## Packing with biased

## random-key genetic

## algorithms

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## Summary

- Metaheuristics and basic concepts of genetic algorithms
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
- Encoding / Decoding
- Initial population
- Evolutionary mechanisms
- Problem independent / problem dependent components
- Multi-start strategy
- Specifying a BRKGA
- Application programming interface (API) for BRKGA
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing


## Metaheuristics

Metaheuristics are heuristics to devise heuristics.

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Examples: GRASP and C-GRASP, simulated annealing, genetic algorithms, tabu search, scatter search, ant colony optimization, variable neighborhood search, and biased random-key genetic algorithms (BRKGA).

## Genetic algorithms

## Genetic algorithms Holund (1975)



Adaptive methods that are used to solve search and optimization problems.

Individual: solution


## Genetic algorithms



Individual: solution (chromosome $=$ string of genes)
Population: set of fixed number of individuals

## Genetic algorithms

Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

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A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

Individuals from one generation are combined to produce offspring that make up next generation.

## Genetic algorithms

Probability of selecting individual to mate is proportional to its fitness: survival of the fittest.

## Genetic algorithms



Parents drawn from
generation K

## Crossover and mutation



## Crossover and mutation



Crossover: Combines parents ... passing along to offspring characteristics of each parent ...

Intensification of search

## Crossover and mutation



Mutation: Randomly changes chromosome of offspring ... Driver of evolutionary process ...

Diversification of search

## Evolution of solutions

## Evolution of solutions



## Evolution of solutions



## Evolution of solutions



## Evolution of solutions



## Evolution of solutions



## Evolution of solutions



## Evolution of solutions



## Reference



## J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:
http://www.research.att.com/~mgcr/doc/srkga.pdf

## Encoding solutions with random keys

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## Encoding with random keys

- A random key is a real random number in the continuous interval $[0,1$ ).
- A vector $X$ of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.


## Encoding with random keys: Sequencing

Encoding

$$
\begin{array}{rrrrr}
{\left[\begin{array}{rrr}
{[ } & 2, & 3,
\end{array} 4,\right.} & 5] \\
X & {[0.099,} & 0.216, & 0.802, & 0.368,0.658]
\end{array}
$$

## Encoding with random keys: Sequencing

Encoding

$$
\begin{array}{rrrrr}
{[ } & 1, & 2, & 3, & 4, \\
X & 5] \\
& 0.099, & 0.216, & 0.802, & 0.368,0.658]
\end{array}
$$

Decode by sorting vector of random keys

$$
\left.\begin{array}{rrrrr}
{\left[\begin{array}{rrrr}
{[ } & 2, & 4, & 5,
\end{array}\right]} \\
X & =[0.099, & 0.216, & 0.368, & 0.658, \\
{[0.802}
\end{array}\right]
$$

## Encoding with random keys: Sequencing

Therefore, the vector of random keys:
$X=[0.099,0.216,0.802,0.368,0.658$ ] encodes the sequence: 1-2-4-5-3

## Encoding with random keys: Subset

 selection (select 3 of 5 elements)Encoding

$$
\begin{array}{rrrrr}
{\left[\begin{array}{rrr}
{[ } & 2, & 3,
\end{array} 4,\right.} & 5] \\
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Decode by sorting vector of random keys

$$
\left[\begin{array}{llllll}
{[ } & 1, & 2, & 4, & 5, & 3]
\end{array}\right.
$$

$X=[0.099,0.216,0.368,0.658,0.802]$

## Encoding with random keys: Subset

 selection (select 3 of 5 elements)Therefore, the vector of random keys:
$X=[0.099,0.216,0.802,0.368,0.658]$
encodes the subset: $\{1,2,4\}$

## Encoding with random keys: Assigning integer

 weights $\in[0,10]$ to a subset of 3 of 5 elementsEncoding

$$
\left.\begin{array}{c}
{\left[\begin{array}{cccccccccr}
{[ } & 1, & 2, & 3, & 4, & 5 \mid & 1, & 2, & 3, & 4, \\
5
\end{array}\right]} \\
X=[0.099,0.216,0.802,0.368,0.658
\end{array} 0.4634,0.5611,0.2752,0.4874,0.0348\right] ~ \$
$$

## Encoding with random keys: Assigning integer

## weights $\in[0,10]$ to a subset of 3 of 5 elements

Encoding

$$
\begin{aligned}
& {\left[\begin{array}{llllllllll}
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\end{array}\right]} \\
& x=[0.099,0.216,0.802,0.368,0.658 \mid 0.4634,0.5611,0.2752,0.4874,0.0348]
\end{aligned}
$$

Decode by sorting the first 5 keys and assign as the weight the value $W_{i}=$ floor $\left[10 X_{5+i}\right]+1$ to the 3 elements with smallest keys $X_{i}$, for $i=1, \ldots, 5$.

## Encoding with random keys: Assigning integer

 weights $\in[0,10]$ to a subset of 3 of 5 elementsTherefore, the vector of random keys:
$X=[0.099,0.216,0.802,0.368,0.658 \mid 0.4634,0.5611,0.2752,0.4874,0.0348]$ encodes the weight vector $W=(5,6,-, 5,-)$

## Genetic algorithms <br> and random keys

## GAs and random keys

- Introduced by Bean (1994) for sequencing problems.


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- Individuals are strings of
real-valued numbers
(random keys) in the interval $[0,1$ ).

$$
\begin{array}{r}
s=\left(\begin{array}{rlll}
0.25, & 0.19, & 0.67, & 0.05, \\
& 0.89
\end{array}\right) \\
s(1) \\
s(2) \\
s(3) \\
s(4) \\
s(5)
\end{array}
$$

## GAs and random keys

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- Individuals are strings of real-valued numbers (random keys) in the interval $[0,1$ ).
- Sorting random keys results

$$
\begin{aligned}
S^{\prime}=\left(\begin{array}{llll}
0.05, & 0.19, & 0.25, & 0.67, \\
s(4) & s(2) & s(1) & s(3) \\
& s(5)
\end{array}\right.
\end{aligned}
$$

Sequence: 4-2-1-3-5

## GAs and random keys

- Mating is done using
parametrized uniform
CrOSSOVEY (Spears \& DeJong, 1990)

$$
\begin{aligned}
& a=(0.25,0.19,0.67,0.05,0.89) \\
& b=(0.63,0.90,0.76,0.93,0.08)
\end{aligned}
$$

## GAs and random keys

- Mating is done using
parametrized uniform CrOSSOVET (Spears \& DeJong, 1990)
- For each gene, flip a biased

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& a=(0.25,0.19,0.67,0.05,0.89) \\
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\end{aligned}
$$ coin to choose which parent passes the allele (key, or value of gene) to the child.

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& c=(0.25,0.90
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& b=(0.63,0.90,0.76,0.93,0.08) \\
& c=(0.25,0.90,0.76
\end{aligned}
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## GAs and random keys

Initial population is made up of $P$ random-key vectors, each with N keys, each having a value generated uniformly at random in the interval $[0,1)$.


## GAs and random keys

Population K
At the K-th generation, compute the cost of each solution ...


## GAs and random keys

Population K
At the K-th generation, compute the cost of each solution and partition the solutions into two sets:


## GAs and random keys

Population K
At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions.


## GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.


## GAs and random keys

Population K

## Evolutionary dynamics



## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$

Population K


Population K+1


Elite solutions

## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population $K$ to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$

Population K


## GAs and random keys

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$
- While $\mathrm{K}+1$-th population $<\mathrm{P}$
- RANDOM-KEY GA: Use any two solutions in population K to produce child in population $\mathrm{K}+1$. Mates are chosen at random.

Population K


## Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).


## Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.


## How RKGA \& BRKGA differ

## RKGA

BRKGA
both parents chosen at random from entire population

## How RKGA \& BRKGA differ

RKGA
both parents chosen at both parents chosen at random from entire population

## BRKGA

 random but one parent chosen from population of elite solutions
## How RKGA \& BRKGA differ

RKGA
both parents chosen at random from entire population

## BRKGA

both parents chosen at random but one parent chosen from population of elite solutions
either parent can be
parent $A$ in parametrized
uniform crossover
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## How RKGA \& BRKGA differ

RKGA
both parents chosen at random from entire population

## BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

## Biased random key GA

BRKGA: Probability child inherits key of elite

## Evolutionary dynamics

- Copy elite solutions from population K to population $\mathrm{K}+1$
- Add $R$ random solutions (mutants) to population $\mathrm{K}+1$
- While $\mathrm{K}+1$-th population $<\mathrm{P}$
- RANDOM-KEY GA: Use any two solutions in population K to produce child in population $\mathrm{K}+1$. Mates are chosen at random.
- BIASED RANDOM-KEY GA: Mate elite solution with other solution of population K to produce child in population $\mathrm{K}+1$. Mates are chosen at
 random.


## Paper comparing BRKGA and Bean's

## Method

Gonçalves, R., and Toso, "Biased and unbiased random-key genetic algorithms: An experimental analysis", Proceedings of the $10^{\text {th }}$ Metaheuristics International Conference, Singapore, August 2013.






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## Observations

- Random method: keys are randomly generated so solutions are always vectors of random keys


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- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent >0.5 Not so in the RKGA of Bean.
- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs ... help escape local optima)


## Framework for biased random-key genetic algorithms



## Framework for biased random-key genetic algorithms

Generate $P$ vectors of random keys

Decode each vector of random keys

Classify solutions as elite or non-elite



> Combine elite and non-elite solutions and add children to next population

## Framework for biased random-key genetic algorithms

Problem dependent

Decode each vector of random keys

Classify solutions as elite or non-elite


## Decoding of random key vectors can be done in parallel



## Is a BRKGA any different from applying

 the decoder to random keys?- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to $\mathrm{P}-1$
- Each iteration, best solution is maintained in elite set and $\mathrm{P}-1$ random key vectors are generated as mutants ... no mating is done since population already has P individuals

Network monitor location problem (opt $=23$ ) solution
n18日-i2-n180-b18日: GA and randon multi-start iterates


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## BRKGA in multi-start strategy



Packing with a BRKGA



In most of the independent runs, the algorithm finds the target solution in relatively few iterations:


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In most of the independent runs, the algorithm finds the target solution in relatively few iterations: $50 \%$ of the runs take fewer than 192 iterations


In most of the independent runs, the algorithm finds the target solution in relatively few iterations: $75 \%$ of the runs take fewer than 345 iterations


However, some runs take much longer: $10 \%$ of the runs take over 1000 iterations


However, some runs take much longer: 5\% of the runs take over 2000 iterations


However, some runs take much longer: $2 \%$ of the runs take over 9715 iterations


However, some runs take much longer: the longest run took 11607 iterations


Probability that algorithm will take over 345 iterations: $25 \%=1 / 4$


Probability that algorithm will take over 345 iterations: $25 \%=1 / 4$

By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: $25 \%$ = 1/4

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 X probability of taking over 690 iterations given it took over $345=$ $1 / 4 \times 1 / 4=1 / 4^{2}$


Probability that algorithm will still be running after K periods of 345 iterations: $1 / 4^{\mathrm{K}}$



Probability that algorithm will still be running after K periods of 345 iterations: $1 / 4^{\mathrm{K}}$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1 / 4^{5} \cong$ 0.0977\%

This is much less than the $5 \%$ probability that the algorithm without restart will take over 2000 iterations.

## Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S=\left\{\tau_{1}, \tau_{2}, T_{3}, \ldots\right\}$ which define epochs $\tau_{1}, \quad \tau_{1}+\tau_{2}, \tau_{1}+\tau_{2}+\tau_{3}, \ldots$ when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $T_{1}=T_{2}=T_{3}=\cdots=T^{*}$, where $T^{*}$ is a constant.


## Restart strategies

- Luby et al. (1993)
- Kautz et al. (2002)
- Palubeckis (2004)
- Sergienko et al. (2004)
- Nowicki \& Smutnicki (2005)
- D'Apuzzo et al. (2006)
- Shylo et al. (2011a)
- Shylo et al. (201 1b)
- Resende \& Ribeiro (2011)


## Restart strategy for BRKGA

- Recall the restart strategy of Luby et al. where equal time intervals $\tau_{1}=\tau_{2}=\tau_{3}=\cdots=\tau^{*}$ pass between restarts.
- Strategy requires $\mathrm{T}^{*}$ as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
- choosing $\mathrm{T}^{*}$ too small: restart variant may take long to converge
- choosing $T^{*}$ too big: restart variant may become like no-restart variant


## Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy restart(K)


## Example of restart strategy for BRKGA: Load balancing

Given an unordered sequence of 1024 integers p[0], p[1], ..., p[1023]

## Example of restart strategy for BRKGA: Load balancing

Place consecutive numbers in 32 buckets $b[0], b[1], \ldots, b[31]$


## Example of restart strategy for BRKGA: Load balancing

Add the numbers in each bucket $b[0], b[1], \ldots, b[31]$


## Example of restart strategy for BRKGA: Load balancing

Place the buckets in 16 bins $\mathrm{B}[0], \mathrm{B}[1], \ldots, \mathrm{B}[15]$



## Example of restart strategy for BRKGA: Load balancing

Add up the numbers in each bin $\mathrm{B}[0], \mathrm{B}[1], \ldots, \mathrm{B}[15]$

$\mathrm{T}[0]=\Sigma(\Sigma \mathrm{p})$
$\mathrm{T}[1]=\Sigma(\Sigma \mathrm{p})$
$T[2]=\Sigma(\Sigma \mathrm{p})$
$T[15]=\Sigma(\Sigma \mathrm{p})$

## Example of restart strategy for BRKGA: Load balancing

OBJECTIVE: Minimize \{ Maximum (T[0], T[1], ..., T[15]) \}

$T[0]=\Sigma(\Sigma \mathrm{p})$
$T[1]=\sum\left(\sum \mathrm{p}\right)$
$T[2]=\Sigma(\Sigma \mathrm{p})$
$T[15]=\Sigma\left(\sum \mathrm{p}\right)$

## Example of restart strategy for BRKGA: Load balancing

## Encoding

$X=[x[1], x[2], \ldots, x[32] \quad x[32+1], x[32+2], \ldots, x[32+16]]$ Decoding
$x[1], x[2], \ldots, x[32]$ are used to define break points for buckets
$\mathrm{x}[32+1], \mathrm{x}[32+2], \ldots, x[32+16]$ are used to determine to which bins the buckets are assigned

## Example of restart strategy for BRKGA: Load balancing

## Encoding

$X=[x[1], x[2], \ldots, x[32] \quad x[32+1], x[32+2], \ldots, x[32+16]]$
Decoding
$x[1], x[2], \ldots, x[32]$ are used to define break points for buckets
Size of bucket $\mathrm{i}=$ floor $(1024 \times x[i] /(x[1]+x[2]+\cdots+x[32])), i=1, \ldots, 15$

Size of bucket $16=1024$ - sum of sizes of first 15 buckets

## Example of restart strategy for BRKGA: Load balancing

## Encoding

$X=[x[1], x[2], \ldots, x[32] \quad x[32+1], x[32+2], \ldots, x[32+16]]$ Decoding
$x[1], x[2], \ldots, x[32]$ are used to define break points for buckets
$\mathrm{x}[32+1], \mathrm{x}[32+2], \ldots, \mathrm{x}[32+16]$ are used to determine to which bins the buckets are assigned

Bin that bucket i is assigned to $=$ floor $(16 \times x[32+i])+1$

## Example of restart strategy for BRKGA: Load balancing

Decoding (Local search phase)

- while (there exists a bucket in the most loaded bin that can be moved to another bin and not increase the maximum load) then
- move that bucket to that bin
- end while

Make necessary chromosome adjustments to last 16 random keys of vector of random keys to reflect changes made in local search phase: Add or subtract an integer value from chromosome of bucket that moved to new bin.

## Example of restart strategy for BRKGA: Load balancing



## Specifying a BRKGA

## Specifying a biased random-key GA

- Encoding is always done the same way, i.e. with a vector of N random-keys (parameter N must be specified)


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- Decoder that takes as input a vector of N random-keys and outputs the corresponding solution of the combinatorial optimization problem and its cost (this is usually a heuristic)


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- Decoder that takes as input a vector of N random-keys and outputs the corresponding solution of the combinatorial optimization problem and its cost (this is usually a heuristic)
- Parameters


## Specifying a biased random-key GA

## Parameters:

- Size of population
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion


## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of $N$, say $N$ or $2 N$
- Size of elite partition
- Size of mutant set
- Child inheritance probability
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- Stopping criterion


## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion


## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set: 5-15\% of population
- Child inheritance probability
- Restart strategy parameter
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## Specifying a biased random-key GA

## Parameters:

- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set: $5-15 \%$ of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter
- Stopping criterion


## Specifying a biased random-key GA

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- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set: $5-15 \%$ of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N , say 2 N or 10 N
- Stopping criterion


## Specifying a biased random-key GA

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- Size of population: a function of N , say N or 2 N
- Size of elite partition: 15-25\% of population
- Size of mutant set: $5-15 \%$ of population
- Child inheritance probability: > 0.5 , say 0.7
- Restart strategy parameter: a function of N , say 2 N or 10 N
- Stopping criterion: e.g. time, \# generations, solution quality, \# generations without improvement


## brkgaAPI: A C++ API for BRKGA

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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
- population management
- evolutionary dynamics
- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.


## brkgaAPI: A C++ API for BRKGA

# Paper: Rodrigo F. Toso and M.G.C.R., "A C++ <br> Application Programming Interface for Biased Random-Key Genetic Algorithms," 

 AT\&T Labs Technical Report, Florham Park, 2012.Software: http://www.research.att.com/~mgcr/src/brkgaAPI

## An example BRKGA:

## Packing weighted

rectangles

## Reference



# J.F. Gonçalves and M.G.C.R., "A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal <br> packing problem," Journal of Combinatorial Optimization, vol. 22, pp. 180-201, 2011. 

Tech report:
http://www.research.att.com/~mgcr/doc/pack2d.pdf

## Constrained orthogonal packing

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0 \leq P[i] \leq r[i] \leq Q[i]
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Suppose $5 \leq r[1] \leq 12$

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Suppose $5 \leq r[1] \leq 12$

## Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

$$
v[1] r[1]+v[2] r[2]+\cdots+v[N] r[N]
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## Applications

Problem arises in several production processes, e.g.

- Textile
- Glass
- Wood
- Paper
where rectangular figures are cut from large rectangular sheets of materials.

2D-HopperTP12-1-49-3576.txt: 3576


Hopper \& Turton, 2001 Instance 4-1 $60 \times 60$ Value: 3576

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

2D-HopperTP12-1-49-3585.txt: 3585


2D-HopperTP12-1-49-3586.txt: 3586


2D-HopperTP12-1-49-3591.txt: 3591


2D-HopperTP12-1-49-3591.txt: 3591


## BRKGA for

## constrained 2-dim

## orthogonal packing

## Encoding

- Solutions are encoded as vectors K of

$$
2 N^{\prime}=2\{\mathrm{Q}[1]+\mathrm{Q}[2]+\cdots+\mathrm{Q}[\mathrm{~N}]\}
$$

random keys, where $\mathrm{Q}[\mathrm{i}]$ is the maximum number of rectangles of type $i$ (for $i=1, \ldots, N$ ) that can be packed.

- $K=\left(k[1], \ldots, k\left[N^{\prime}\right], \quad k\left[N^{\prime}+1\right], \ldots, k\left[2 N^{\prime}\right]\right)$


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Rectangle type
packing sequence
(RTPS)

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Rectangle type
packing sequence (RTPS)

Vector of placement procedures (VPP)

## Decoding

- Simple heuristic to pack rectangles:
- Make $\mathrm{Q}[\mathrm{i}]$ copies of rectangle i , for $\mathrm{i}=1, \ldots, \mathrm{~N}$.
- Order the $\mathrm{N}^{\prime}=\mathrm{Q}[1]+\mathrm{Q}[2]+\cdots+\mathrm{Q}[\mathrm{N}]$ rectangles in some way.
- Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order.


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## Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
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## Decoding

- If BL is used, ERSs are ordered such that $\operatorname{ERS}[i]<\operatorname{ERS}[j]$ if $y[i]<y[j]$ or $y[i]=y[j]$ and $x[i]<x[j]$.



## ERS[i] < ERS[j]



# BL can run into problems even on small instances (Liu \& Teng, 1999). 

Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.


We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



## Decoding

- If LB is used, ERSs are ordered such that $E R S[i]<E R S[j]$ if $x[i]<x[j]$ or $x[i]=x[j]$ and $y[i]<y[j]$.


ERS[i] < ERS[j]



## Etsiz










4 does fit in $\operatorname{ERS}[2]$.



Optimal solution!

## Experimental results

## Design

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- PH: population-based heuristic of Beasley (2004)
- GA: genetic algorithm of Hadjiconsantinou \& Iori (2007)
- GRASP: greedy randomized adaptive search procedure of Alvarez-Valdes et al. (2005)
- TABU: tabu search of Alvarez-Valdes et al. (2007)


## Number of best solutions / total instances

| Problem | PH | GA | GRASP | TABU | BRKGA <br> BL-LBL-LANR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| From literature (optimal) | 13/21 | 21/21 | 18/21 | 21/21 | 21/21 |
| Large random | 0/21 | 0/21 | 5/21 | 8/21 | 20/21 |
| Zero-waste |  |  | 5/31 | 17/31 | 30/31 |
| Doubly constrained | 11/21 |  | 12/21 | 17/21 | 19/21 |

[^0]Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

| Problem | Min solution <br> time <br> (secs) | Avg solution <br> time | (secs) | Max solution <br> time (secs) |
| :--- | :--- | :--- | :--- | :--- |
| From literature <br> (optimal) | 0.00 | 0.05 | 0.55 |  |
| Large random | 1.78 | 23.85 | 72.70 |  |
| Zero-waste | 0.01 | 82.21 | 808.03 |  |
| Doubly <br> constrained | 0.00 | 1.16 | 16.87 |  |

2D-ngcutcon18-20678.txt: 20678



## Some remarks

We have extended this to 3D packing:
J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers \& Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: http://www.research.att.com/~mgcr/doc/brkga-pack3d.pdf


## 3D bin packing


J.F. Gonçalves and M.G.C.R., "A biased random-key genetic algorithm for 2D and 3D bin packing problems," International J. of Production Economics, vol. 15, pp. 500-510, 2013.
http://www.research.att.com/~mgcr/doc/brkga-binpacking.pdf

## 3D bin packing problem

 Container (bin) of

Minimize number of containers


Boxes of different dimensions

## 3D bin packing constraints

- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)


## Six possible orientations for each box



## Difference process - DP

 (Lai \& Chan, 1997)

When box is placed in container ... use DP to keep track of maximal free spaces

## Encoding

Solutions are encoded as vectors of $3 n$ random keys, where n is the number of boxed to be packed.

$$
X=\left(x_{1}, x_{2}, \ldots, x_{n}, x_{n+1}, x_{n+2}, \ldots, x_{2 n}, x_{2 n+1}, x_{2 n+2}, \ldots, x_{3 n}\right)
$$

Box packing sequence
Placement heuristic
Box orientation

## Decoding

1) Sort first $n$ keys of $X$ to produce sequence boxes will be packed;
2) Use second $n$ keys of $X$ to determine which placement heuristic to use (back-bottom-left or back-left-bottom):

- if $x_{n+i}<1 / 2$ then use back-bottom-left to pack i-th box
- if $x_{n+i} \geq 1 / 2$ then use back-left-bottom to pack i-th box

3) Use third $n$ keys of $X$ to determine which of six orientations to use when packing box:

- $x_{2 n+i} \in[0,1 / 6)$ : orientation 1 ;
- $x_{2 n+i} \in[1 / 6,2 / 6)$ : orientation $2 ; \ldots$
- $x_{2 n+i} \in[5 / 6,1]:$ orientation 6 .


## Decoding

## For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation


## Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness: aNB
$-\mathrm{aNB}=\mathrm{NB}+($ LeastLoad $/$ BinVolume $)$, where
$\times$ LeastLoad is load on least loaded bin
$\times$ BinVolume is volume of bin: $\mathrm{H} \times \mathrm{W} \times \mathrm{L}$


## Experiment

- Parameters:
- population size: $p=30 n$
- size of elite partition: $\mathrm{P}_{\mathrm{e}}=.10 \mathrm{p}$
- number of of mutans: $p_{m}=.15 p$
- crossover probability: 0.7
- stopping criterion: 300 generations


## Experiment

- Instances:
- 320 instances of Martello et al. (2000)
- generator is available at http://www.diku.dk/ $/$ pisinger/codes/html $^{\text {m }}$
- 8 classes
- 40 instances per class
-10 instances for each value of $n \in\{50,100,150$, 200)


## Experiment

- We compare BRKGA with:
- TS3, the tabu search of Lodi et al. (2002)
- GLS, the guided local search of Faroe et al. (2003)
- TS2PACK, the tabu search of Crainic et al. (2009)
- GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)


## Class 1 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 12.5 | $\mathbf{1 3 . 4}$ | $\mathbf{1 3 . 4}$ | $\mathbf{1 3 . 4}$ | $\mathbf{1 3 . 4}$ | $\mathbf{1 3 . 4}$ |
| 100 | 25.1 | $\mathbf{2 6 . 6}$ | 26.6 | $\mathbf{2 6 . 6}$ | 26.7 | 26.7 |
| 150 | 34.7 | 36.4 | 36.4 | 36.7 | 37.0 | 37.0 |
| 200 | 48.4 | 50.9 | $\mathbf{5 0 . 9}$ | 51.2 | 51.1 | 51.2 |
| Sum: | 120.7 | $\mathbf{1 2 7 . 3}$ | $\mathbf{1 2 7 . 3}$ | $\mathbf{1 2 7 . 9}$ | 128.2 | 128.3 |

## Class 2 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 12.7 | 13.8 | 13.8 | $\mathbf{1 3 . 8}$ |  |  |
| 100 | 24.1 | 25.6 | 25.7 | 25.7 | Did not run |  |
| 150 | 35.1 | 36.7 | 36.9 | 37.2 |  |  |
| 200 | 47.5 | 49.4 | 49.4 | 50.1 |  |  |
| Sum: | 119.4 | $\mathbf{1 2 5 . 5}$ | 125.8 | 126.8 |  |  |

## Class 3 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 12.3 | 13.3 | 13.3 | 13.3 |  |  |
| 100 | 24.7 | 25.9 | 26.0 | 26.0 | Did not run |  |
| 150 | 36.0 | 37.5 | 37.6 | 37.7 |  |  |
| 200 | 47.8 | 49.8 | 50.0 | 50.5 |  |  |
| Sum: | 120.8 | $\mathbf{1 2 6 . 5}$ | 126.9 | 127.5 |  |  |

## Class 4 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 28.7 | 29.4 | 29.4 | $\mathbf{2 9 . 4}$ | 29.4 | 29.4 |
| 100 | 57.6 | 59.0 | 59.0 | 59.0 | 58.9 | 59.0 |
| 150 | 85.2 | 86.8 | 86.8 | 86.8 | 86.8 | 86.8 |
| 200 | 116.3 | 118.8 | 118.8 | 118.8 | 118.8 | 119.0 |
| Sum: | 287.8 | 294.0 | 294.0 | 294.0 | $\mathbf{2 9 3 . 9}$ | 294.2 |

## Class 5 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 7.3 | 8.3 | 8.3 | 8.4 | 8.3 | 8.3 |
| 100 | 12.9 | 15.0 | 15.0 | 15.0 | 15.2 | 15.1 |
| 150 | 17.4 | 20.0 | 20.1 | 20.4 | 20.1 | 20.2 |
| 200 | 24.4 | 27.1 | 27.1 | 27.6 | 27.4 | 27.2 |
| Sum: | 62.0 | $\mathbf{7 0 . 4}$ | 70.5 | 71.4 | 71.0 | 70.8 |

## Class 6 - Bin size: $10 \times 10 \times 10$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 8.7 | 9.8 | 9.8 | 9.9 | 9.8 | 9.8 |
| 100 | 17.5 | 18.8 | 19.0 | 19.1 | 19.1 | 19.1 |
| 150 | 26.9 | 29.2 | 29.2 | 29.4 | 29.2 | 29.4 |
| 200 | 35.0 | 37.2 | 37.4 | 37.7 | 37.7 | 37.7 |
| Sum: | 88.1 | $\mathbf{9 5 . 0}$ | 95.4 | 96.1 | 95.8 | 96.0 |

## Class 7 - Bin size: $40 \times 40 \times 40$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 6.3 | 7.4 | 7.4 | 7.5 | 7.4 | 7.4 |
| 100 | 10.9 | 12.2 | 12.5 | 12.5 | 12.3 | 12.3 |
| 150 | 13.7 | 15.2 | 16.0 | 16.1 | 15.8 | 15.8 |
| 200 | 21.0 | 23.4 | 23.5 | 23.9 | 23.5 | 23.5 |
| Sum: | 51.9 | $\mathbf{5 8 . 2}$ | 59.4 | 60.0 | 59.0 | 59.0 |

## Class 8 - Bin size: $100 \times 100 \times 100$

| Boxes | LB | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 50 | 0.8 | 9.2 | 9.2 | 9.3 | 9.2 | 9.2 |
| 100 | 17.5 | 18.9 | 18.9 | 18.9 | 18.8 | 18.9 |
| 150 | 21.3 | 23.5 | 24.1 | 24.1 | 23.9 | 23.9 |
| 200 | 26.7 | 29.3 | 29.8 | 30.3 | 30.0 | 29.9 |
| Sum: | 66.3 | $\mathbf{8 0 . 9}$ | 82.0 | 82.6 | 81.9 | 81.9 |

## Summary

| Class | Bin size | BRKGA | GRASP | TS3 | TS2PACK | GLS |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | $100^{3}$ | 127.3 | 127.3 | 127.9 | 128.2 | 128.3 |
| 2 | $100^{3}$ | 125.5 | 125.8 | 126.8 |  |  |
| 3 | $100^{3}$ | 126.5 | 126.9 | 127.5 |  |  |
| 4 | $100^{3}$ | 294.0 | 294.0 | 294.0 | 293.9 | 294.2 |
| 5 | $100^{3}$ | 70.4 | 70.5 | 71.4 | 71.0 | 70.8 |
| 6 | $10^{3}$ | 95.0 | 95.4 | 96.1 | 95.8 | 96.0 |
| 7 | $40^{3}$ | 58.2 | 59.4 | 60.0 | 59.0 | 59.0 |
| 8 | $100^{3}$ | 80.9 | 82.0 | 82.6 | 81.9 | 81.9 |
| Sum(rows 1, 4-8): | $\mathbf{7 2 5 . 8}$ | 728.6 | 732.0 | 729.8 | 730.2 |  |
| Sum(rows 1-8): | $\mathbf{9 7 7 . 8}$ | 981.3 | 986.3 |  |  |  |

## Concluding remarks

- Reviewed BRKGA framework
- Applied framework to
- 2D/3D packing to maximize value packed
- 2D/3D bin packing to minimize number of bins
- All decoders were simple heuristics
- BRKGA "learned" how to "operate" the heuristics
- In all cases, several new best known solutions were produced


These slides and all of the papers cited in this lecture can be downloaded from my homepage:
http://www.research.att.com/~~mgcr


[^0]:    * For large random: number of best average solutions / total instance classes

