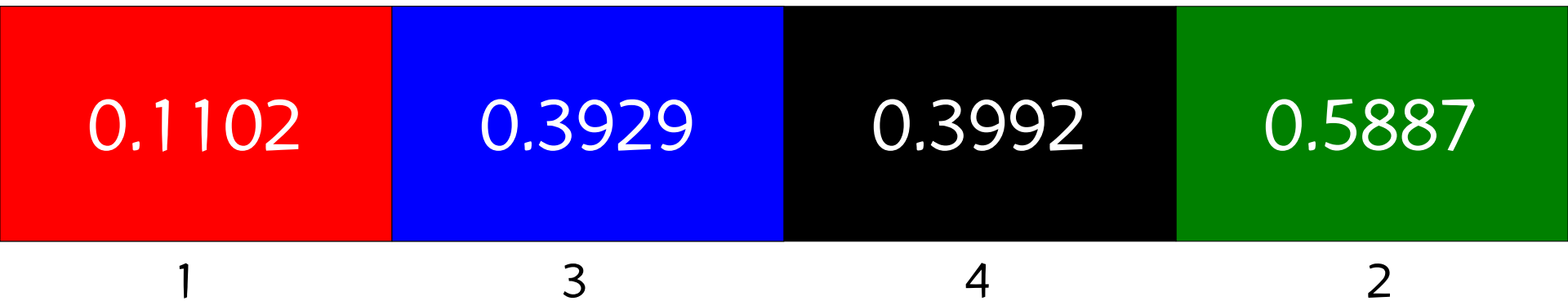
# Random-key genetic algorithms



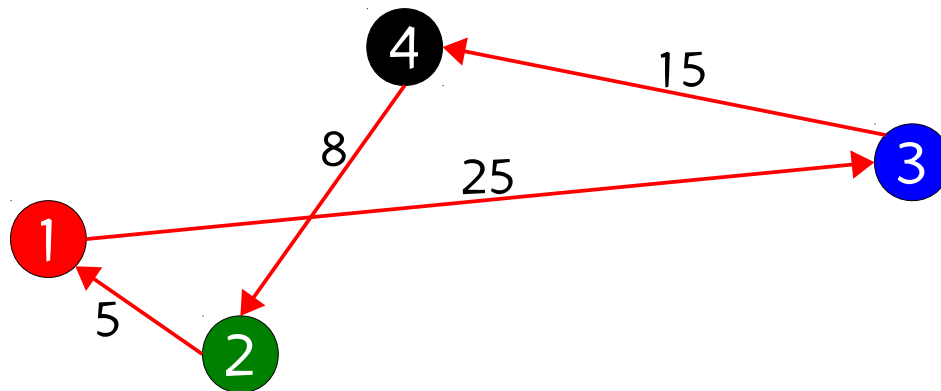
TSP tour length: 63



# Random-key genetic algorithms



TSP tour length: 53



# Random-key genetic algorithms

0.0143

4

0.2384

1

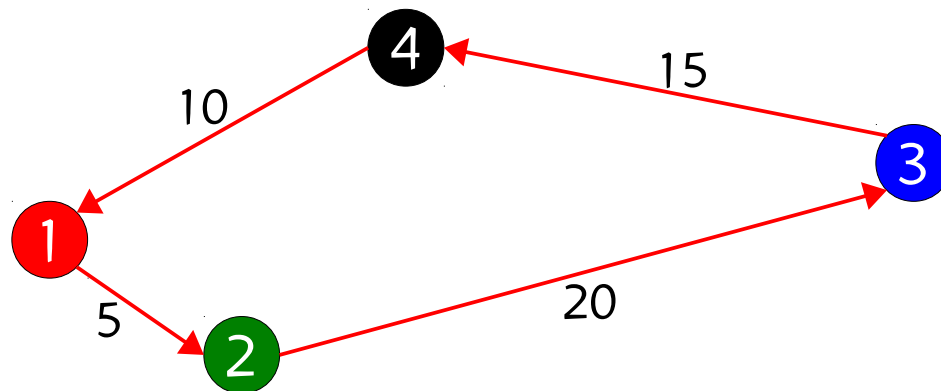
0.8394

2

0.9234

3

TSP tour length: 50  
Optimal tour!



# Random-key genetic algorithms

0.1029

2

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1

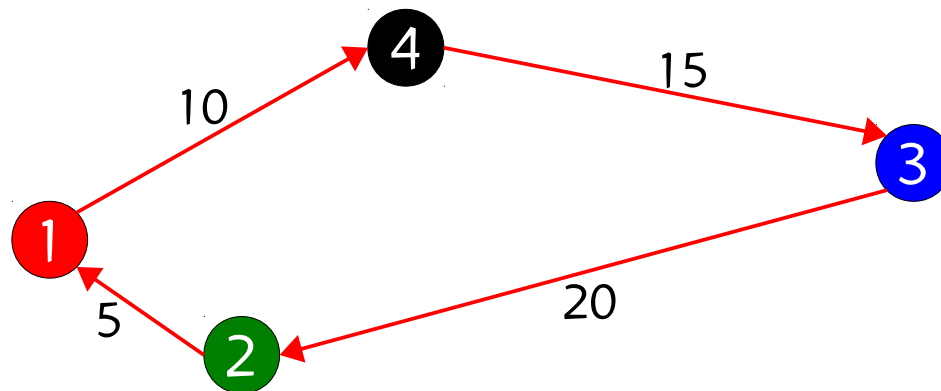
0.3233

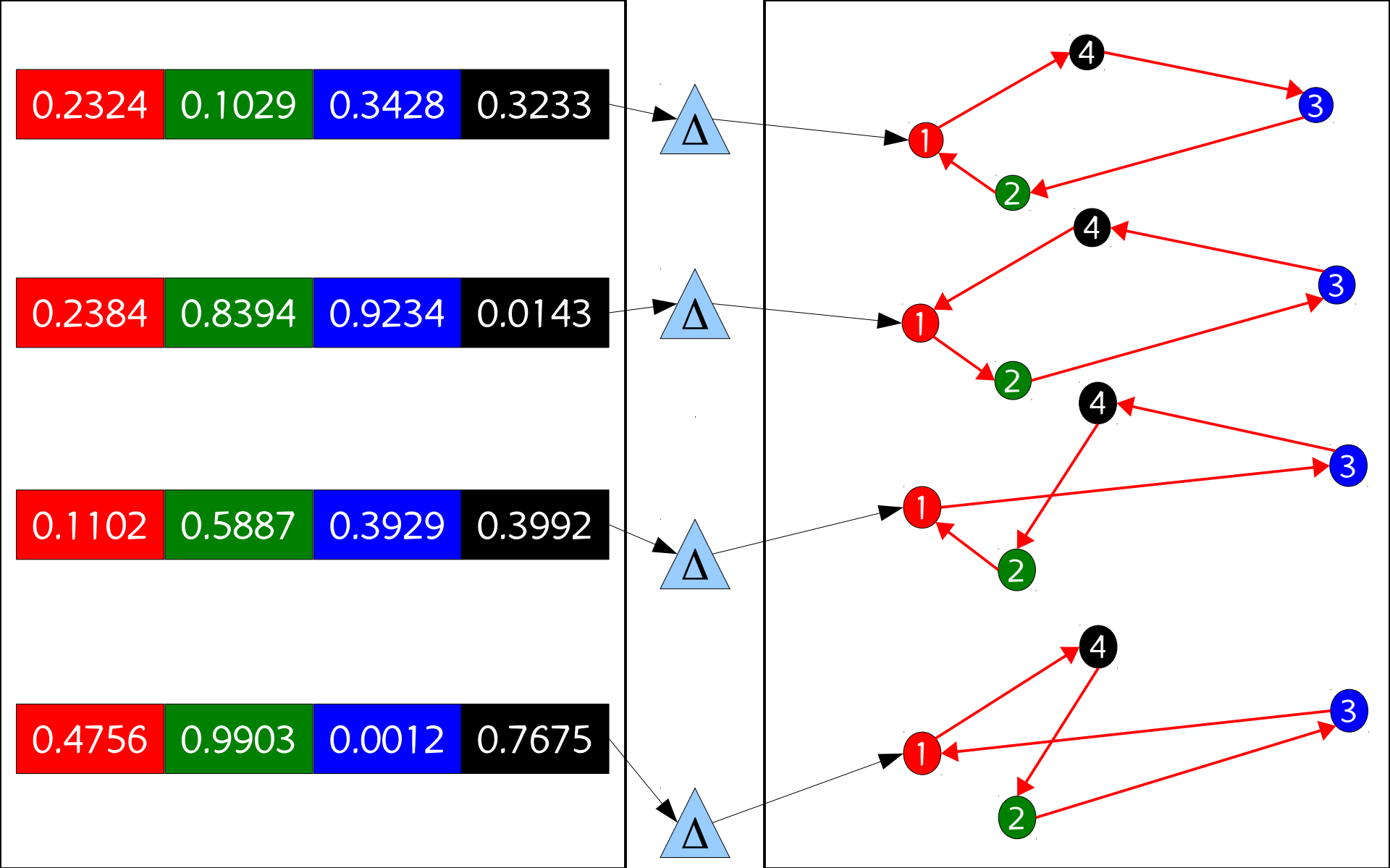
4

0.3428

3

TSP tour length: 50  
Another optimal tour!





$n$ -dim continuous unit hypercube

decoder

solution space

# Feasibility-preserving property

Given a valid decoder and two random-key vectors,  
each corresponding to a feasible solution ...

tour length: 50

0.2324

0.1029

0.3428

0.3233

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0.7675

Any combination (flipping coin) of the two

tour length: 63

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0.9903

0.3428

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also corresponds to a feasible solution.

tour length: 53

# Parameterized uniform crossover

[ Spears & DeJong, 1991]

Flip biased coin (probability  $p$  of resulting in heads)  $n$  times  
for random-key vector of size  $n$

tour length: 50

0.2324	0.1029	0.3428	0.3233
0.4756	0.9903	0.0012	0.7675

tour length: 63

$p = 0.7$

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0.6872 (H)

0.2324

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0.6872 (H)	0.7802 (T)
0.2324	0.9903

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tour length: 63

$p = 0.7$

0.6872 (H)	0.7802 (T)	0.1234 (H)	0.9278 (T)
0.2324	0.9903	0.3428	0.7675

tour length: 53



# Bean's algorithm

- Start with population of  $P$  vectors of  $n$  random keys
- Evolve population until stopping criterion is satisfied
- Best decoded solution from vectors in final population is output as solution of the algorithm

# Evolution in Bean's algorithm

- Decode and evaluate all new vectors of random keys
- Partition population into a small set of  $P_E$  elite solutions and  $P - P_E$  non-elite solutions
- Copy all  $P_E$  elite vectors to new population
- Generate  $P_M$  vectors of random keys (mutants) in new population
- Apply parameterized uniform crossover on  $P - P_E - P_M$  pairs of vectors of random keys chosen at random from entire population and add each resulting vector to new population

# Biased random key genetic algorithm

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- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.

# How RKGA & BRKGA differ

## RKGA

## BRKGA

both parents chosen at random  
from entire population

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both parents chosen at random  
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both parents chosen at random  
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either parent can be associated  
with heads in parametrized  
uniform crossover coin flip

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best fit parent is associated  
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## BRKGA

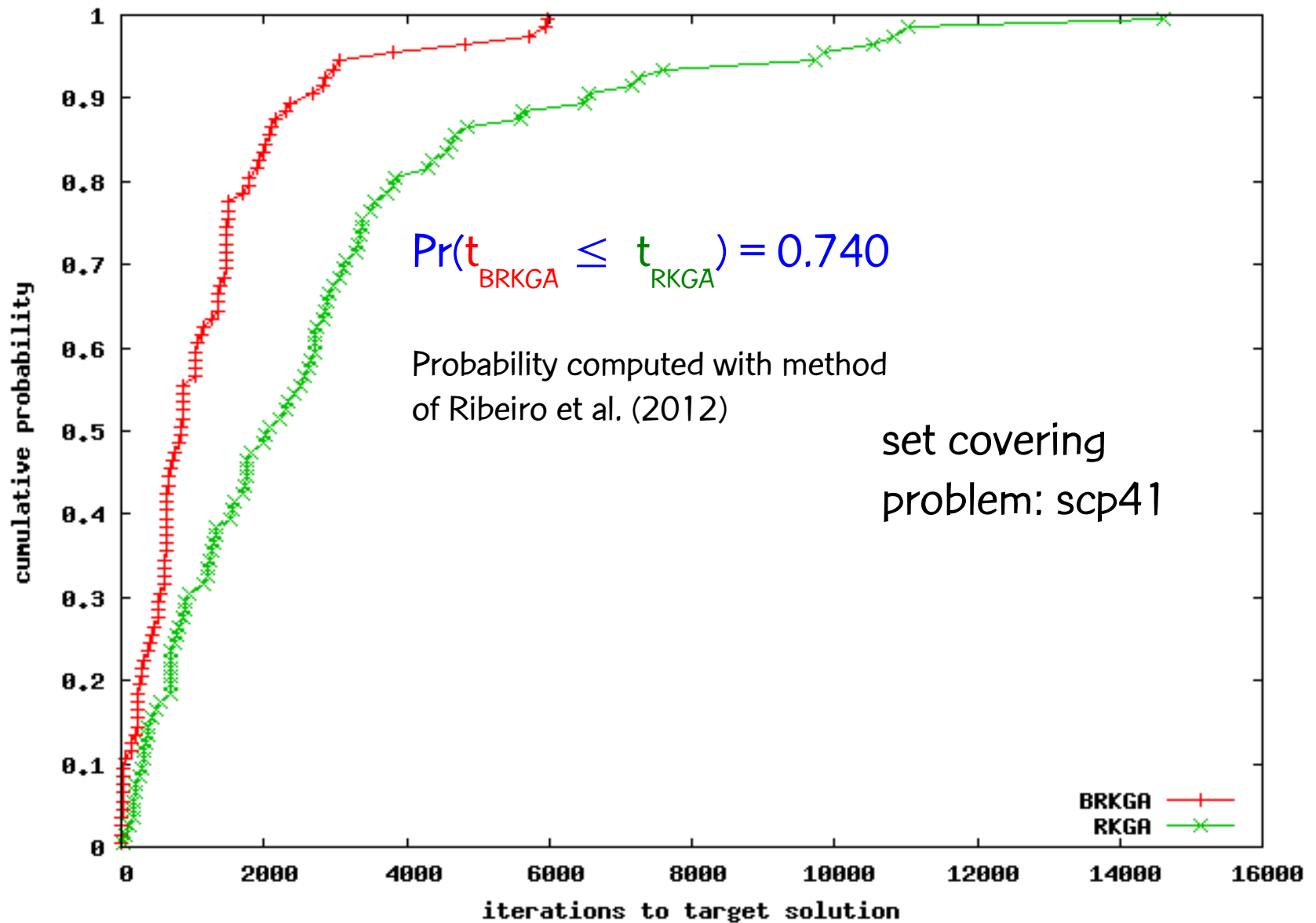
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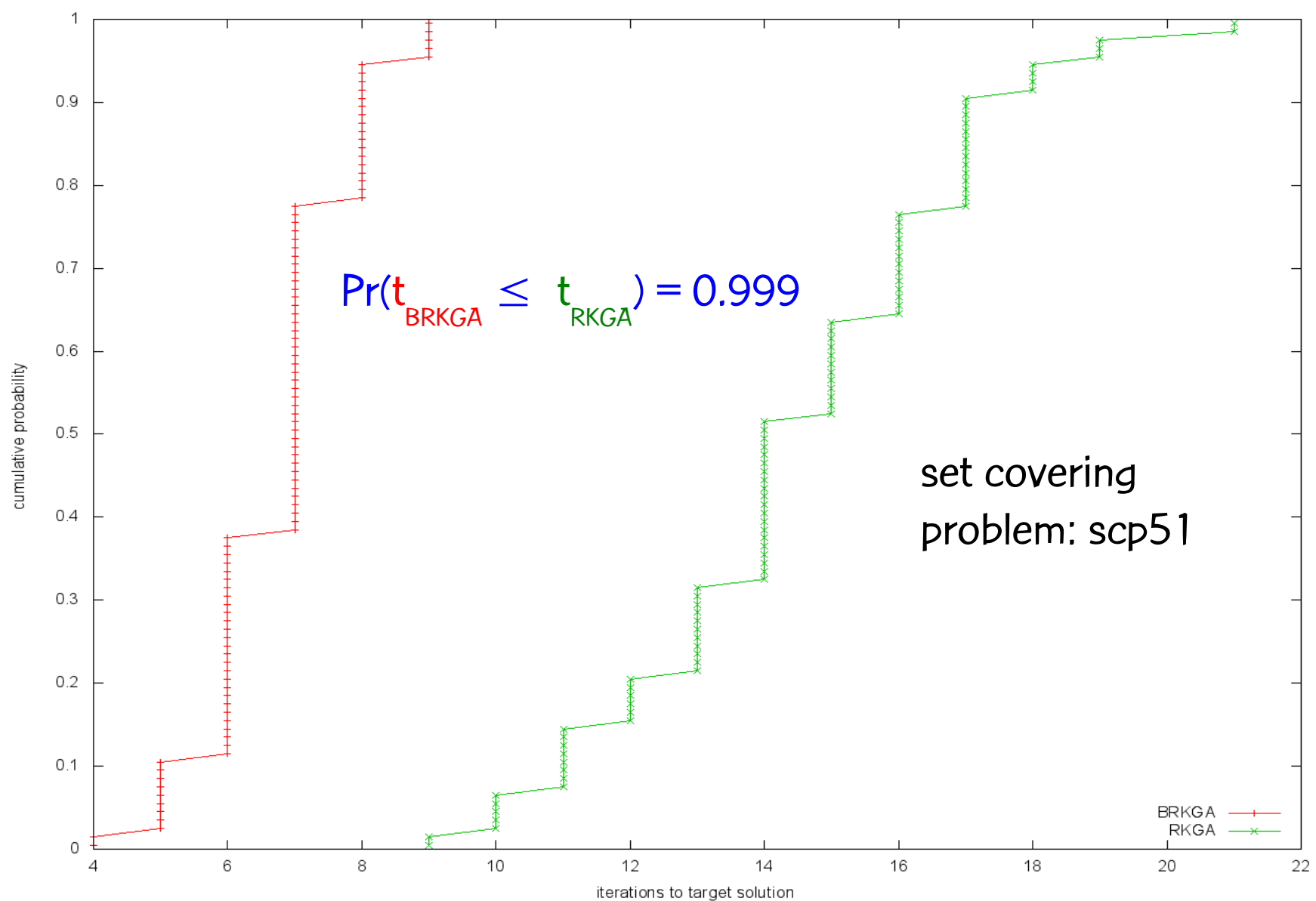
best fit parent is associated  
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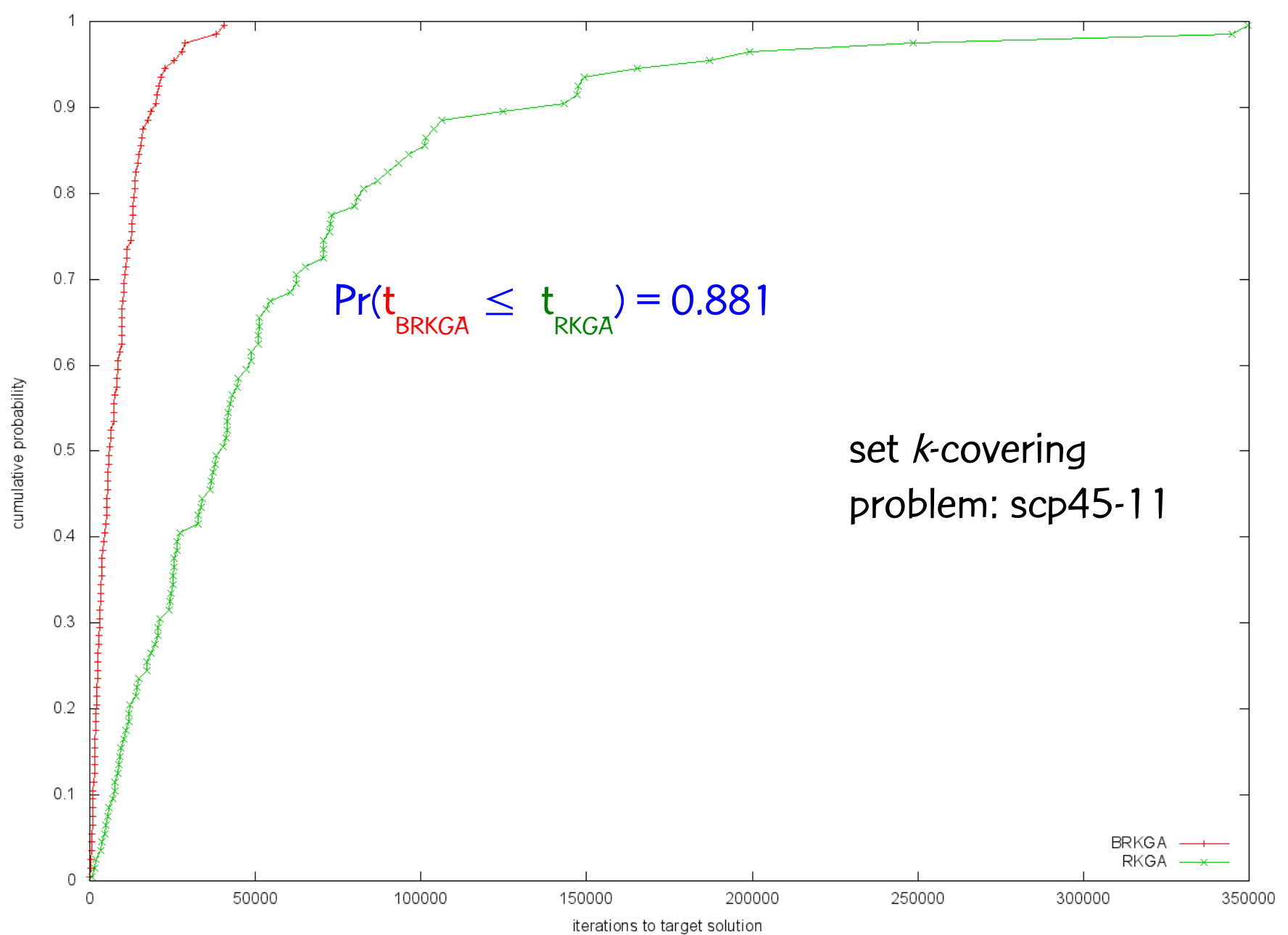
This adds "survival of the fittest"  
to RKGA

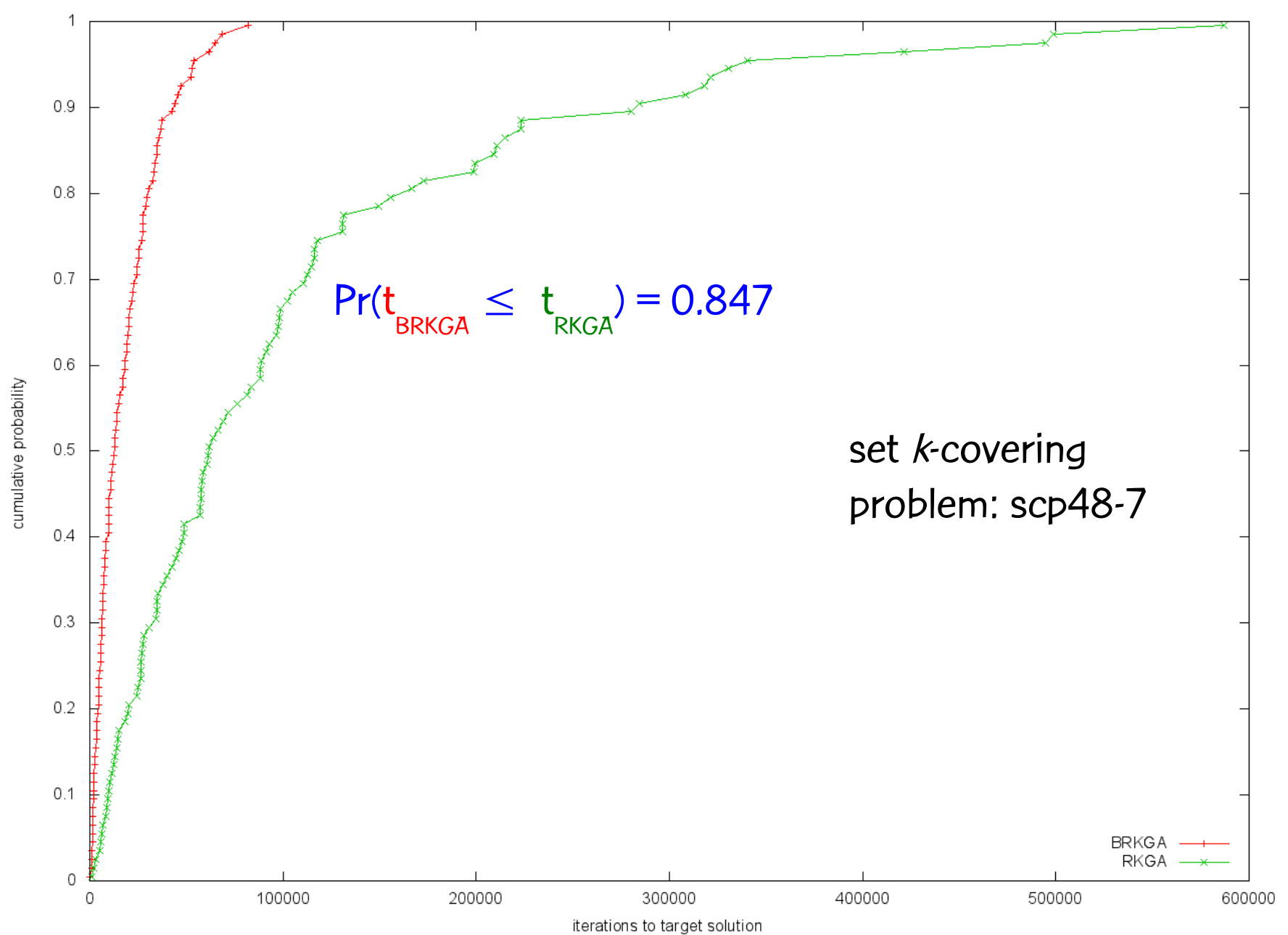
# Paper comparing BRKGA and Bean's Method

Gonçalves, M.G.C.R., and Toso, “Biased and unbiased random-key genetic algorithms: An experimental analysis”, *Proceedings of the 10<sup>th</sup> Metaheuristics International Conference, Singapore, August 2013.*

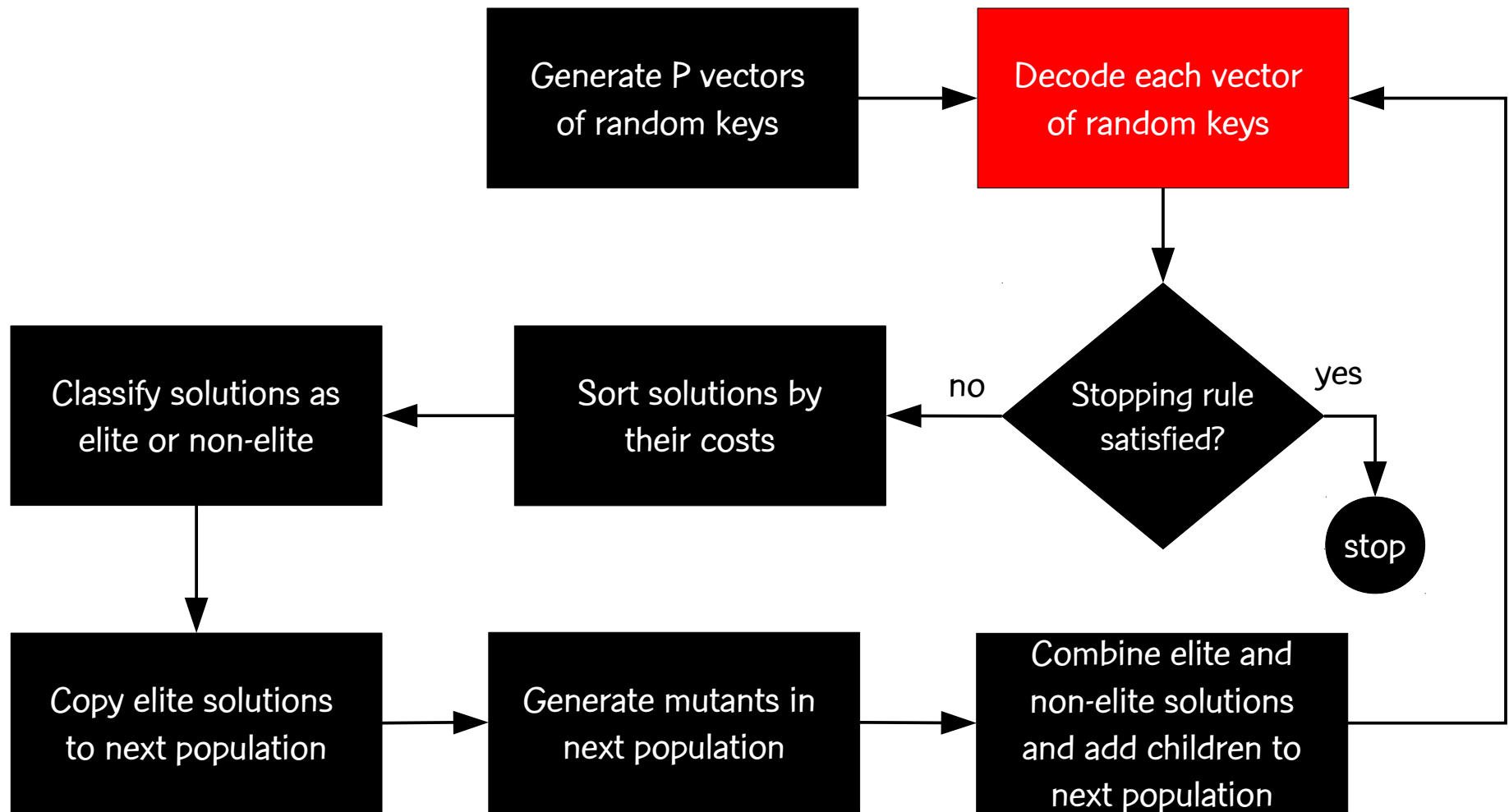




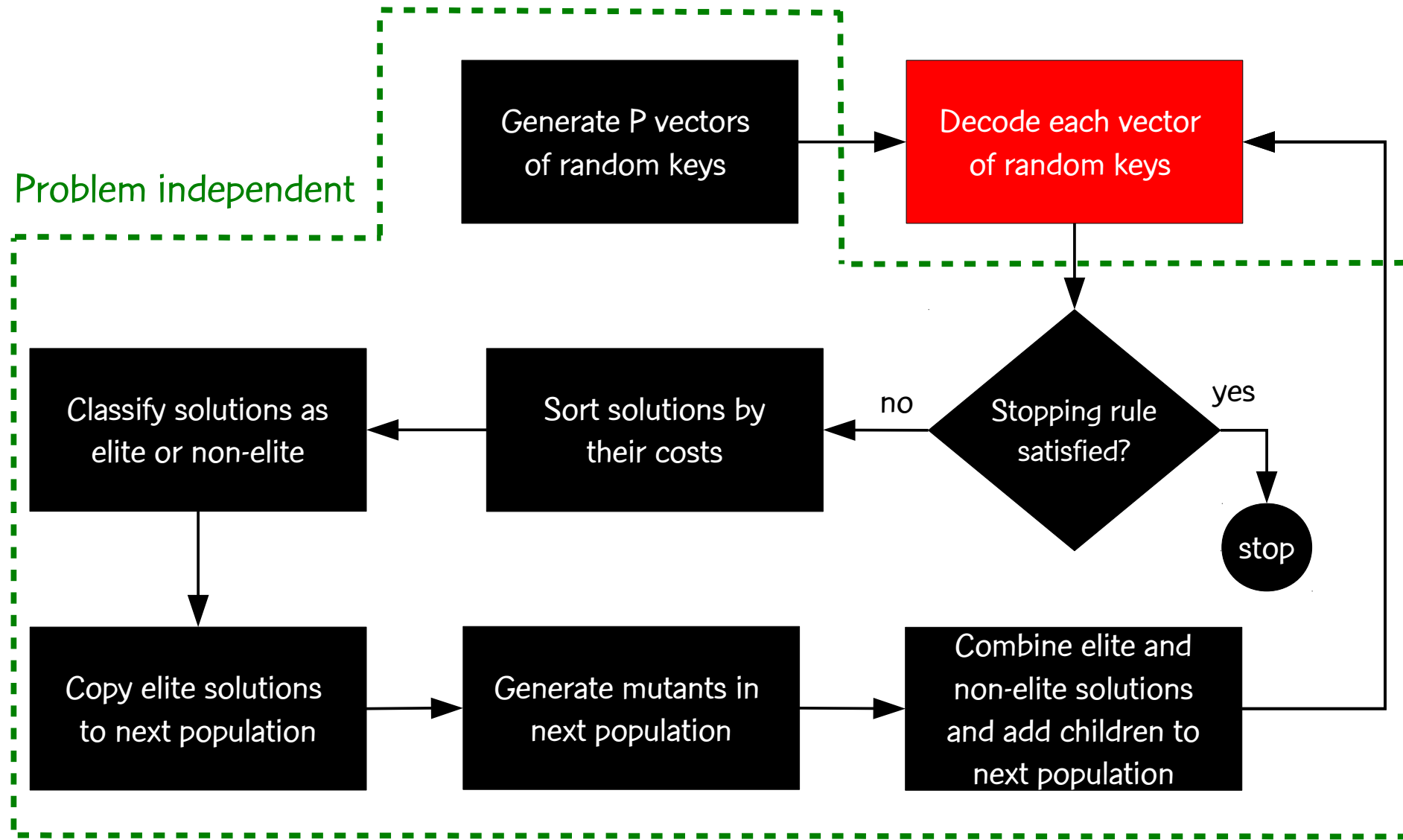




# Framework for biased random-key genetic algorithms

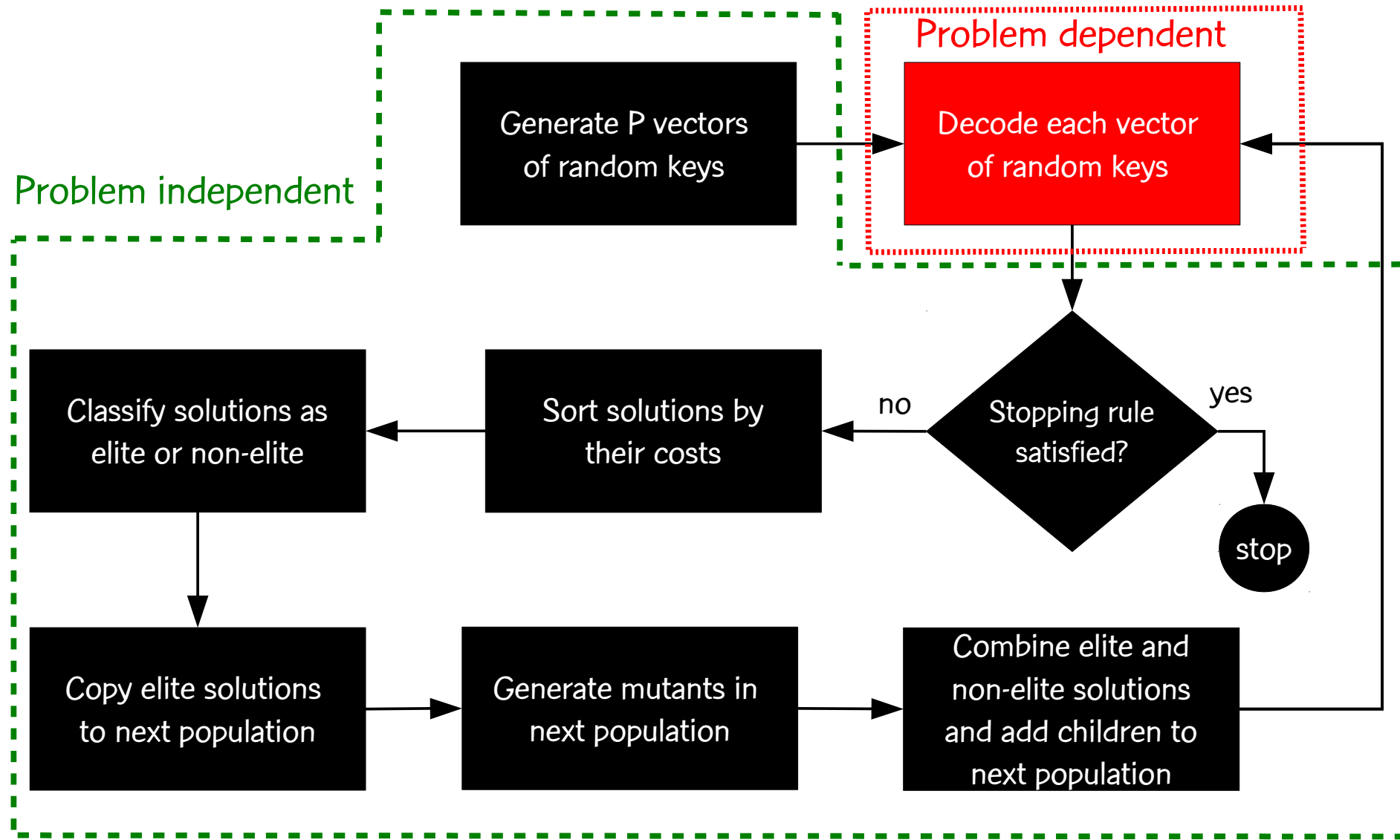


# Framework for biased random-key genetic algorithms

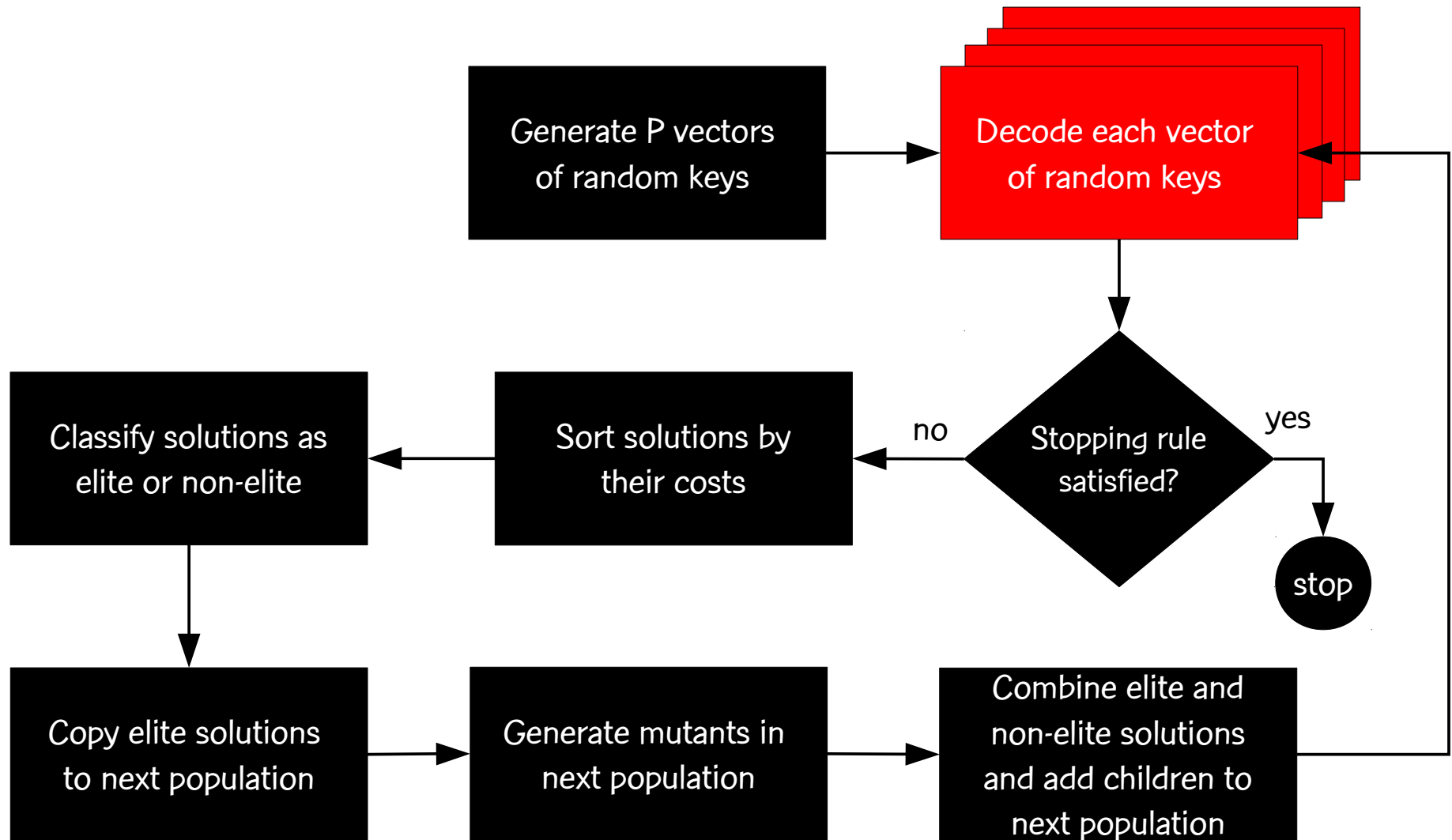




# Framework for biased random-key genetic algorithms



# Decoding of random key vectors can be done in parallel



# Specifying a BRKGA

# Specifying a biased random-key GA

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- Parameters

# Specifying a biased random-key GA

## Parameters:

- Size of population
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Stopping criterion

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# Specifying a biased random-key GA

## Parameters:

- Size of population: a function of  $N$ , say  $N$  or  $2N$
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- Size of mutant set
- Child inheritance probability
- Stopping criterion

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- Stopping criterion: e.g. time, # generations, solution quality, # generations without improvement

# brkgaAPI: A C++ API for BRKGA

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- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.



# brkgaAPI: A C++ API for BRKGA

Paper: Rodrigo F. Toso and M.G.C.R., "A C++  
Application Programming Interface for  
Biased Random-Key Genetic Algorithms,"  
*Optimization Methods & Software*, published online 13 Mar 2014.

Software: <http://github.com/rfrancotoso/brkgaAPI>

# Packing weighted rectangles with a BRKGA

# Reference



J.F. Gonçalves and M.G.C.R., “A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal packing problem,” *Journal of Combinatorial Optimization*, vol. 22, pp. 180-201, 2011.

Tech report:

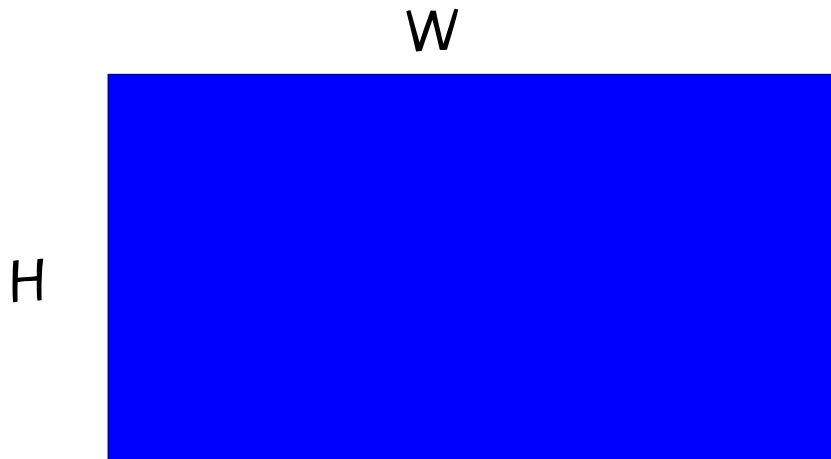
<http://www.research.att.com/~mgcr/doc/pack2d.pdf>

# Constrained orthogonal packing

- Given a large planar stock rectangle ( $W$ ,  $H$ ) of width  $W$  and height  $H$ ;

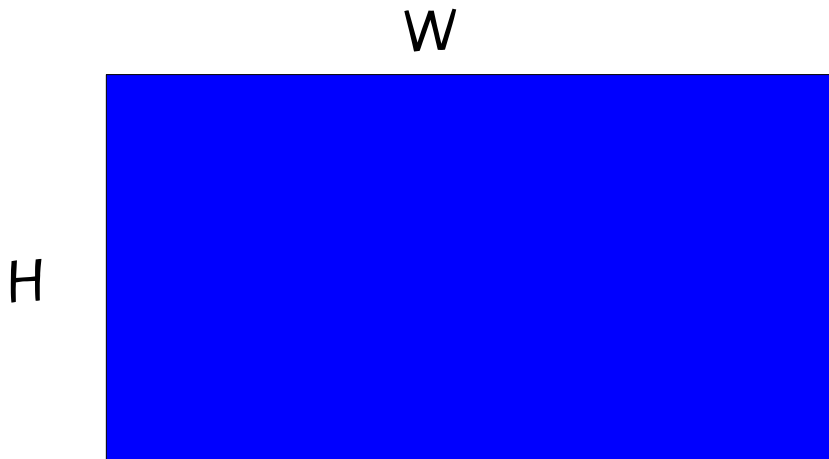
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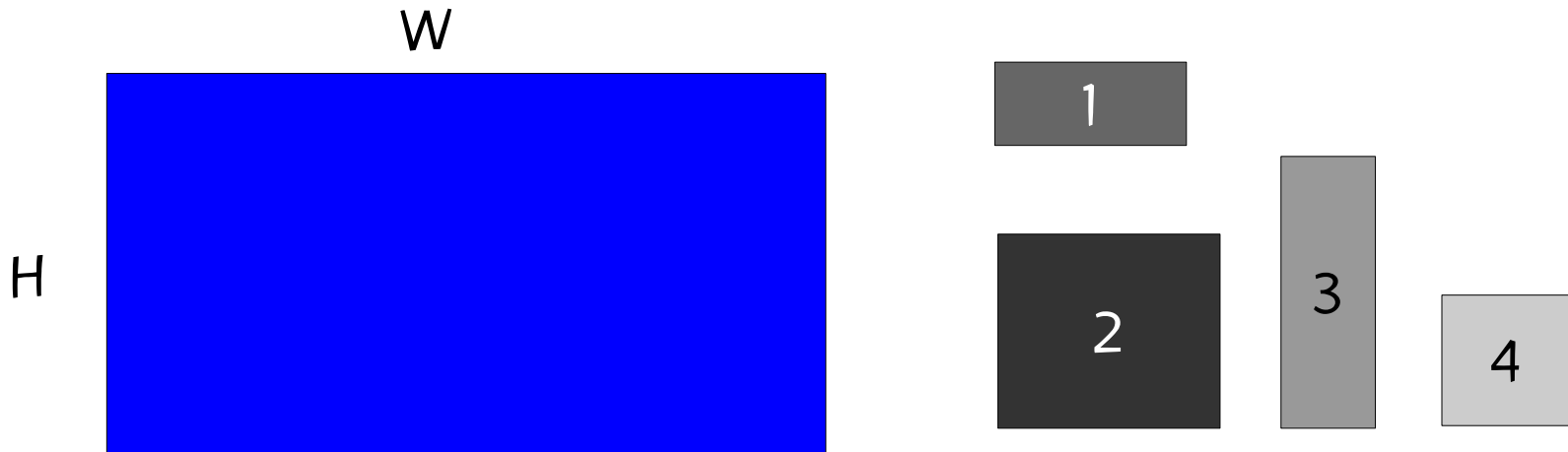
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- Given a large planar stock rectangle ( $W$ ,  $H$ ) of width  $W$  and height  $H$ ;
- Given  $N$  smaller rectangle types ( $w[i]$ ,  $h[i]$ ),  $i = 1, \dots, N$ , each of width  $w[i]$ , height  $h[i]$ , and value  $v[i]$ ;



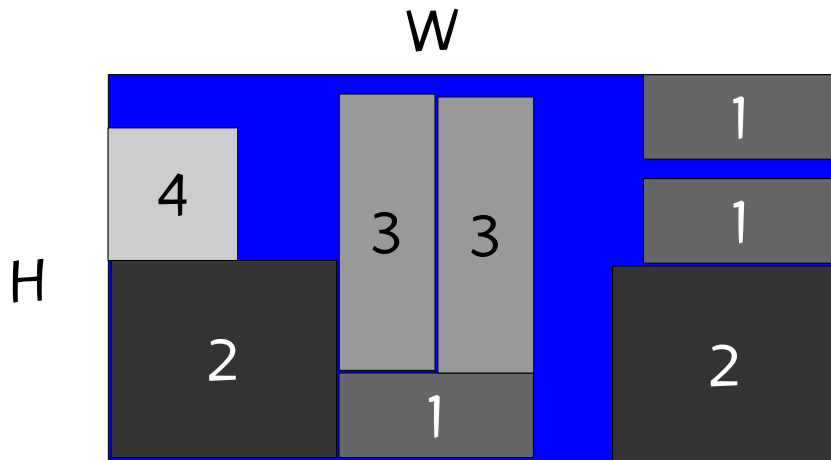
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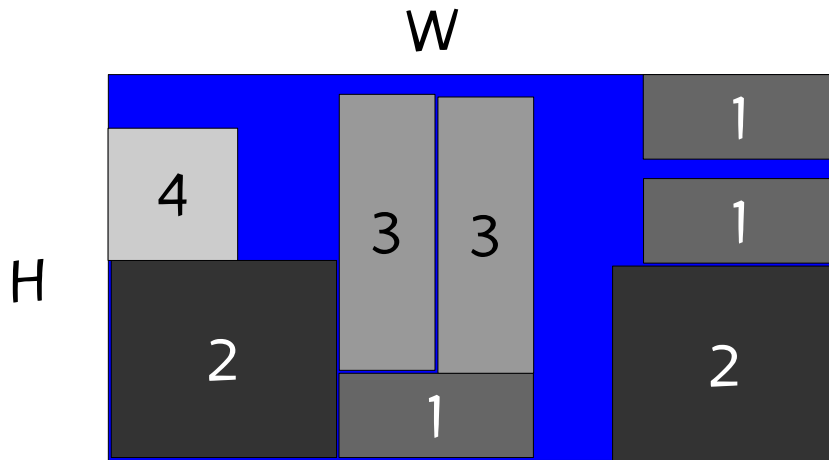




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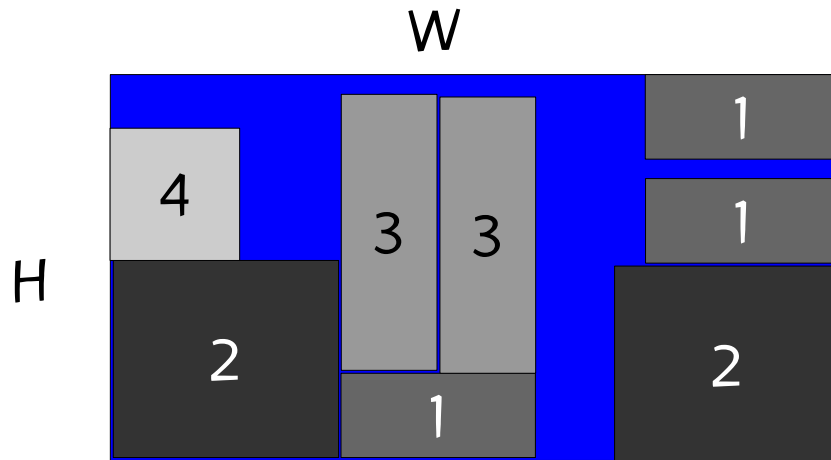
$$0 \leq P[i] \leq r[i] \leq Q[i]$$



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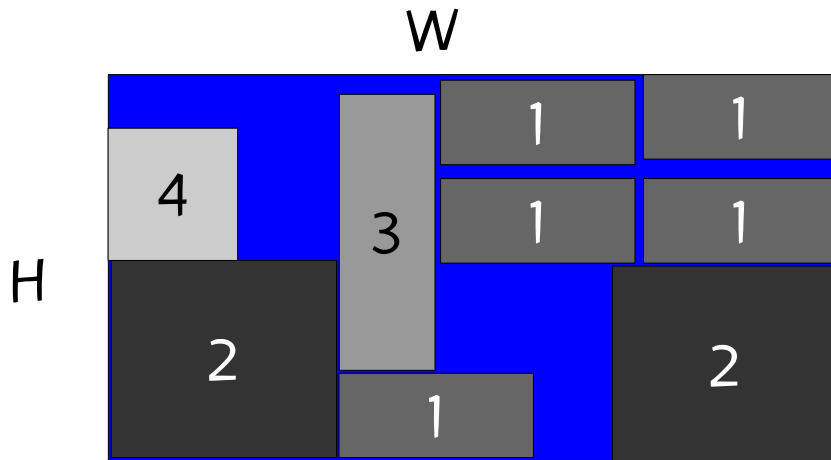


Suppose  $5 \leq r[1] \leq 12$

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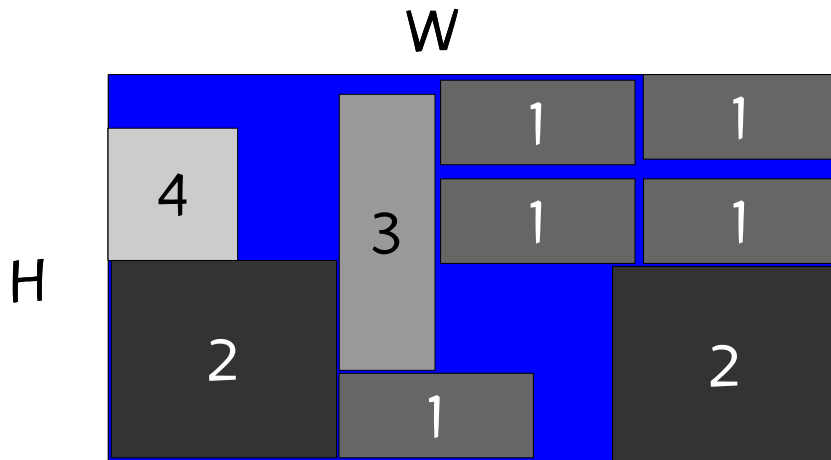


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# Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

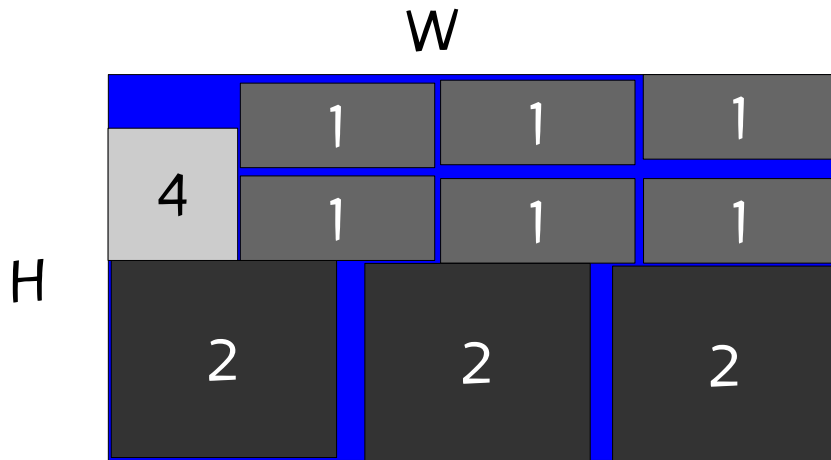
$$v[1] r[1] + v[2] r[2] + \dots + v[N] r[N]$$



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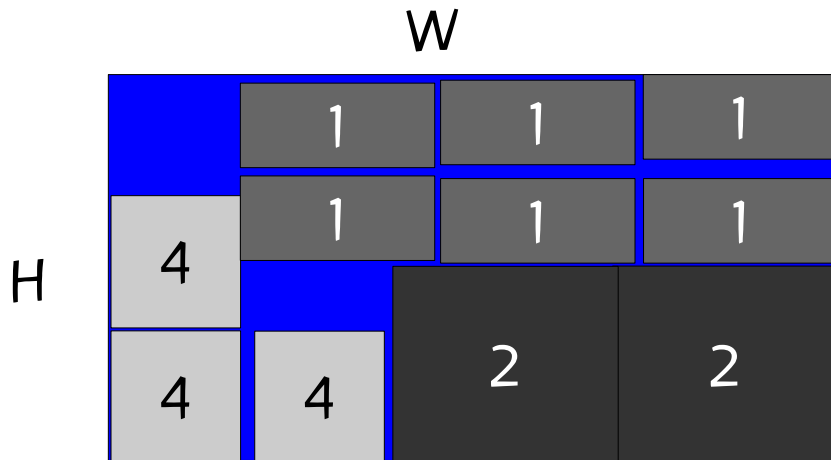
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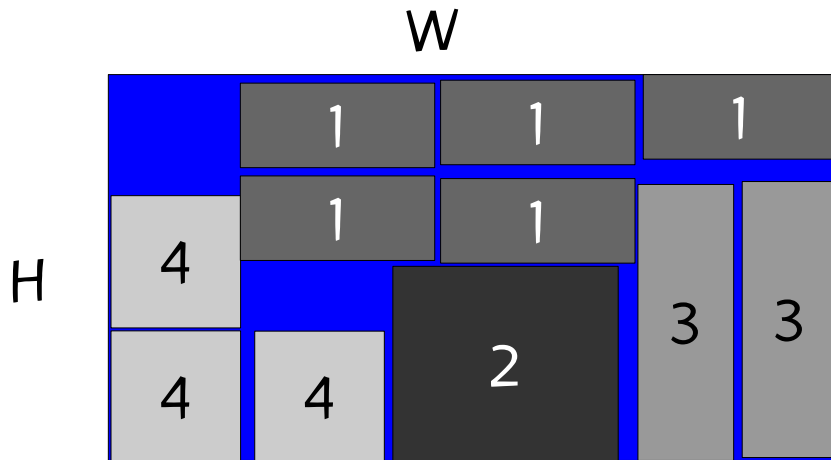
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# Applications

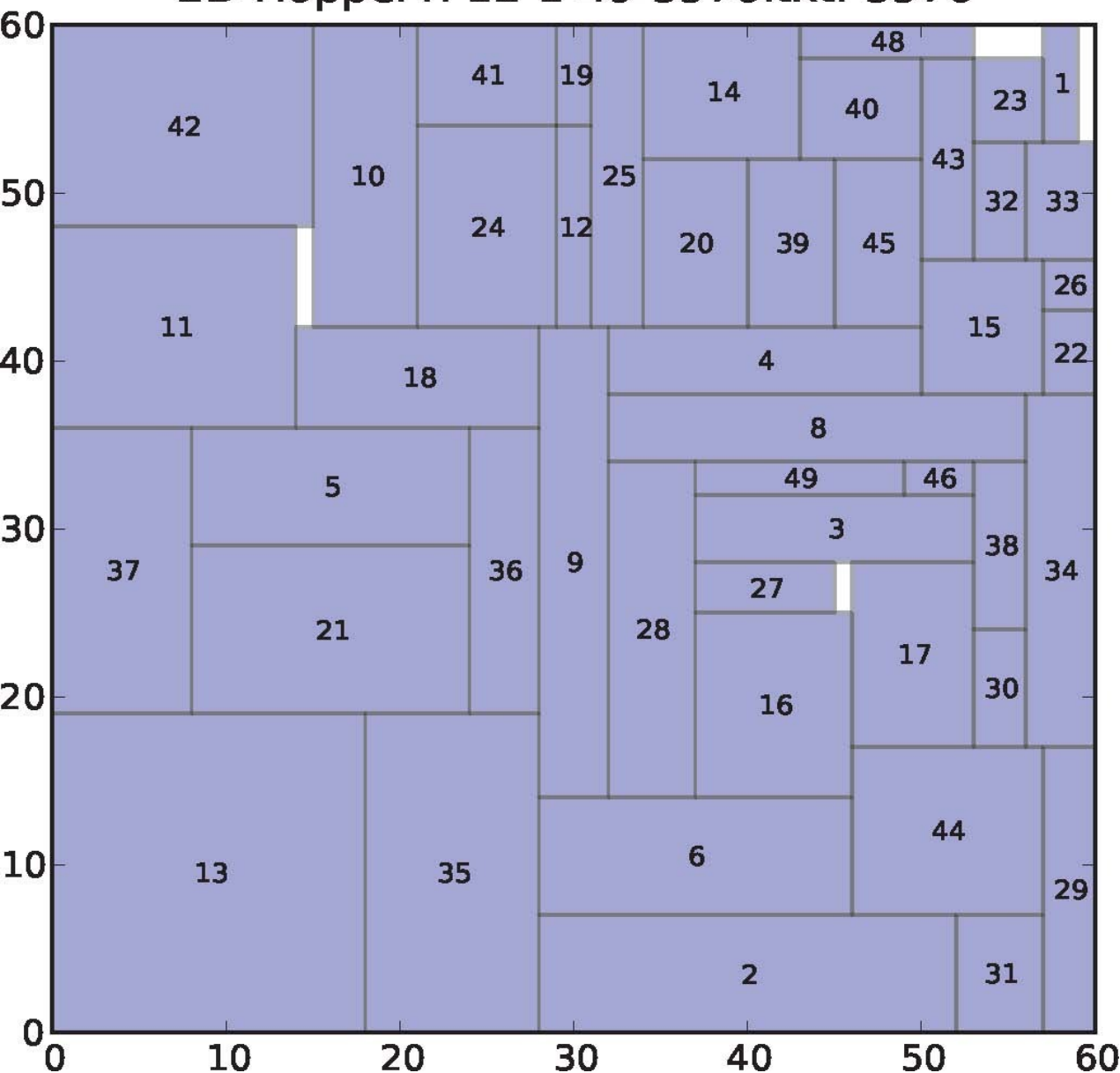
Problem arises in several production processes, e.g.

- Textile
- Glass
- Wood
- Paper

where rectangular figures are cut from large rectangular sheets of materials.



## 2D-HopperTP12-1-49-3576.txt: 3576



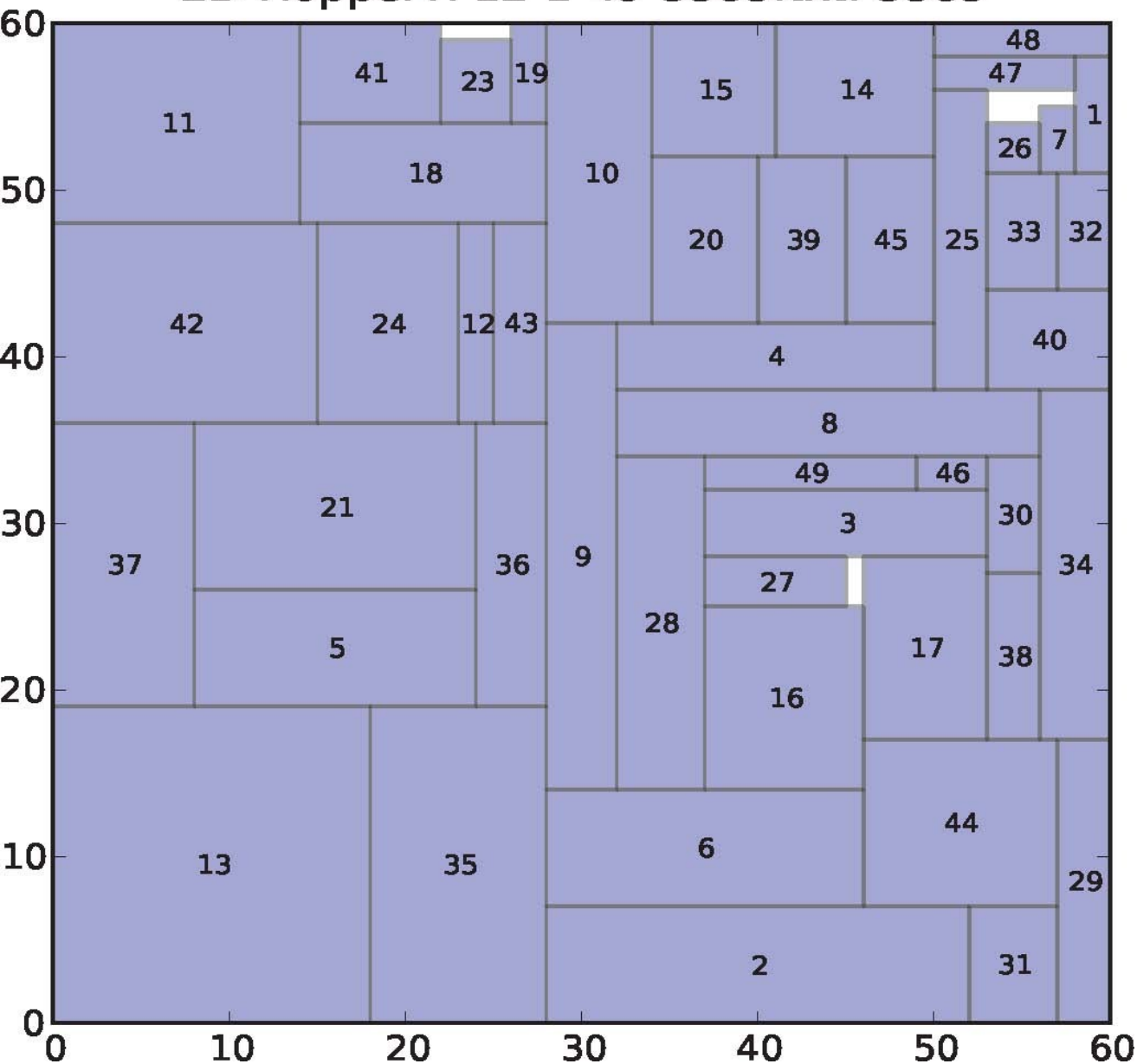
Hopper & Turton, 2001

Instance 4-1 60 x 60

Value: 3576

Previous best: 3580 by a  
Tabu Search heuristic  
(Alvarez-Valdes et al., 2007)

## 2D-HopperTP12-1-49-3585.txt: 3585



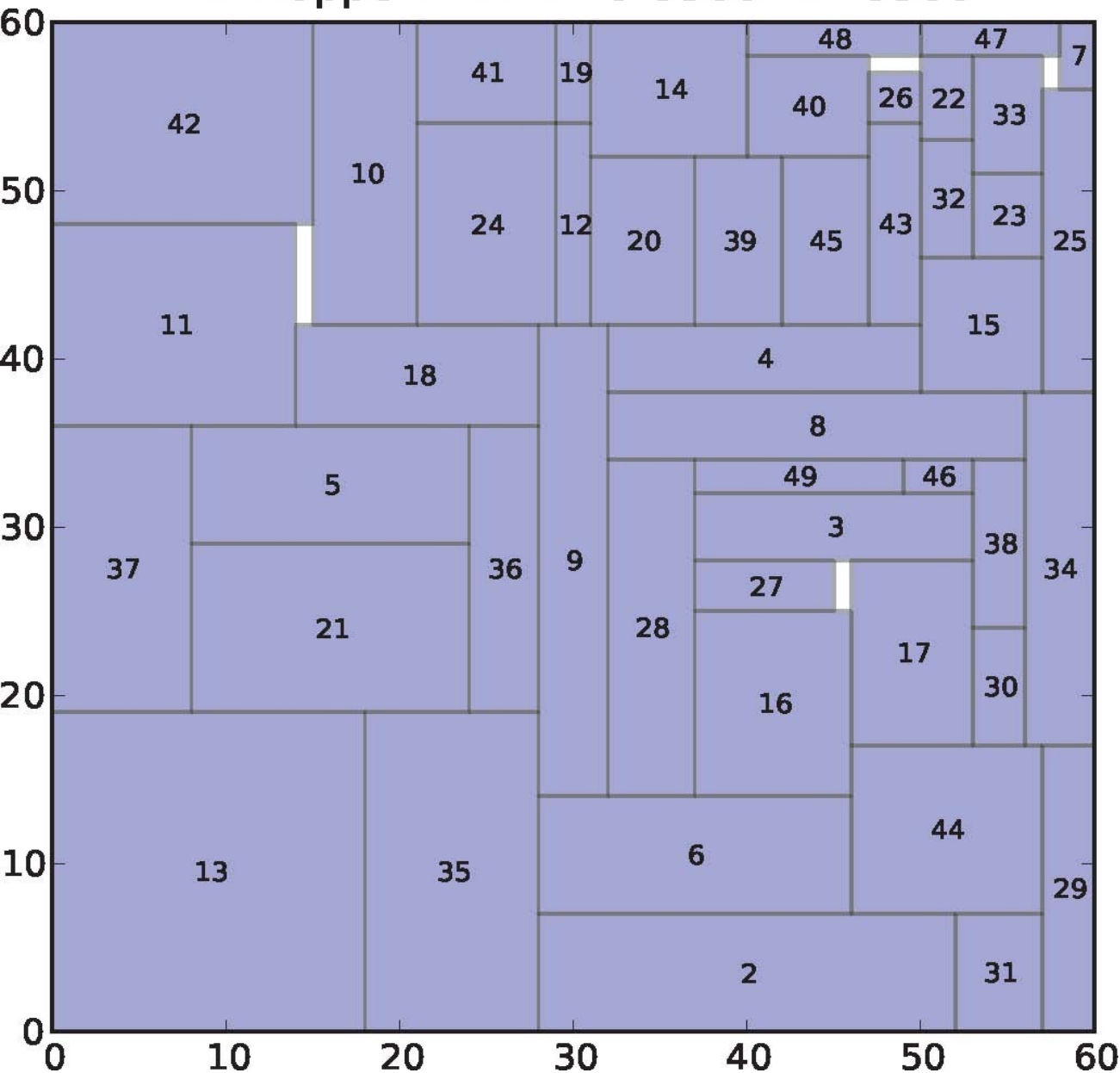
Hopper & Turton, 2001

Instance 4-2 60 x 60

Value: 3585

Previous best: 3580 by a  
Tabu Search heuristic  
(Alvarez-Valdes et al., 2007)

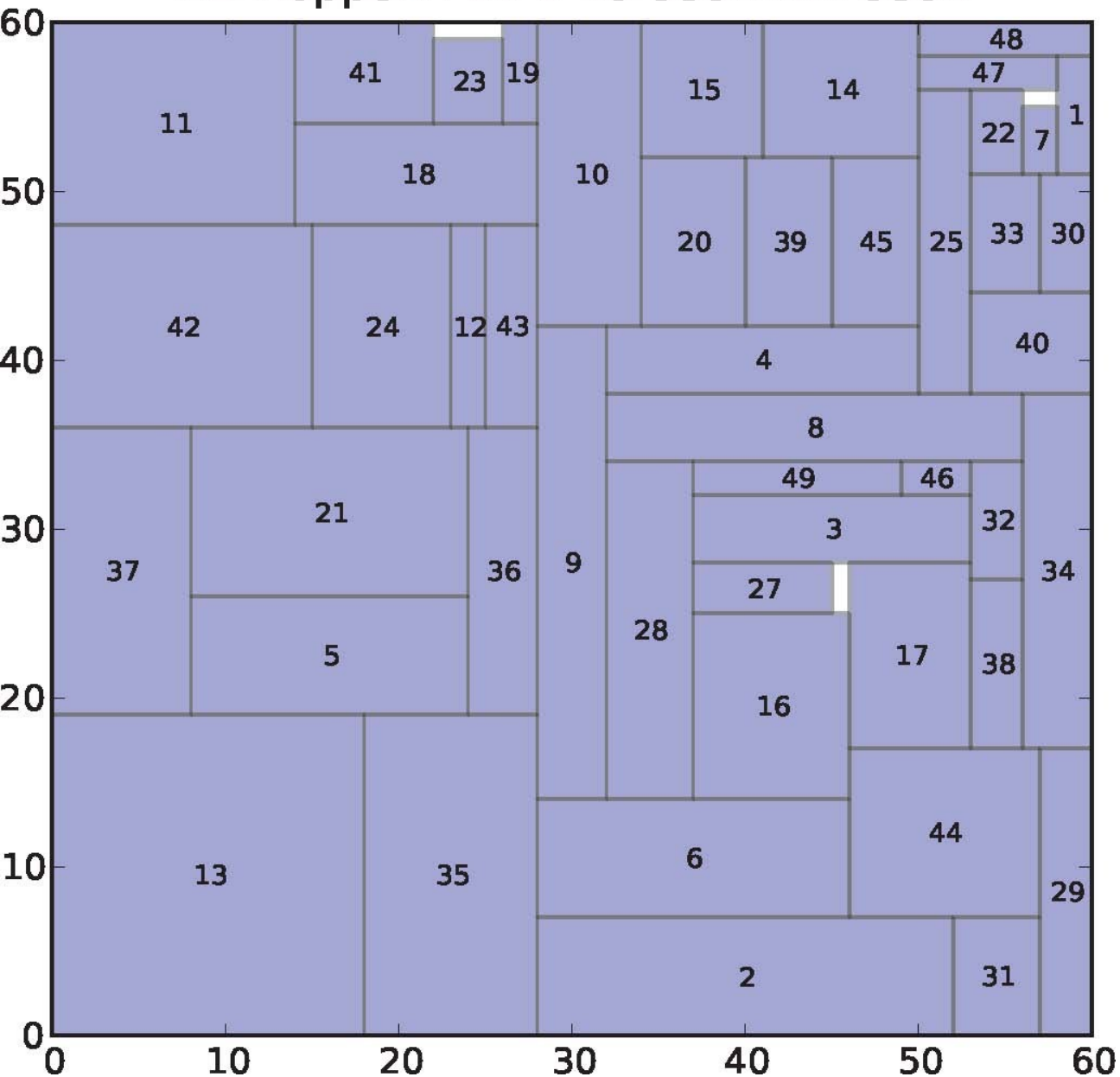
## 2D-HopperTP12-1-49-3586.txt: 3586



Hopper & Turton, 2001  
Instance 4-2 60 x 60  
Value: 3586

Previous best: 3580 by a  
Tabu Search heuristic  
(Alvarez-Valdes et al., 2007)

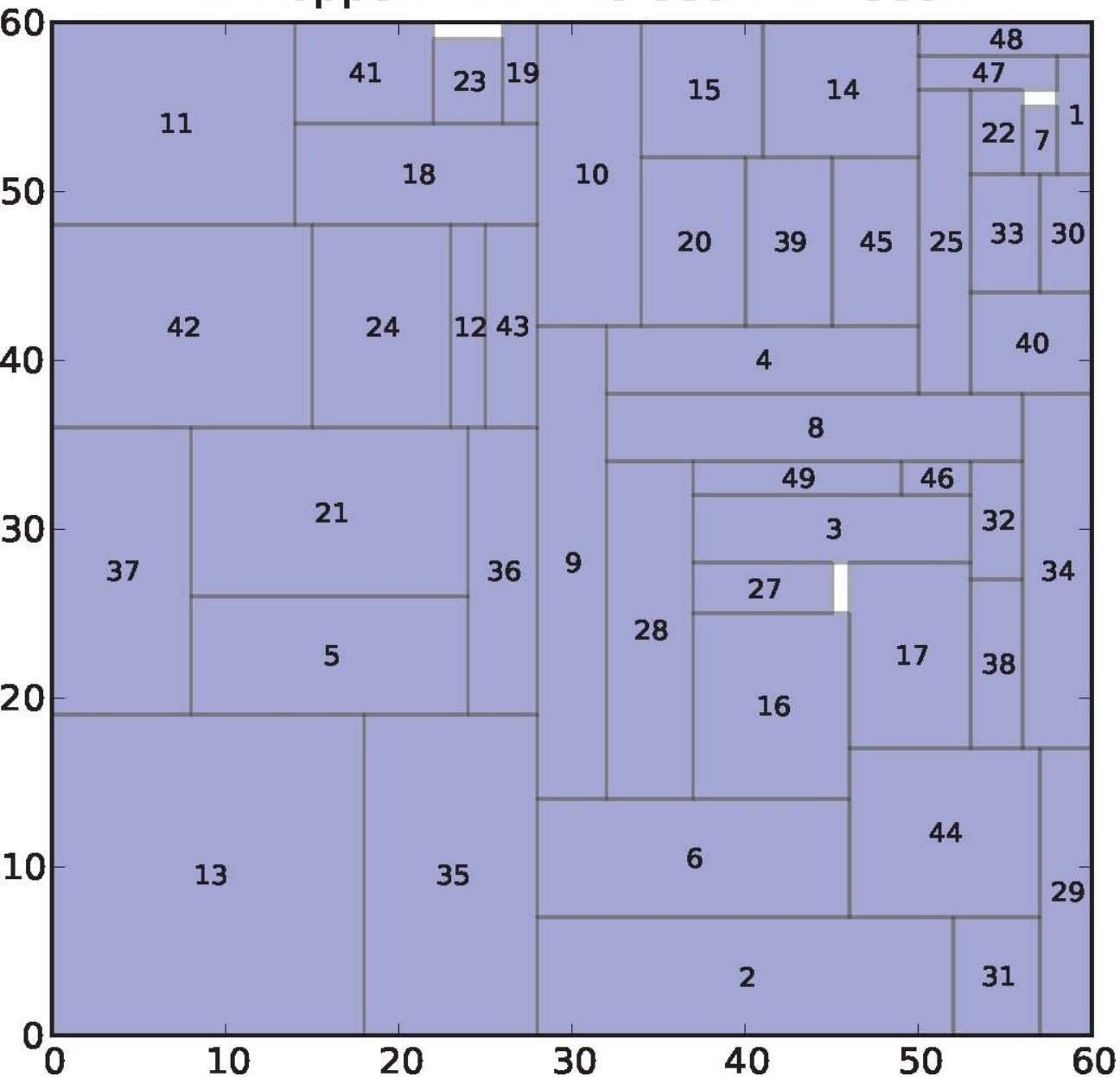
## 2D-HopperTP12-1-49-3591.txt: 3591



Hopper & Turton, 2001  
Instance 4-2 60 x 60  
Value: 3591

Previous best: 3580 by a  
Tabu Search heuristic  
(Alvarez-Valdes et al., 2007)

## 2D-HopperTP12-1-49-3591.txt: 3591



Hopper & Turton, 2001

Instance 4-2 60 x 60

Value: 3591

New best known solution!

Previous best: 3580 by a  
Tabu Search heuristic  
(Alvarez-Valdes et al., 2007)

# BRKGA for constrained 2-dim orthogonal packing

# Encoding

- Solutions are encoded as vectors  $X$  of  
$$2N' = 2 \{ Q[1] + Q[2] + \dots + Q[N] \}$$
random keys, where  $Q[i]$  is the maximum number of rectangles of type  $i$  (for  $i = 1, \dots, N$ ) that can be packed.
- $X = ( X[1], \dots, X[N'], \quad X[N'+1], \dots, X[2N'] )$

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# Decoding

- Simple heuristic to pack rectangles:
  - Make  $Q[i]$  copies of rectangle  $i$ , for  $i = 1, \dots, N$ .
  - Order the  $N' = Q[1] + Q[2] + \dots + Q[N]$  rectangles in some way.
  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: **bottom-left (BL)** or **left-bottom (LB)**. If **rectangle cannot be positioned, discard it** and go on to the next rectangle in the order.

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# Decoding

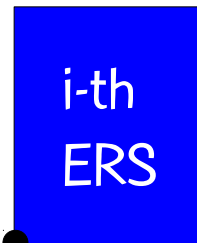
- Simple heuristic to pack rectangles:
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  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: **bottom-left (BL)** or **left-bottom (LB)**. If **rectangle cannot be positioned, discard it** and go on to the next rectangle in the order. **Use the last  $N'$  keys to determine which heuristic to use. If  $X[N'+i] > 0.5$  use LB, else use BL.**

# Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
- Let  $(x[i], y[i])$  be the coordinates of the bottom left corner of the  $i$ -th ERS.

# Decoding

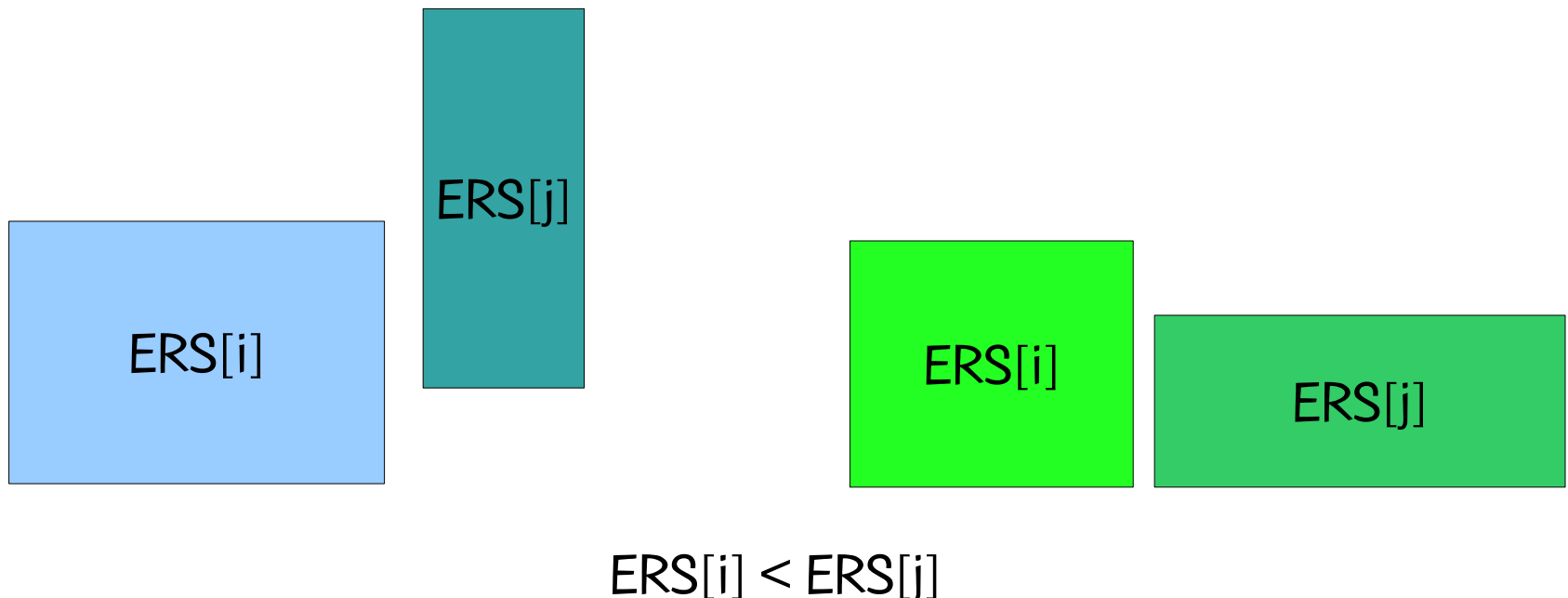
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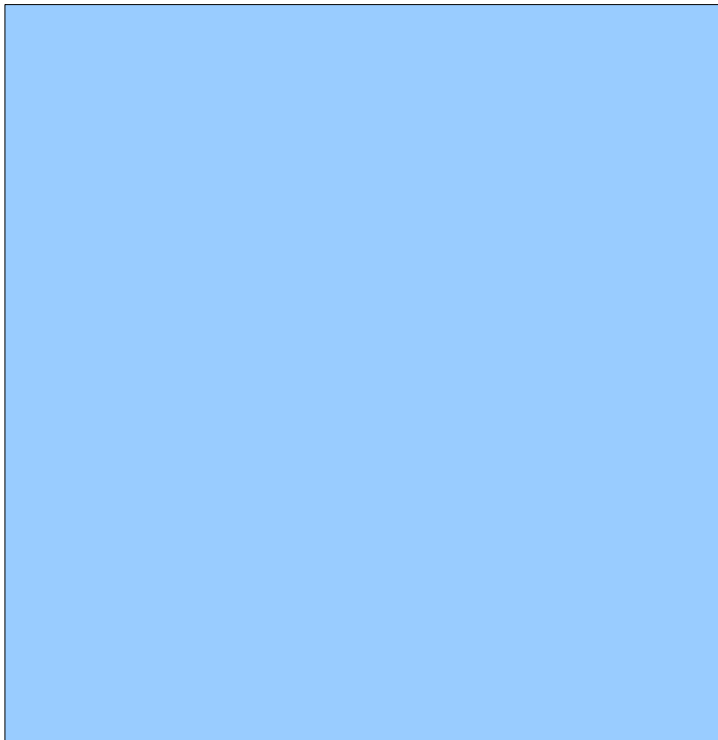
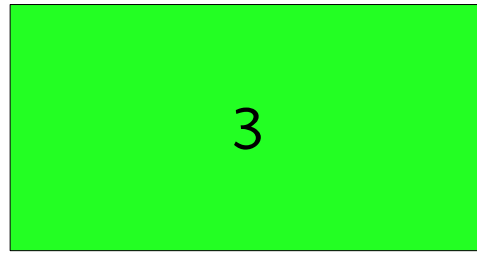
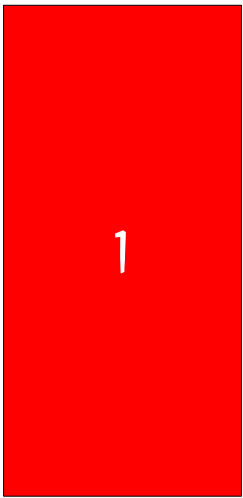


$(x[i], y[i])$

# Decoding

- If BL is used, ERSs are ordered such that  $ERS[i] < ERS[j]$  if  $y[i] < y[j]$  or  $y[i] = y[j]$  and  $x[i] < x[j]$ .



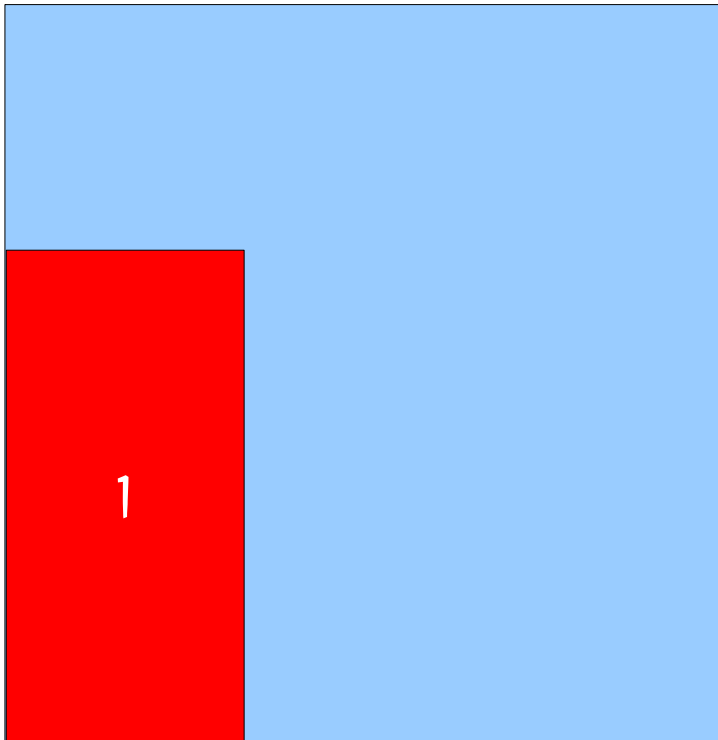
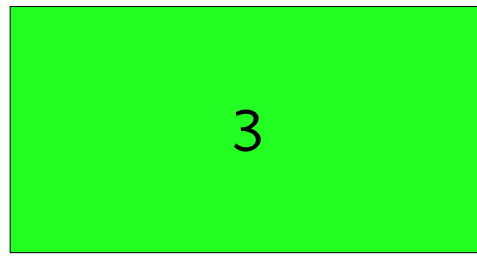


BL can run into problems even on small instances (Liu & Teng, 1999).

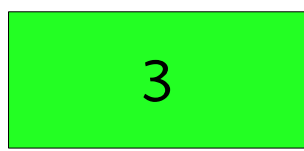
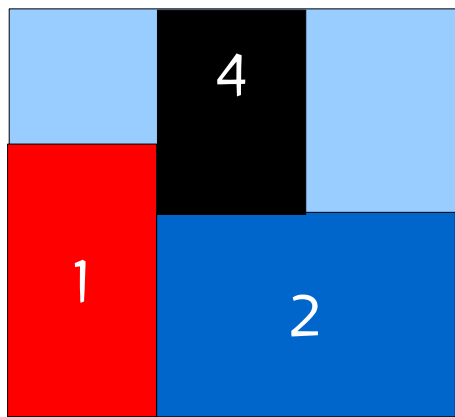
Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.

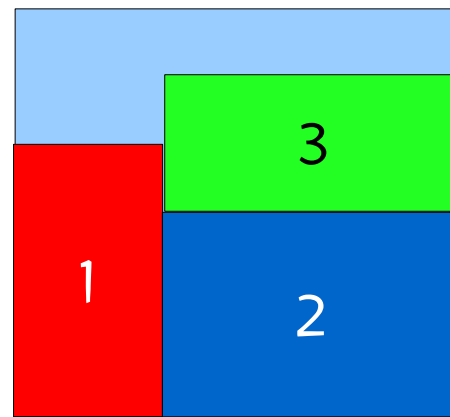




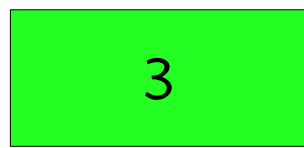
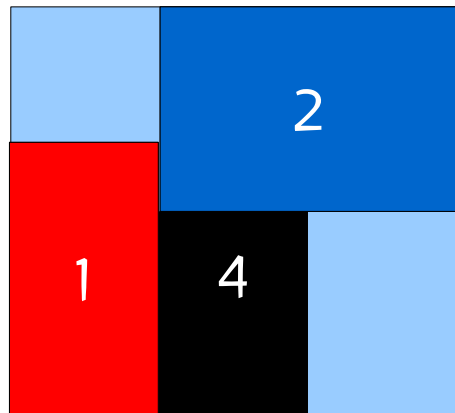
We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



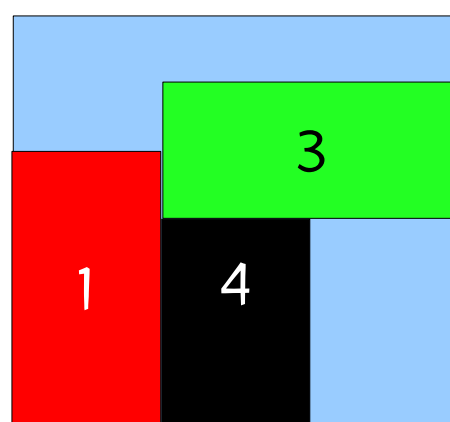
RTPS: 1-2-4-3



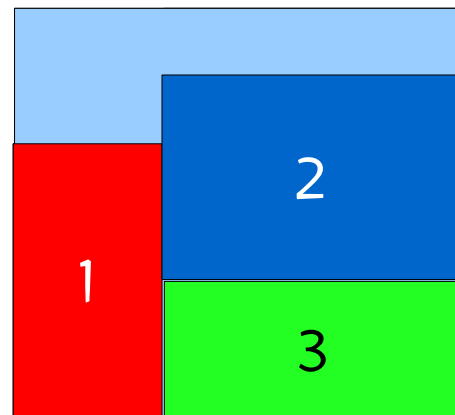
RTPS: 1-2-3-4



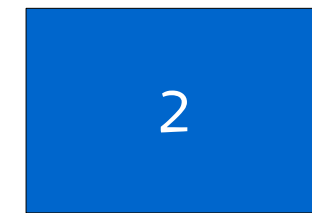
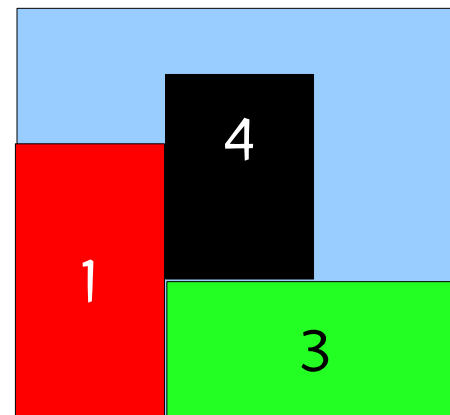
RTPS: 1-4-2-3



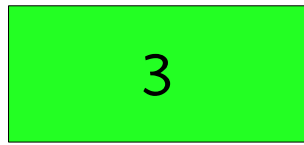
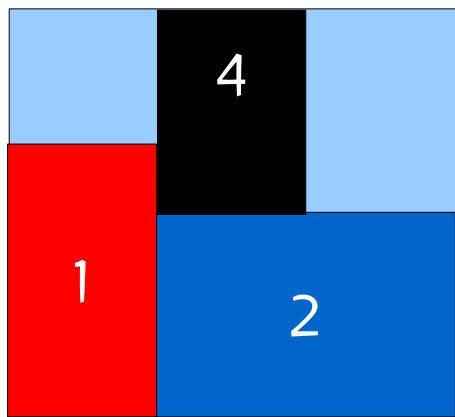
RTPS: 1-4-3-2



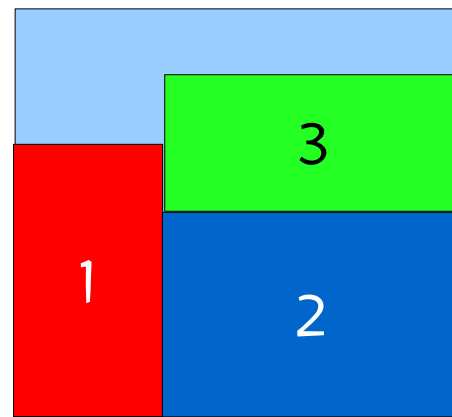
RTPS: 1-3-2-4



RTPS: 1-3-4-2

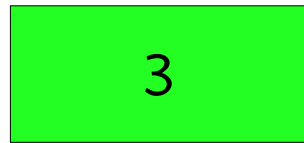
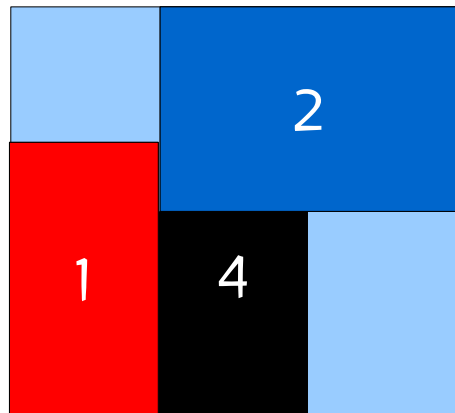


RTPS: 1-2-4-3

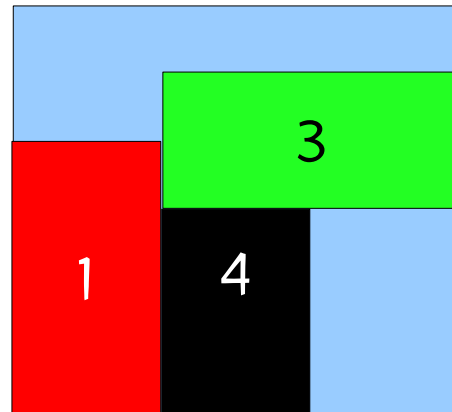


RTPS: 1-2-3-4

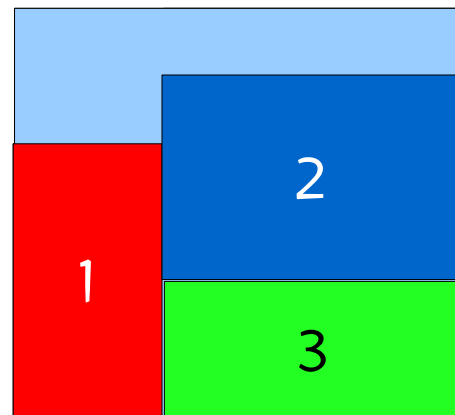
Similar infeasibilities are observed if 2, 3, or 4 is the first rectangle in the RTPS.



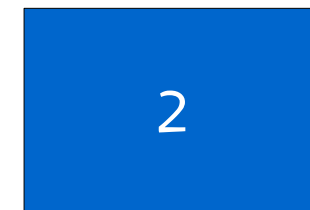
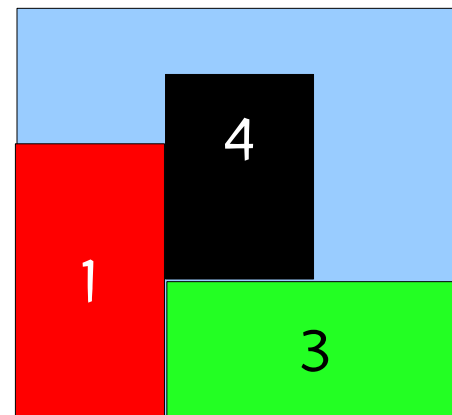
RTPS: 1-4-2-3



RTPS: 1-4-3-2



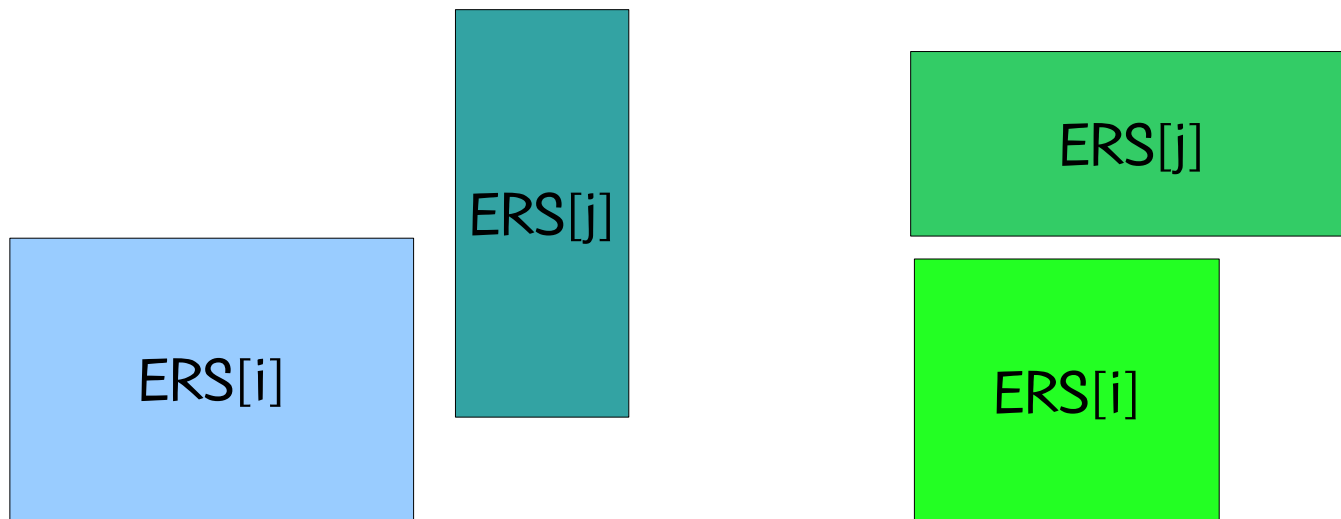
RTPS: 1-3-2-4



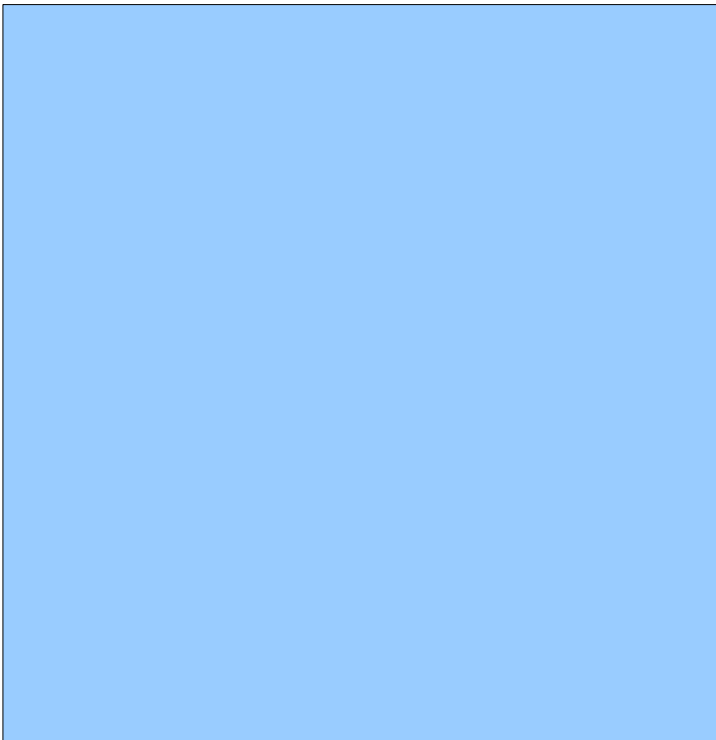
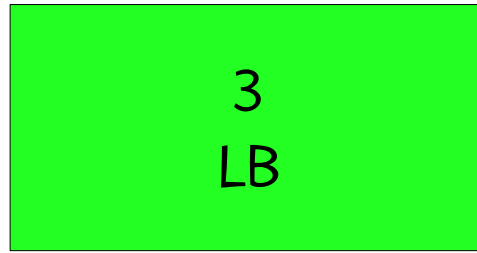
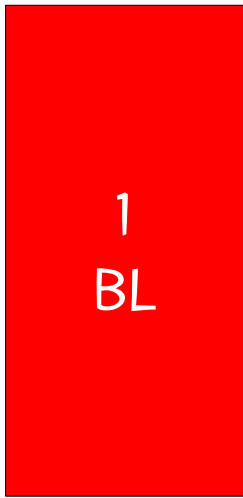
RTPS: 1-3-4-2

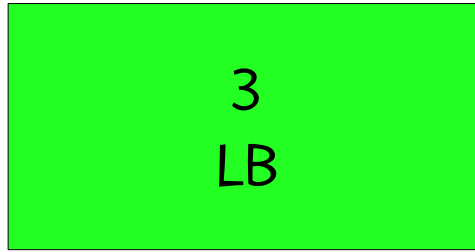
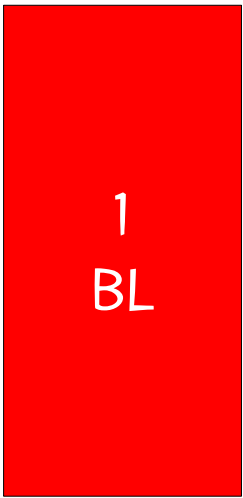
# Decoding

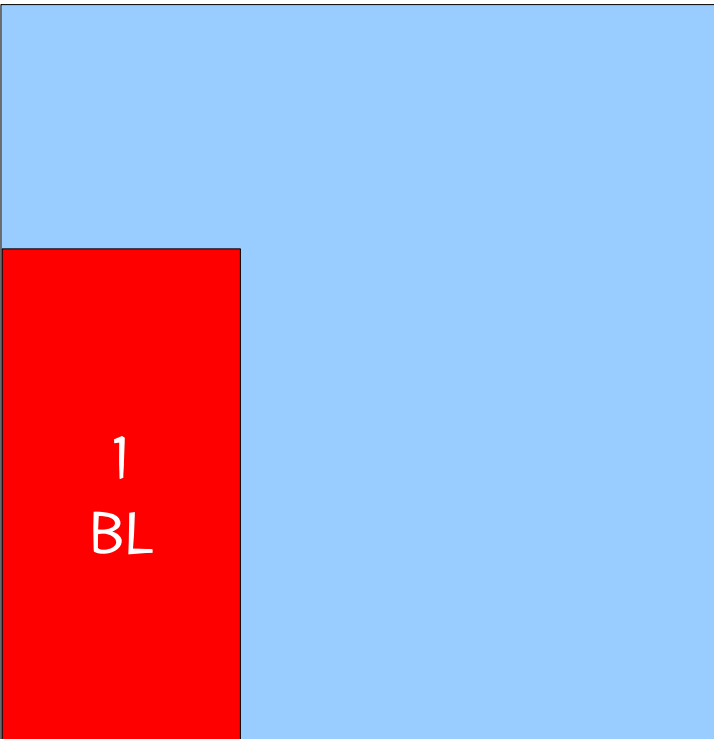
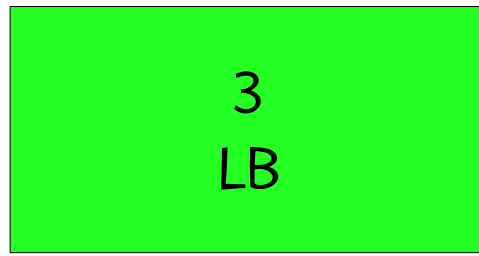
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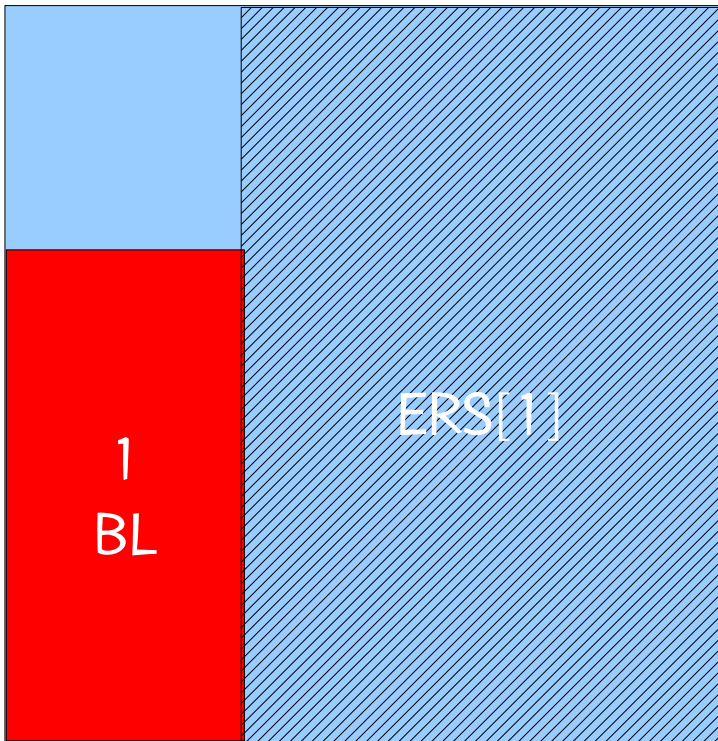
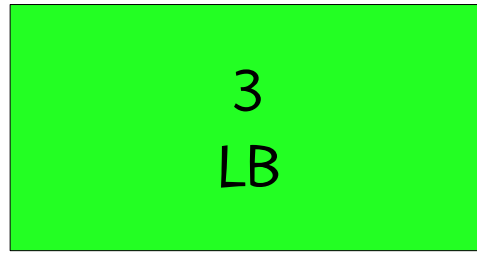


$ERS[i] < ERS[j]$

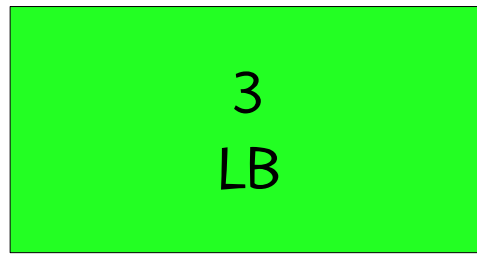


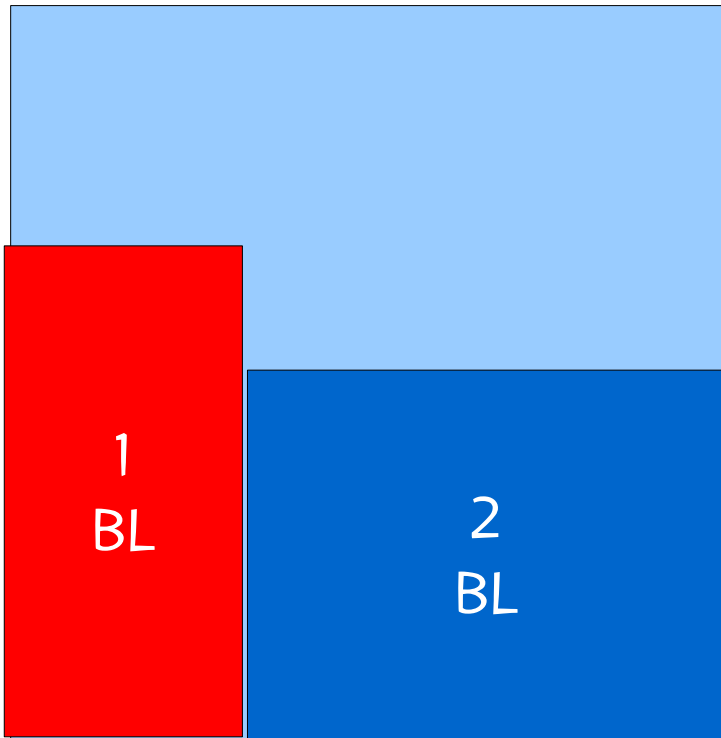
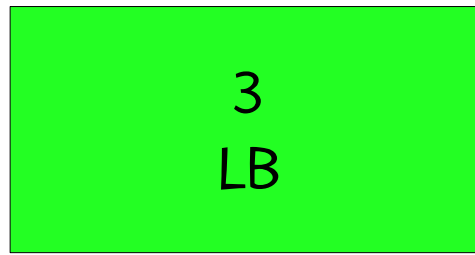


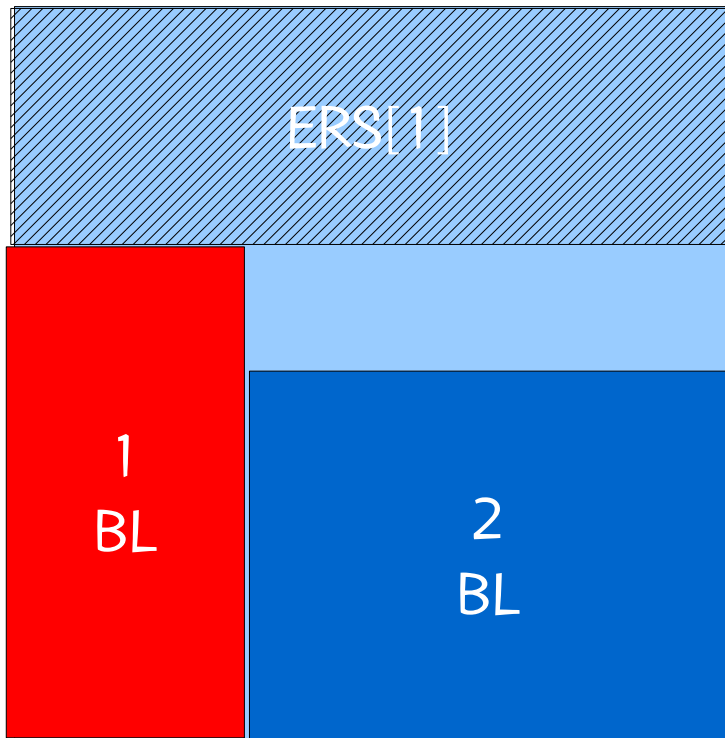
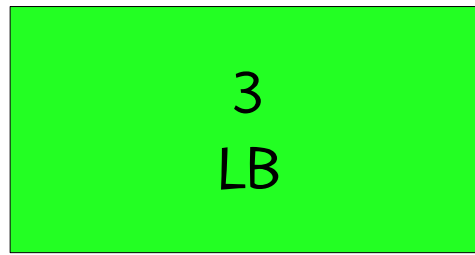


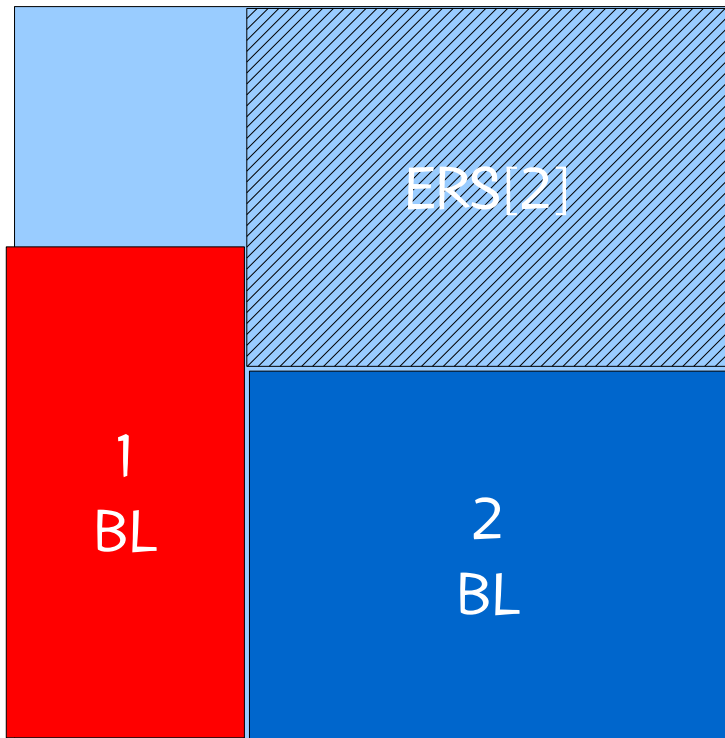
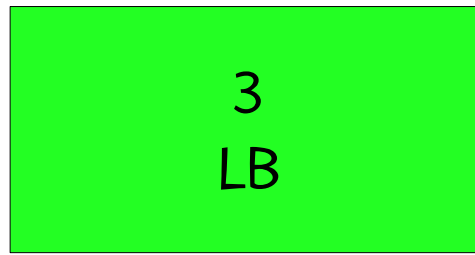


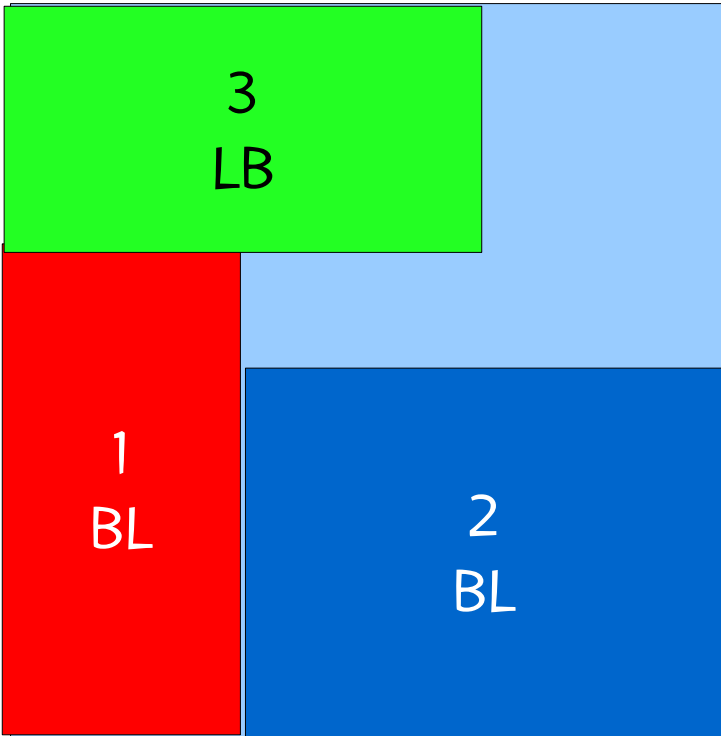


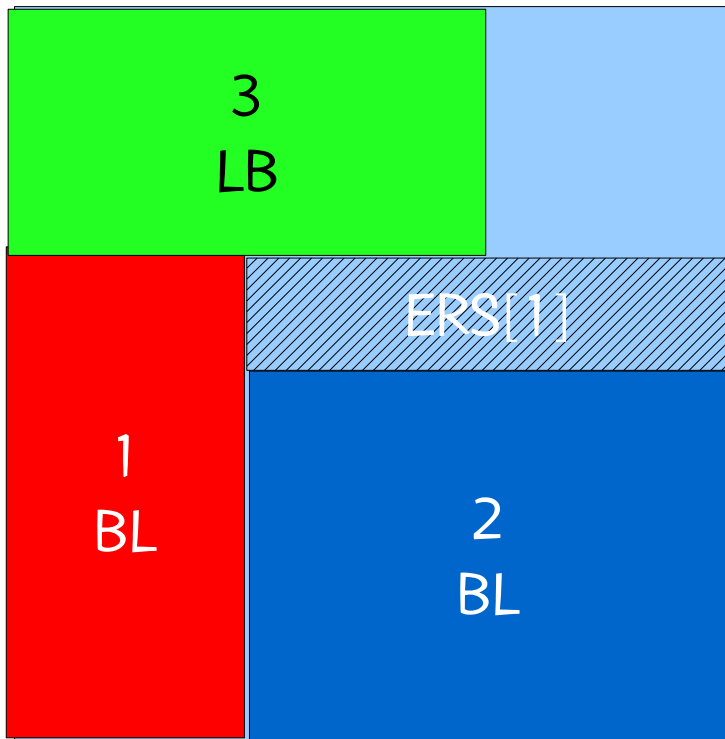






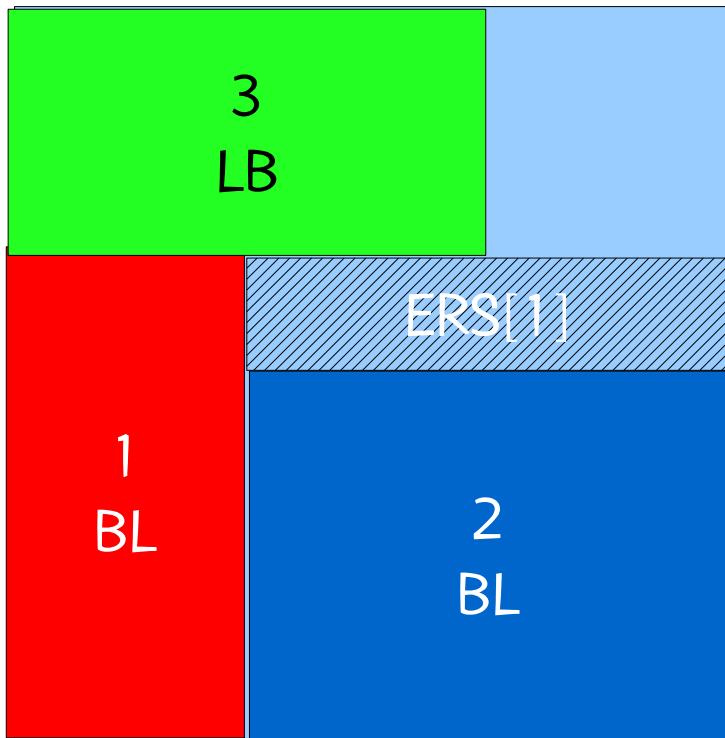






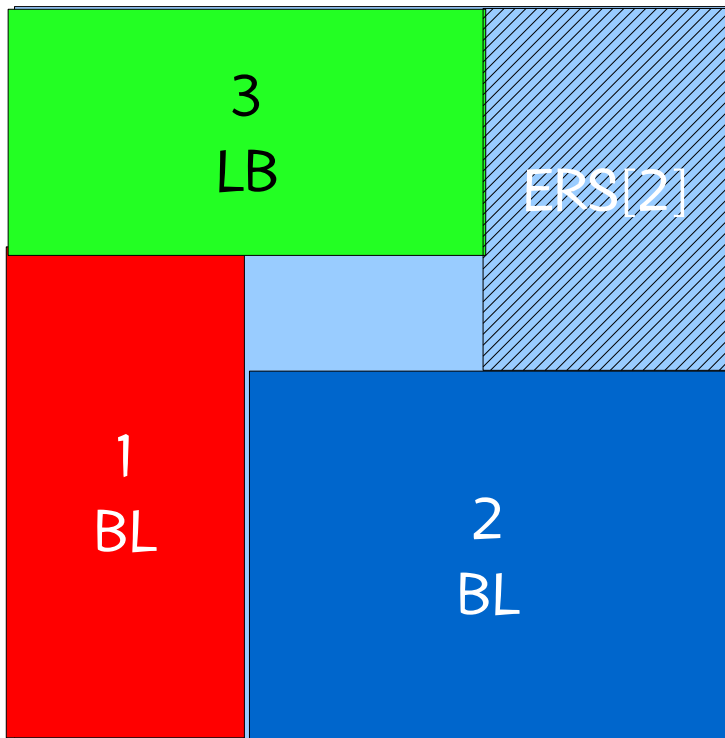


4 does not fit  
in ERS[1].

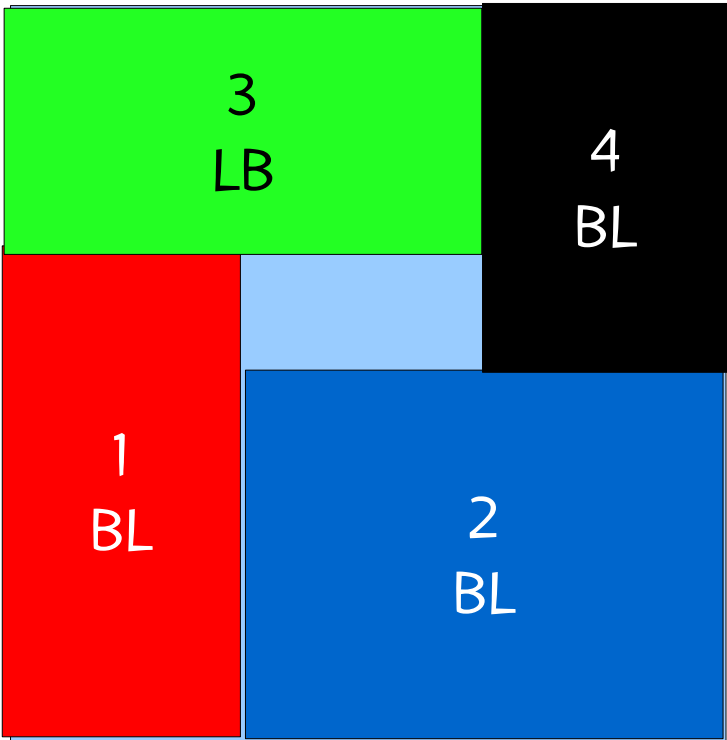




4 does fit  
in ERS[2].







Optimal solution!

# Experimental results

# Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:

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  - GRASP: greedy randomized adaptive search procedure of Alvarez-Valdes et al. (2005)
  - TABU: tabu search of Alvarez-Valdes et al. (2007)

# Number of best solutions / total instances

Problem	PH	GA	GRASP	TABU	BRKGA BL-LB-L-4NR
From literature (optimal)	13/21	<b>21/21</b>	18/21	<b>21/21</b>	<b>21/21</b>
Large random*	0/21	0/21	5/21	8/21	<b>20/21</b>
Zero-waste			5/31	17/31	<b>30/31</b>
Doubly constrained	11/21		12/21	17/21	<b>19/21</b>

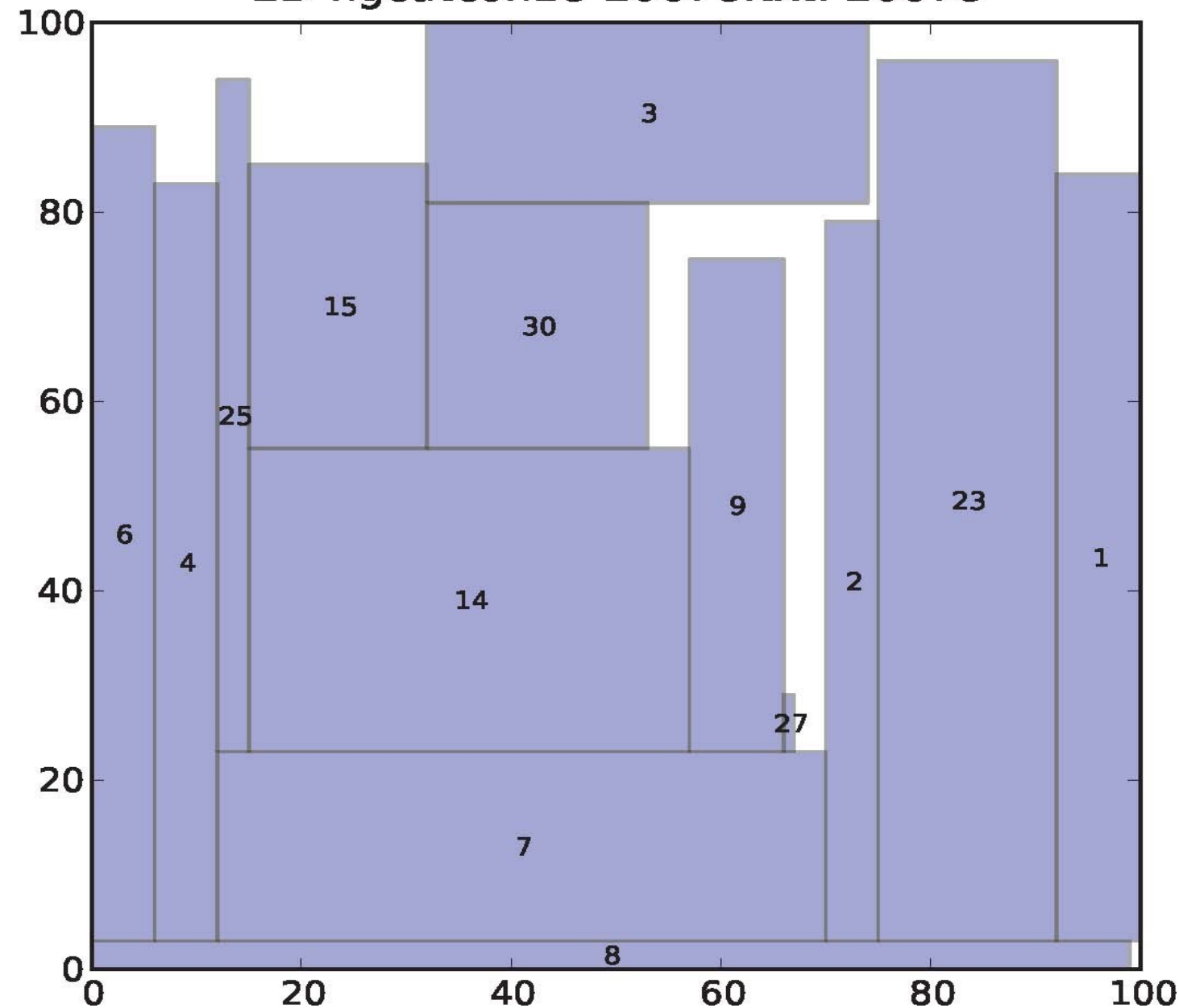
\* For large random: number of best average solutions / total instance classes



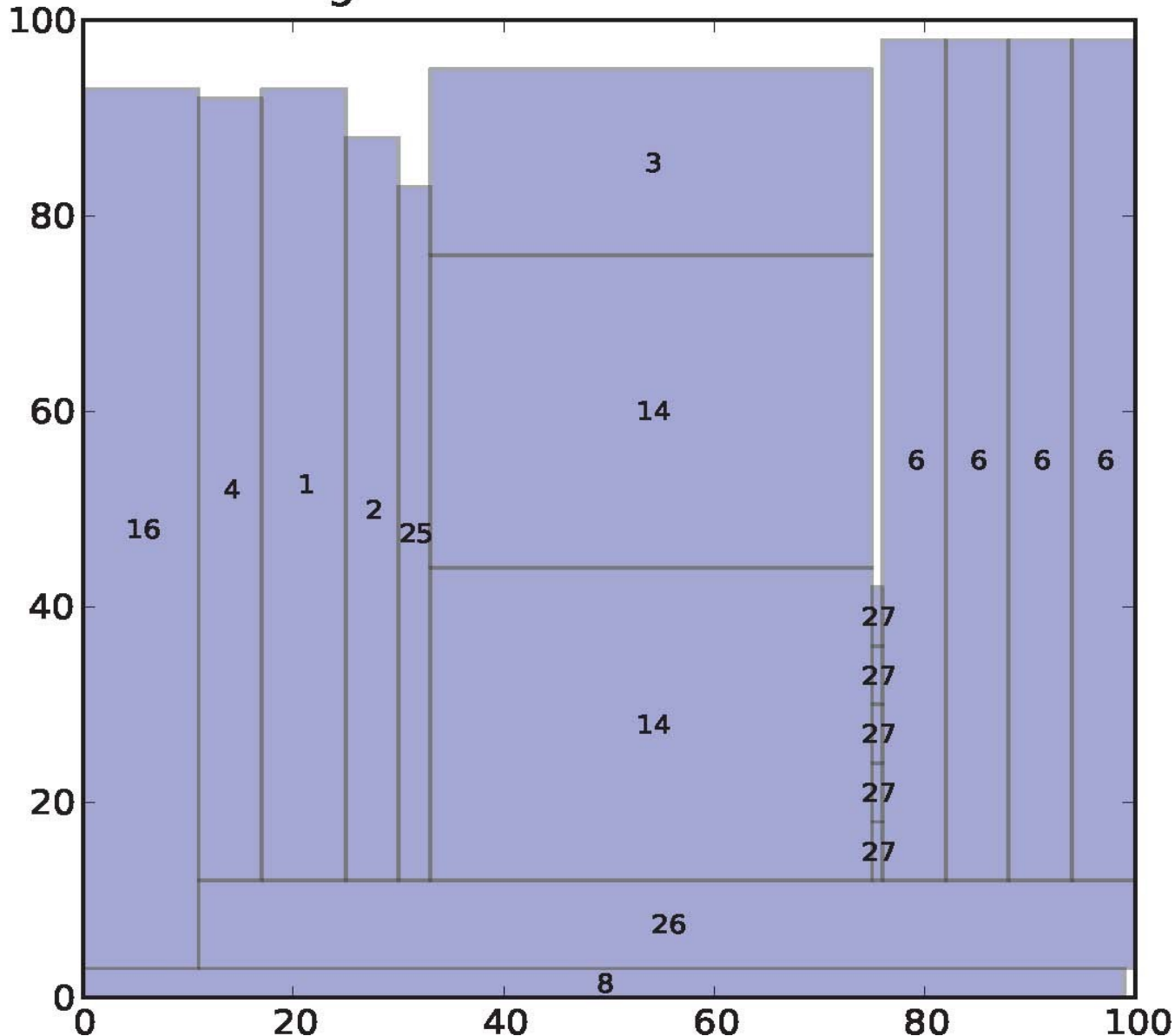
# 2D-ngcutcon18-20678.txt: 20678

New BKS  
for a 100 x100  
doubly  
constrained  
instance of  
Fekete &  
Schepers (1997)  
of value **20678**.  
Previous best  
was **19657** by  
tabu search of  
Alvarez-Valdes et  
al., (2007).

30 types  
30 rectangles



2D-ngcutcon21-22140-1.txt: 22140



New BKS for a 100 x 100 doubly constrained instance Fekete & Schepers (1997) of value **22140**.

Previous BKS was **22011** by tabu search of Alvarez-Valdes et al. (2007).

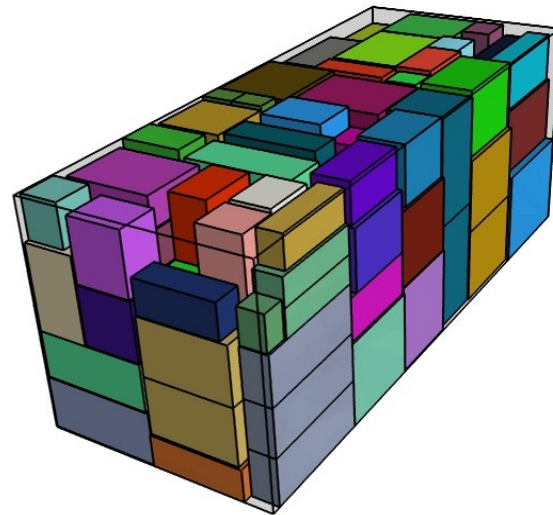
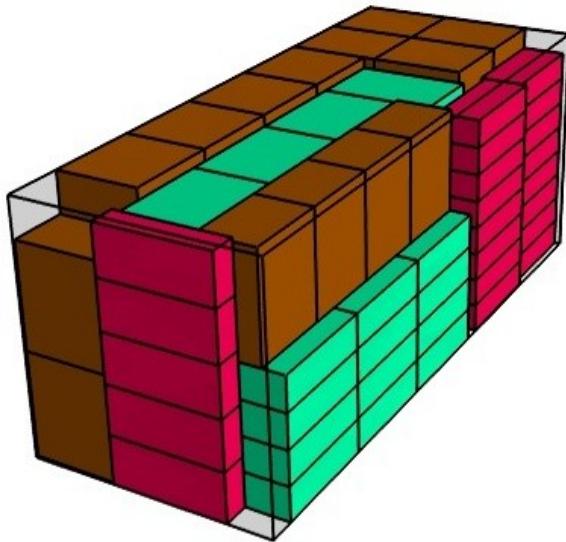
29 types  
97 rectangles

# 3D packing

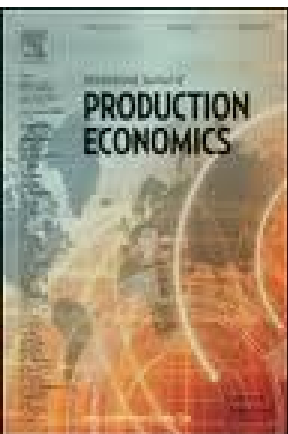
We have extended this to 3D packing:

J.F. Gonçalves and M.G.C.R., “A parallel multi-population biased random-key genetic algorithm for a container loading problem,” Computers & Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: <http://www.research.att.com/~mgcr/doc/brkga-pack3d.pdf>



# 3D bin packing



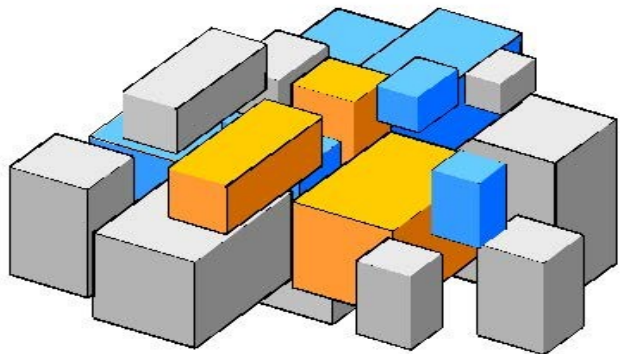
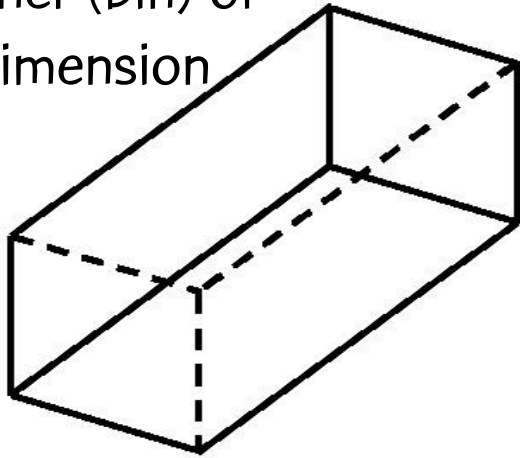
J.F. Gonçalves and M.G.C.R., “A biased  
random-key genetic algorithm for 2D  
and 3D bin packing problems,”

International J. of Production Economics, vol. 15, pp.  
500–510, 2013.

<http://www.research.att.com/~mgcr/doc/brkga-binpacking.pdf>

# 3D bin packing problem

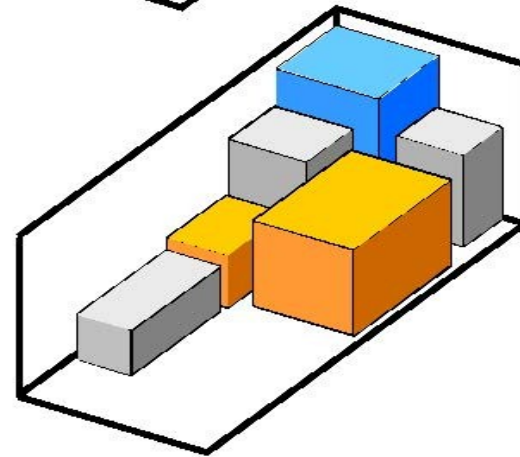
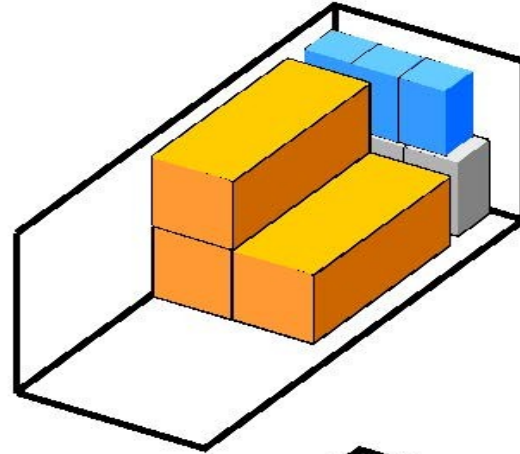
Container (bin) of  
fixed dimension



Boxes of different dimensions



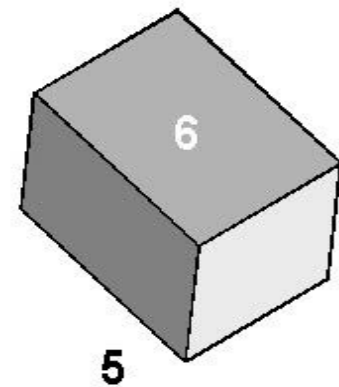
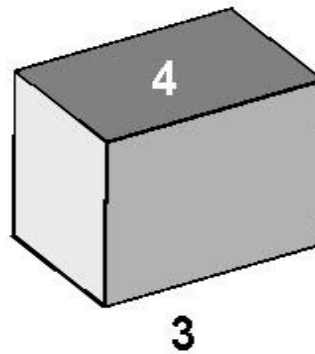
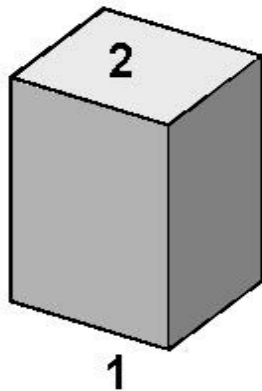
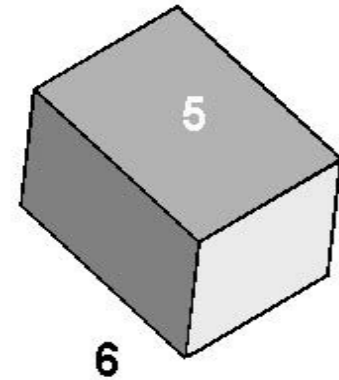
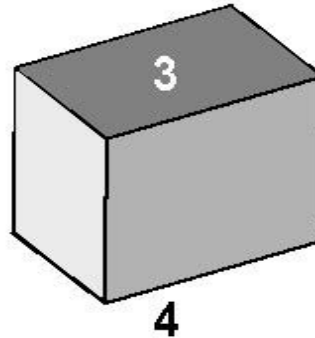
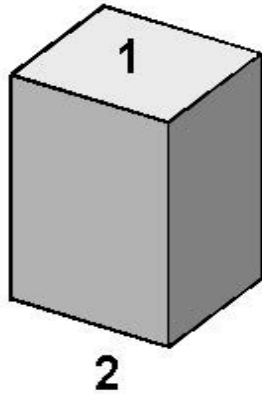
Minimize number of containers  
(bins) needed to pack all boxes



# 3D bin packing constraints

- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)

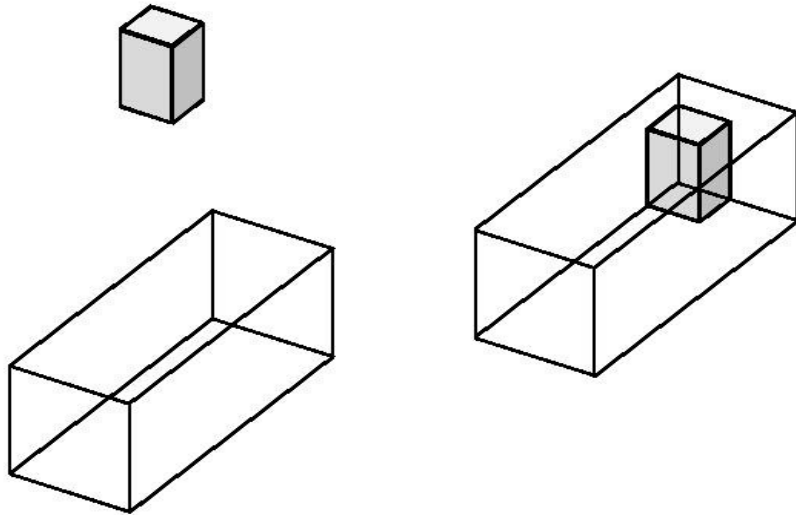
# Six possible orientations for each box



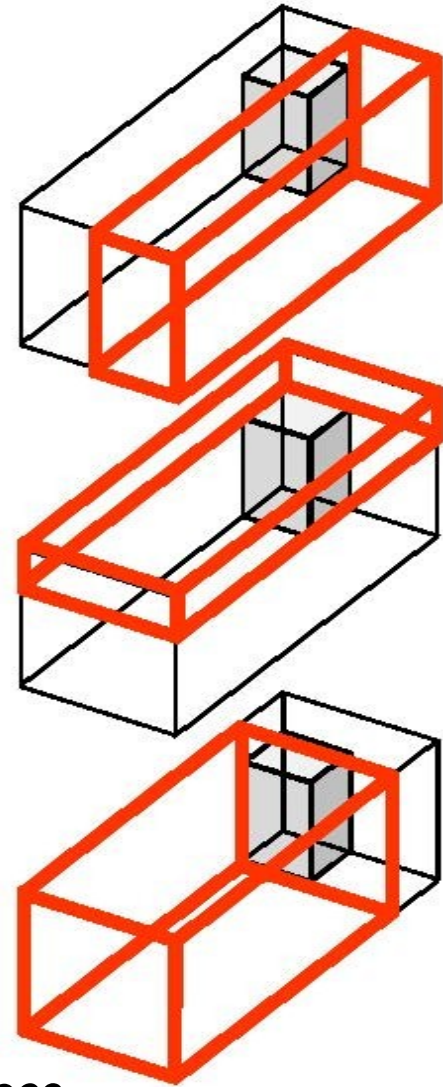


# Difference process - DP

(Lai & Chan, 1997)



When box is placed in container ...  
use DP to keep track of maximal free spaces



# Encoding

Solutions are encoded as vectors of  $3n$  random keys, where  $n$  is the number of boxes to be packed.

$$X = ( \underbrace{x_1, x_2, \dots, x_n}_{\text{Box packing sequence}}, \underbrace{x_{n+1}, x_{n+2}, \dots, x_{2n}}_{\text{Placement heuristic}}, \underbrace{x_{2n+1}, x_{2n+2}, \dots, x_{3n}}_{\text{Box orientation}} )$$

# Decoding

- 1) Sort first  $n$  keys of  $X$  to produce sequence boxes will be packed;
- 2) Use second  $n$  keys of  $X$  to determine which placement heuristic to use (back-bottom-left or back-left-bottom):
  - if  $x_{n+i} < 1/2$  then use back-bottom-left to pack  $i$ -th box
  - if  $x_{n+i} \geq 1/2$  then use back-left-bottom to pack  $i$ -th box
- 3) Use third  $n$  keys of  $X$  to determine which of six orientations to use when packing box:
  - $x_{2n+i} \in [0, 1/6)$ : orientation 1;
  - $x_{2n+i} \in [1/6, 2/6)$ : orientation 2; ...
  - $x_{2n+i} \in [5/6, 1]$ : orientation 6.

# Decoding

## For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation

# Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness:  $aNB$
- $aNB = NB + ( \text{LeastLoad} / \text{BinVolume} )$ , where
  - ×  $\text{LeastLoad}$  is load on least loaded bin
  - ×  $\text{BinVolume}$  is volume of bin:  $H \times W \times L$

# Experiment

- Instances:
  - 320 instances of Martello et al. (2000)
  - generator is available at <http://www.diku.dk/~pisinger/codes/html>
  - 8 classes
  - 40 instances per class
  - 10 instances for each value of  $n \in \{50, 100, 150, 200\}$

# Experiment

- We compare BRKGA with:
  - TS3, the tabu search of Lodi et al. (2002)
  - GLS, the guided local search of Faroe et al. (2003)
  - TS2PACK, the tabu search of Crainic et al. (2009)
  - GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)

# Summary

Average number of bin in each class of 40 instances

Class	Bin size	BRKGA	GRASP	TS3	TS2PACK	GLS
1	100 <sup>3</sup>	127.3	127.3	127.9	128.2	128.3
2	100 <sup>3</sup>	125.5	125.8	126.8		
3	100 <sup>3</sup>	126.5	126.9	127.5		
4	100 <sup>3</sup>	294.0	294.0	294.0	293.9	294.2
5	100 <sup>3</sup>	70.4	70.5	71.4	71.0	70.8
6	10 <sup>3</sup>	95.0	95.4	96.1	95.8	96.0
7	40 <sup>3</sup>	58.2	59.4	60.0	59.0	59.0
8	100 <sup>3</sup>	80.9	82.0	82.6	81.9	81.9
Sum(rows 1, 4-8):		725.8	728.6	732.0	729.8	730.2
Sum(rows 1-8):		977.8	981.3	986.3		



# Concluding remarks of technical part of talk

- Reviewed BRKGA framework
- Applied framework to
  - 2D/3D packing to maximize value packed
  - 2D/3D bin packing to minimize number of bins
- All decoders were simple heuristics
- BRKGA “learned” how to “operate” the heuristics
- In all cases, several new best known solutions were produced

# My vision for the Department of ISE at UF

# Department of Industrial & Systems Engineering at the U. of Florida

Internationally renowned center of excellence in operations research and industrial & systems engineering.

Focused on education and inter-disciplinary research on the big problems of the 21st century, including theoretical and applied issues in:

- Big data analytics
- Cloud computing
- Mobility
- Network science, including social networks
- Renewable energy
- Supply chain & manufacturing
- Health care
- Telecommunications
- Transportation

# Raise department's U.S. News & World Report ranking

Raise ranking from 13<sup>th</sup> (tied with Columbia & NC State) to a top-10 (on a par with Cornell, Wisconsin, VA Tech & Purdue).

A higher ranking has the potential to result in a number of favorable outcomes:

- Attract more top-notch students
- Attract star faculty, both junior & senior
- Increase research production & funding
- Increased geographical diversity as a consequence of increase in number of
  - out-of-state undergraduate applicants
  - domestic graduate applicants

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# Raise department's U.S. News & World Report ranking

To achieve this ambitious goal, we envision a multi-faceted plan, including

- Undergraduate education
- Graduate education
- Research
- Administration
- Alumni relations
- Fund raising

# Undergraduate education

Goal: Expose undergrads to an intellectually-rich environment so that upon graduation they best serve Florida, the Nation & the world.

The department will provide rigorous coursework and opportunities for research and practical experience.

Outcome: Graduates will be placed in industry & business where they can design, implement & manage complex systems.

Those wishing to pursue post-graduate education will have the needed skills to work toward graduate degrees in engineering, computer science, business, or applied mathematics.

# Undergraduate students should be exposed ...

... not only to fundamental concepts in:

- Science
- Mathematics
- Computer science and information technology
- Economics
- Statistics
- Industrial and systems engineering
- Operations research

... but also to the new disciplines of:

- Data science
- Machine learning
- Network science



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... but also to the new disciplines of:

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- Machine learning
- Network science

They should learn how to apply these concepts to make data-driven decisions, carry out theoretical and empirical analyses, and manage complex systems in industry and government.

# Graduate education

Goal: Prepare students for lifetime careers in academics, industry, or government.

Success of a graduate program depends heavily on

- Number
- Quality

of its student body.

Grad students have a significant impact on volume and quality of department's research production

- Directly, through student-faculty collaboration
- Indirectly, by assisting faculty and lessening burden imposed on faculty

Outcome: Place graduates in industry, government, or university for careers in

- research
- teaching
- high-level technical position

# Research

Goal: Balance theory and applications and encourage risk taking.

Labs enable research to focus on specific area, e.g.

- CAO – fomented research in biomedicine and energy
- Information lab – could facilitate thrust in big data analytics
- Simulation lab – stimulate simulation optimization

Grad students have a significant impact on volume and quality of department's research production

- Directly, through student-faculty research collaboration
- Indirectly, by assisting faculty with courses and lessening burden on faculty

Outlet: Though scholarly publications should continue to be the main outlet for research produced in department, the pursuit of patents should be encouraged.

# Administration

## Some opportunities:

- Manage a web-based system to aid in advising
- Aid faculty in writing proposals
- Help faculty write initial invention disclosures for possible patent filings

# Alumni relations

Good relations with alumni are important not only because of potential funding opportunities, but also to maintain a strong sense of community.

## Some ideas:

- Keep database of alumni up-to-date
- Graduating classes should be made aware of importance of keeping in touch
- Annual newsletter (as simple as an email) could be sent out to inform and engage alumni
- Department reception at INFORMS and IIE meetings

# Fund raising

Fund raising is an important responsibility of a department chair. A constant effort must be sustained to seek new sources of funding.

## Some potential sources:

- Alumni & non-alumni donations
- Grants from research funding agencies
- Grants from industrial partners
- Patent licensing
- Externally funded graduate students

# What to do?

Hire 5 – 6 new faculty in four critical areas:

- data sciences
- machine learning
- stochastic processes
- simulation

# What to do?

Recruit top-notch graduate students:

- From within the U.S.
- From abroad
- Increase availability of
  - Fellowships
  - Teaching assistantships
  - Research assistantships



# What to do?

Increase alliances with industrial partners:

- Internships
- Research collaborations
- Expose students to real-world problems
- Harvest data for data science research

# What to do?

Update computer infrastructure to support data science research:

- Within department
- In partnerships with other departments
- Commercial cloud computing

# What to do?

Encourage graduate students to collaborate with faculty in the preparation and filing of patents:

- Potential source of funding
- Acquire critical skill

# What to do?

Undergraduate students **should have the opportunity to carry out** independent research during their senior year:

- Senior thesis
- Introduce student to tasks involved in research: literature search, problem statement, empirical studies, and writing and defending the thesis
- Faculty member should supervise student

# What to do?

Students should graduate with the programming skills needed for the technical jobs of the 21<sup>st</sup> century:

- Many traditional ISE jobs are not going to ISE graduates
- CS majors (software engineers) are taking many of these jobs

# What to do?

Increase diversity through Outreach and targeted fellowships:

- Recruit women and minorities to increase gender and ethnic diversity
- Recruit out-of-state & international undergraduate students and domestic graduate students to increase geographical diversity

# Thanks!