Packing with biased random-key genetic algorithms

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Lecture given at XV ONPCE
S. J. do Rio Preto, Brazil ✤ March 21-22, 2013

Summary

- Metaheuristics and basic concepts of genetic algorithms
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
  - Encoding / Decoding
  - Initial population
  - Evolutionary mechanisms
  - Problem independent / problem dependent components
  - Multi-start strategy
  - Specifying a BRKGA
  - Application programming interface (API) for BRKGA
- BRKGA for 2-dim and 3-dim packing
- BRKGA for 3-dim bin packing

Metaheuristics

Metaheuristics are heuristics to devise heuristics.

Metaheuristics

Metaheuristics are high level procedures that coordinate simple heuristics, such as local search, to find solutions that are of better quality than those found by the simple heuristics alone.

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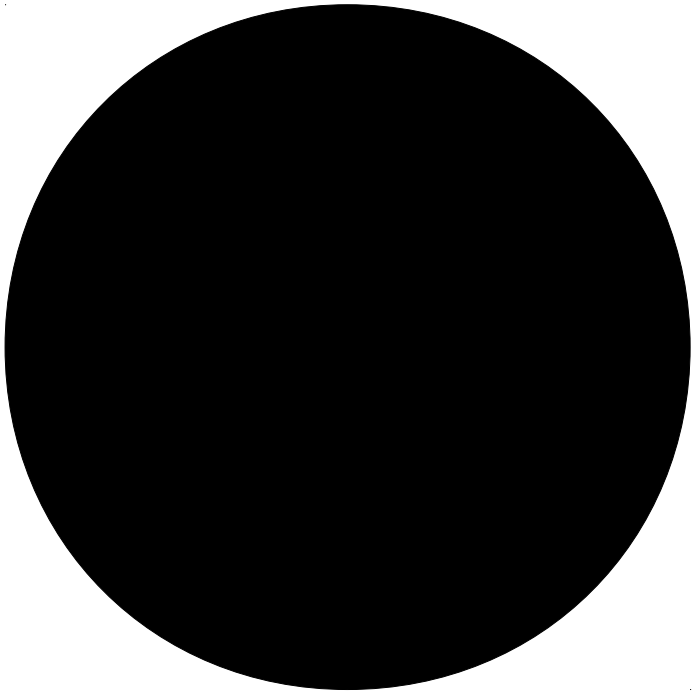
Examples: GRASP and C-GRASP, simulated annealing, genetic algorithms, tabu search, scatter search, ant colony optimization, variable neighborhood search, and biased random-key genetic algorithms (BRKGA).

Genetic algorithms

Genetic algorithms

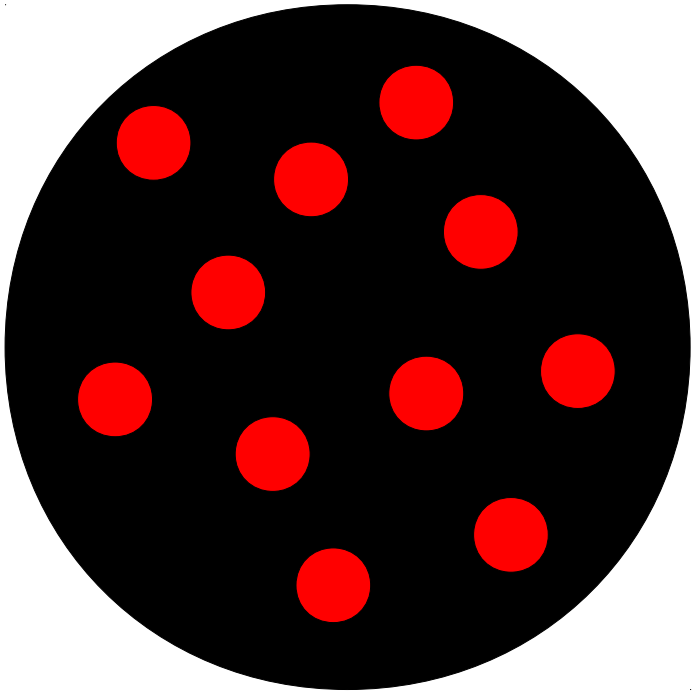
Holland (1975)



Adaptive methods that are used to solve search and optimization problems.

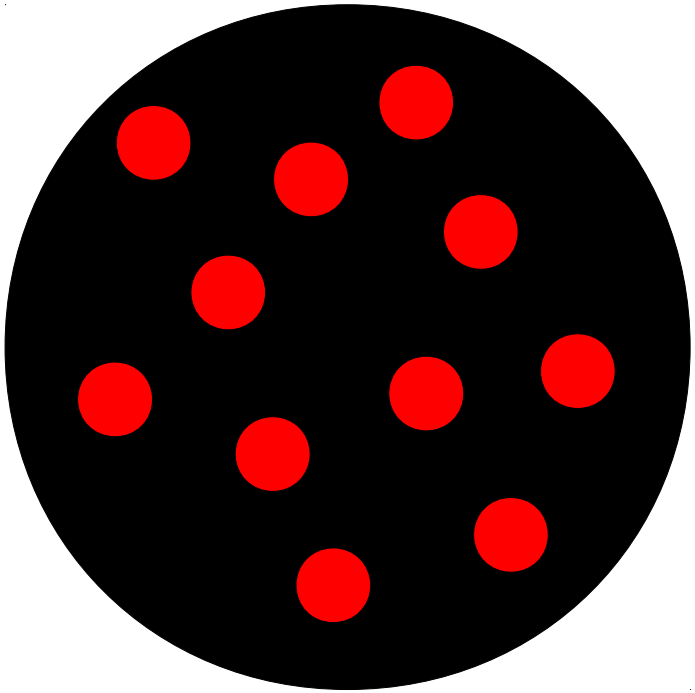
Individual: solution 

Genetic algorithms



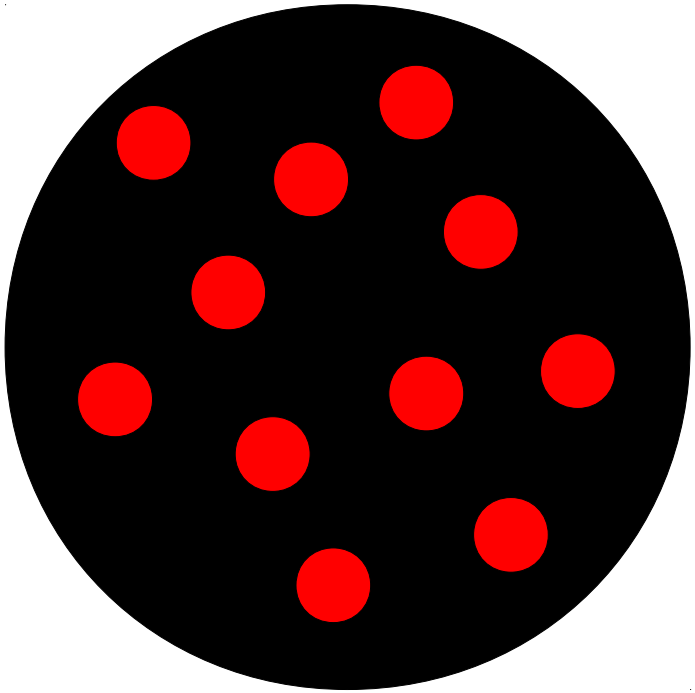
Individual: solution (chromosome = string of genes)
Population: set of fixed number of individuals

Genetic algorithms



Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

Genetic algorithms



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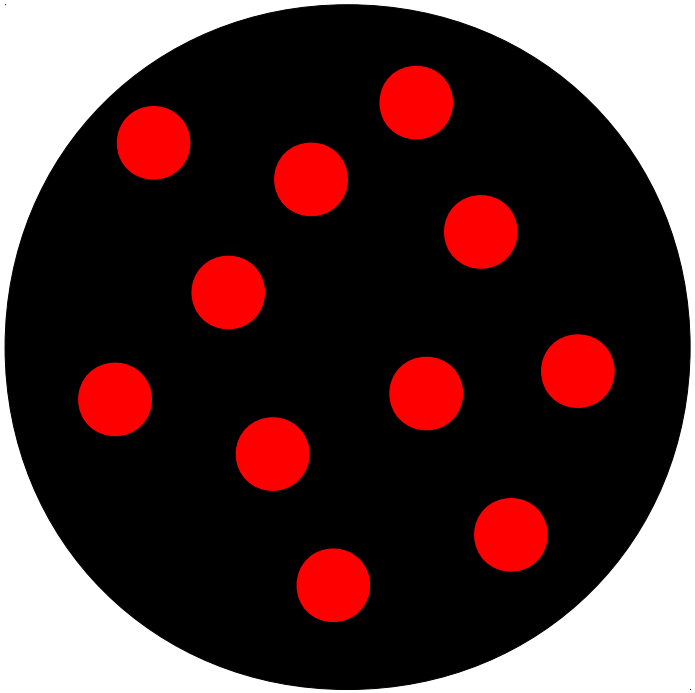
A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

Genetic algorithms

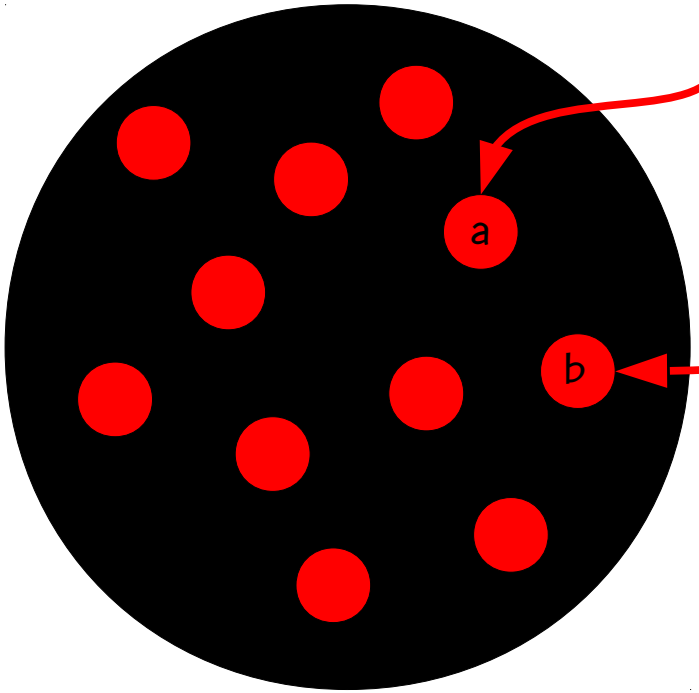


Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

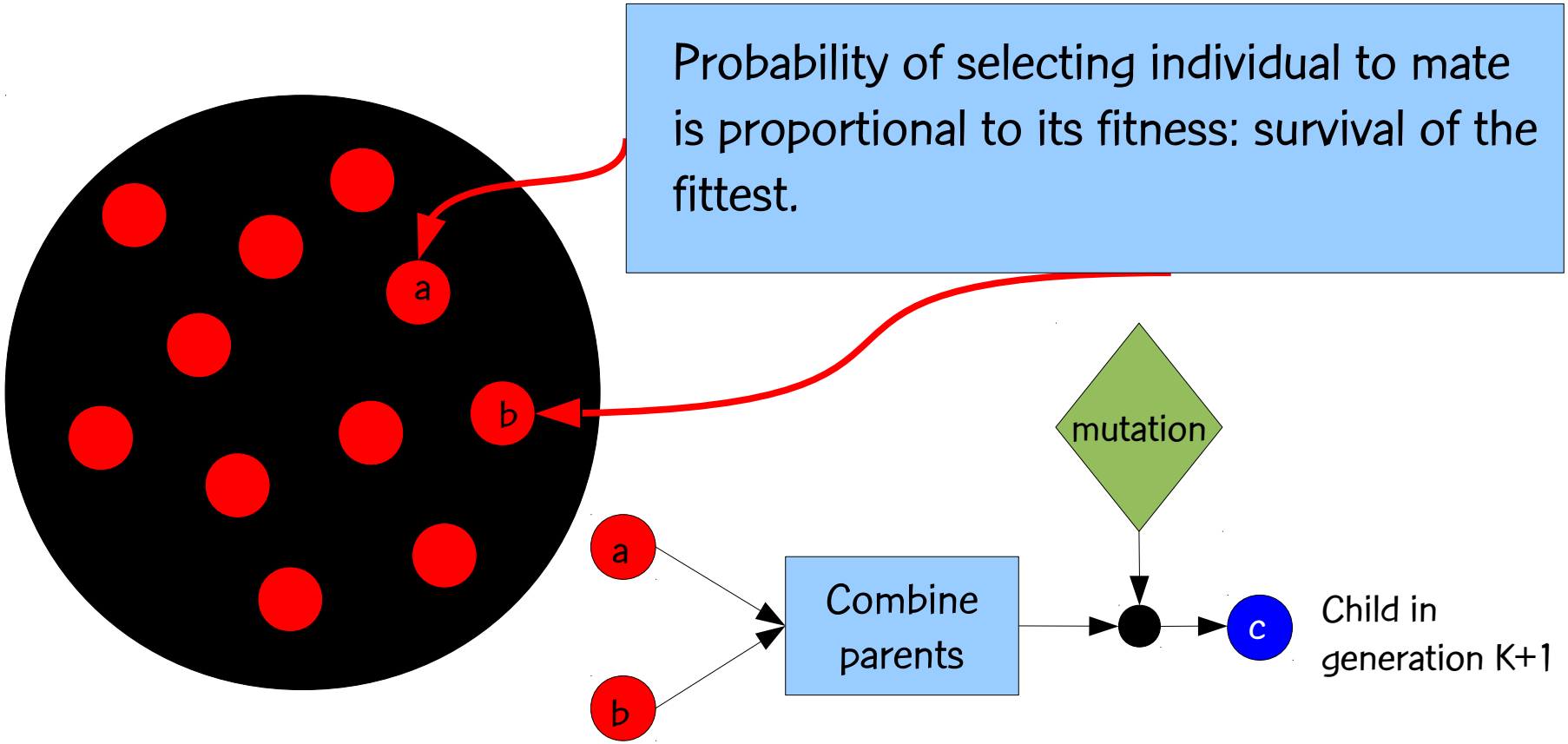
Individuals from one generation are combined to produce offspring that make up next generation.

Genetic algorithms

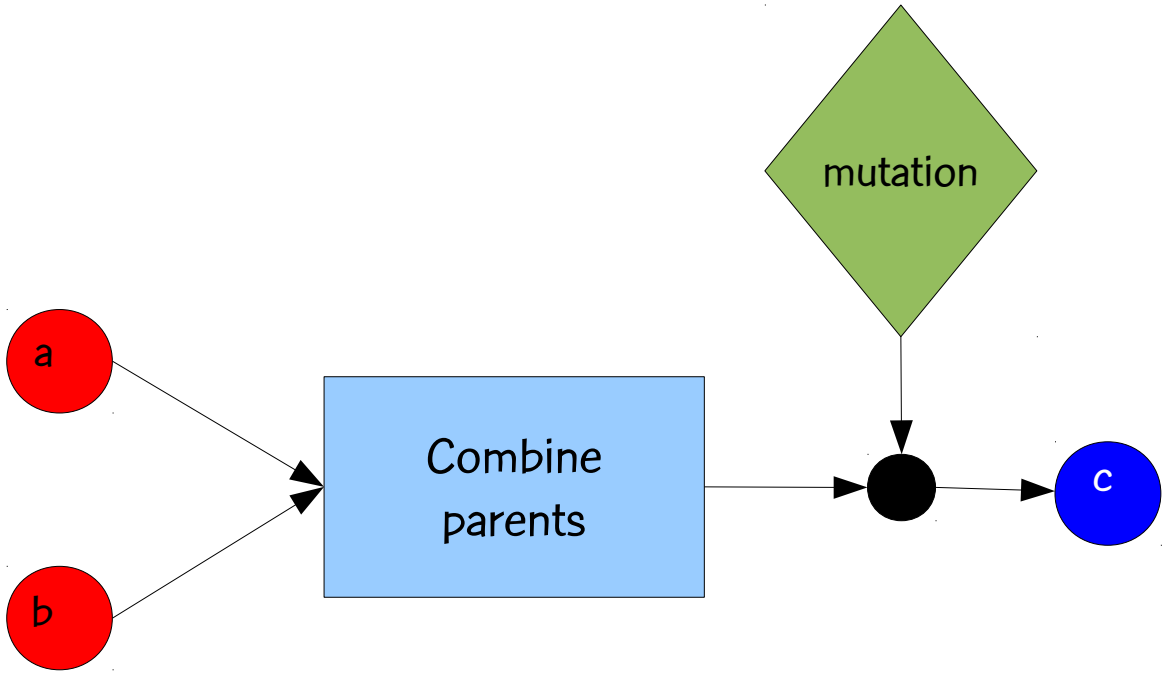


Probability of selecting individual to mate is proportional to its fitness: survival of the fittest.

Genetic algorithms



Crossover and mutation



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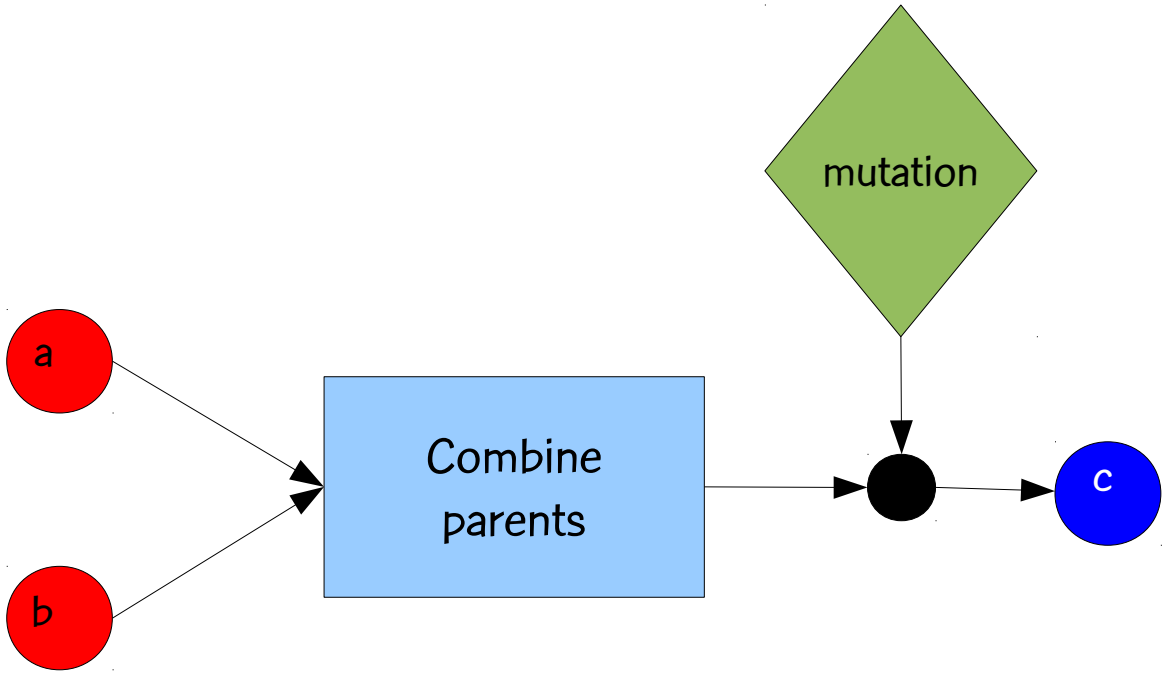
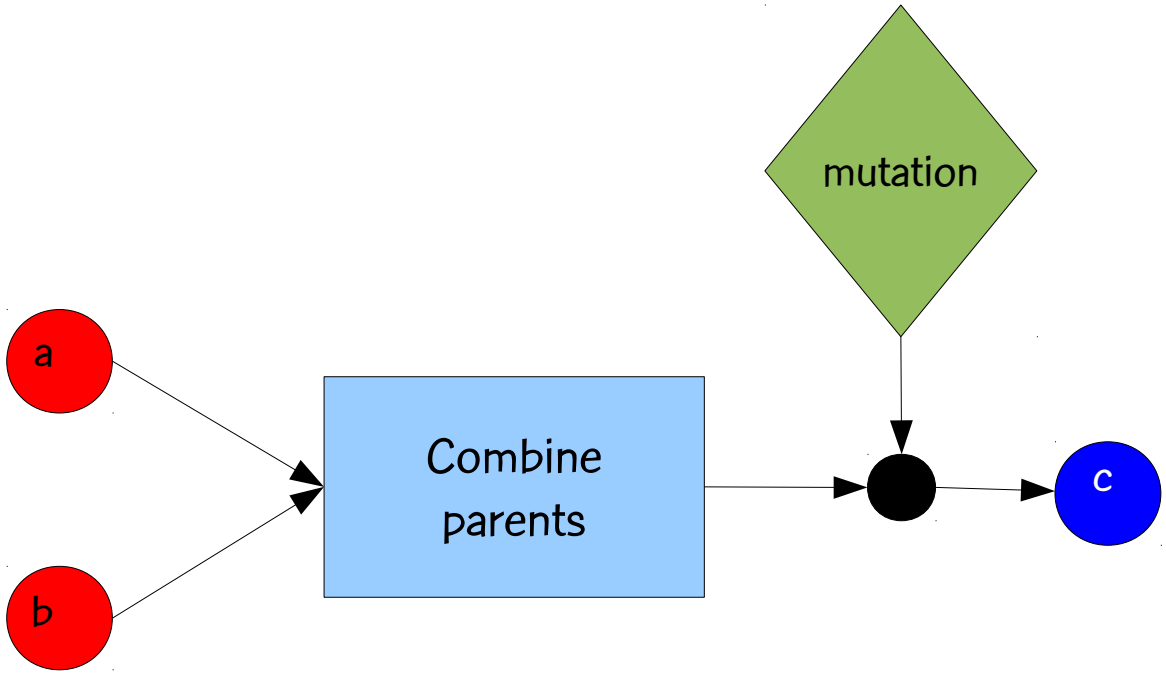


Crossover: Combines parents ... passing along to offspring characteristics of each parent ...

Intensification of search

Crossover and mutation

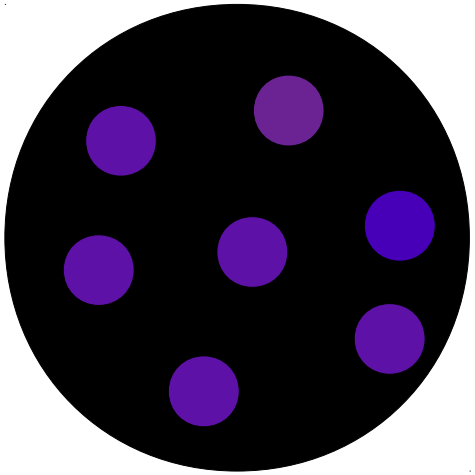


Mutation: Randomly changes chromosome of offspring ...

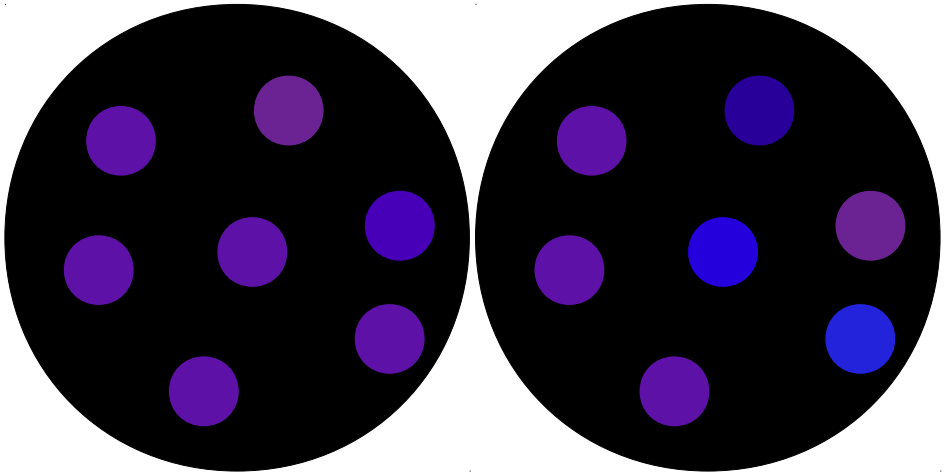
Driver of evolutionary process ...

Diversification of search

Evolution of solutions



Evolution of solutions



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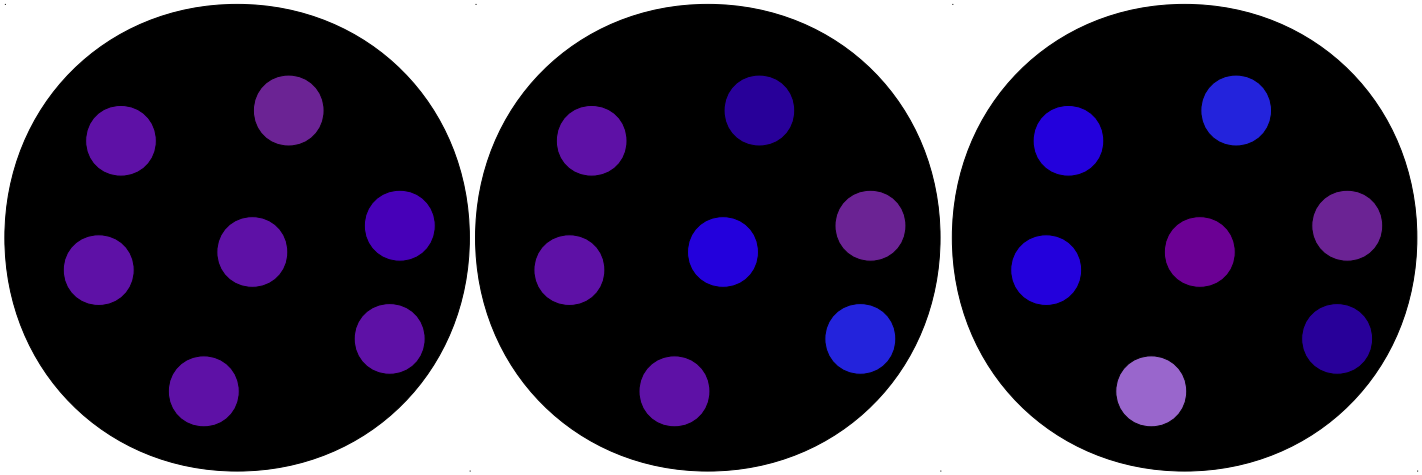
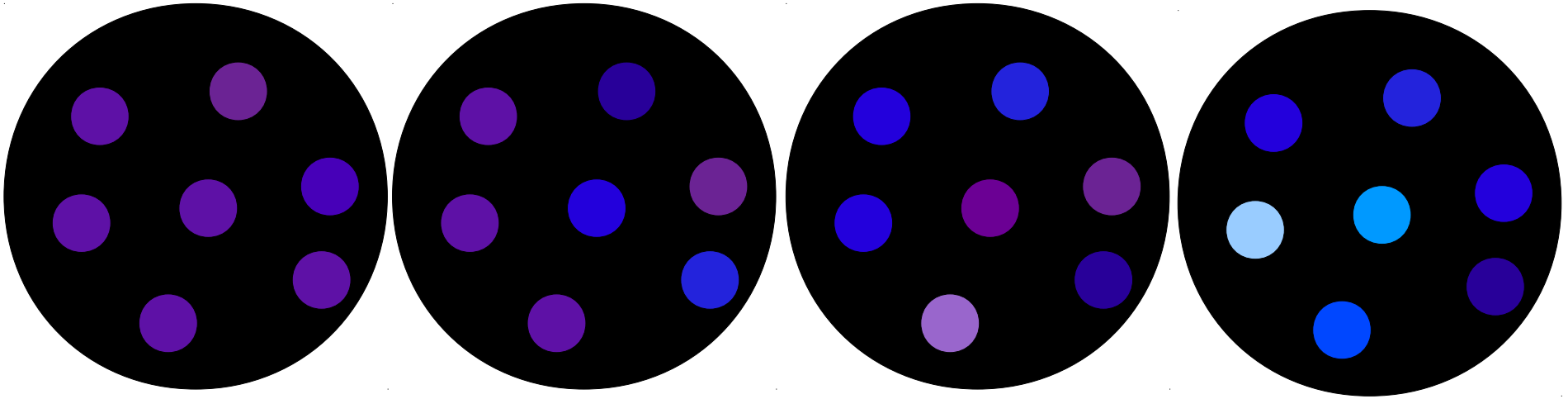
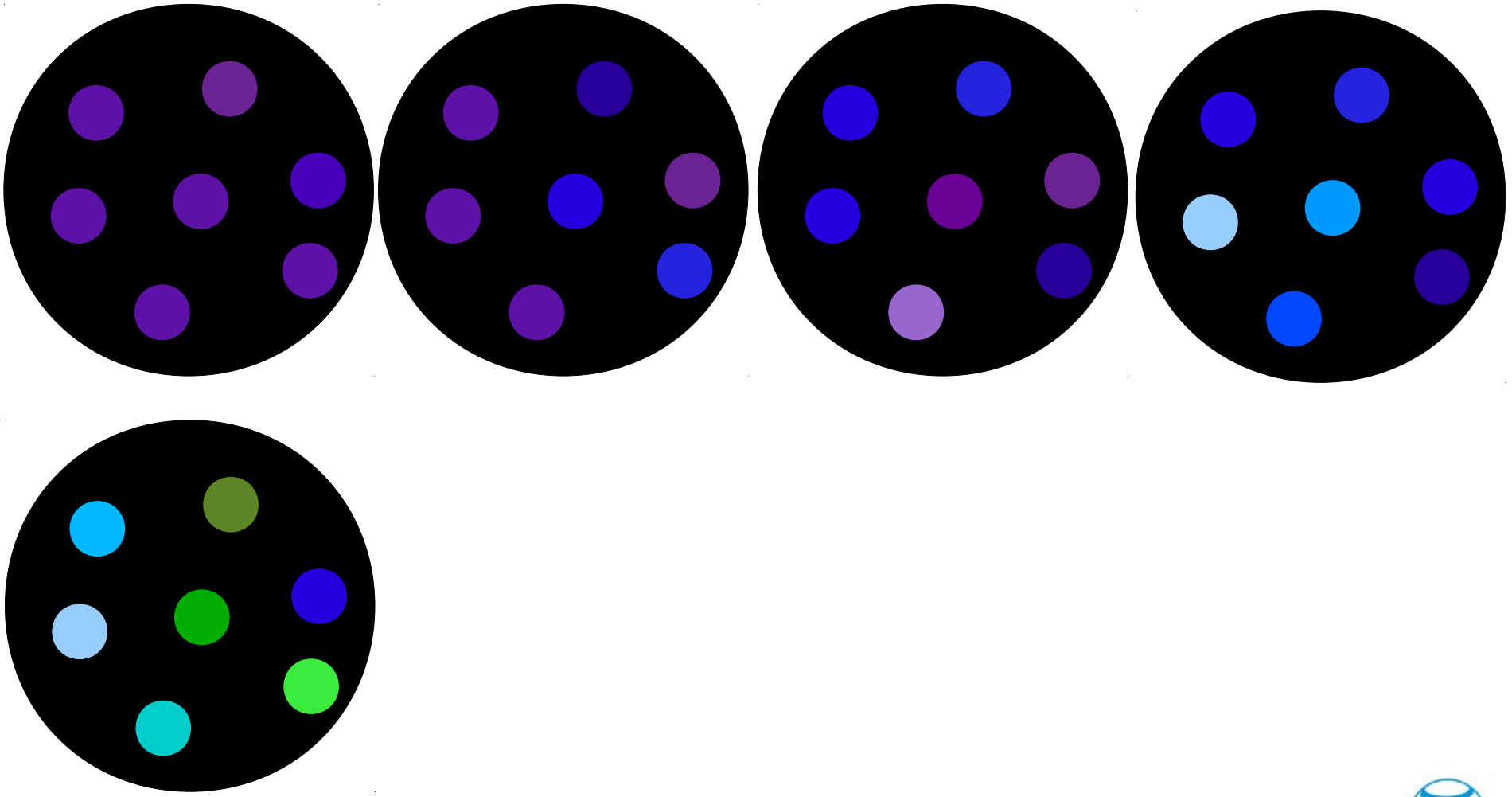


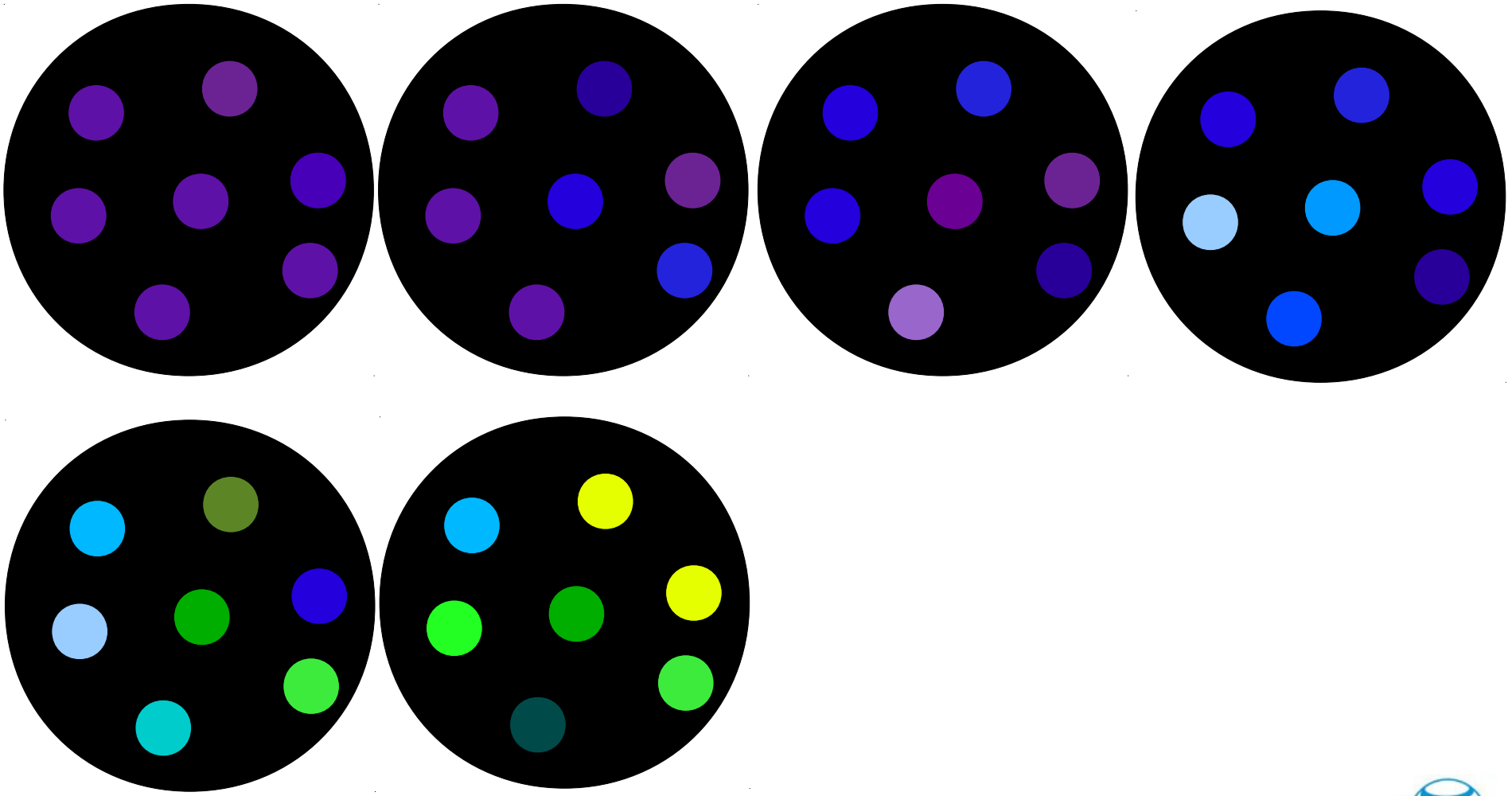
Evolution of solutions



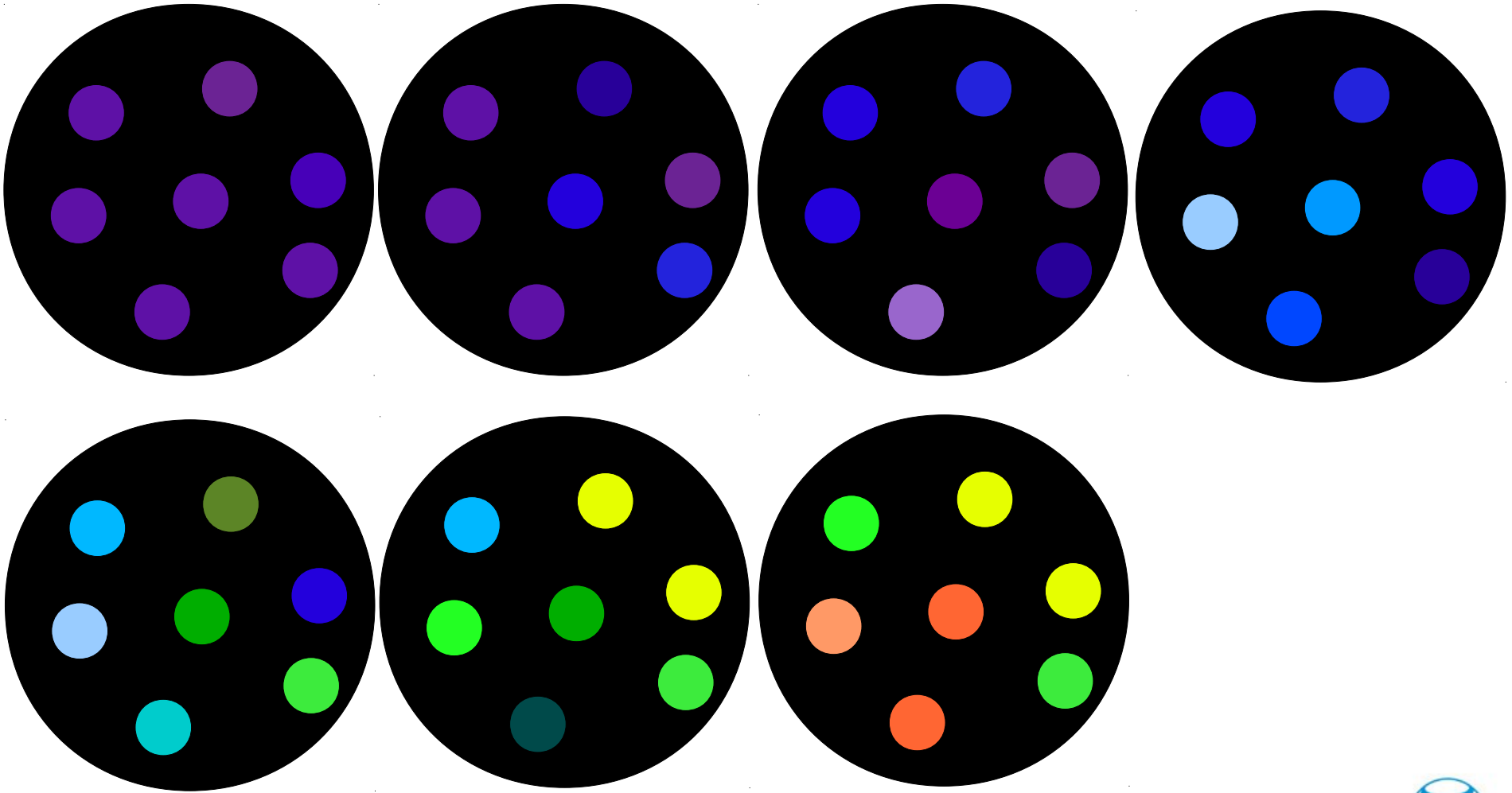
Evolution of solutions



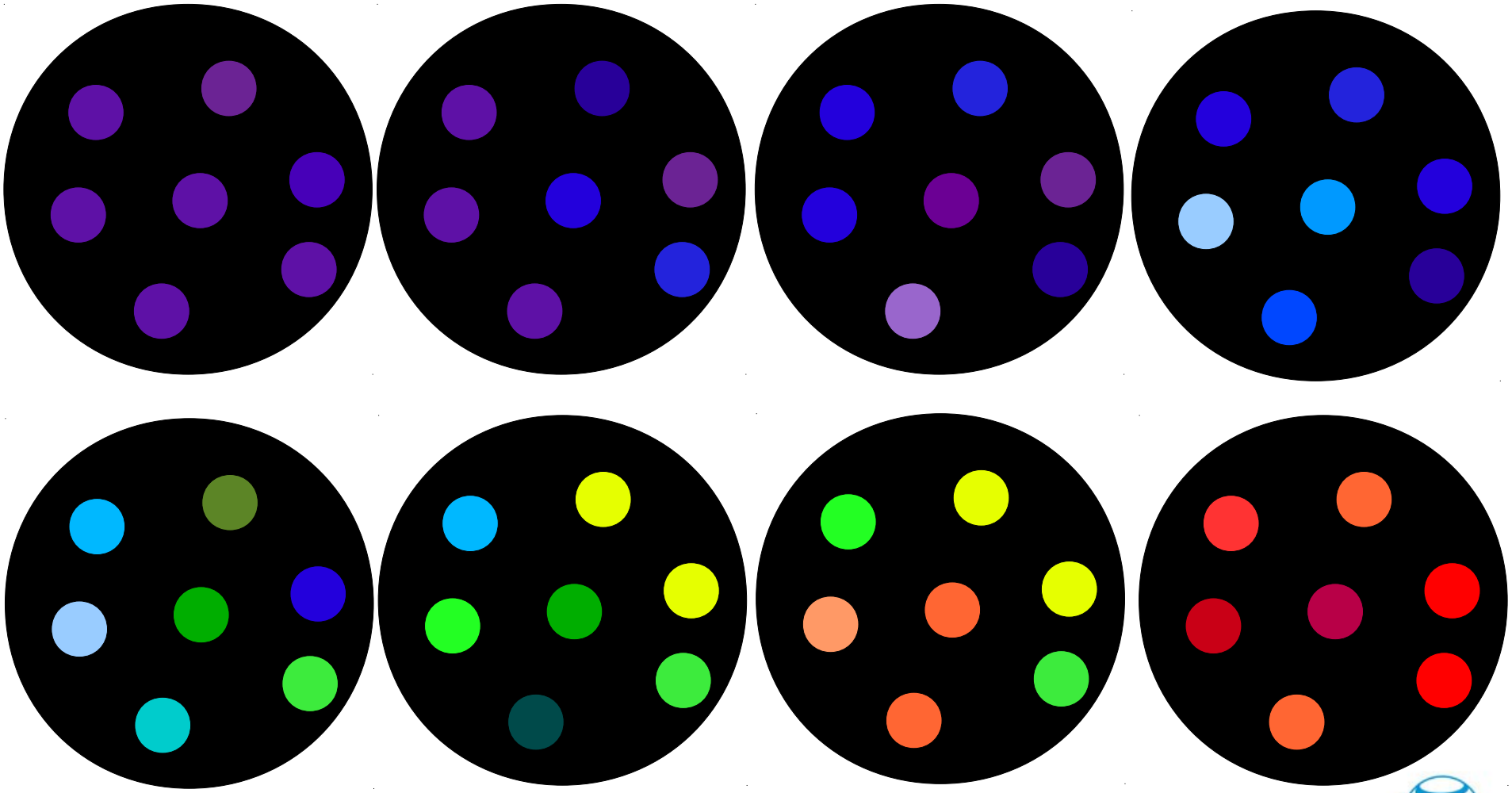
Evolution of solutions



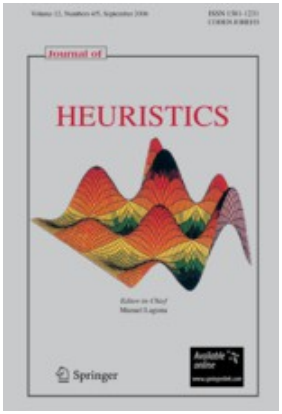
Evolution of solutions



Evolution of solutions



Reference



J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:

http://www.research.att.com/~mgcr/doc/srkga.pdf

Encoding solutions with random keys

Encoding with random keys

- A random key is a real random number in the continuous interval [0,1).

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- A random key is a real random number in the continuous interval [0,1).
- A vector X of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.

Encoding with random keys: Sequencing

Encoding

[1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658]

Encoding with random keys: Sequencing

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Decode by sorting vector of random keys

[1, 2, 4, 5, 3]

X = [0.099, 0.216, 0.368, 0.658, 0.802]

Encoding with random keys: Sequencing

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658]

encodes the sequence: 1 – 2 – 4 – 5 – 3

Encoding with random keys: Subset selection (select 3 of 5 elements)

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Encoding with random keys: Subset selection (select 3 of 5 elements)

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658]

encodes the subset: {1, 2, 4 }

Encoding with random keys: Assigning integer weights ∈ [0,10] to a subset of 3 of 5 elements

Encoding

[1, 2, 3, 4, 5 | 1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]

Encoding with random keys: Assigning integer weights ∈ [0,10] to a subset of 3 of 5 elements

Encoding

[1, 2, 3, 4, 5 | 1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]

Decode by sorting the first 5 keys and assign as the weight the value $W_i =$ **floor** $[\ 10\ X_{5+i}\ ] + 1$ to the 3 elements with smallest keys X_i, for $i = 1,\ldots,5$.

Encoding with random keys: Assigning integer weights ∈ [0,10] to a subset of 3 of 5 elements

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]

encodes the weight vector W = (5,6,–,5,–)

Genetic algorithms and random keys

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.

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- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).

S = (0.25, 0.19, 0.67, 0.05, 0.89)
s(1) s(2) s(3) s(4) s(5)

GAs and random keys

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- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).
- Sorting random keys results in a sequencing order.

S = (0.25, 0.19, 0.67, 0.05, 0.89)
s(1) s(2) s(3) s(4) s(5)

S' = (0.05, 0.19, 0.25, 0.67, 0.89)
s(4) s(2) s(1) s(3) s(5)

Sequence: 4 – 2 – 1 – 3 – 5

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)

a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)
- For each gene, flip a biased coin to choose which parent passes the allele (key, or value of gene) to the child.

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c = (0.25)

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a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)
c = (0.25, 0.90)

GAs and random keys

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a = (0.25, 0.19, 0.67, 0.05, 0.89)
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c = (0.25, 0.90, 0.76)

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c = (0.25, 0.90, 0.76, 0.05)

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a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)
c = (0.25, 0.90, 0.76, 0.05, 0.89)

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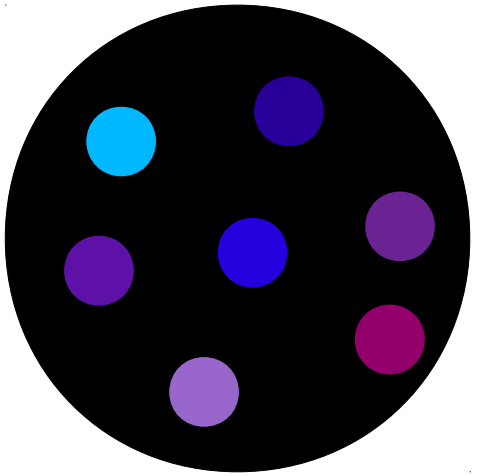
b = (0.63, 0.90, 0.76, 0.93, 0.08)
c = (0.25, 0.90, 0.76, 0.05, 0.89)

If every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

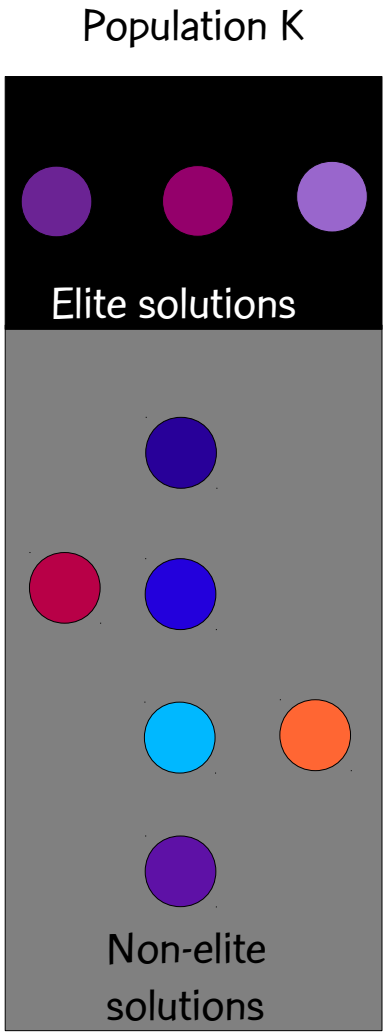
GAs and random keys

Initial population is made up of P random-key vectors, each with N keys, each having a value generated uniformly at random in the interval [0,1).



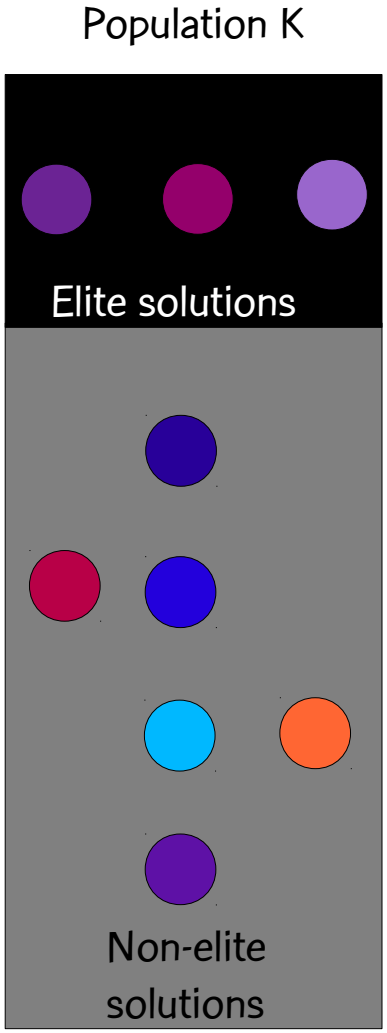
GAs and random keys

At the K-th generation, compute the cost of each solution ...



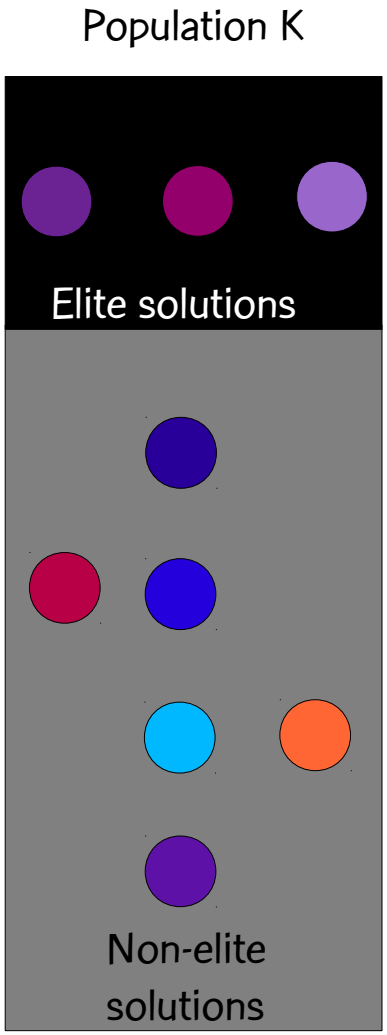
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets:



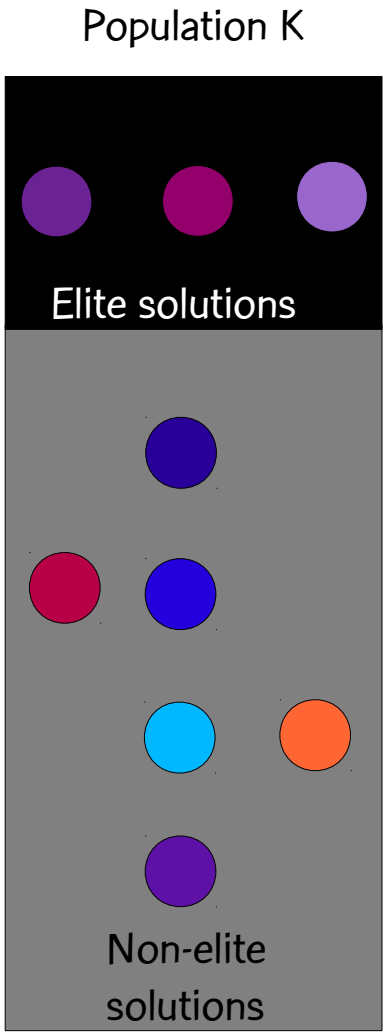
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions.



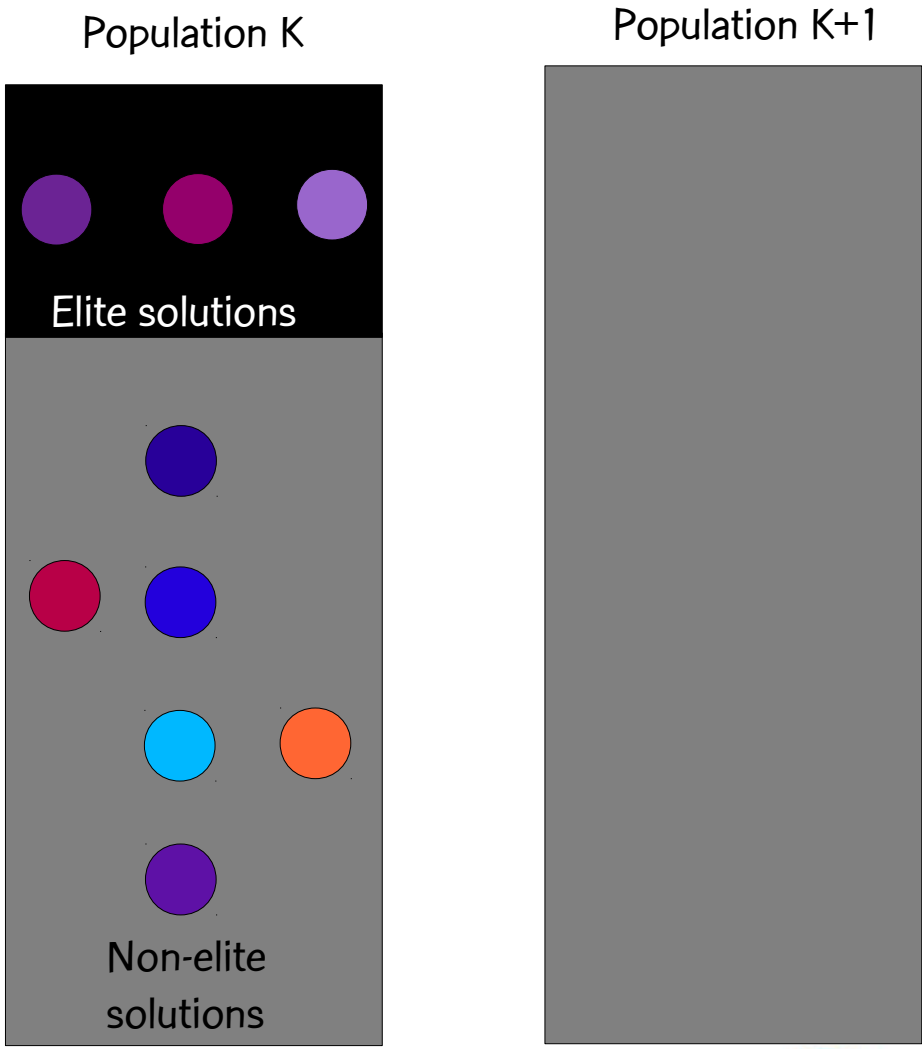
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.



GAs and random keys

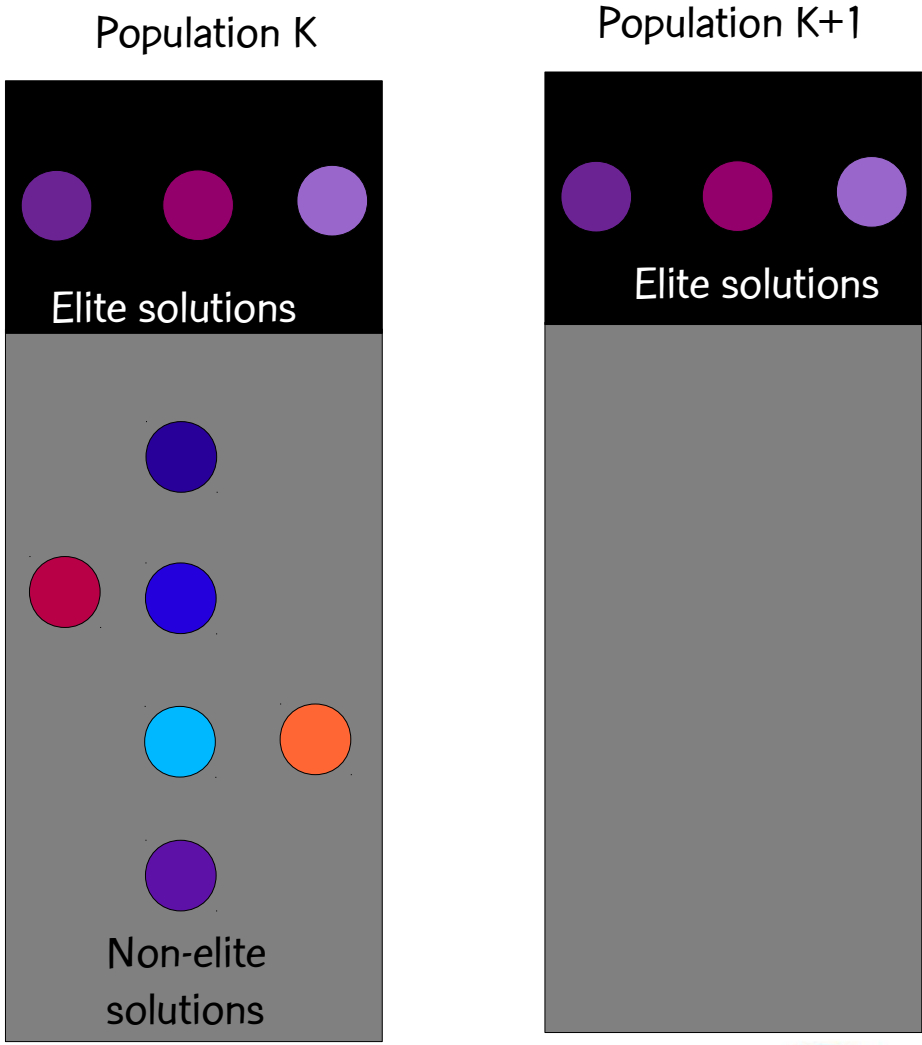
Evolutionary dynamics



GAs and random keys

Evolutionary dynamics

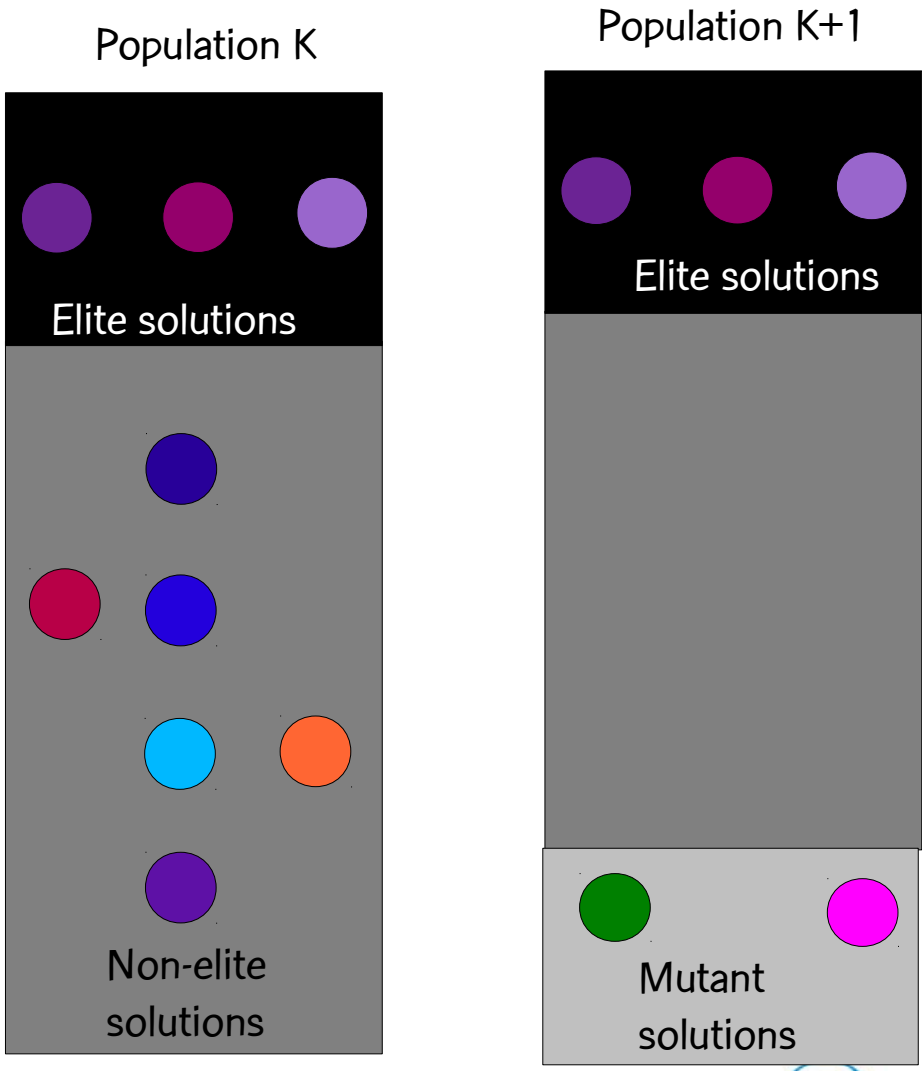
– Copy elite solutions from population K to population K+1



GAs and random keys

Evolutionary dynamics

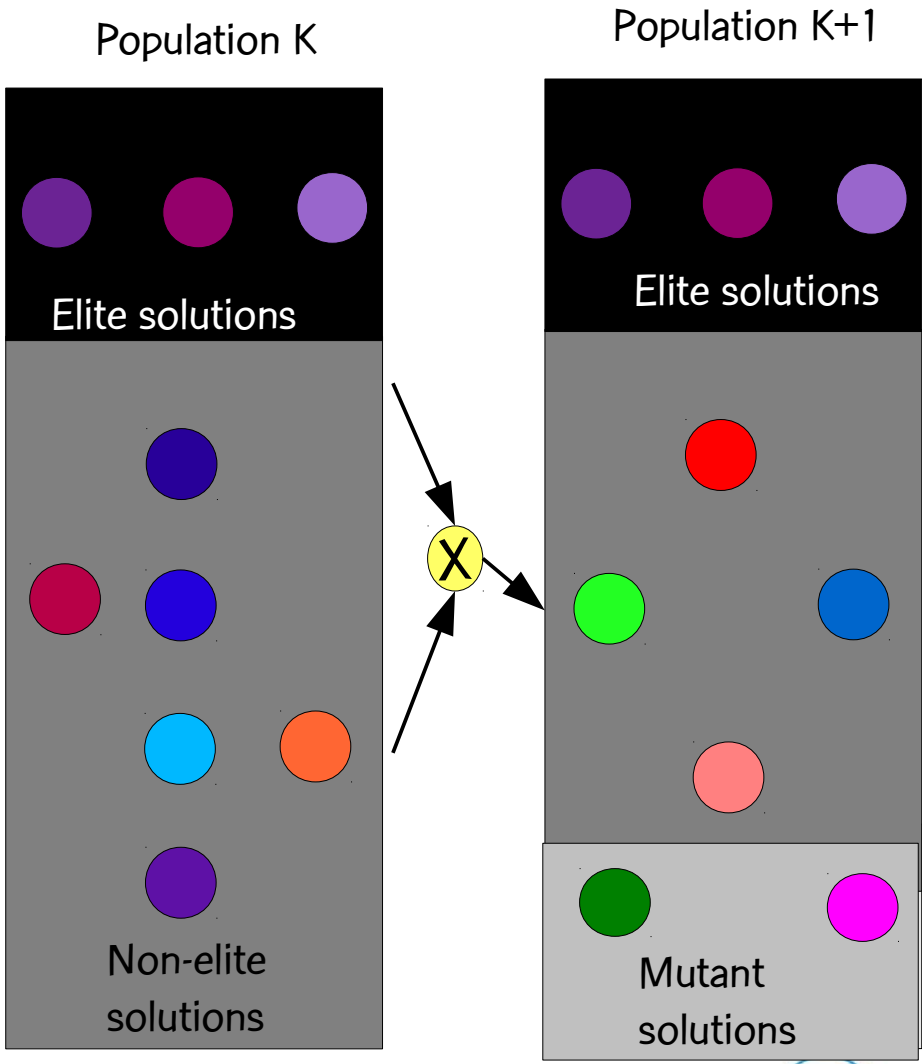
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1



GAs and random keys

Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.



Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).

Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

BRKGA

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

How RKGA & BRKGA differ

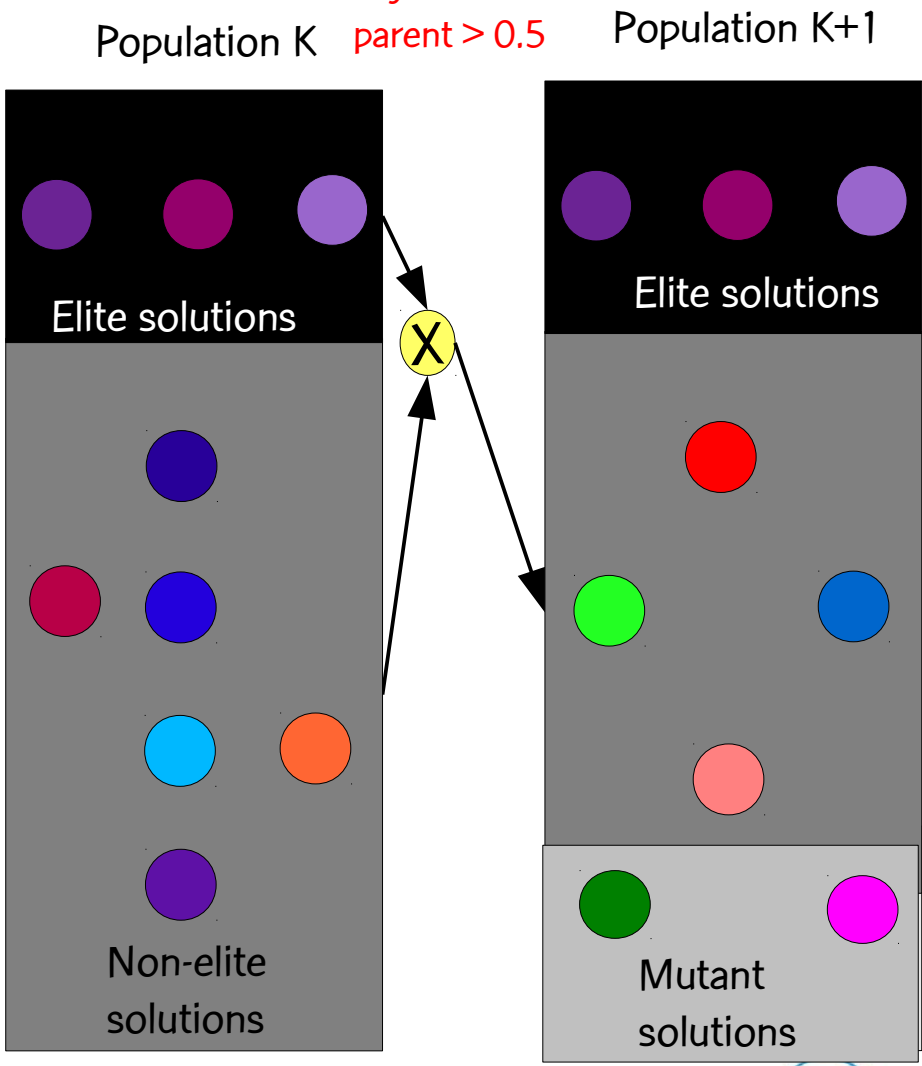
RKGA	BRKGA
both parents chosen at random from entire population	both parents chosen at random but one parent chosen from population of elite solutions
either parent can be parent A in parametrized uniform crossover	best fit parent is parent A in parametrized uniform crossover

Biased random key GA

BRKGA: Probability child inherits key of elite parent > 0.5

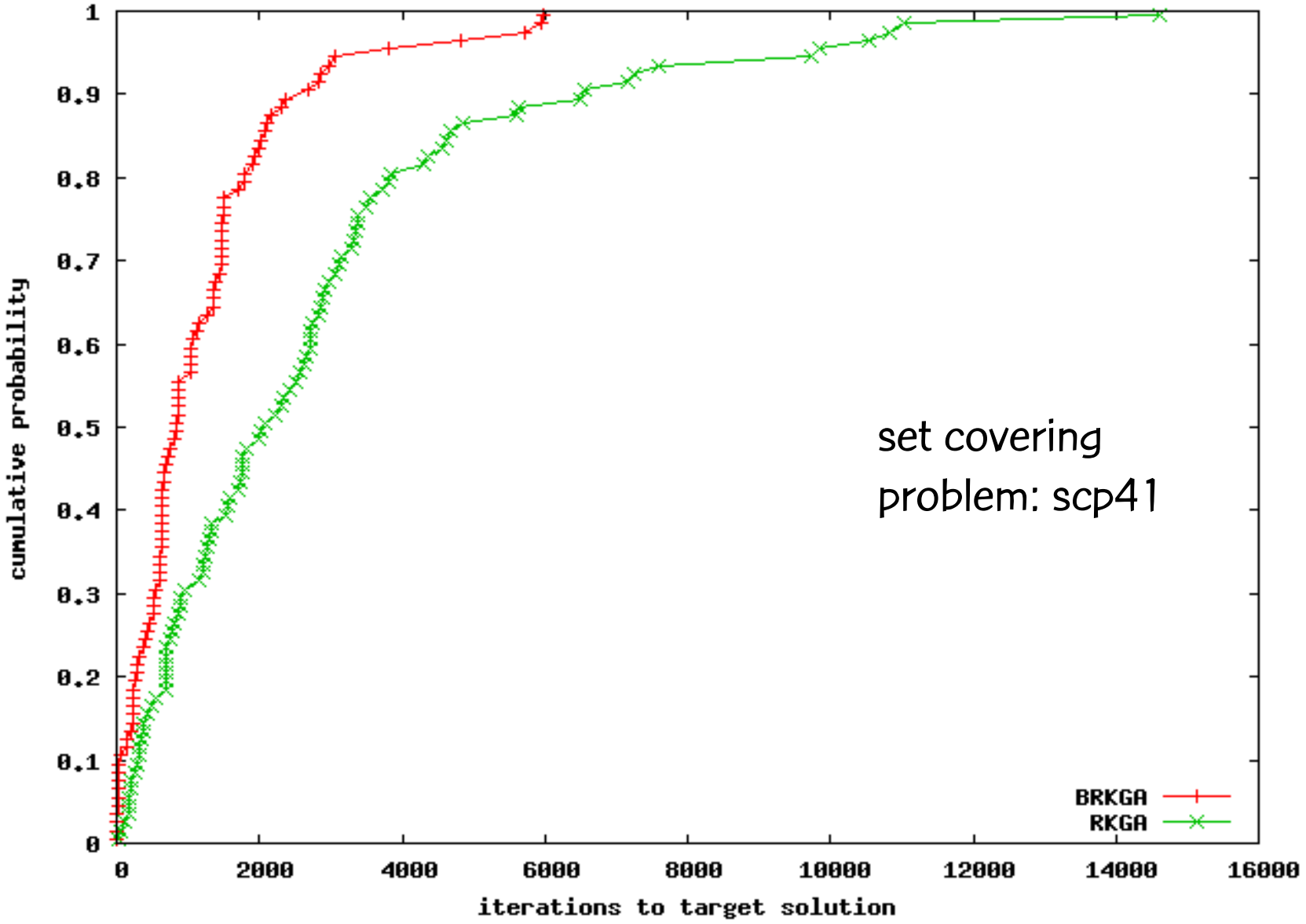
Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.
  - BIASED RANDOM-KEY GA: Mate elite solution with other solution of population K to produce child in population K+1. Mates are chosen at random.

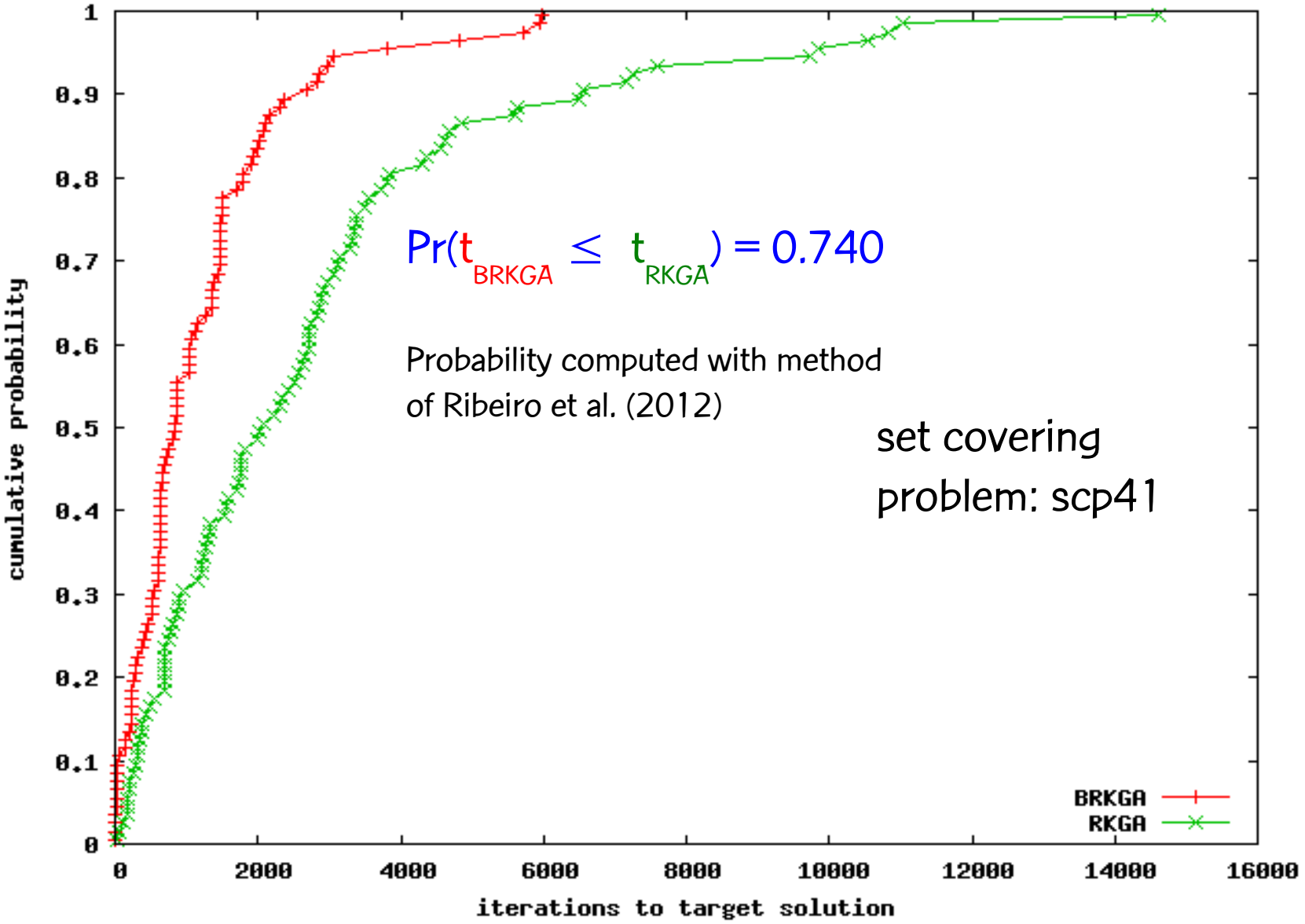


Paper comparing BRKGA and Bean's Method

Gonçalves, R., and Toso, "Biased and unbiased random-key genetic algorithms: An experimental analysis", AT&T Labs Research Technical Report, Florham Park, December 2012.



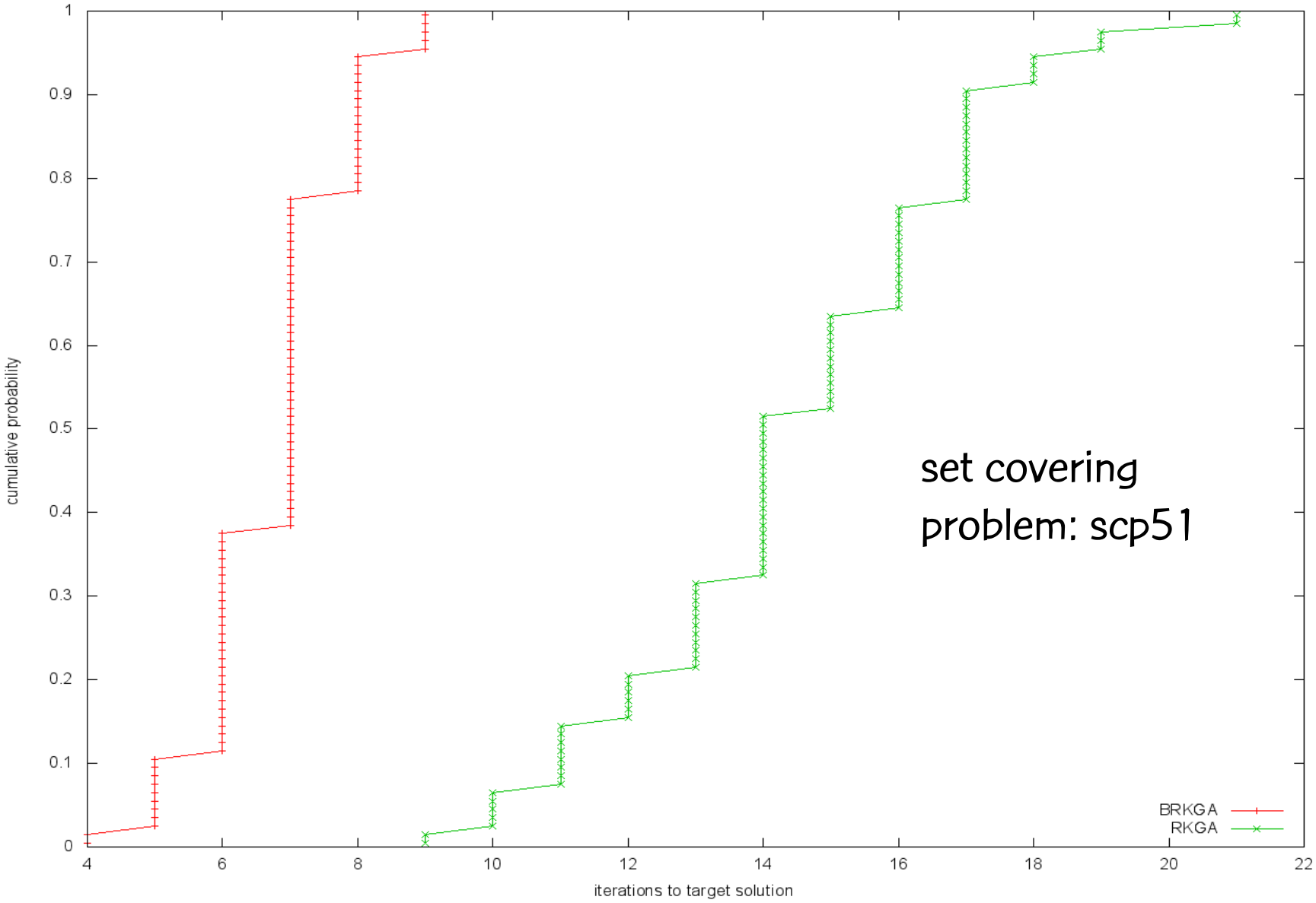
set covering problem: scp41

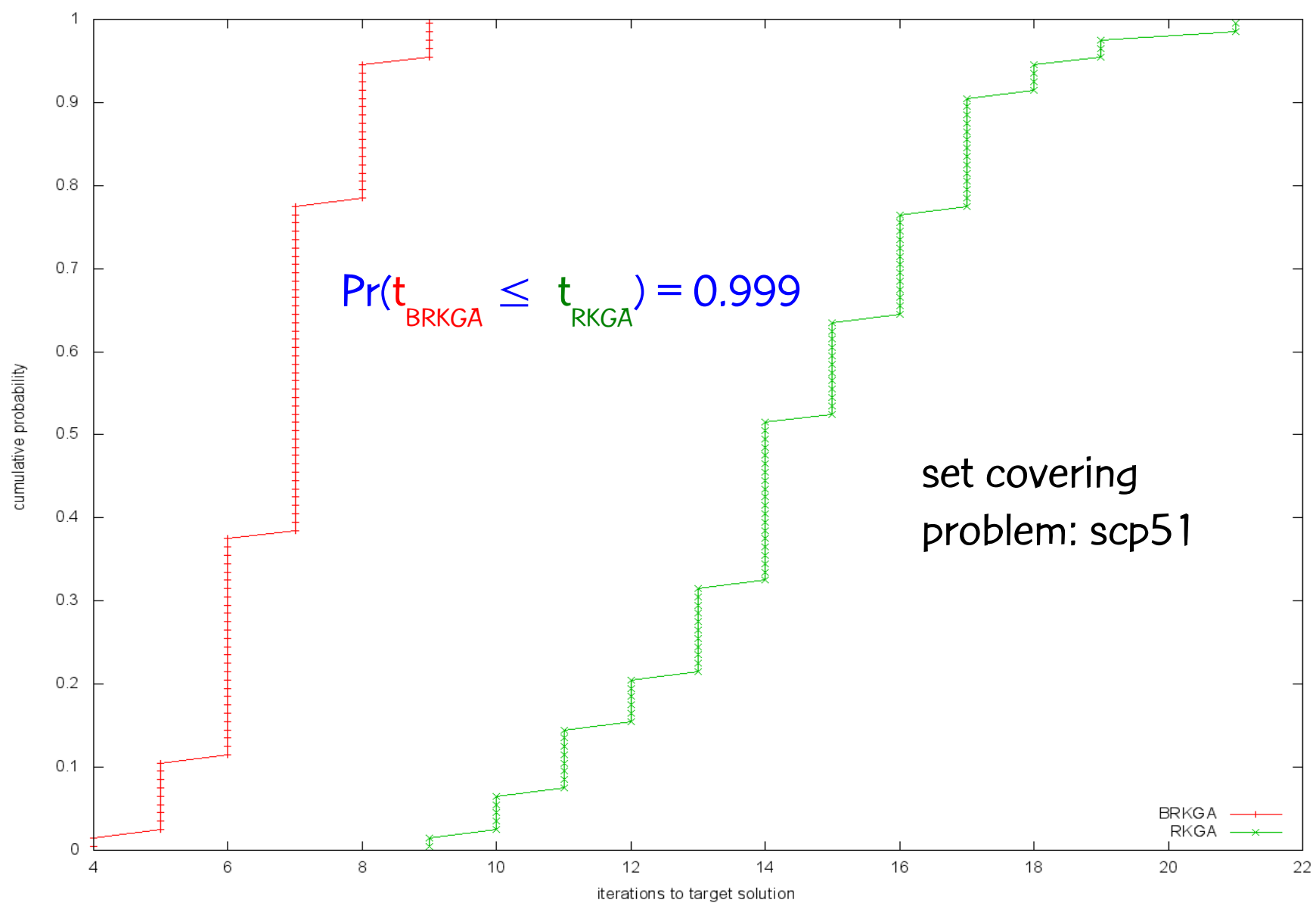


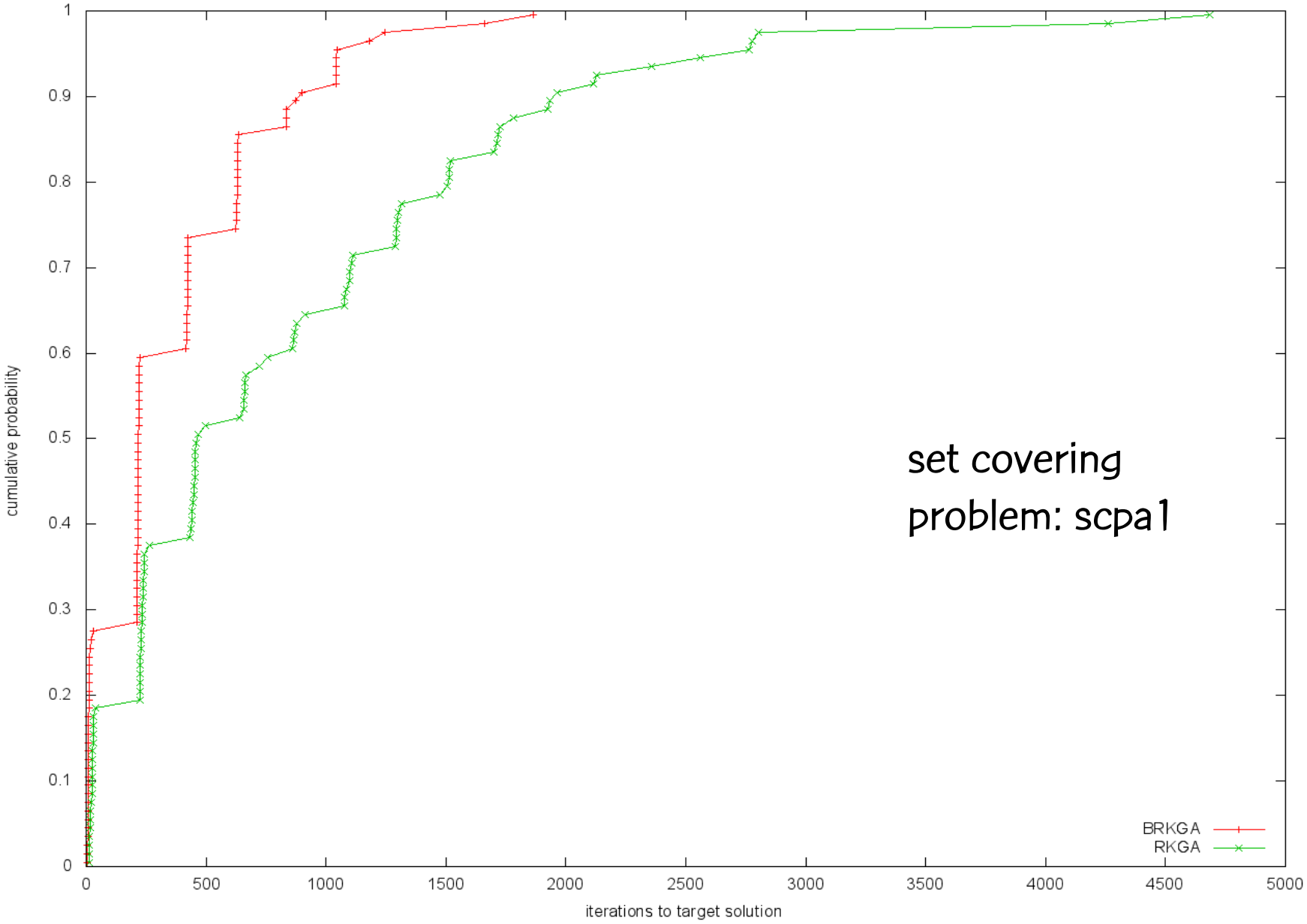
$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.740$

Probability computed with method of Ribeiro et al. (2012)

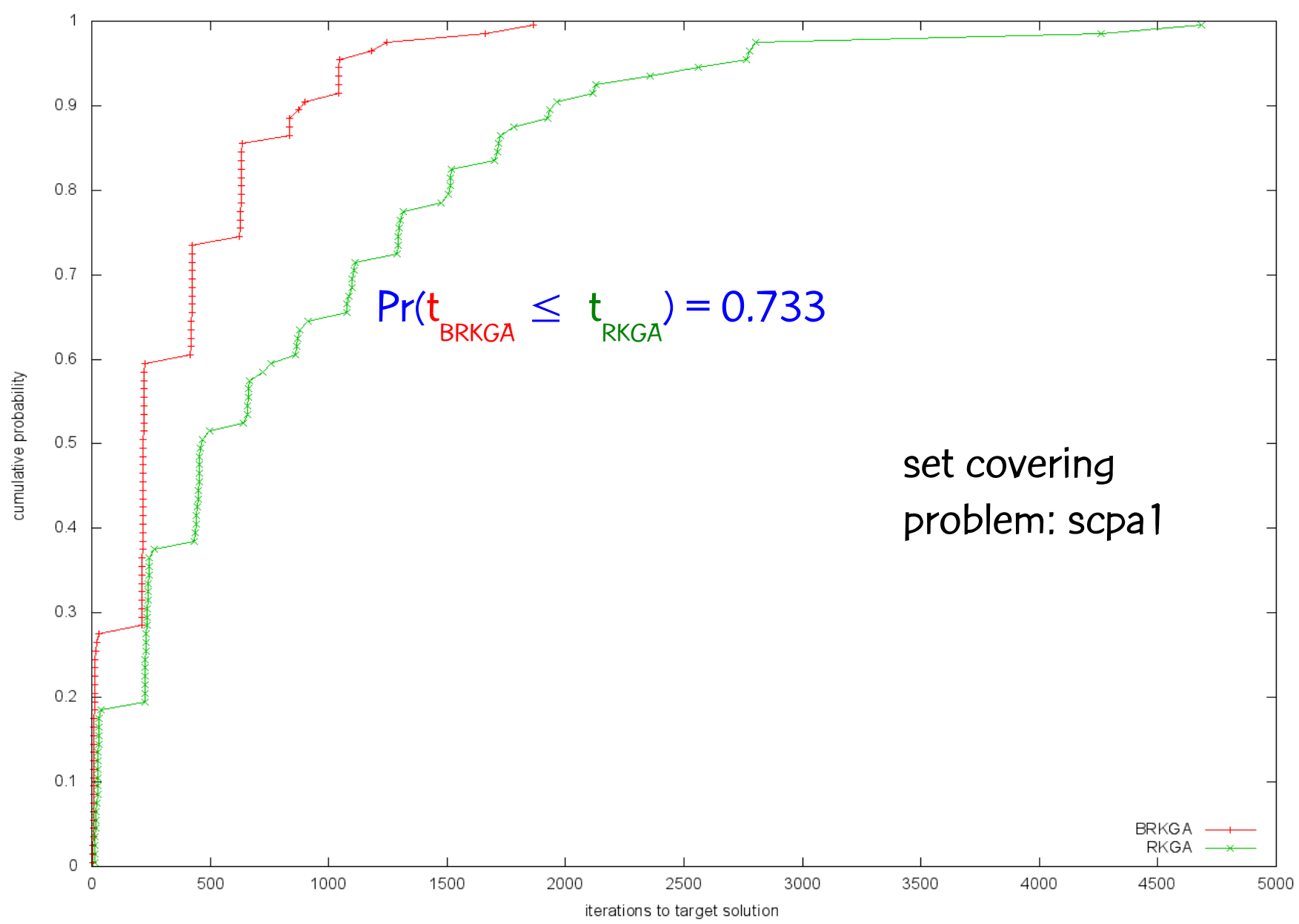
set covering problem: scp41

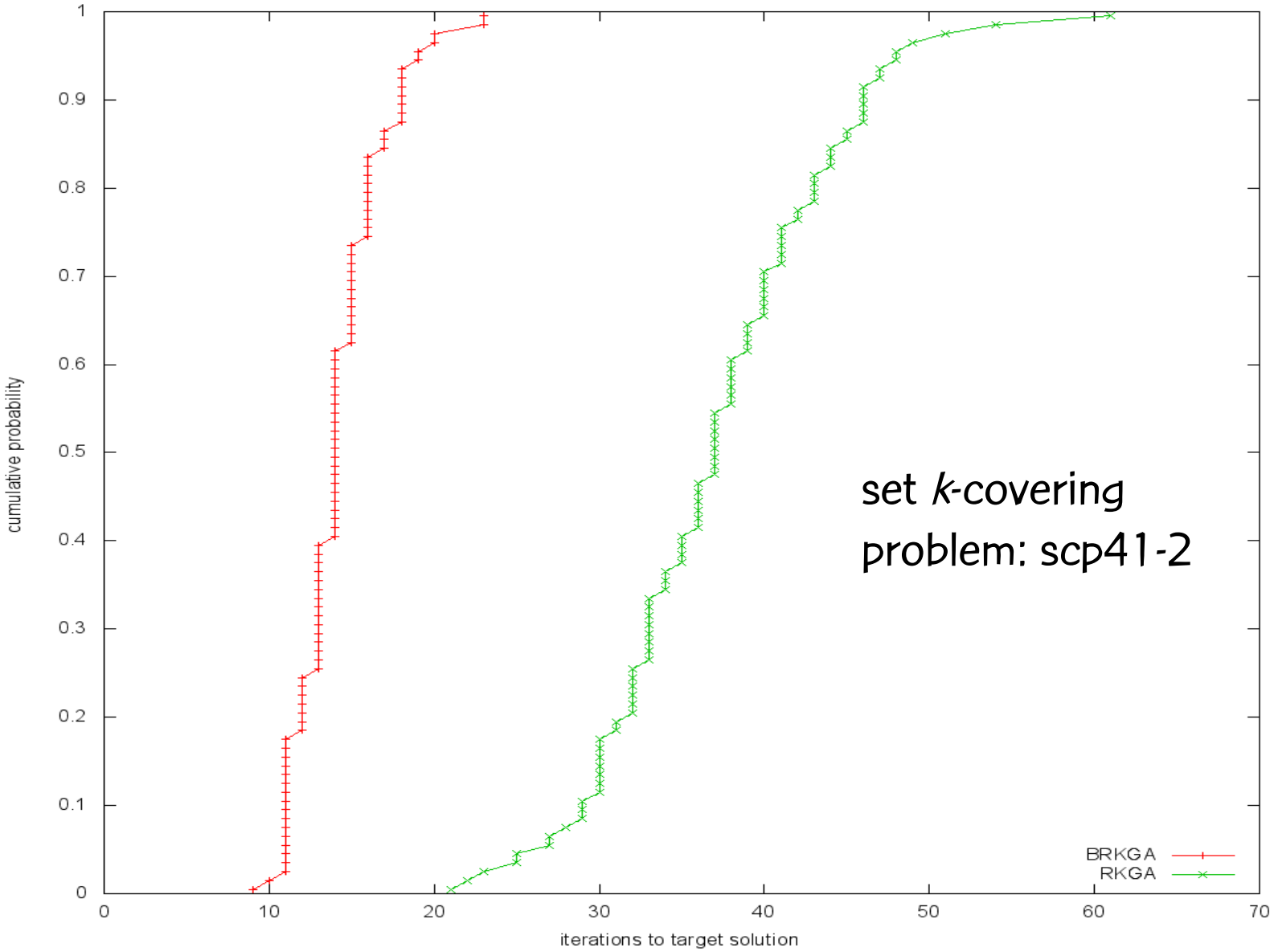




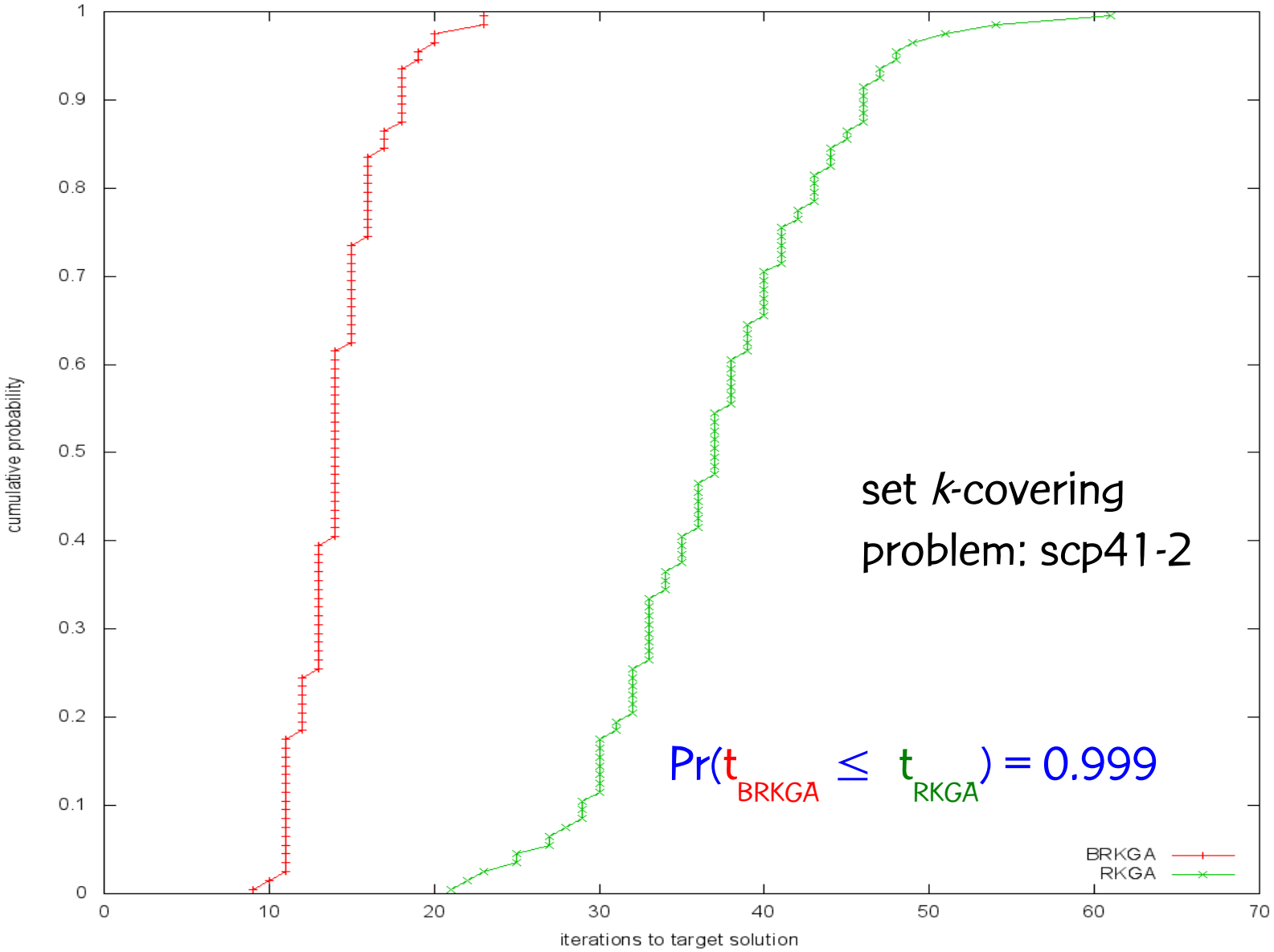


set covering problem: scpa1



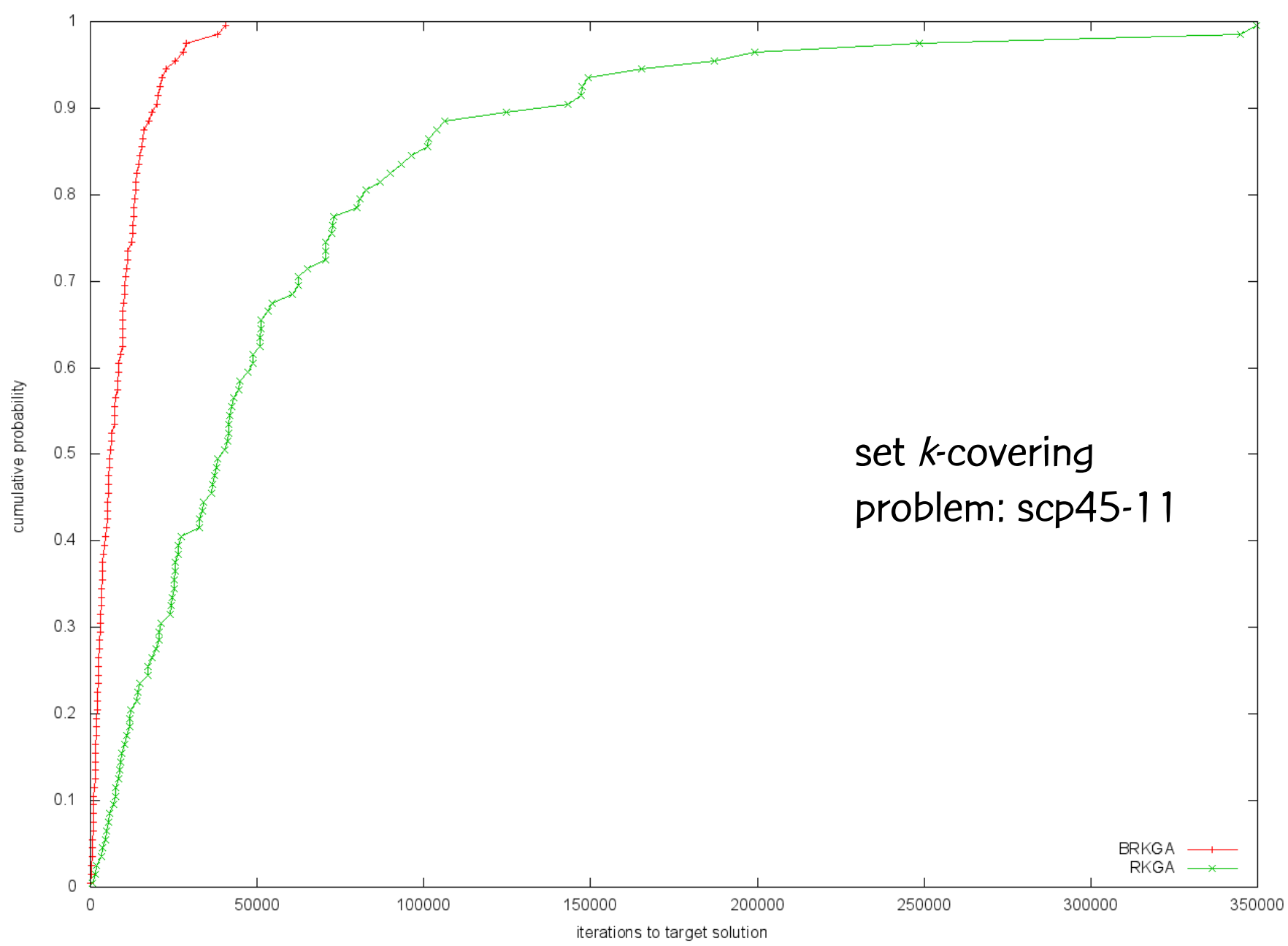


set *k*-covering problem: scp41-2

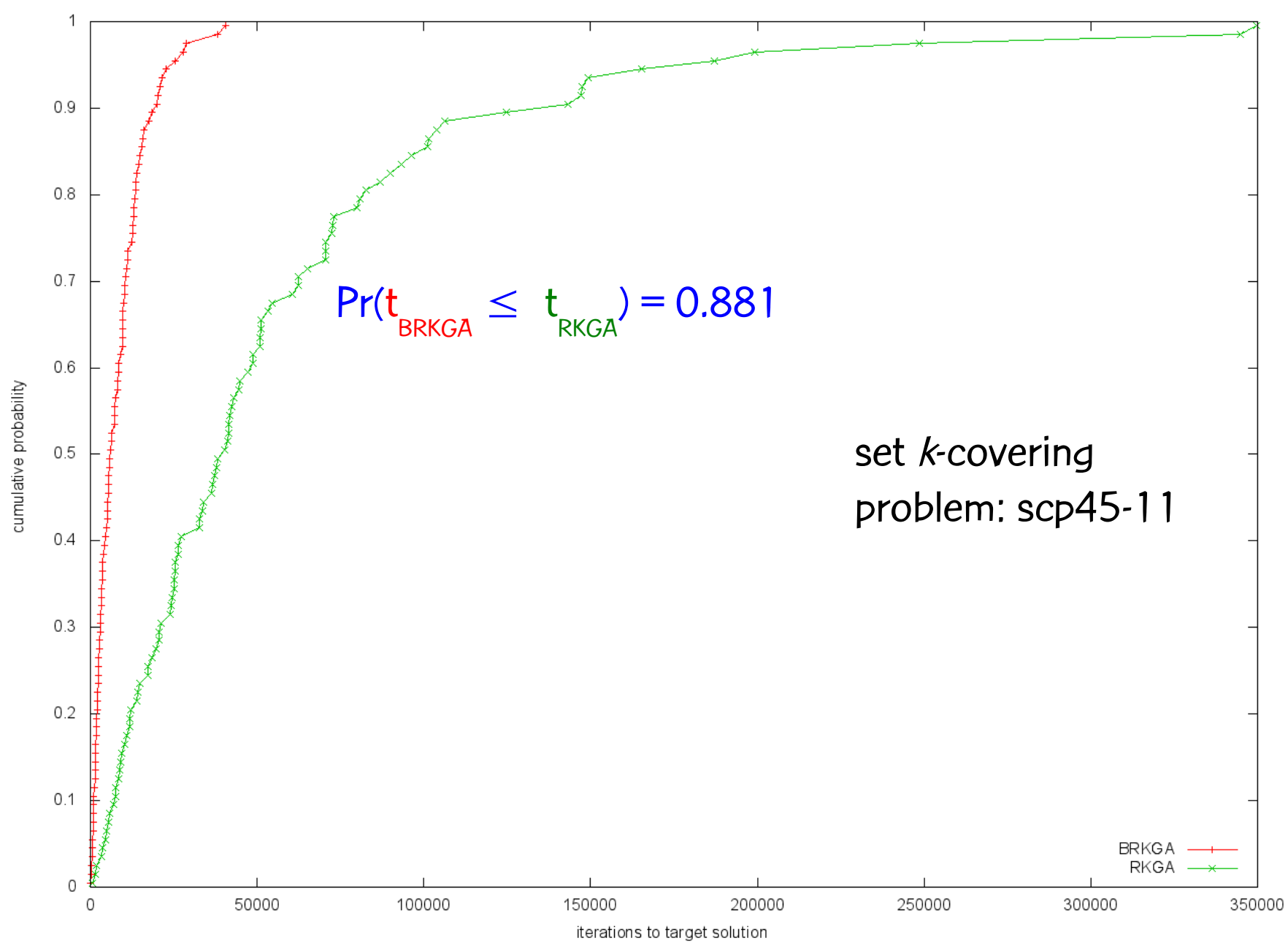


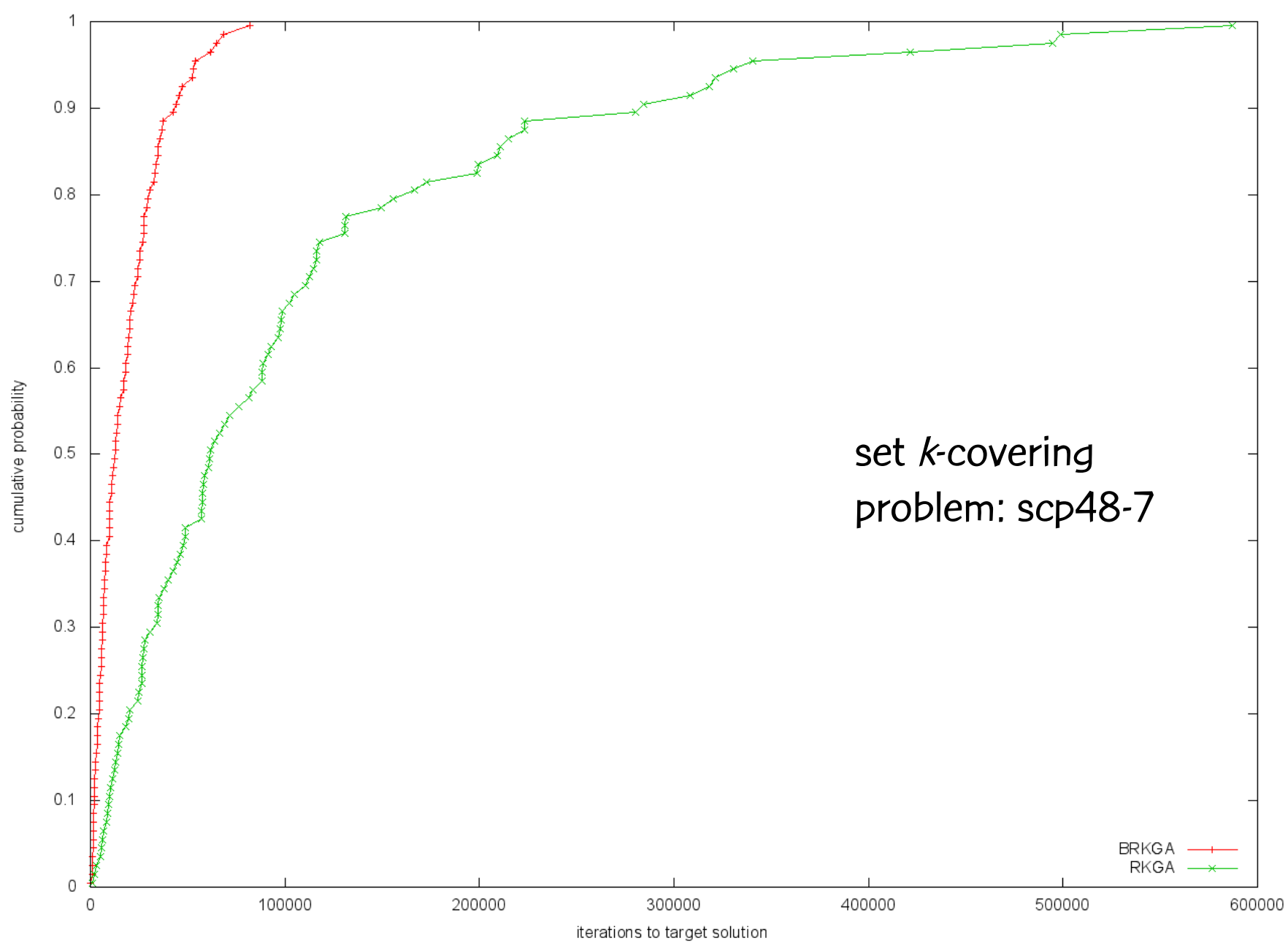
set *k*-covering problem: scp41-2

$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.999$

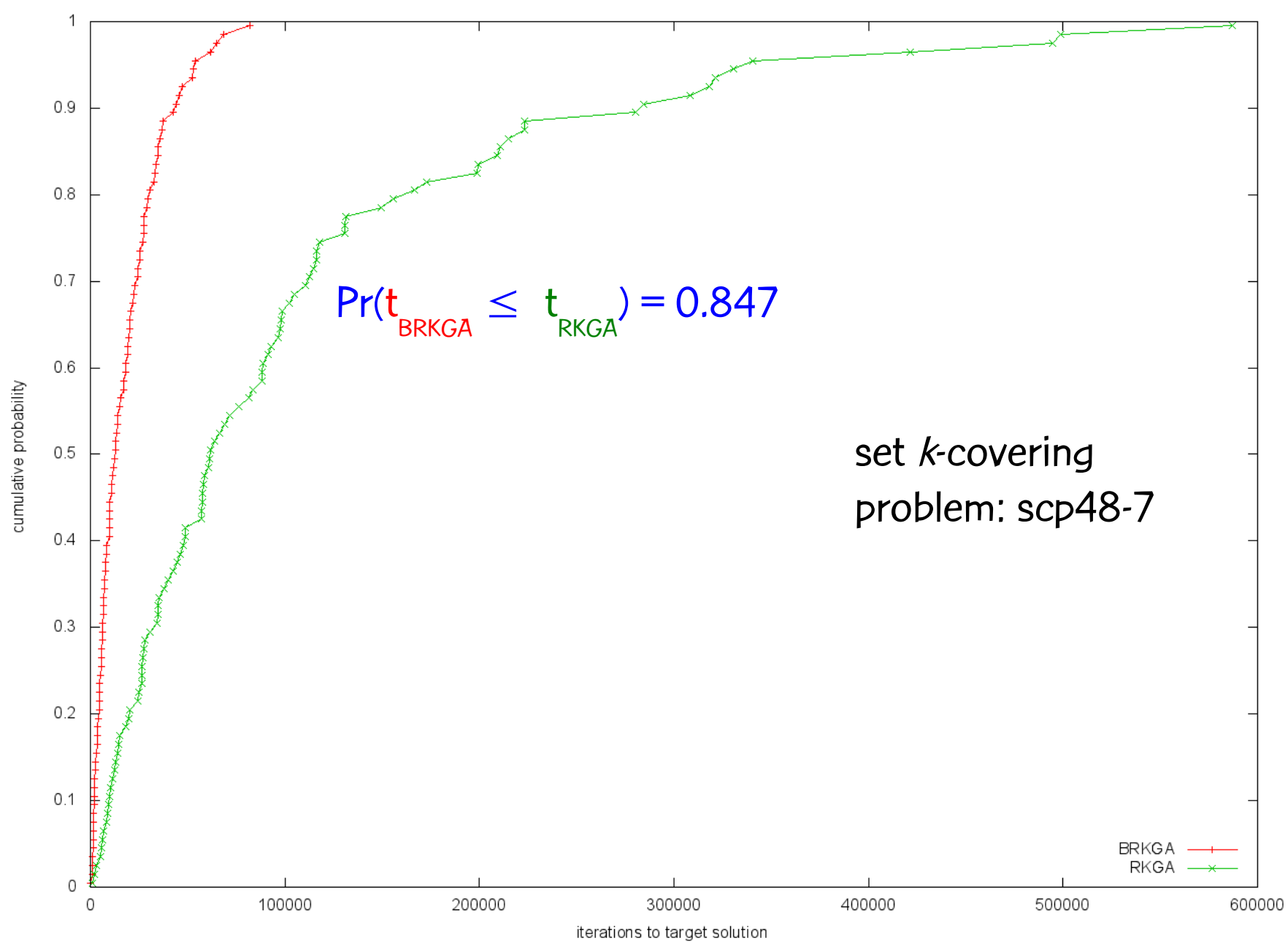


set *k*-covering problem: scp45-11





set *k*-covering problem: scp48-7



Observations

- Random method: keys are randomly generated so solutions are always vectors of random keys

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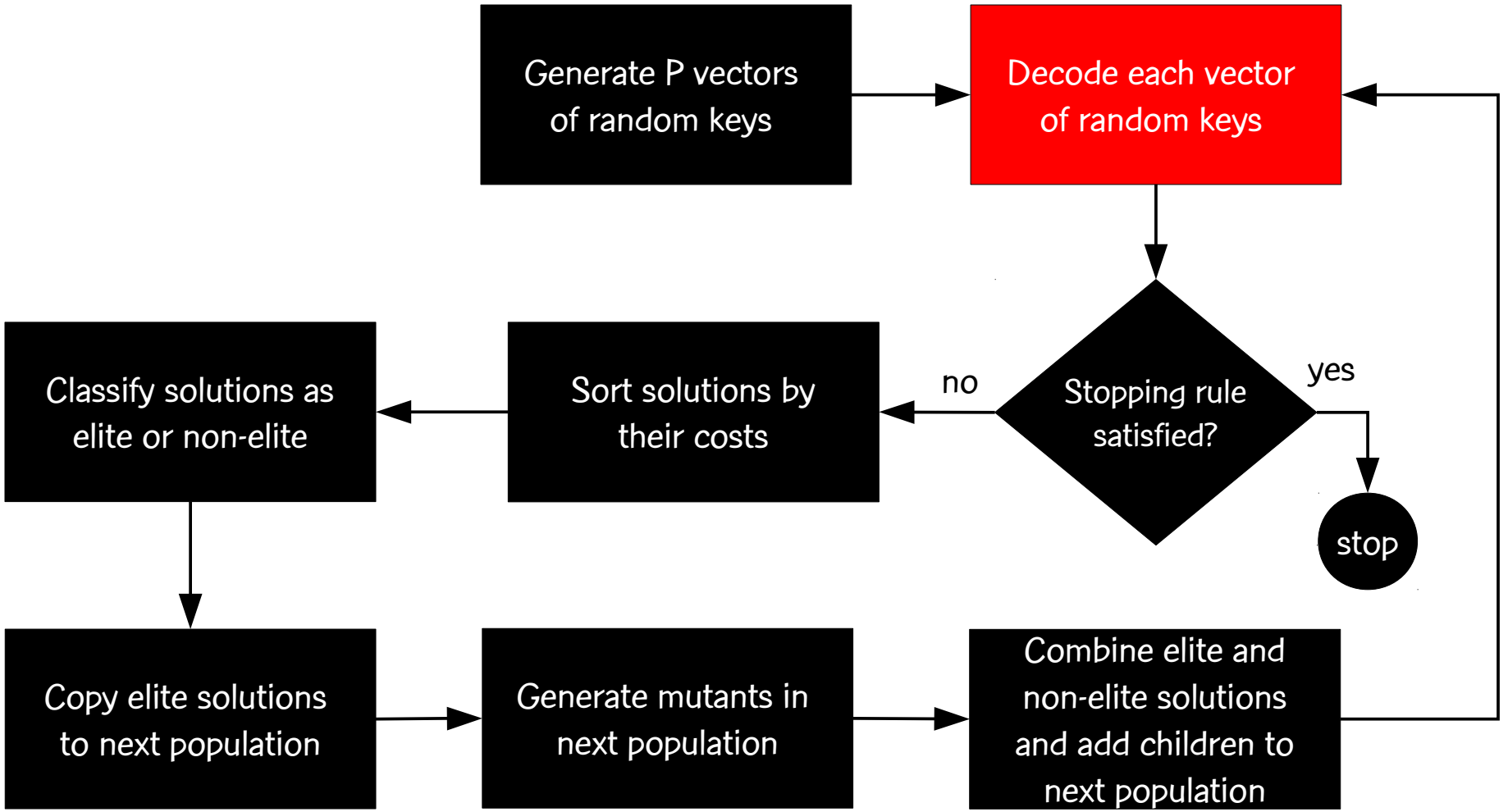
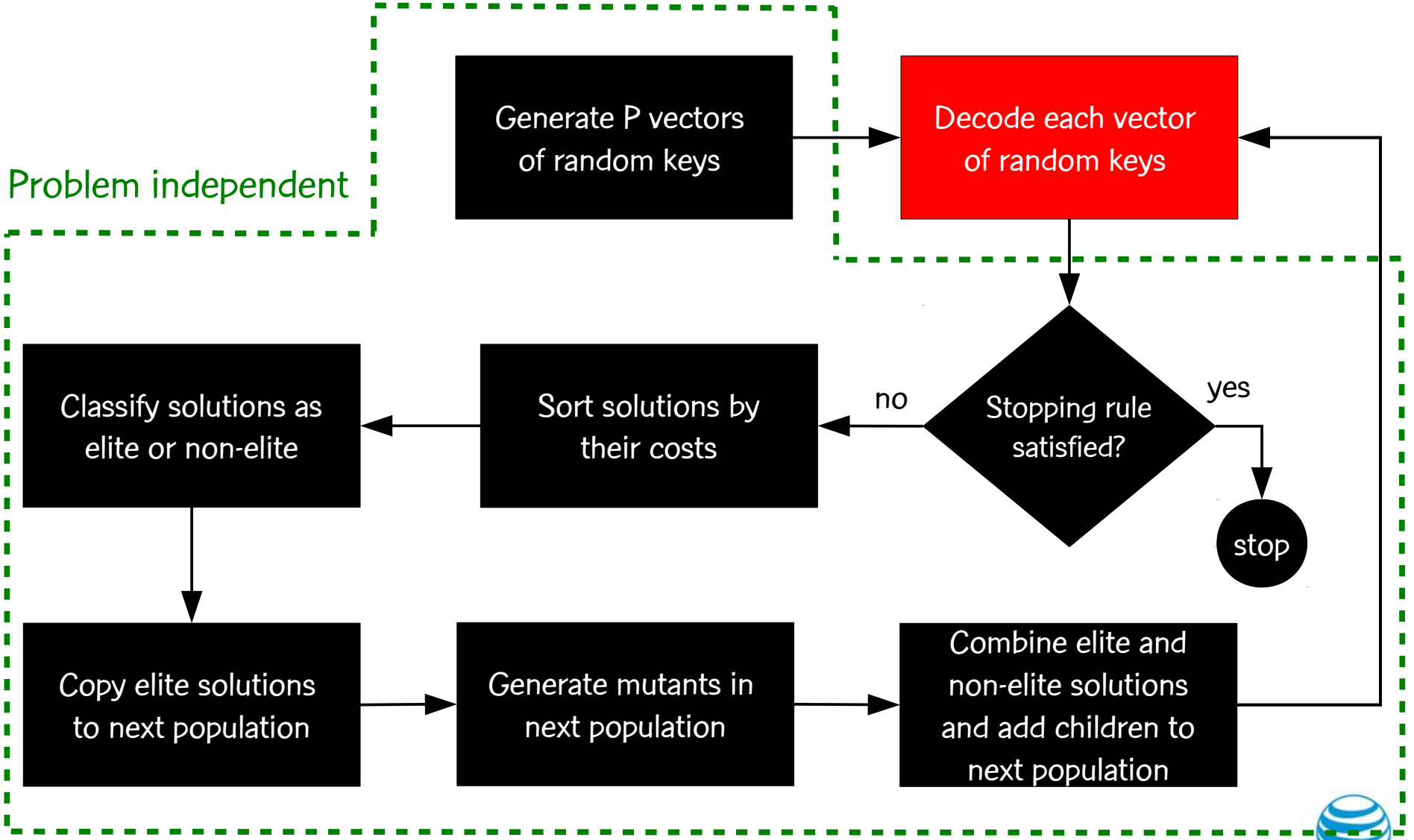
- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs ... help escape local optima)

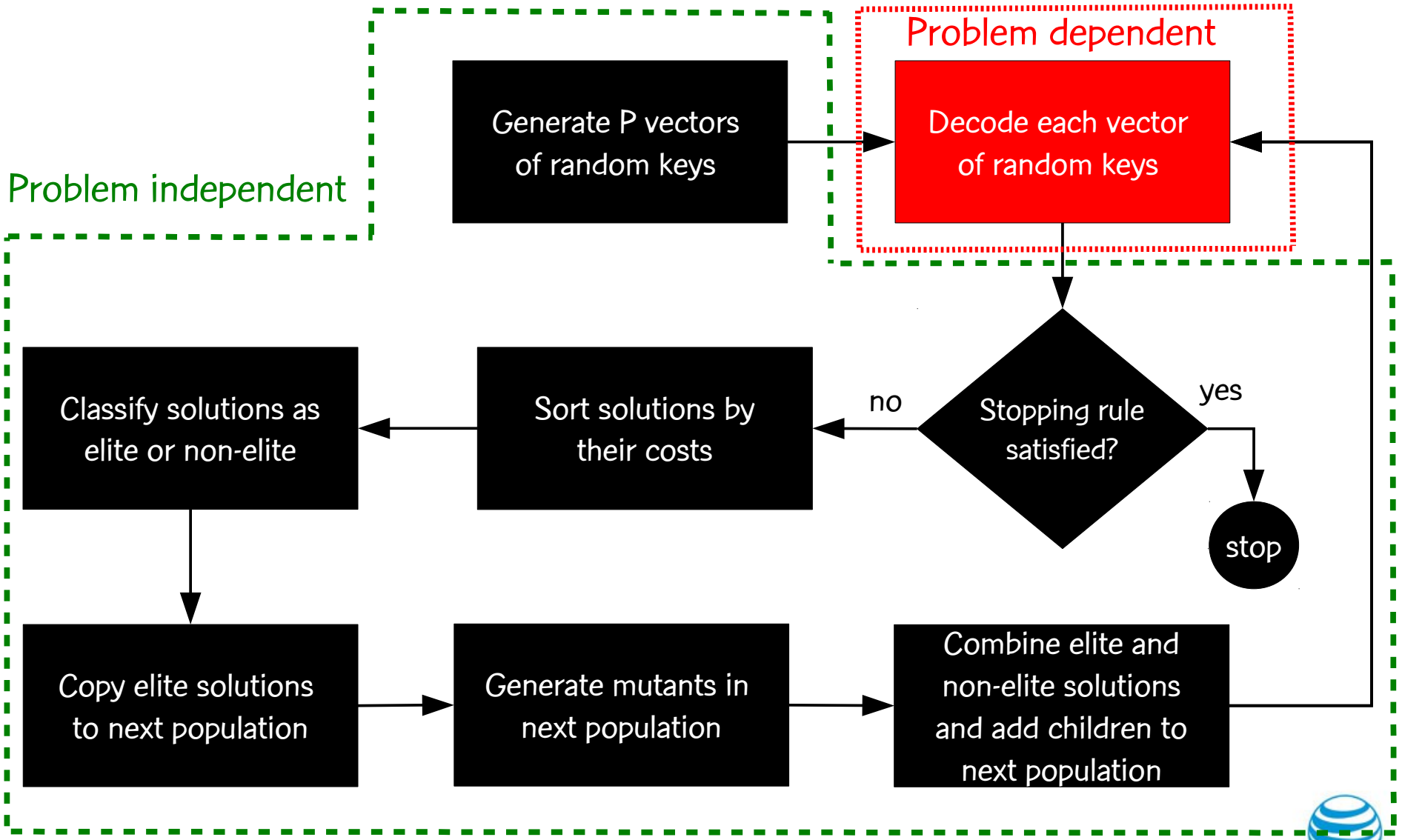
Framework for biased random-key genetic algorithms



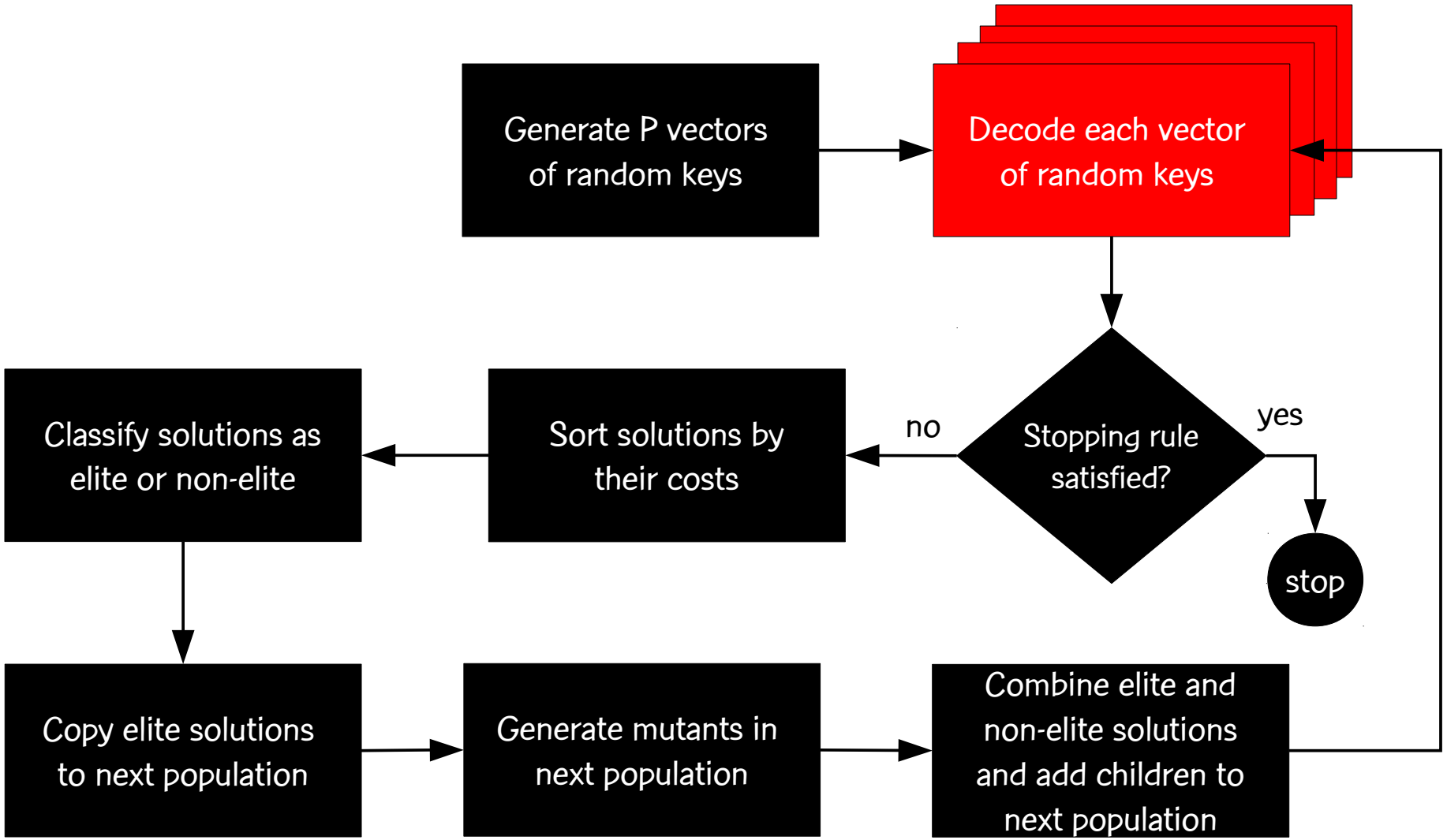
Framework for biased random-key genetic algorithms



Framework for biased random-key genetic algorithms



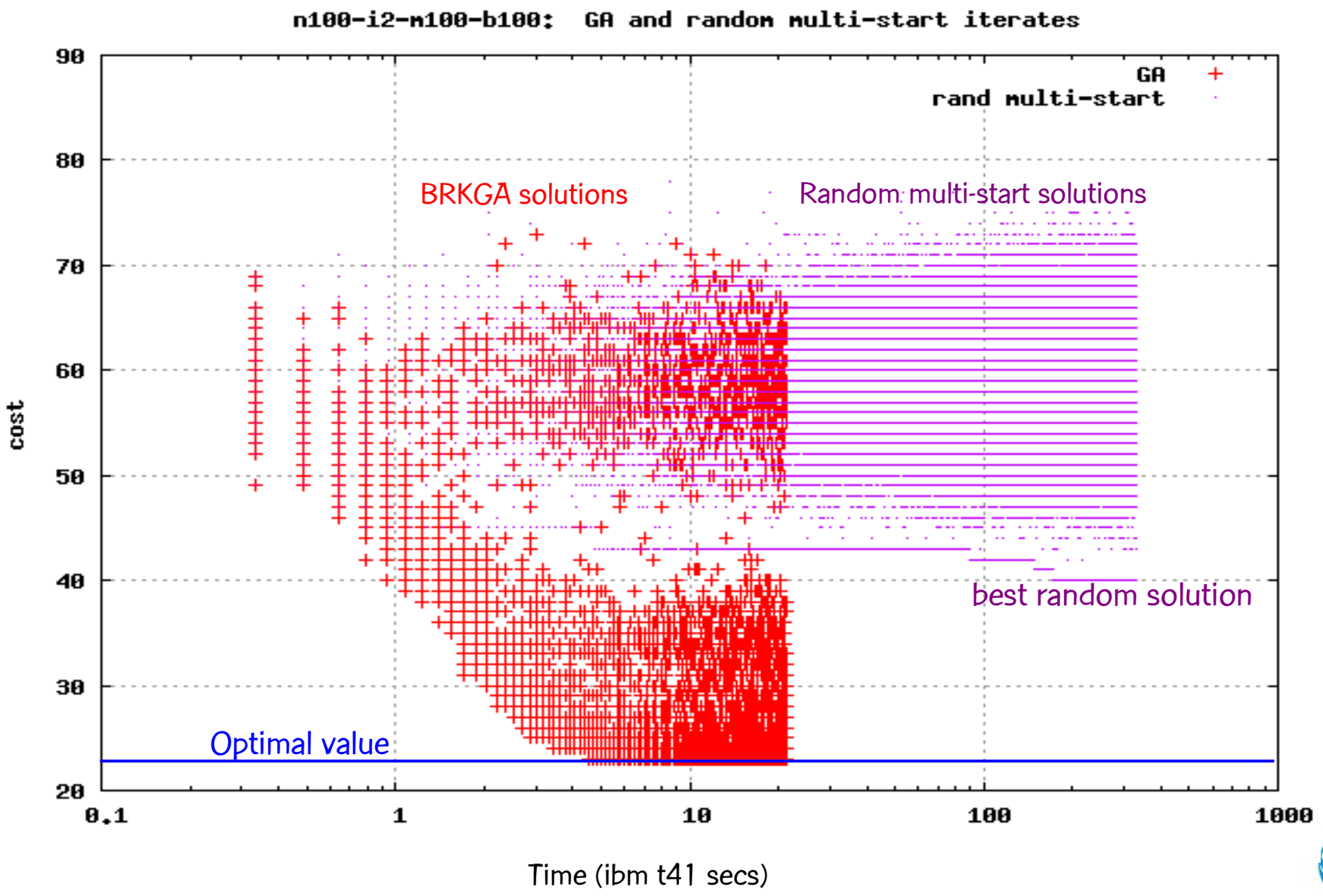
Decoding of random key vectors can be done in parallel



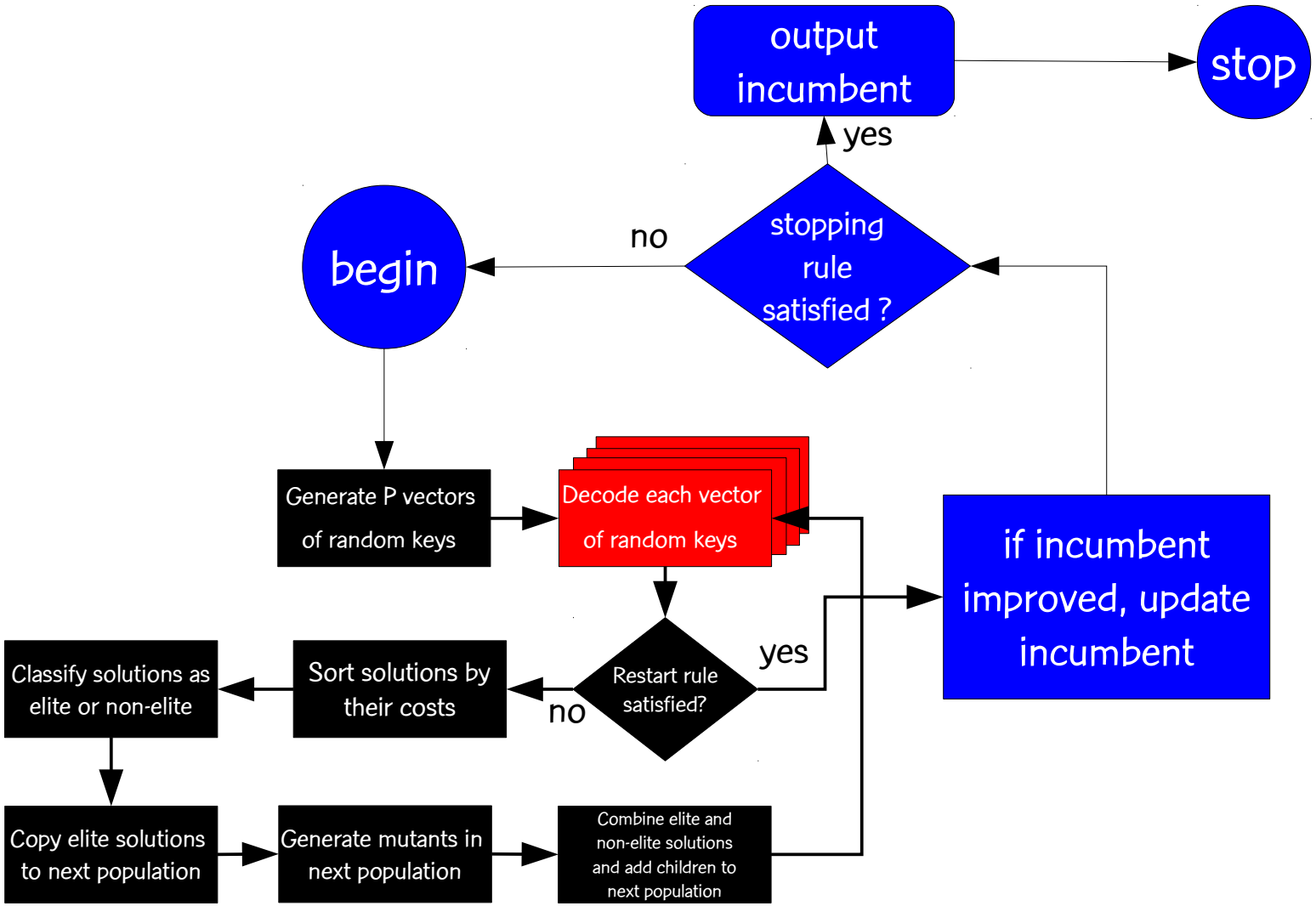
Is a BRKGA any different from applying the decoder to random keys?

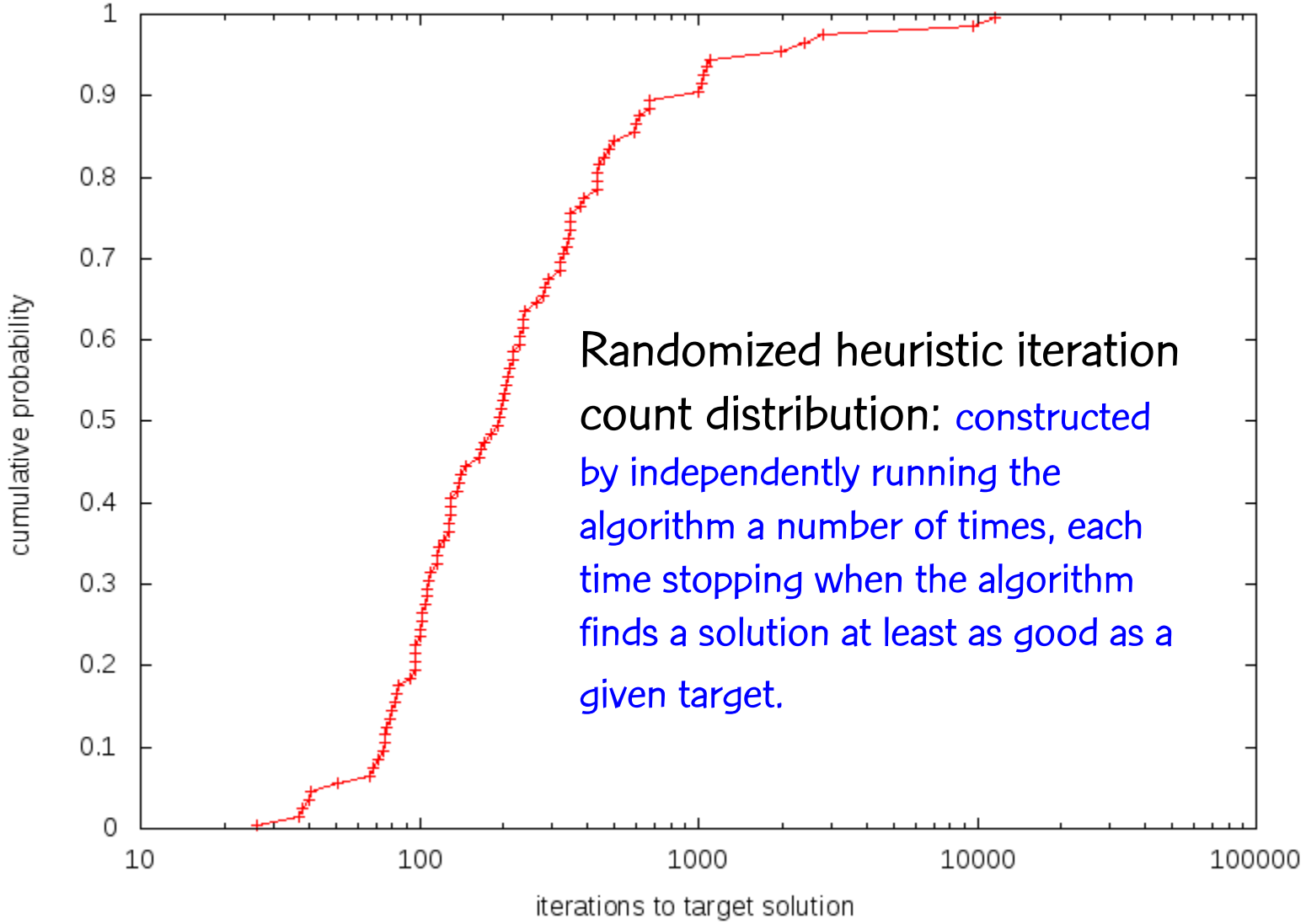
- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to P–1
- Each iteration, best solution is maintained in elite set and P–1 random key vectors are generated as mutants ... no mating is done since population already has P individuals

solution

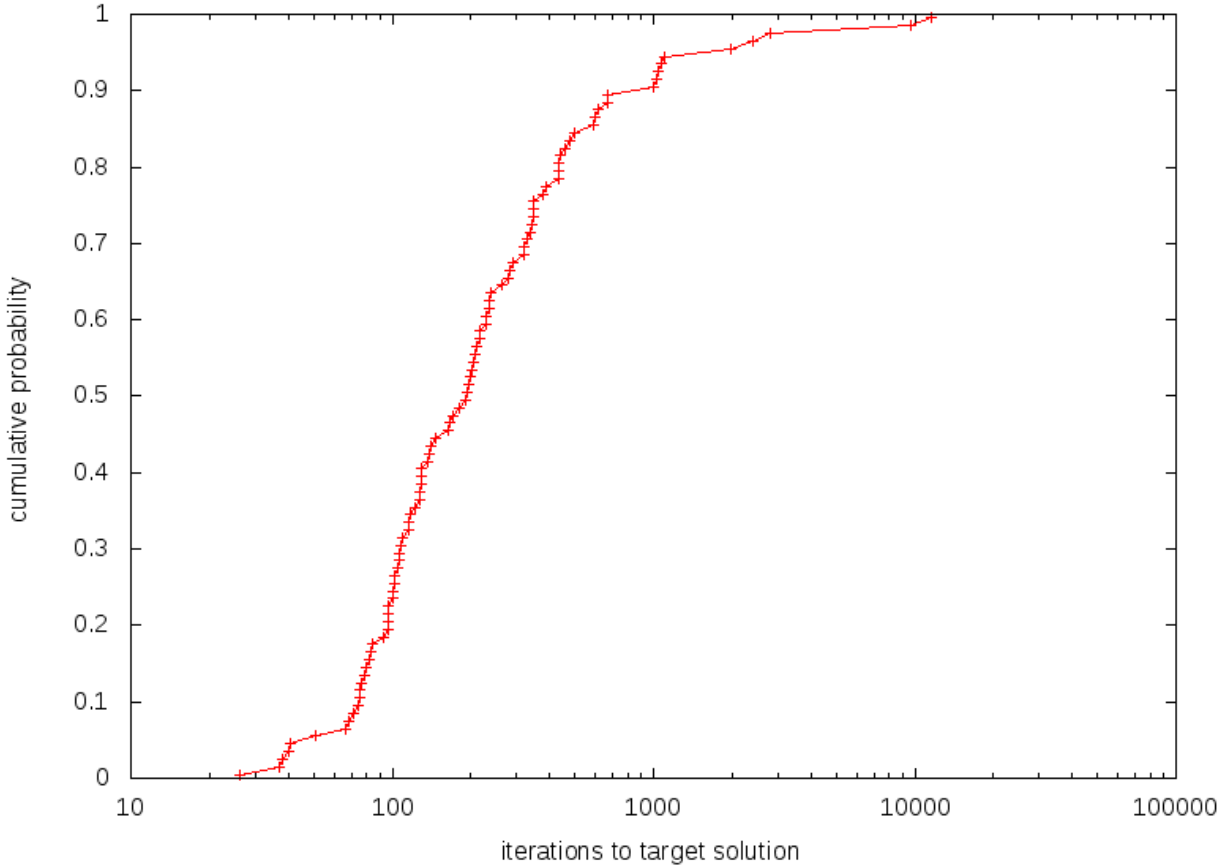


BRKGA in multi-start strategy

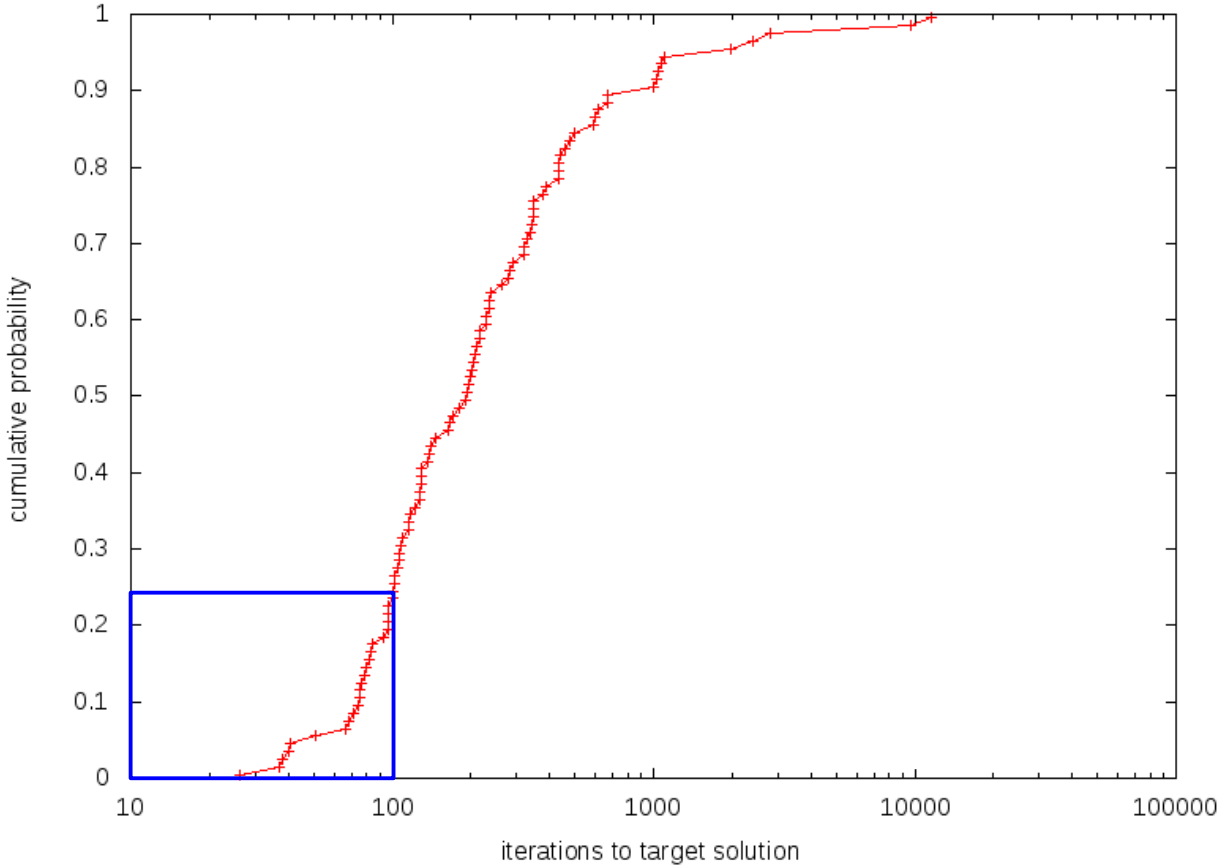




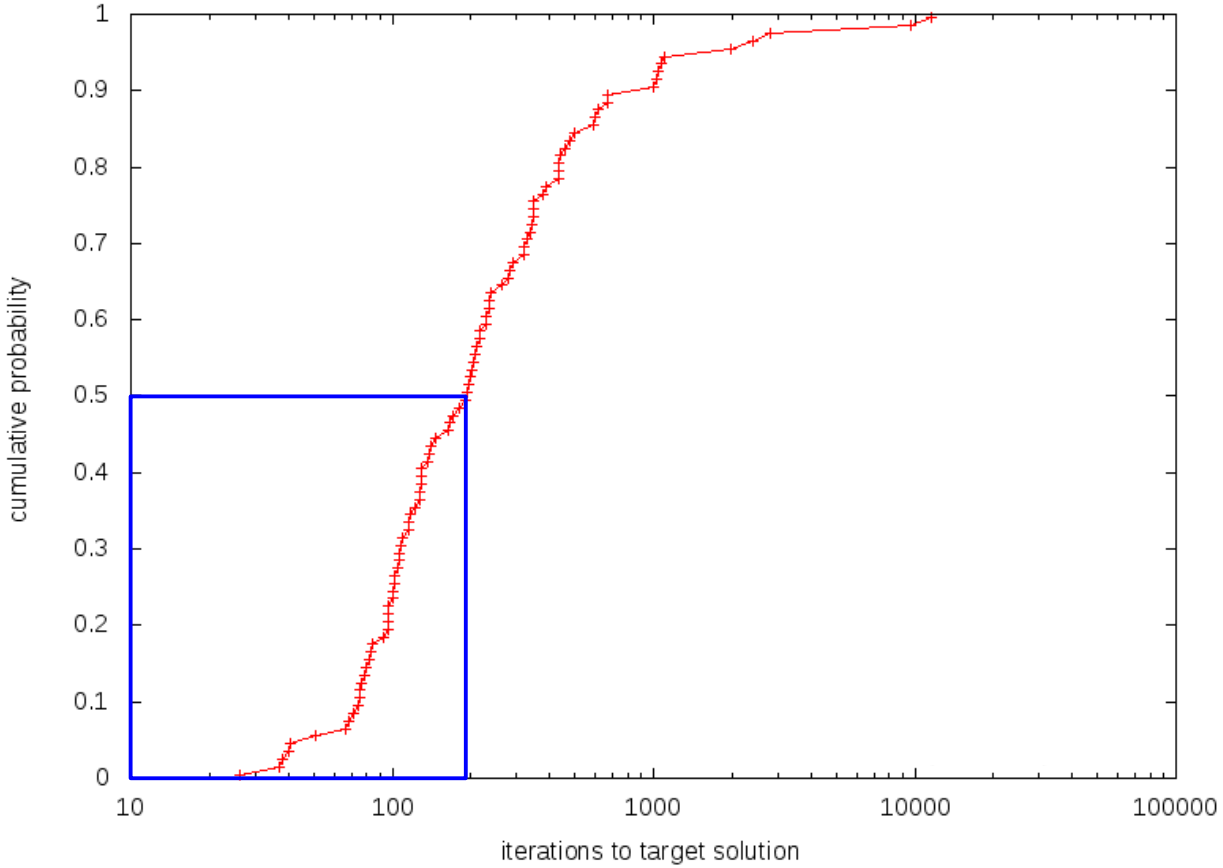
Randomized heuristic iteration count distribution: constructed by independently running the algorithm a number of times, each time stopping when the algorithm finds a solution at least as good as a given target.



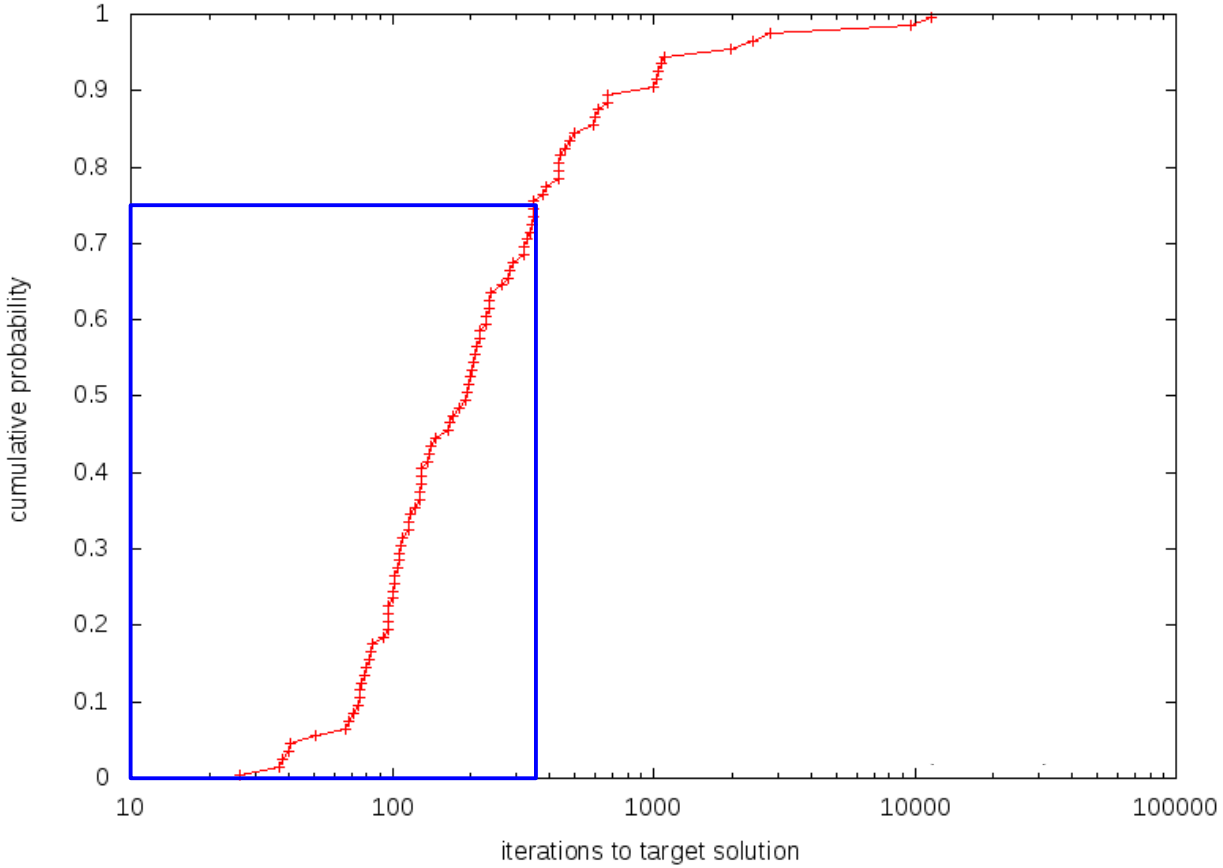
In most of the independent runs, the algorithm finds the target solution in relatively few iterations:



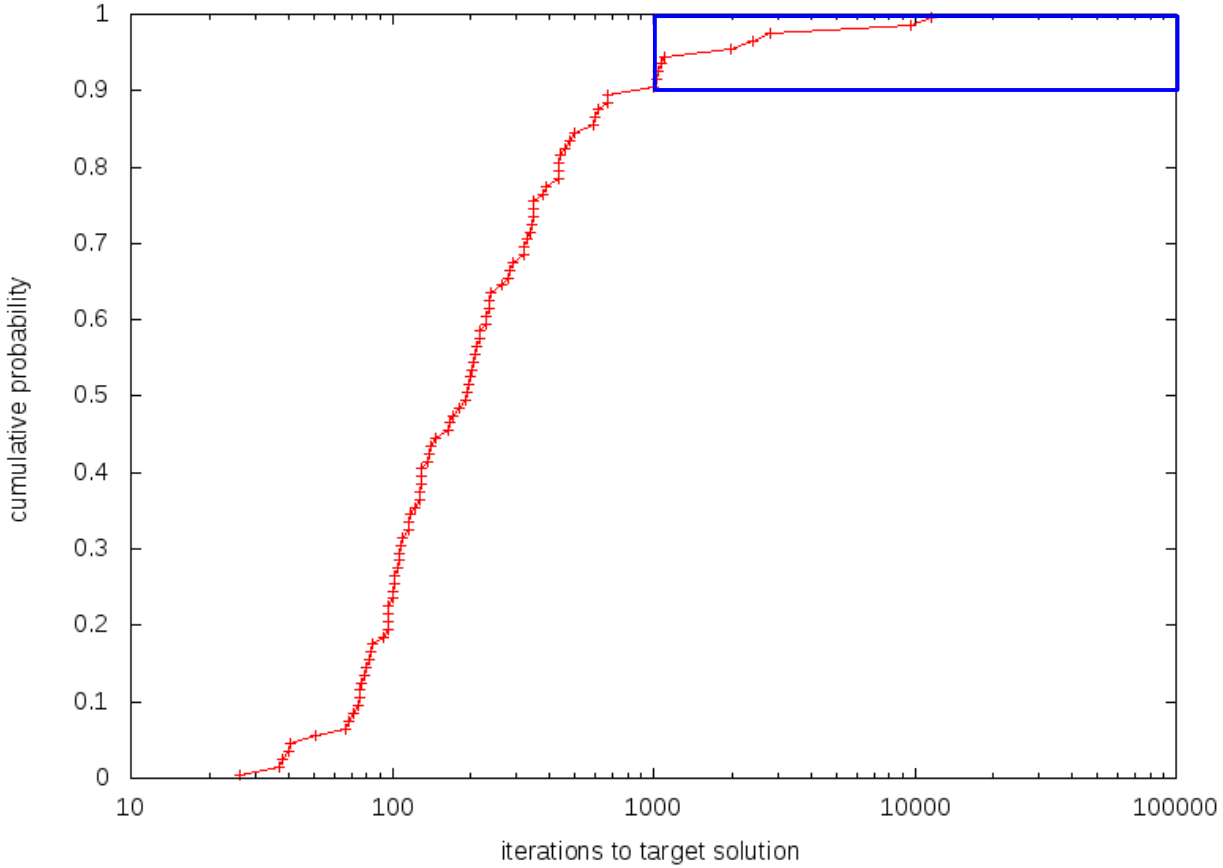
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 25% of the runs take fewer than 101 iterations



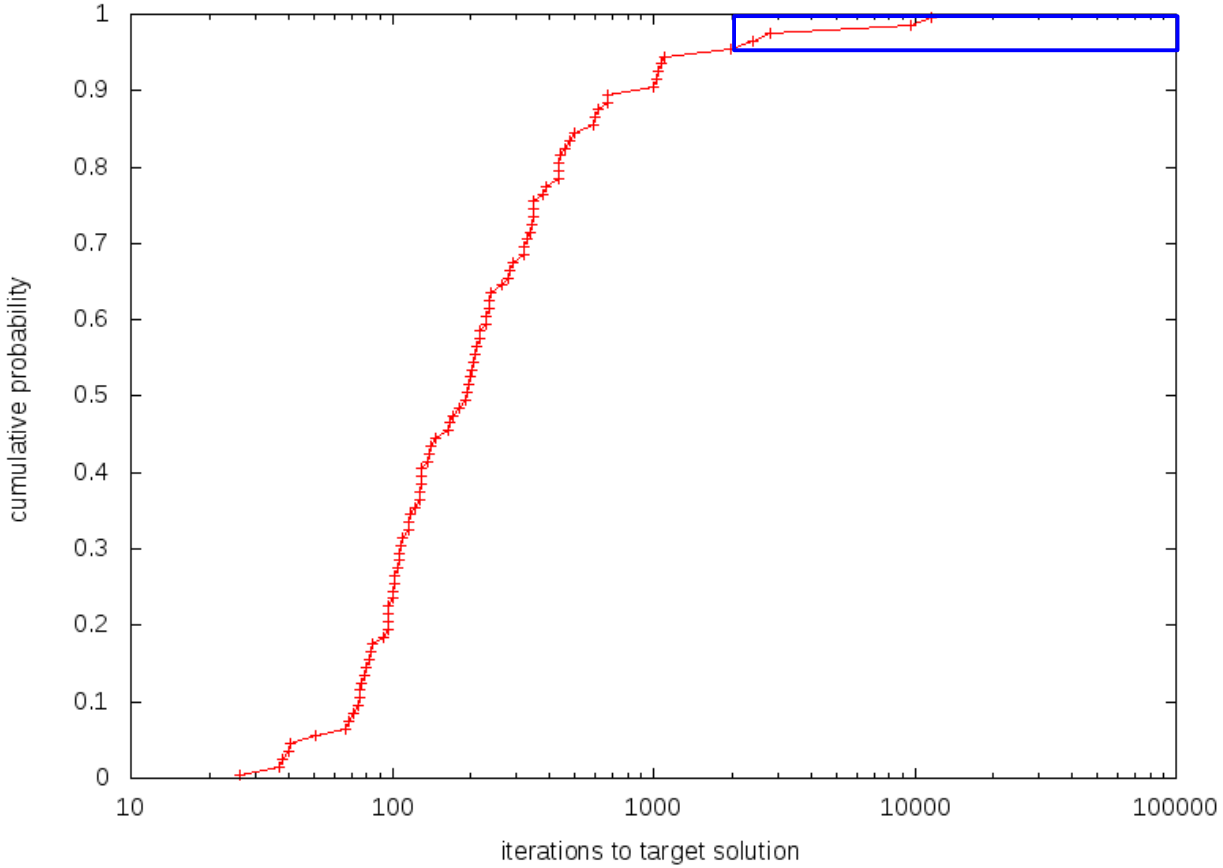
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 50% of the runs take fewer than 192 iterations



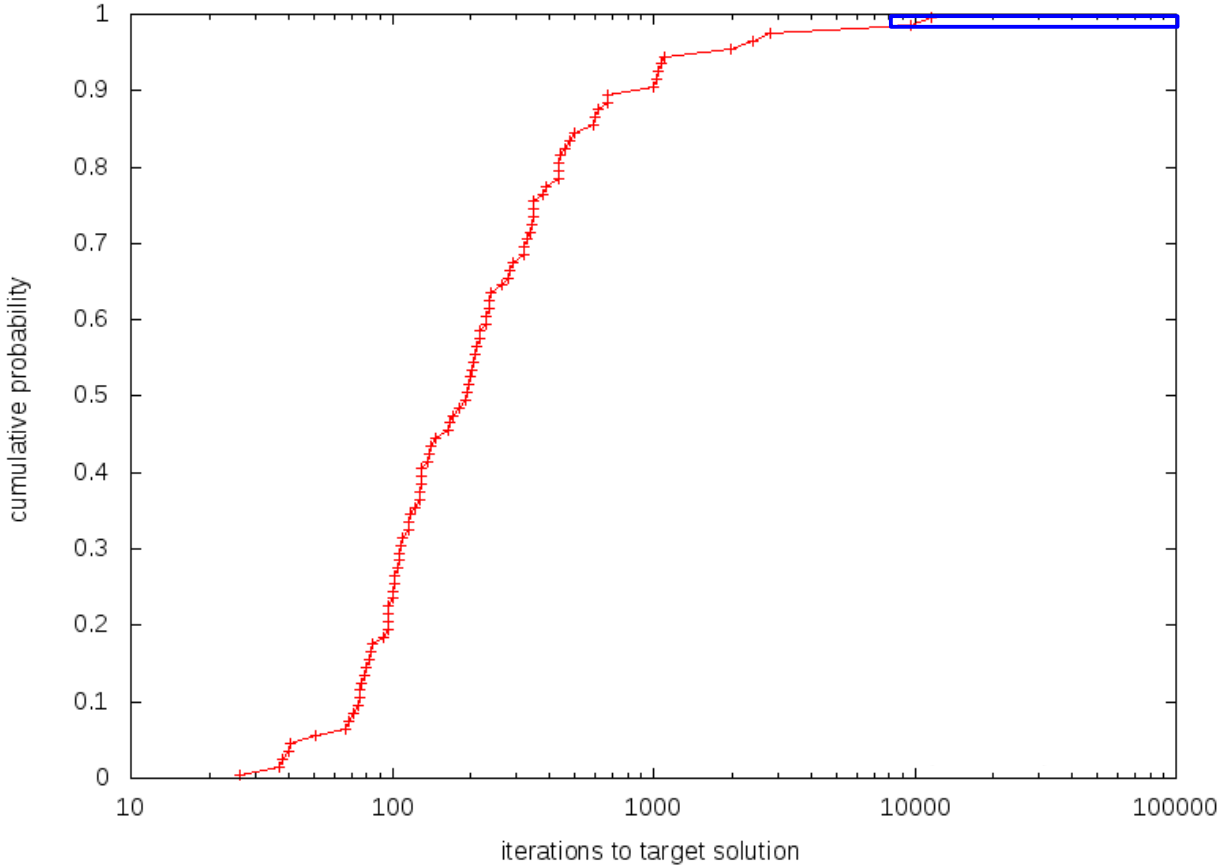
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 75% of the runs take fewer than 345 iterations



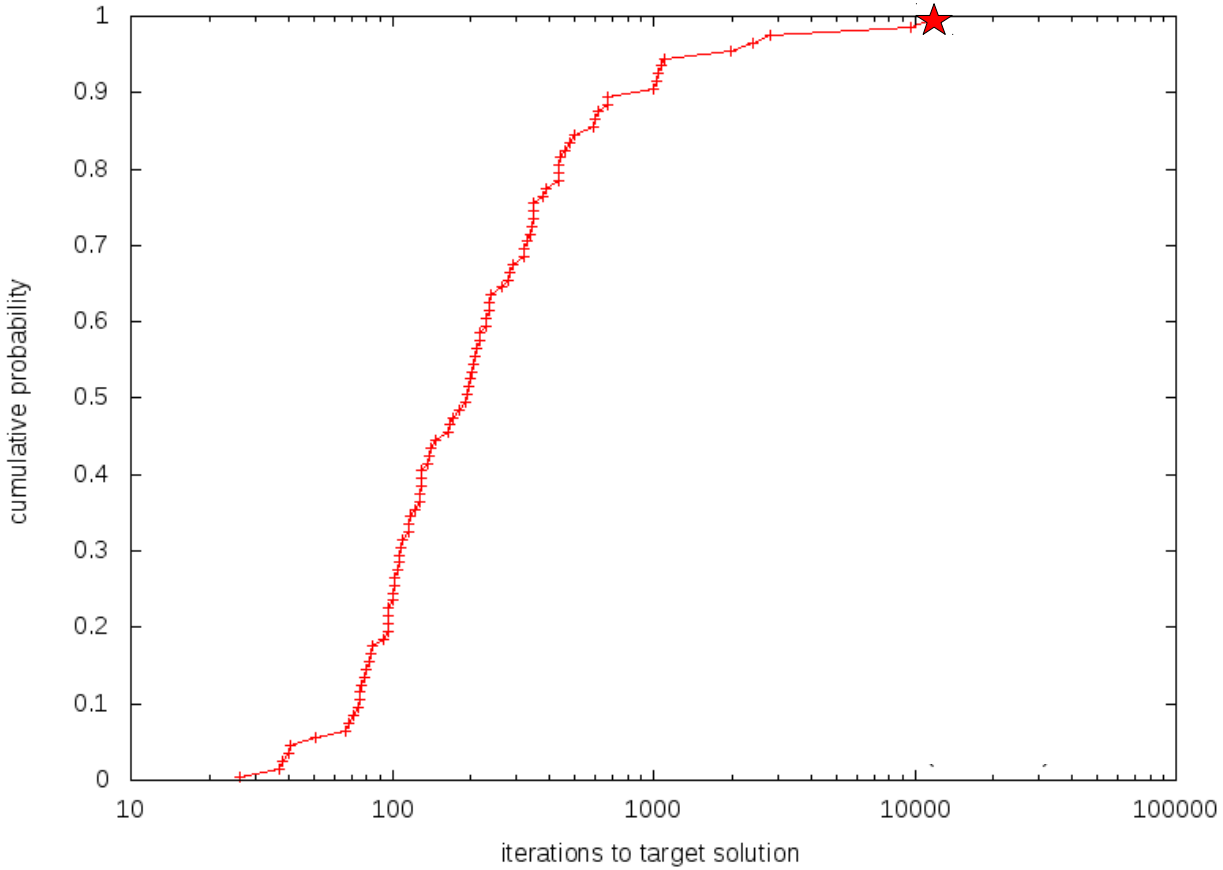
However, some runs take much longer: 10% of the runs take over 1000 iterations



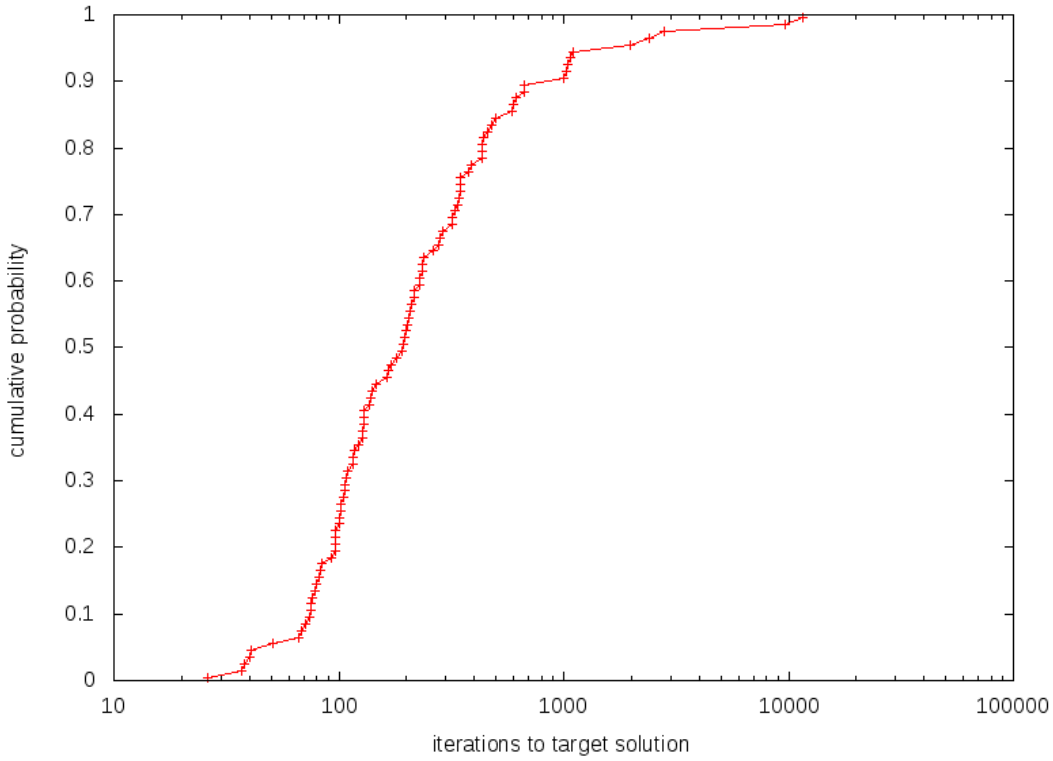
However, some runs take much longer: 5% of the runs take over 2000 iterations



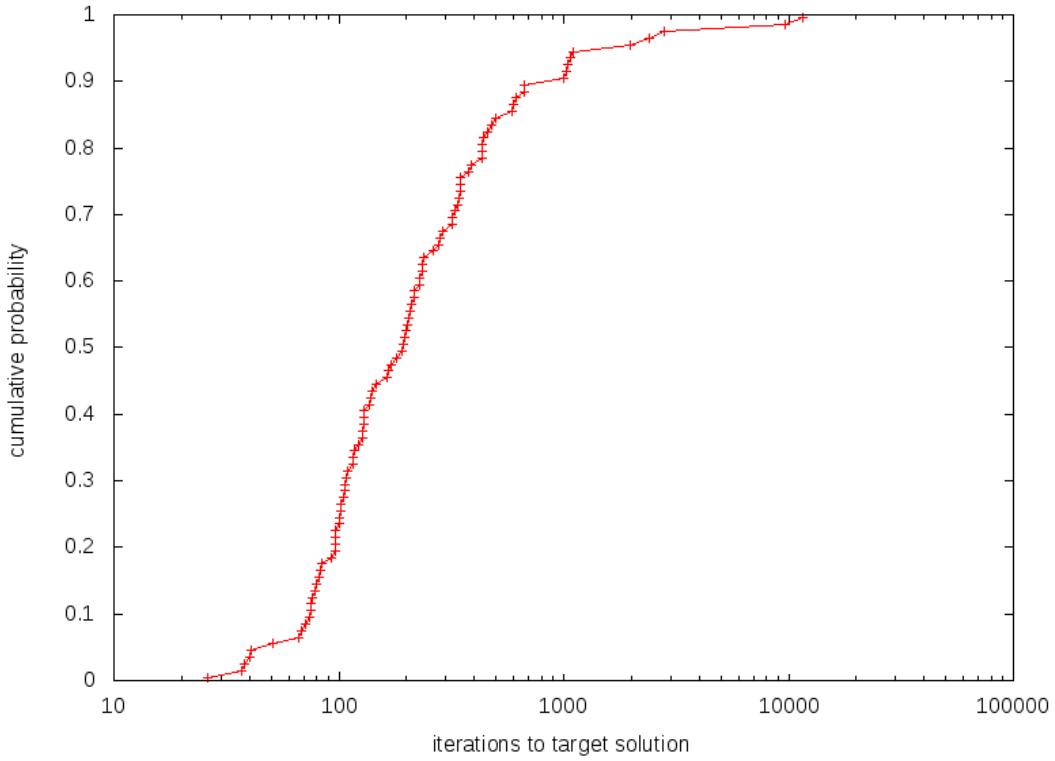
However, some runs take much longer: 2% of the runs take over 9715 iterations



However, some runs take much longer: the longest run took 11607 iterations



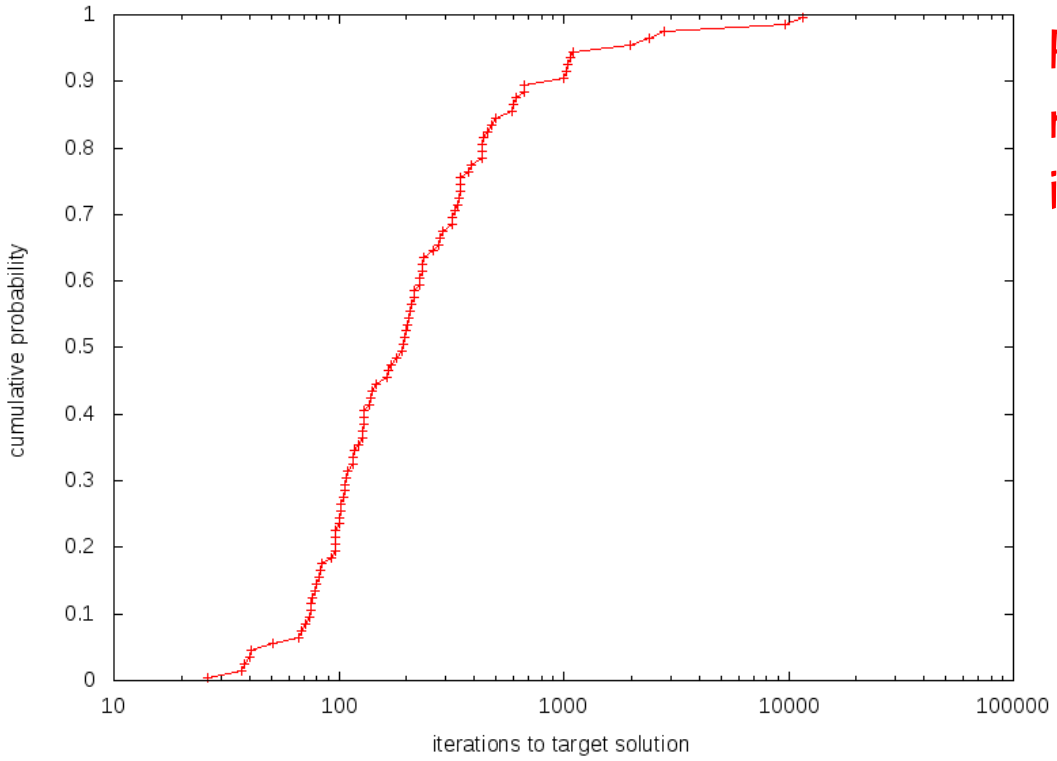
Probability that algorithm will take over 345 iterations: 25% = 1/4



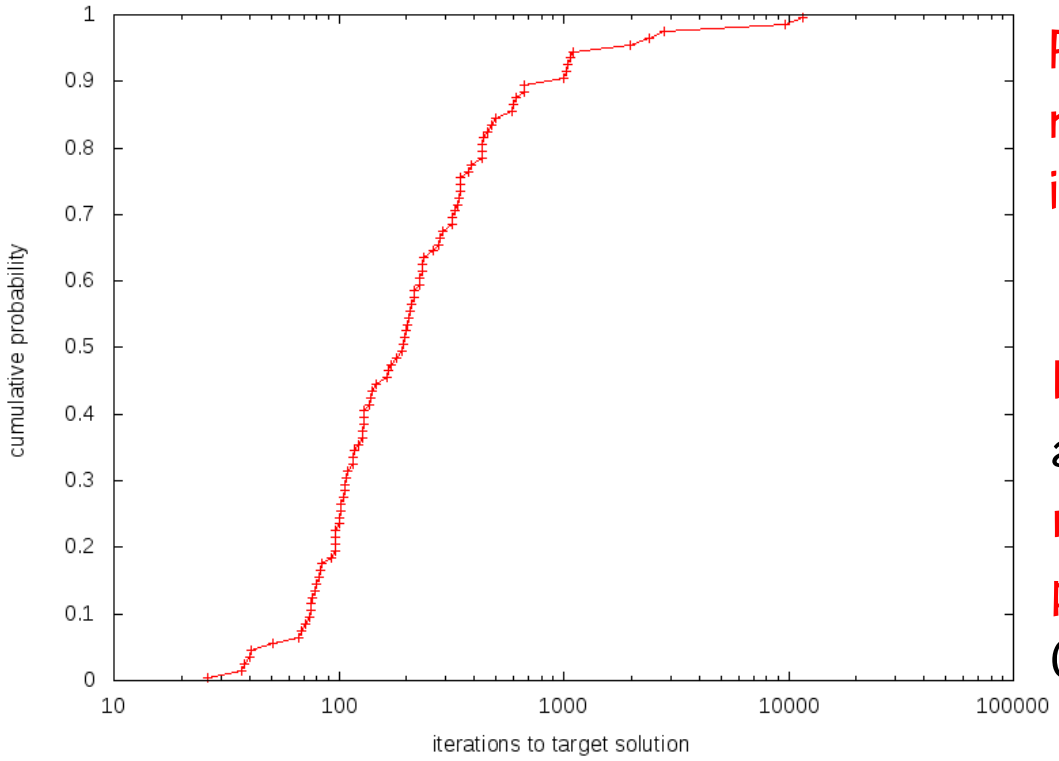
Probability that algorithm will take over 345 iterations: 25% = 1/4

By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: 25% = 1/4

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 X probability of taking over 690 iterations given it took over 345 = ¼ x ¼ = $1/4^2$

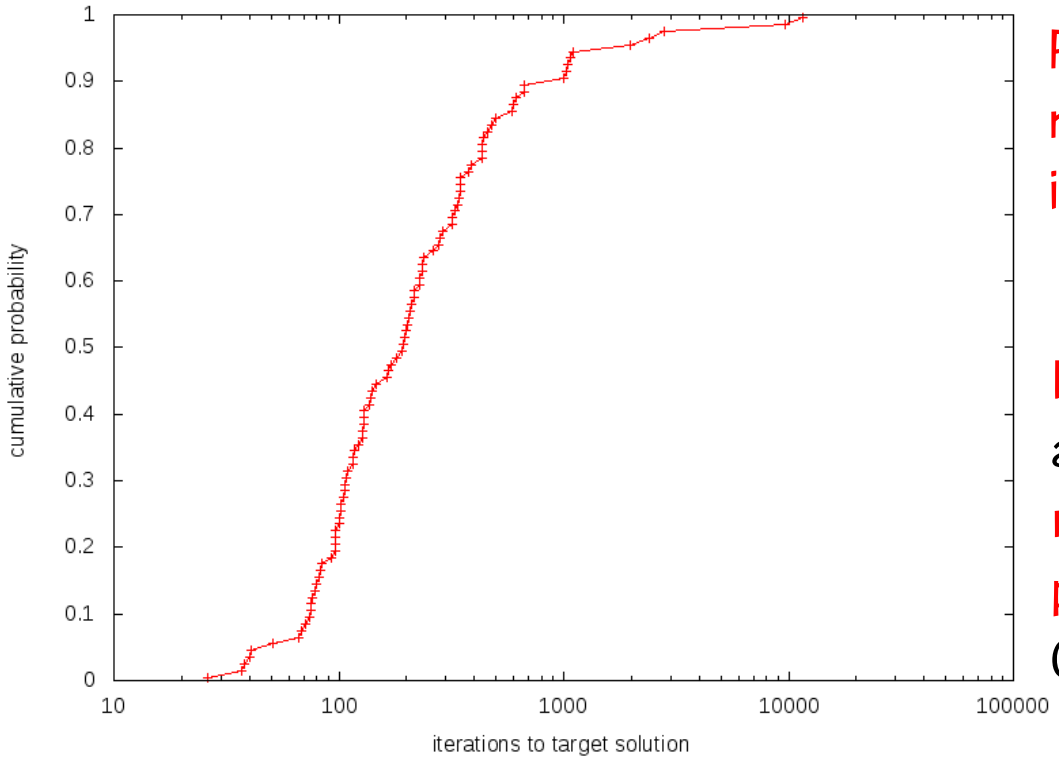


Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$



Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$



Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$

This is much less than the 5% probability that the algorithm without restart will take over 2000 iterations.

Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S = \{\tau_1, \tau_2, \tau_3, \dots\}$ which define epochs τ_1, $\tau_1+\tau_2$, $\tau_1+\tau_2+\tau_3$, ... when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$, where τ^* is a constant.

Restart strategies

- Luby et al. (1993)
- Kautz et al. (2002)
- Palubeckis (2004)
- Sergienko et al. (2004)
- Nowicki & Smutnicki (2005)
- D'Apuzzo et al. (2006)
- Shylo et al. (2011a)
- Shylo et al. (2011b)
- Resende & Ribeiro (2011)

Restart strategy for BRKGA

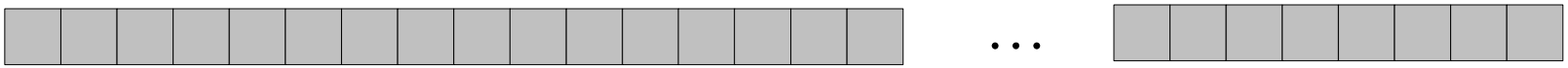
- Recall the restart strategy of Luby et al. where equal time intervals $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$ pass between restarts.
- Strategy requires τ^* as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
  - choosing τ^* too small: restart variant may take long to converge
  - choosing τ^* too big: restart variant may become like no-restart variant

Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy restart(K)

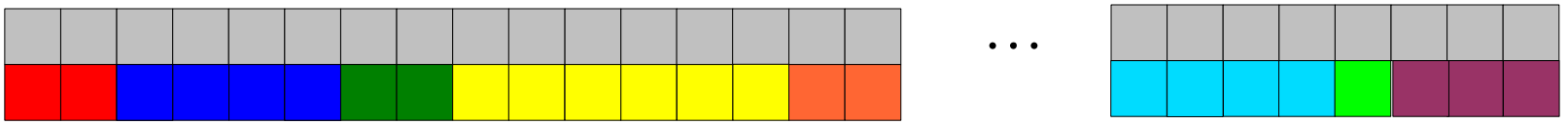
Example of restart strategy for BRKGA: Load balancing

Given an unordered sequence of 1024 integers p[0], p[1], ..., p[1023]



Example of restart strategy for BRKGA: Load balancing

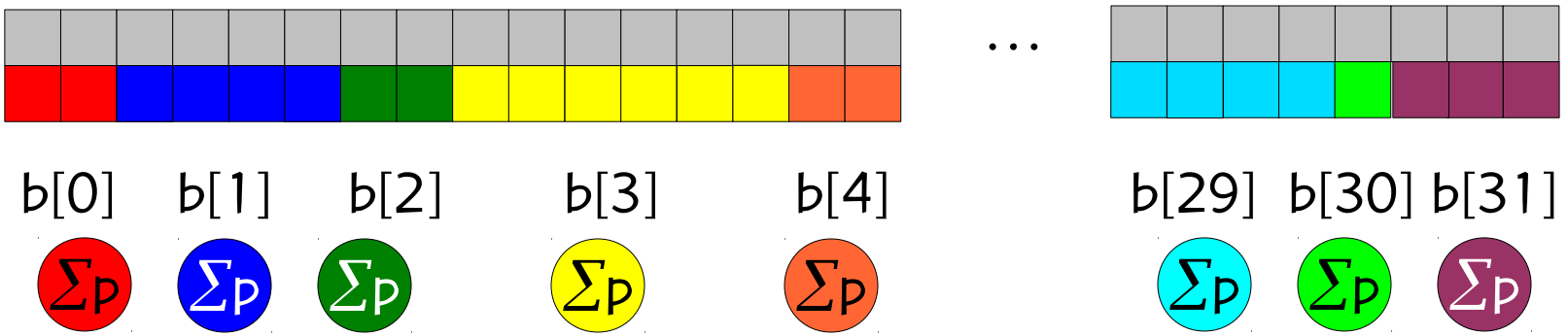
Place consecutive numbers in 32 buckets b[0], b[1], ..., b[31]



b[0] b[1] b[2] b[3] b[4] ... b[29] b[30] b[31]

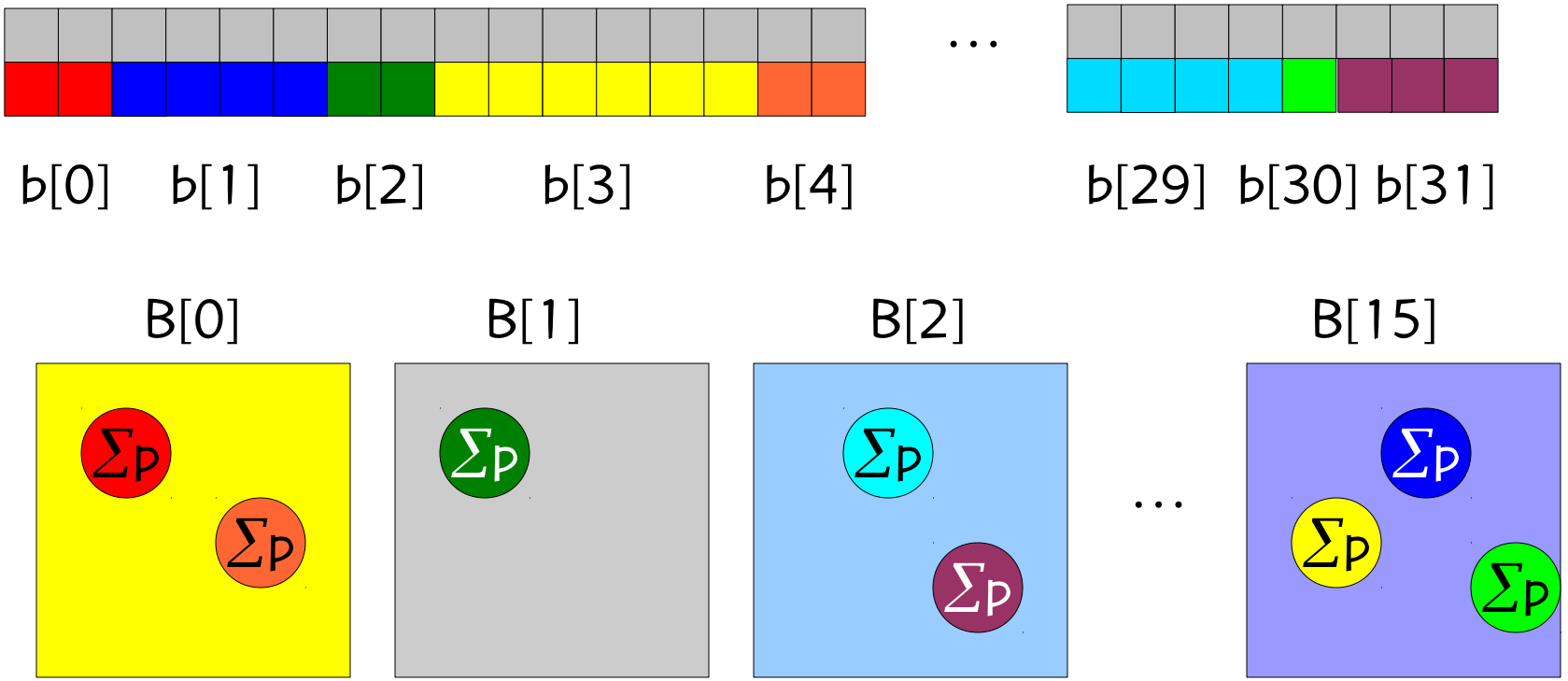
Example of restart strategy for BRKGA: Load balancing

Add the numbers in each bucket b[0], b[1], ..., b[31]



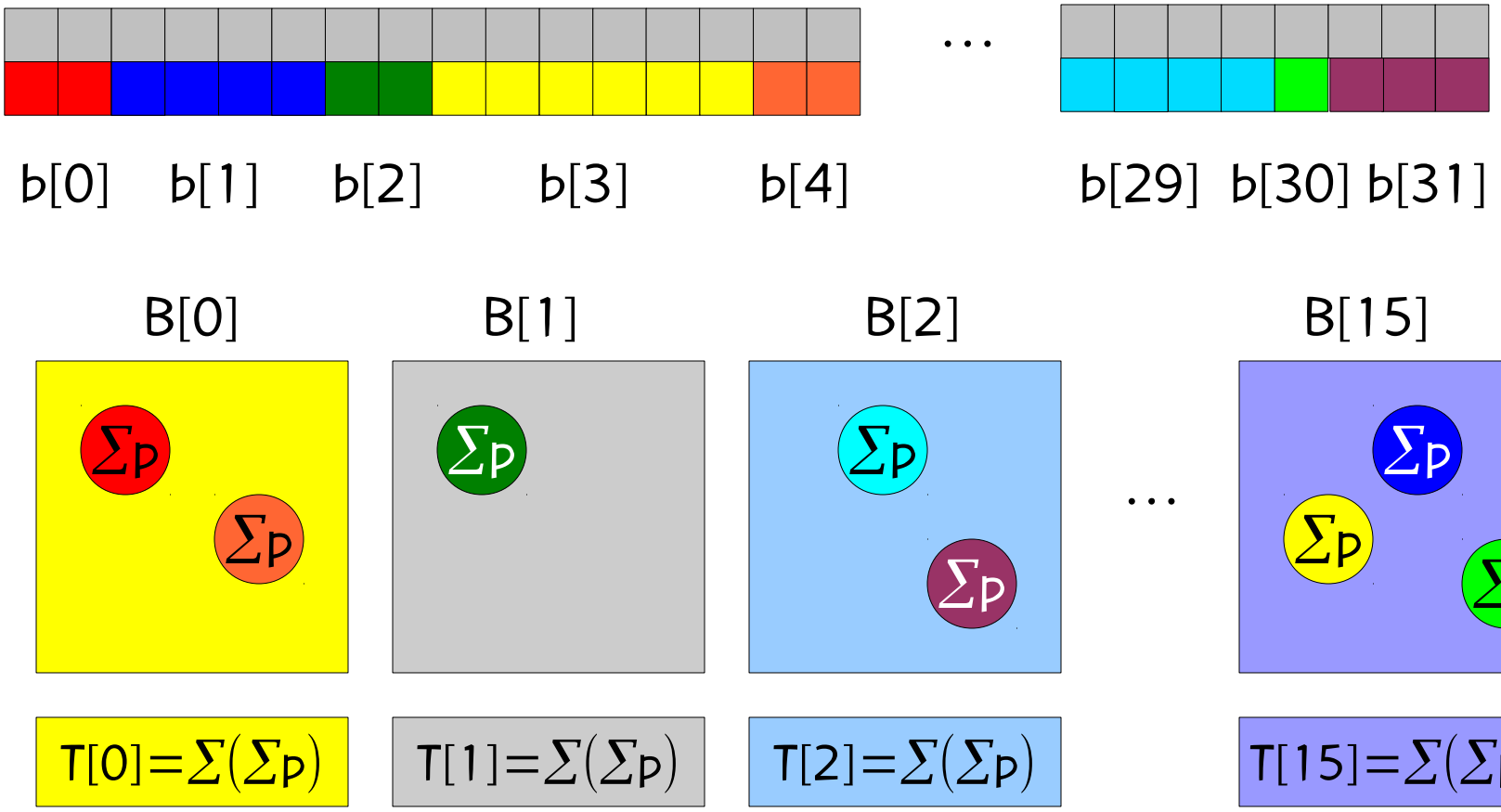
Example of restart strategy for BRKGA: Load balancing

Place the buckets in 16 bins B[0], B[1], ..., B[15]



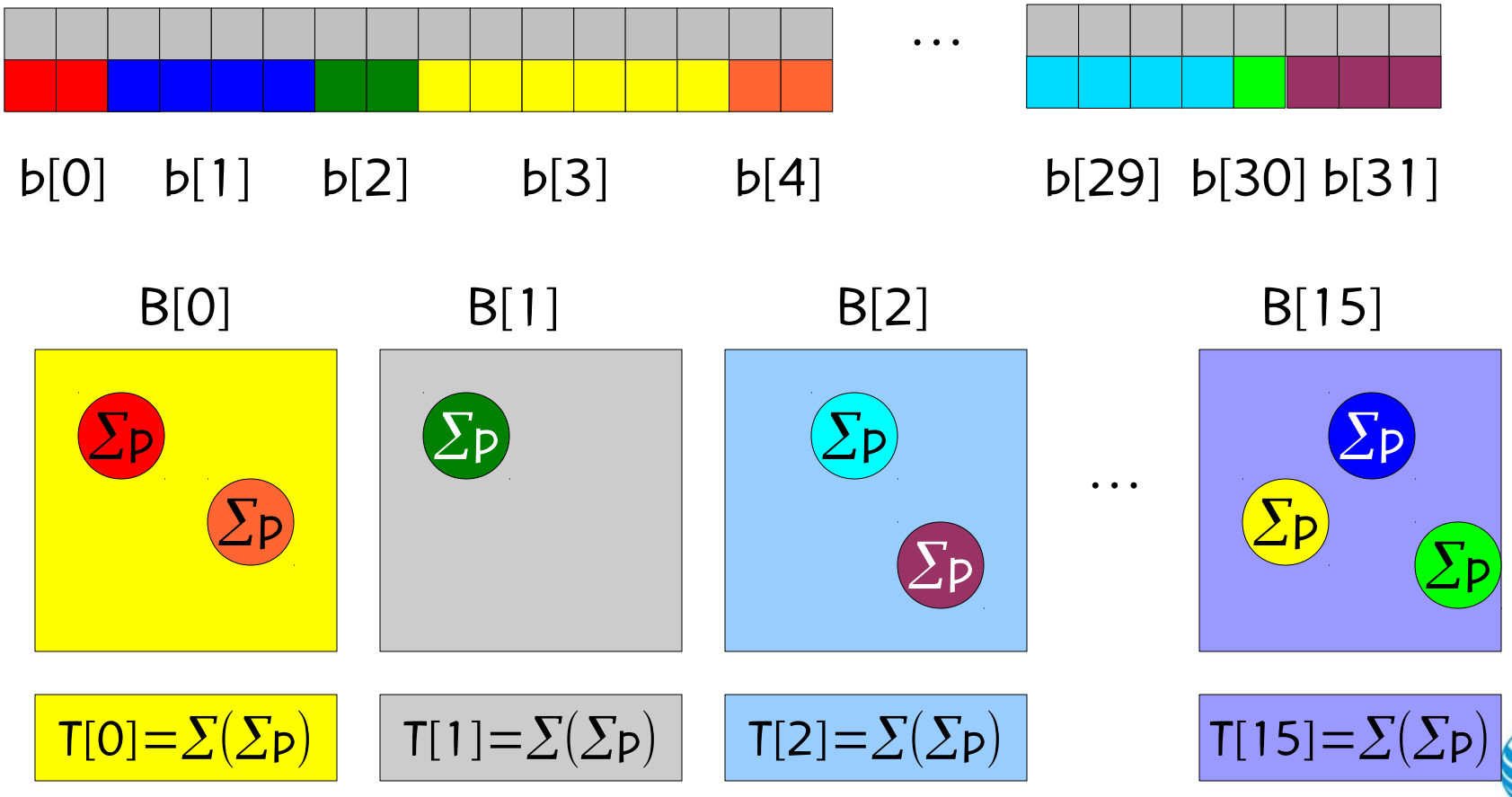
Example of restart strategy for BRKGA: Load balancing

Add up the numbers in each bin B[0], B[1], ..., B[15]



Example of restart strategy for BRKGA: Load balancing

OBJECTIVE: Minimize { Maximum (T[0], T[1], ..., T[15]) }



Example of restart strategy for BRKGA: Load balancing

Encoding

X = [x[1], x[2], ..., x[32] | x[32+1], x[32+2], ..., x[32+16]]

Decoding

x[1], x[2], ..., x[32] are used to define break points for buckets

x[32+1], x[32+2], ..., x[32+16] are used to determine to which bins the buckets are assigned

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Decoding

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Size of bucket i = floor (1024 × x[i]/(x[1]+x[2]+···+x[32])), i=1,...,15

Size of bucket 16 = 1024 – sum of sizes of first 15 buckets

Example of restart strategy for BRKGA: Load balancing

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X = [x[1], x[2], ..., x[32] | x[32+1], x[32+2], ..., x[32+16]]

Decoding

x[1], x[2], ..., x[32] are used to define break points for buckets

x[32+1], x[32+2], ..., x[32+16] are used to determine to which bins the buckets are assigned

Bin that bucket i is assigned to = floor (16 × x[32+i]) + 1

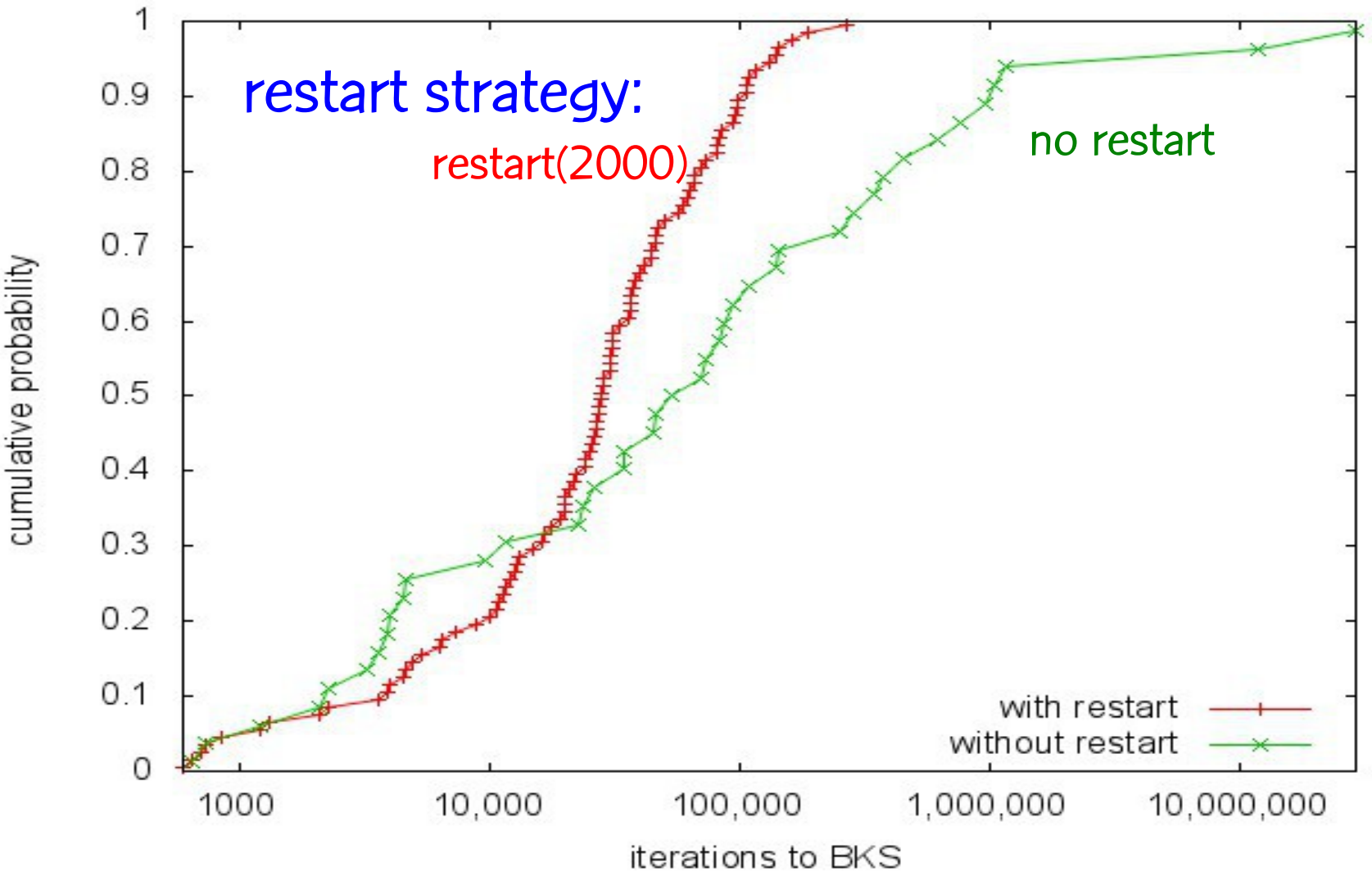
Example of restart strategy for BRKGA: Load balancing

Decoding (Local search phase)

- **while** (there exists a bucket in the most loaded bin that can be moved to another bin and not increase the maximum load) **then**
  - move that bucket to that bin
- **end while**

Make necessary chromosome adjustments to last 16 random keys of vector of random keys to reflect changes made in local search phase: Add or subtract an integer value from chromosome of bucket that moved to new bin.

Example of restart strategy for BRKGA: Load balancing



Specifying a BRKGA

Specifying a biased random-key GA

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- Parameters

Specifying a biased random-key GA

Parameters:

- Size of population
- Parallel population parameters
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters
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Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
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- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N, say 2N or 10N
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N, say 2N or 10N
- Stopping criterion: *e.g.* time, # generations, solution quality, # generations without improvement

brkgaAPI: A C++ API for BRKGA

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.

brkgaAPI: A C++ API for BRKGA

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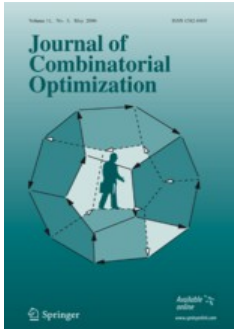
brkgaAPI: A C++ API for BRKGA

Paper: Rodrigo F. Toso and M.G.C.R., "A C++ Application Programming Interface for Biased Random-Key Genetic Algorithms," AT&T Labs Technical Report, Florham Park, August 2011.

Software: http://www.research.att.com/~mgcr/src/brkgaAPI

An example BRKGA: Packing weighted rectangles

Reference



J.F. Gonçalves and M.G.C.R., "A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal packing problem," Journal of Combinatorial Optimization, vol. 22, pp. 180-201, 2011.

Tech report:
http://www.research.att.com/~mgcr/doc/pack2d.pdf

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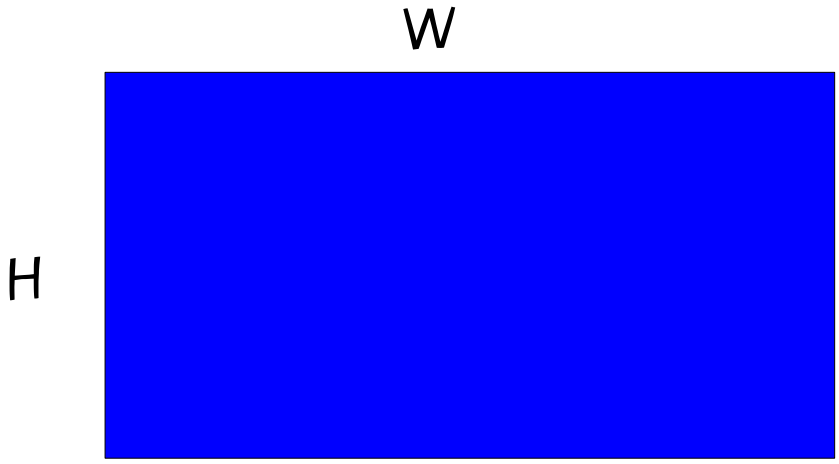

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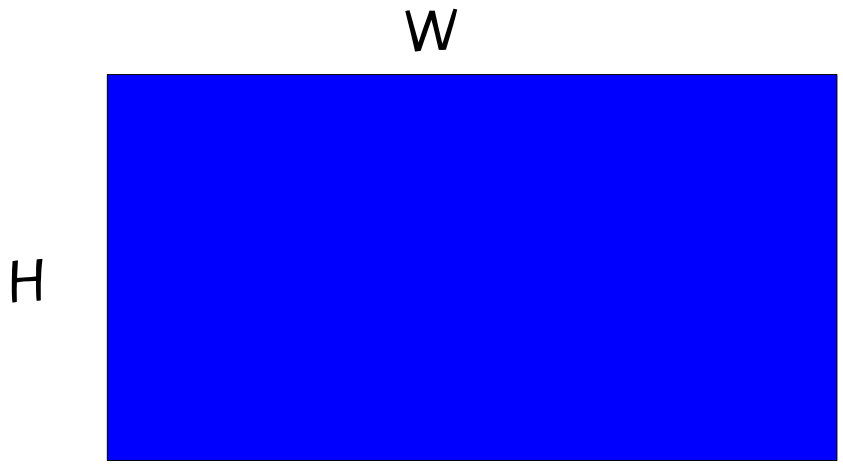

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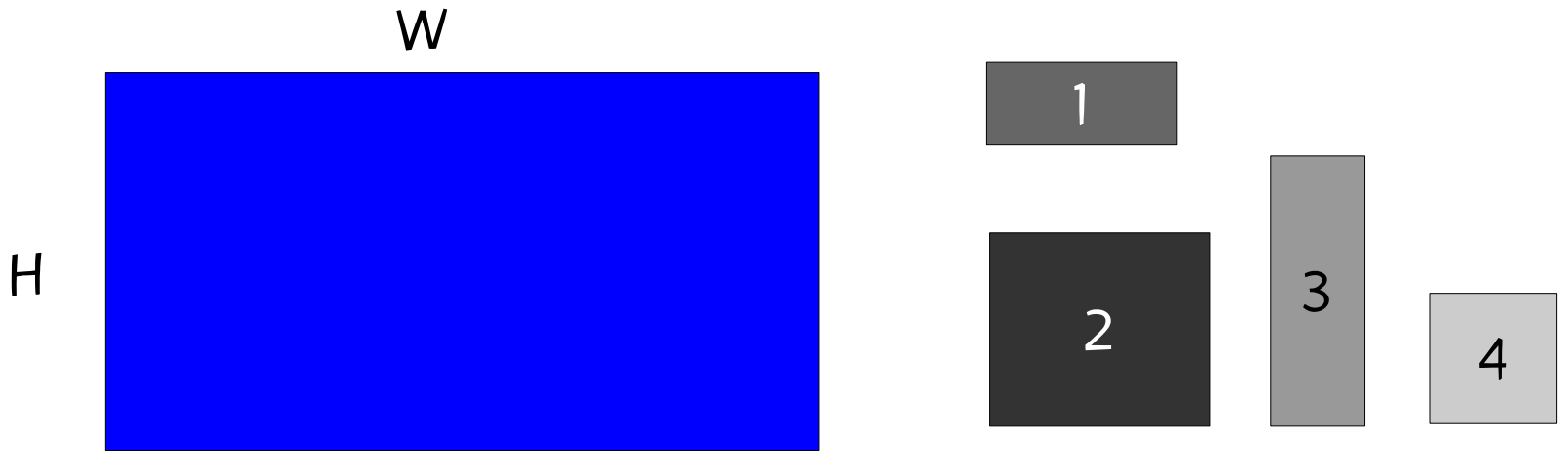

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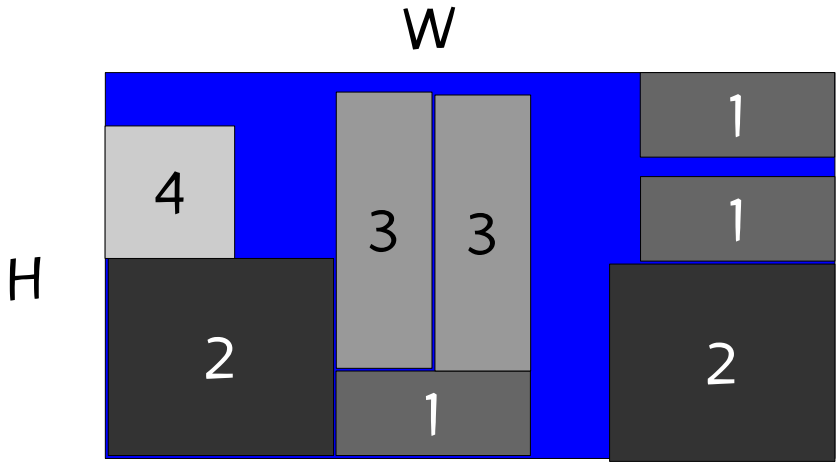
- r[i] rectangles of type i = 1, ..., N are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;



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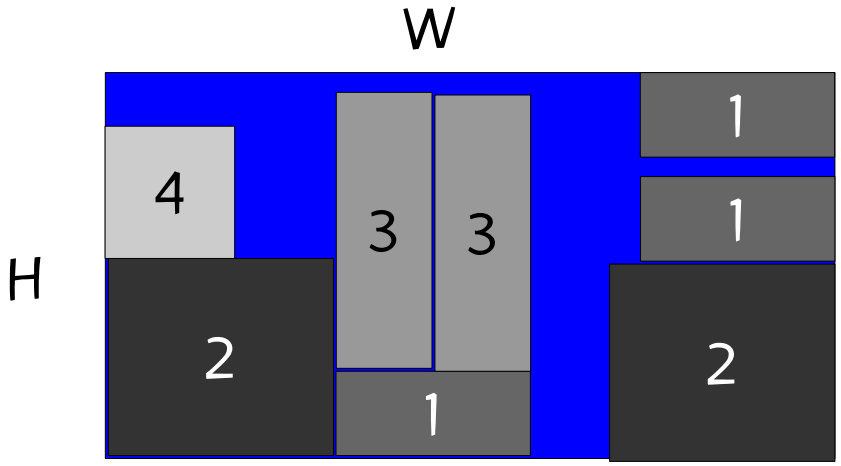
$$0 \leq P[i] \leq r[i] \leq Q[i]$$



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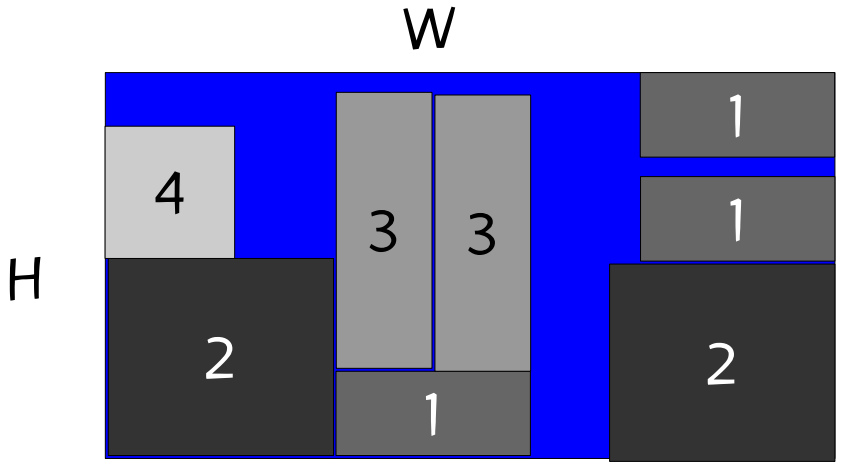

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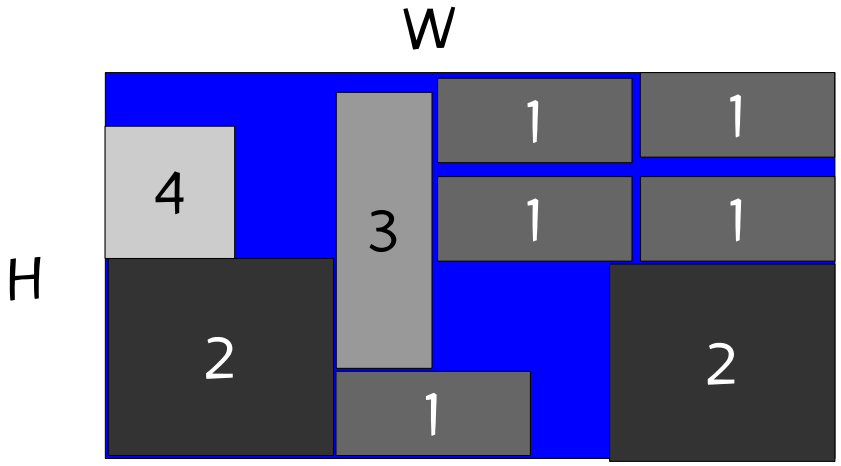

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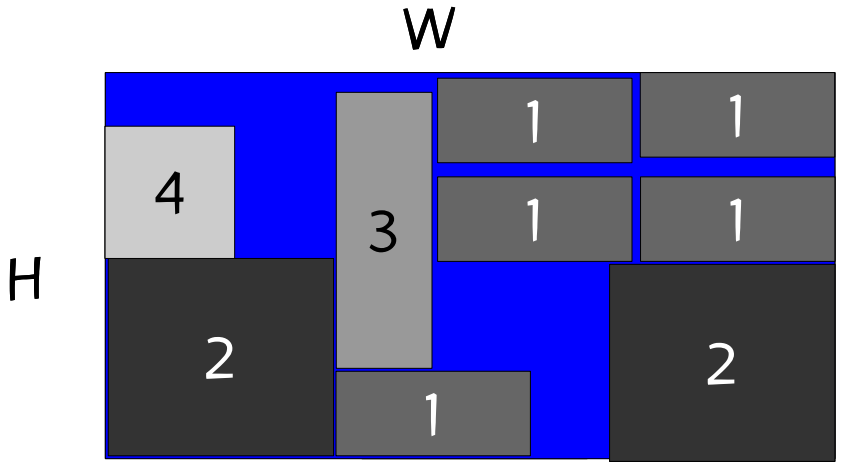
Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

$$v[1]\,r[1] + v[2]\,r[2] + \cdots + v[N]\,r[N]$$



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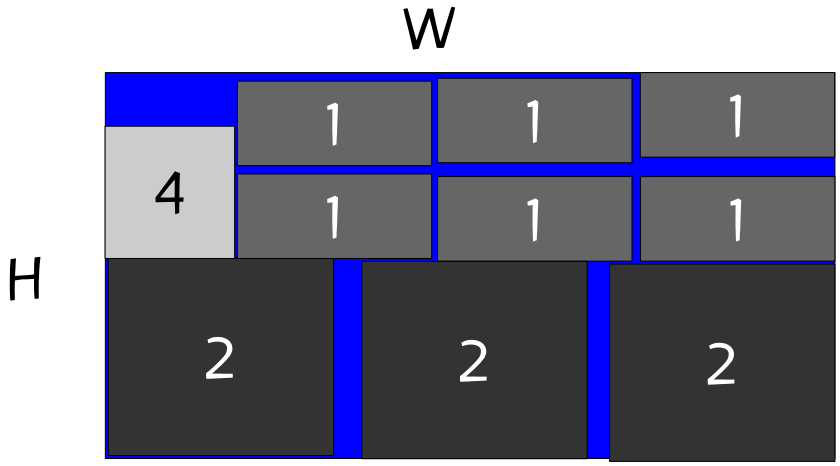

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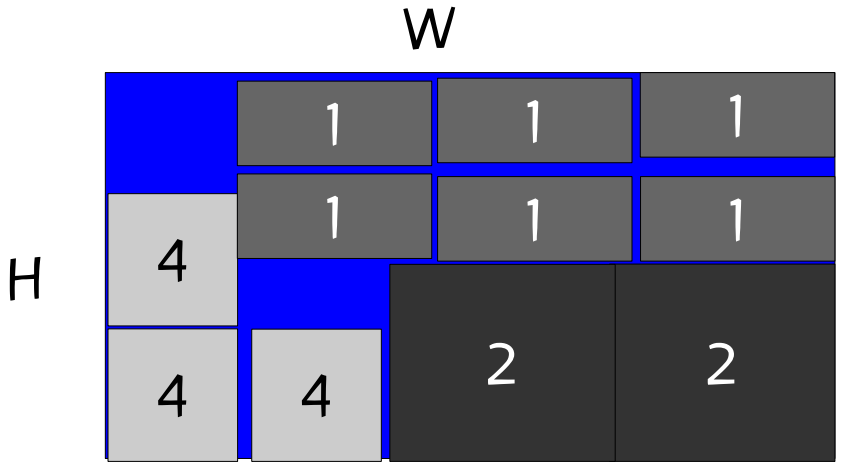

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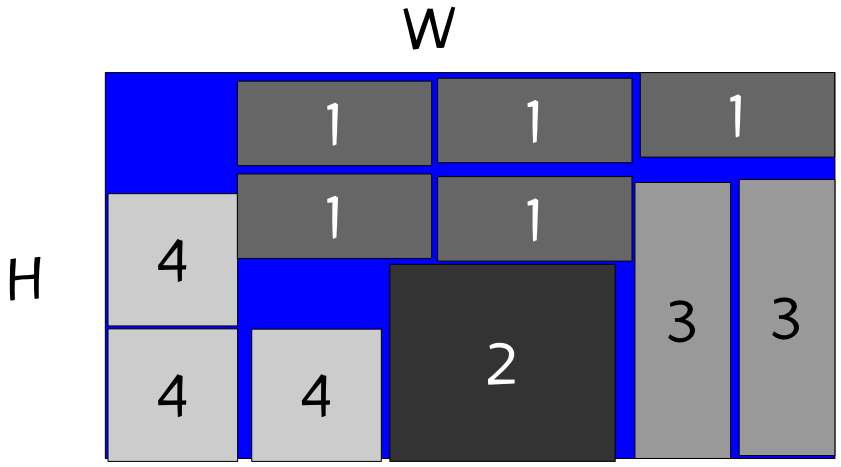
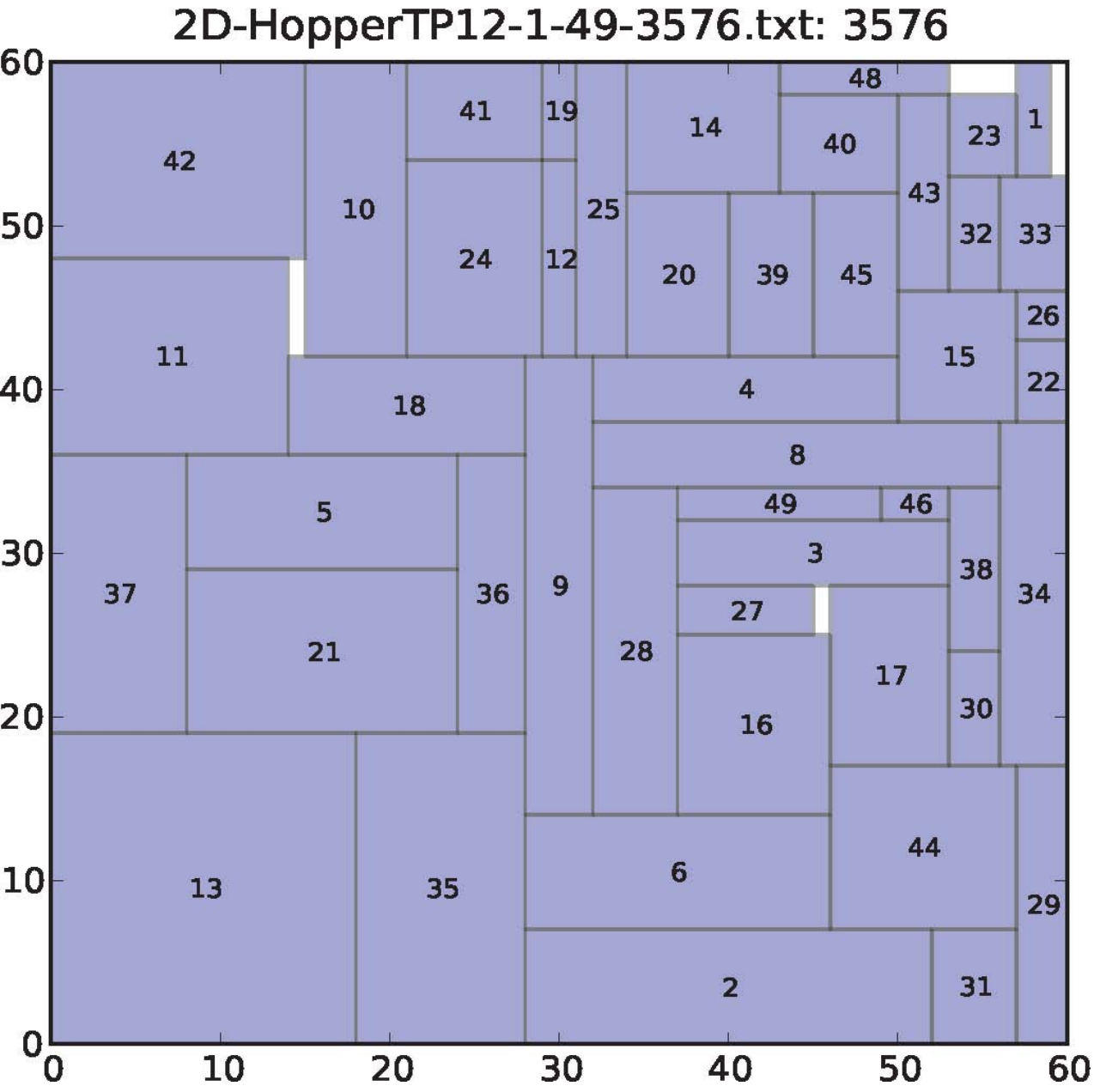


Applications

Problem arises in several production processes, e.g.

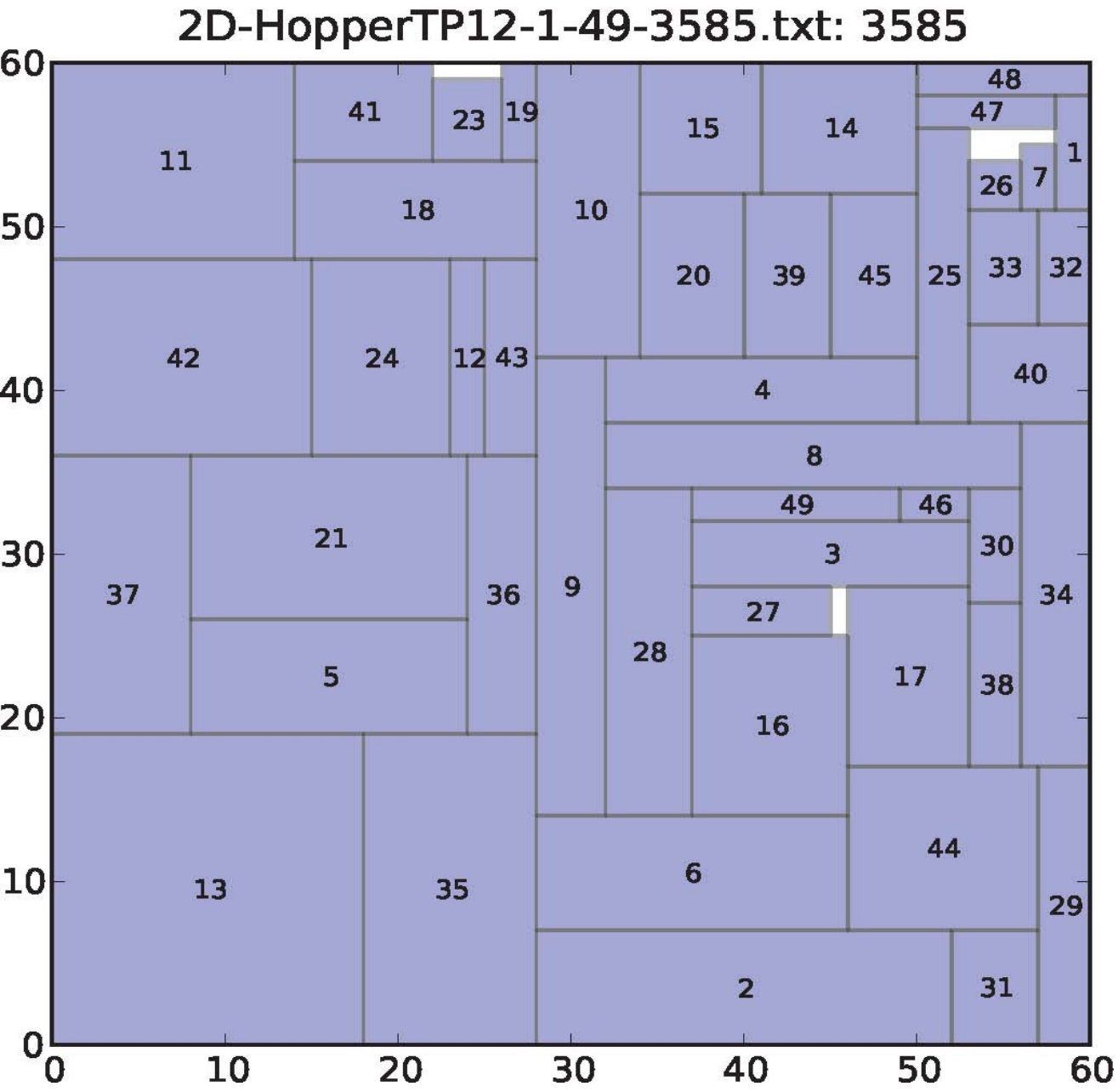
- Textile
- Glass
- Wood
- Paper

where rectangular figures are cut from large rectangular sheets of materials.



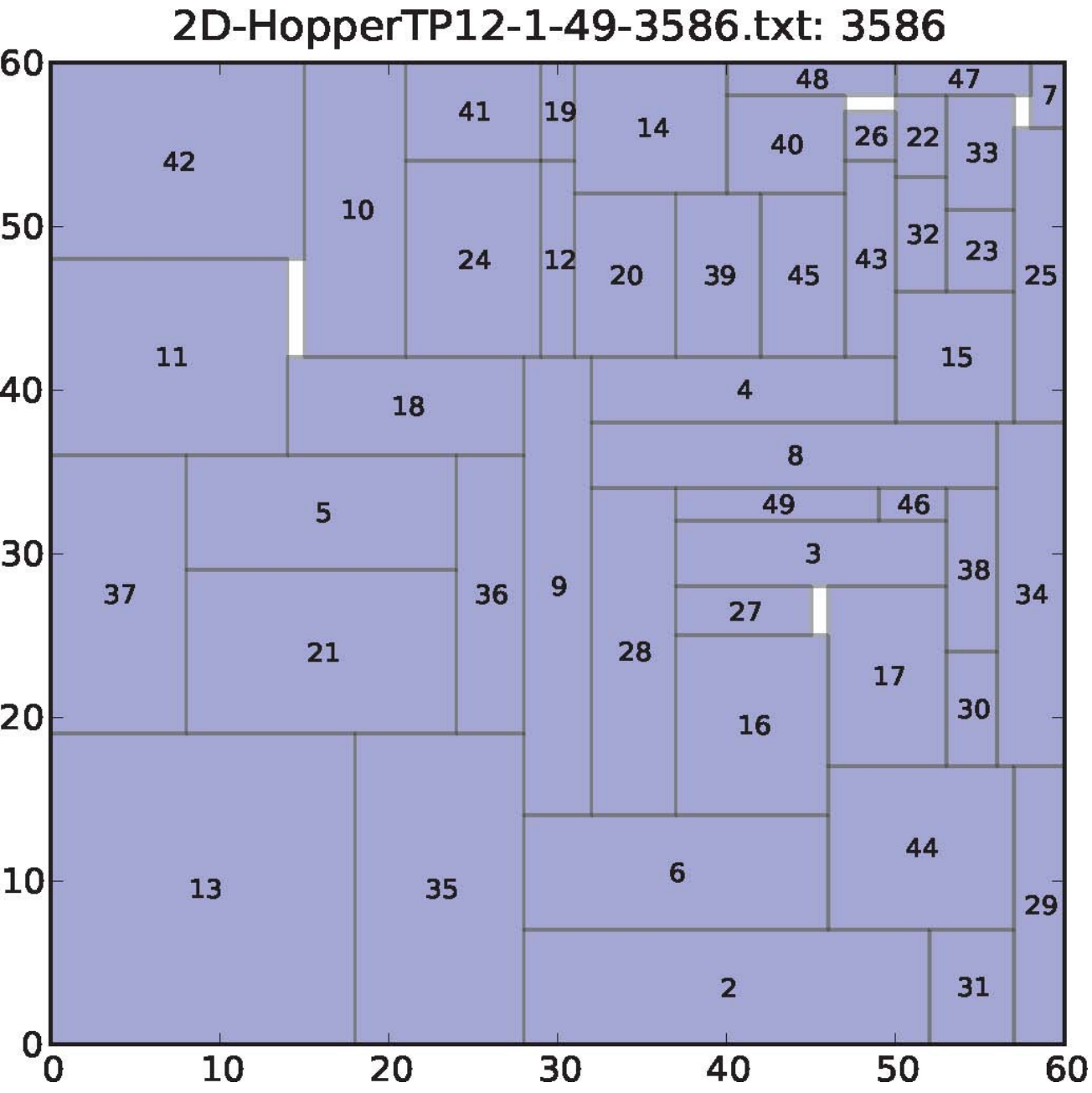
Hopper & Turton, 2001
Instance 4-1 60 x 60
Value: 3576

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)



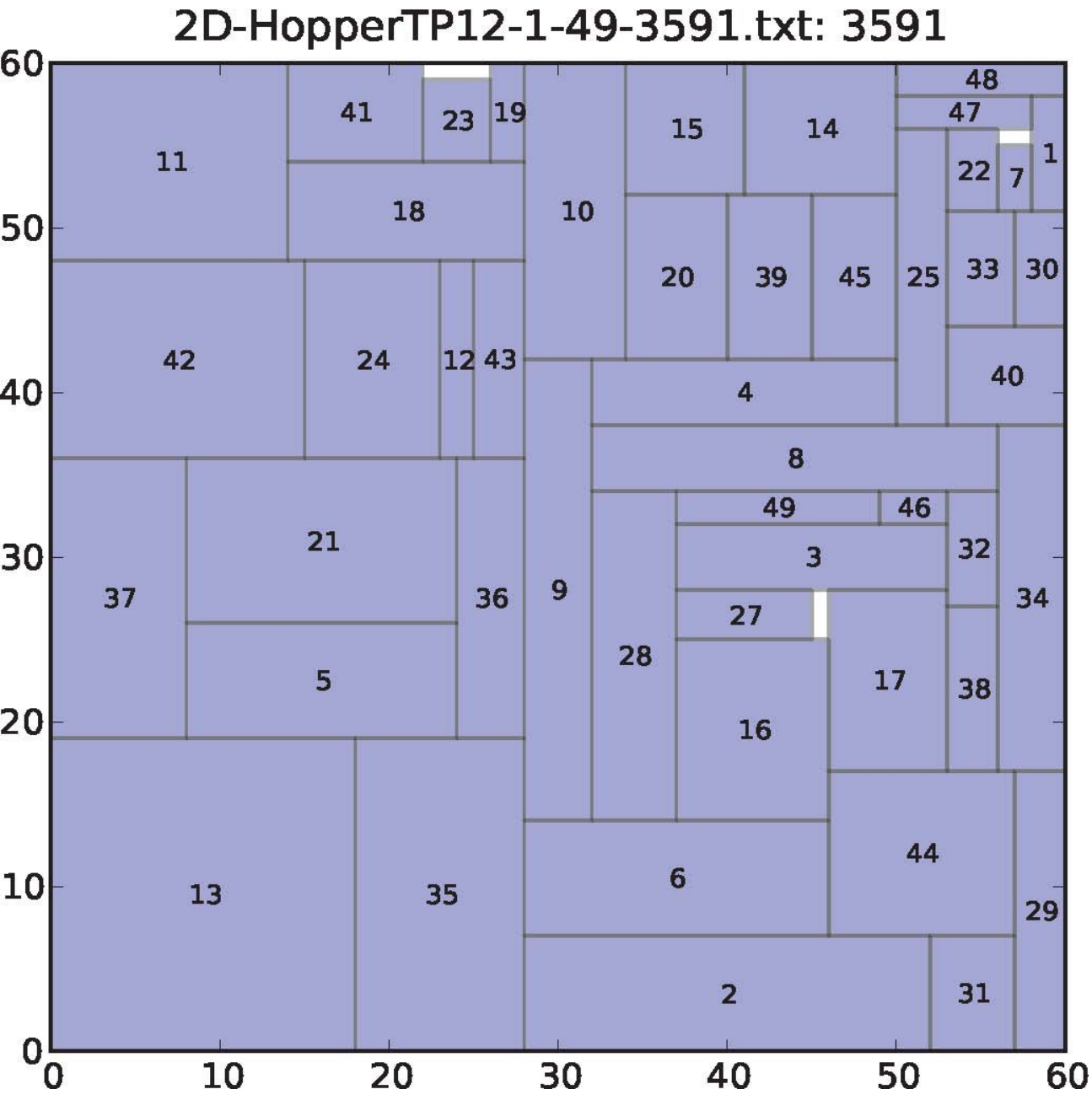
Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3585

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)



Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3586

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)



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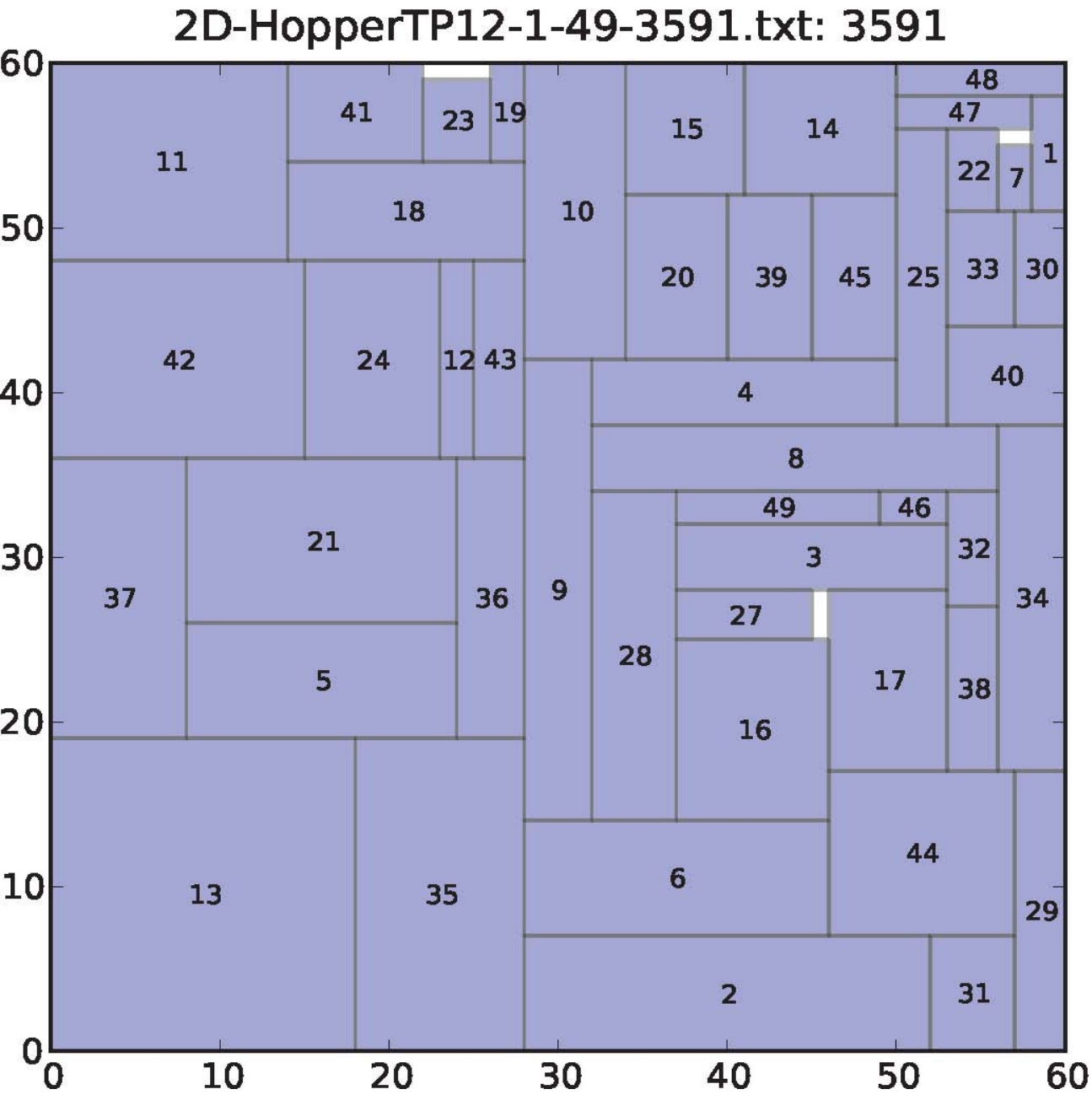

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New best known solution!

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BRKGA for constrained 2-dim orthogonal packing

Encoding

- Solutions are encoded as vectors K of

  $$2N' = 2 \{ Q[1] + Q[2] + \cdots + Q[N] \}$$

  random keys, where Q[i] is the maximum number of rectangles of type i (for i = 1, ..., N) that can be packed.
- K = (k[1], ..., k[N'], k[N'+1], ..., k[2N'])

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Rectangle type packing sequence (RTPS)

Vector of placement procedures (VPP)

Decoding

- Simple heuristic to pack rectangles:
  - Make Q[i] copies of rectangle i, for i = 1, ..., N.
  - Order the N' = Q[1] + Q[2] + ··· + Q[N] rectangles in some way.
  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order.

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Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
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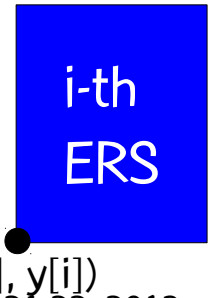

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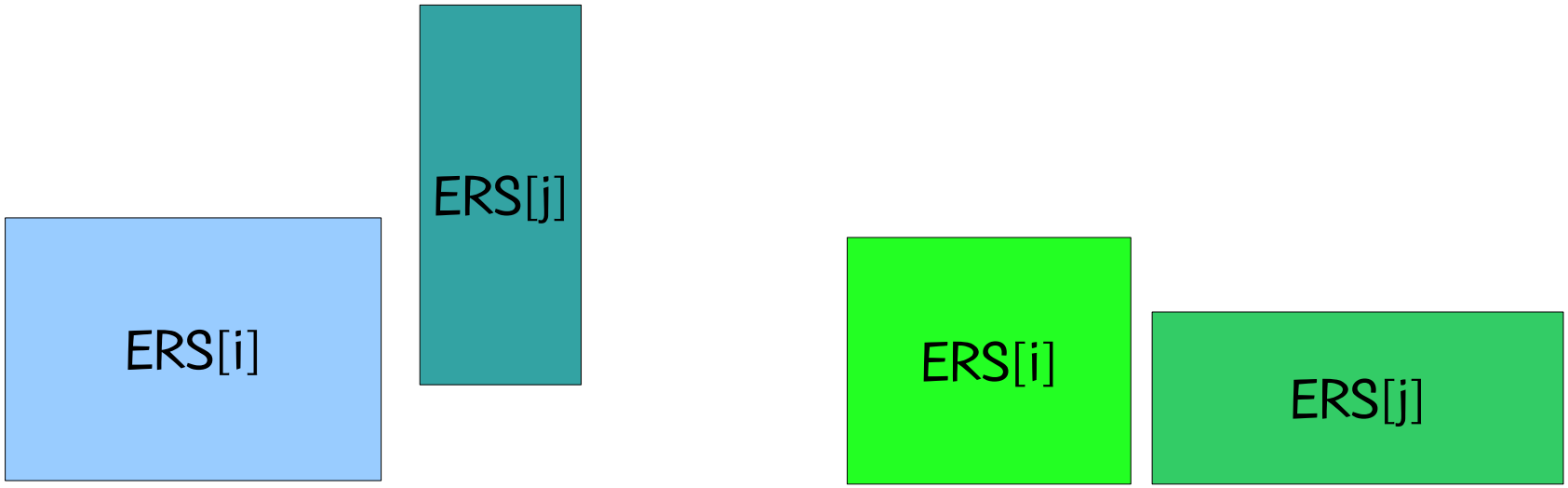
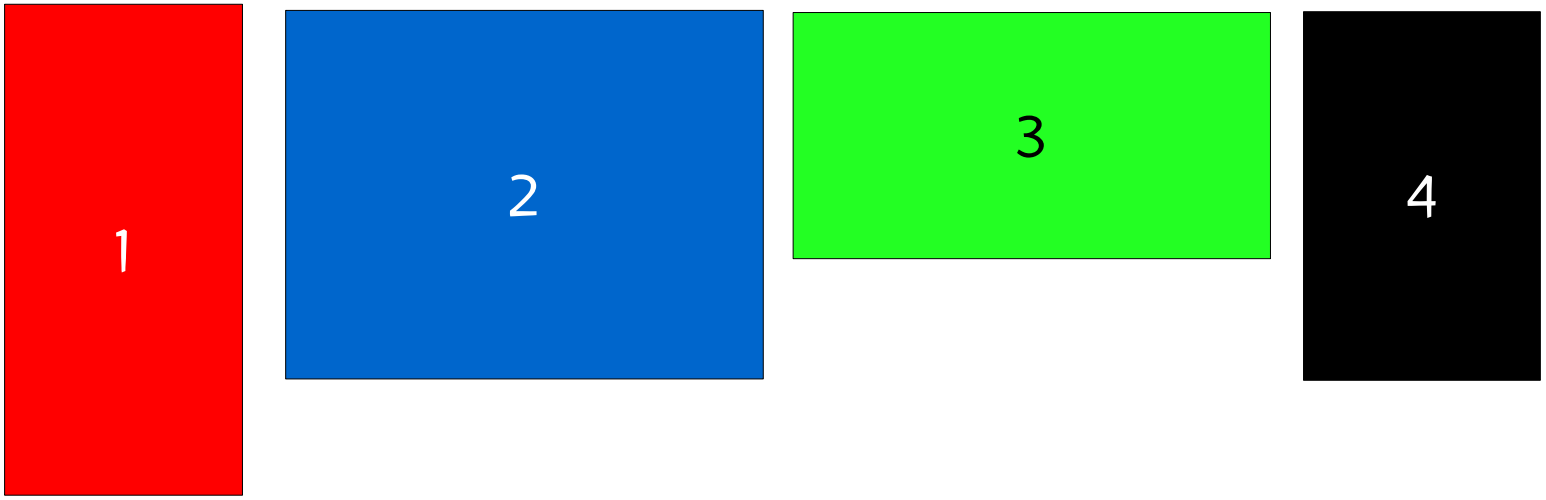
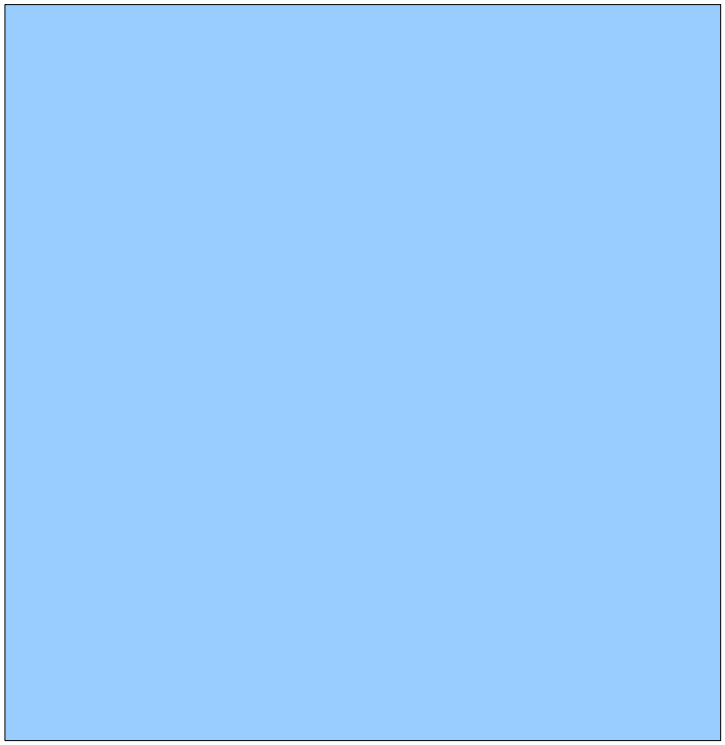


ERS[i] < ERS[j]

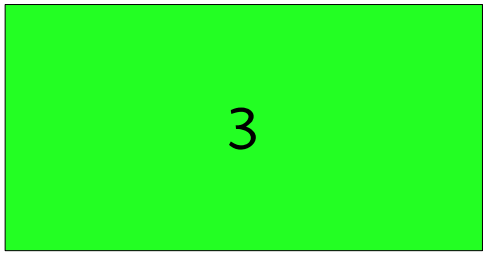


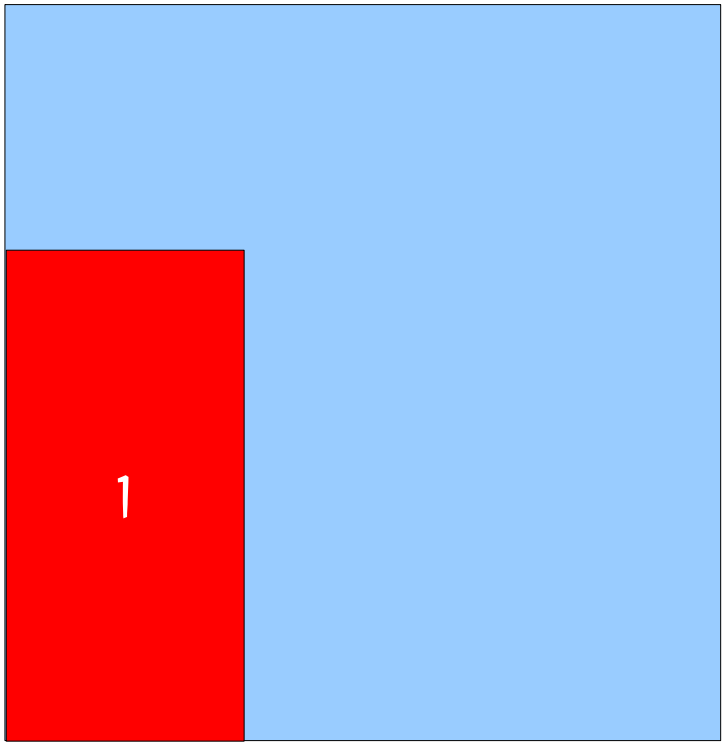


BL can run into problems even on small instances (Liu & Teng, 1999).

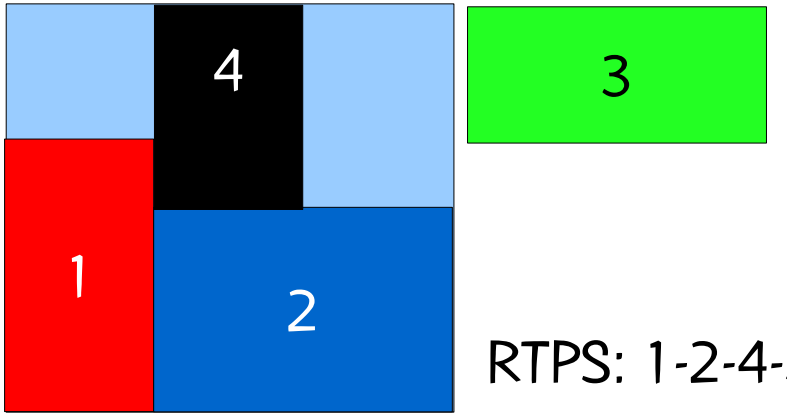
Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.

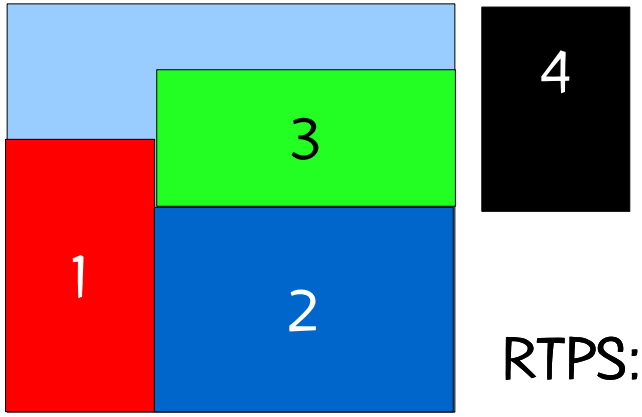




We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



RTPS: 1-2-4-3



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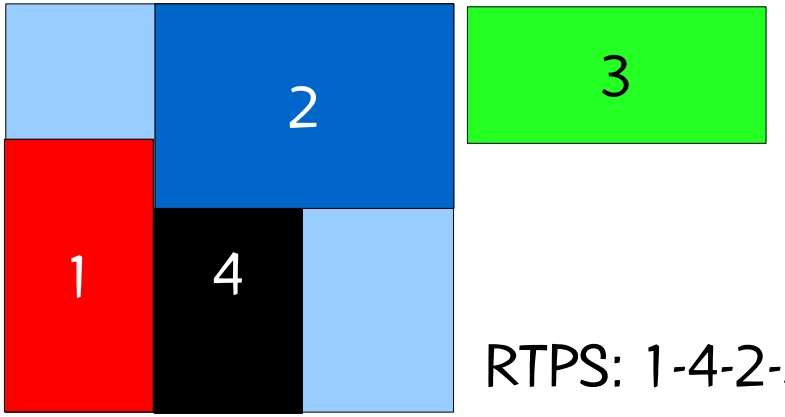
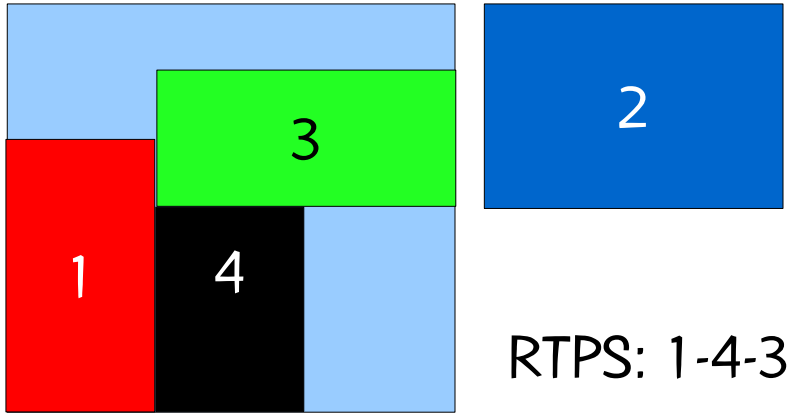
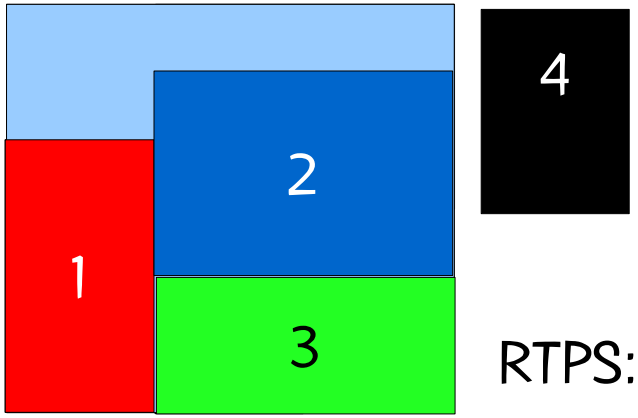


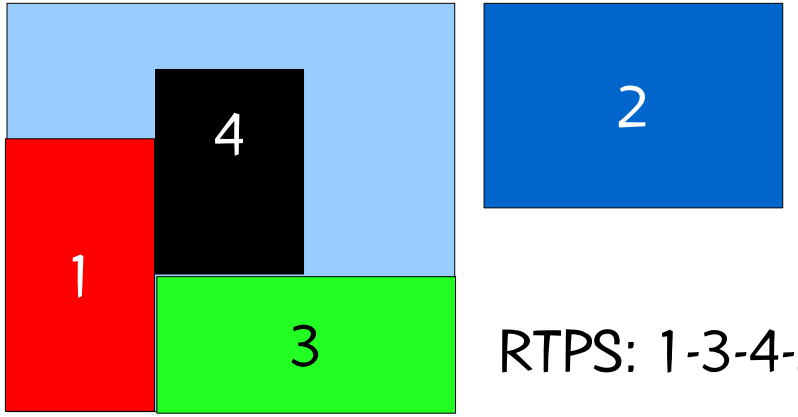
RTPS: 1-4-2-3



RTPS: 1-4-3-2



RTPS: 1-3-2-4



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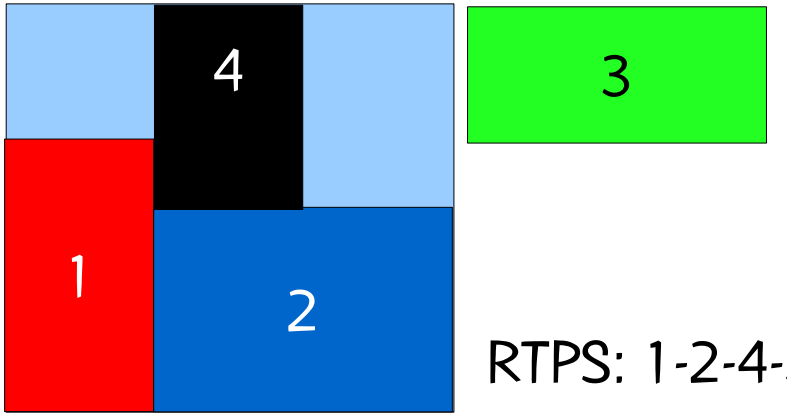

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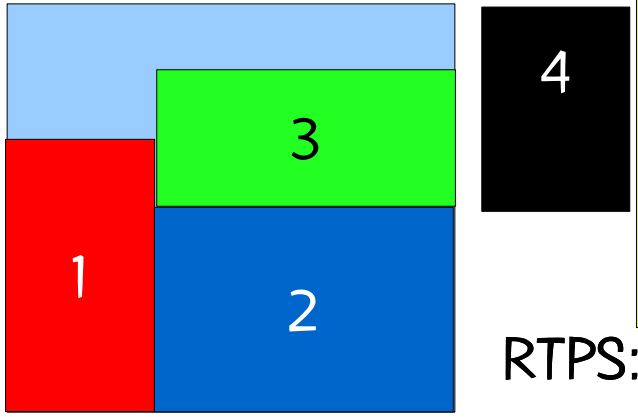
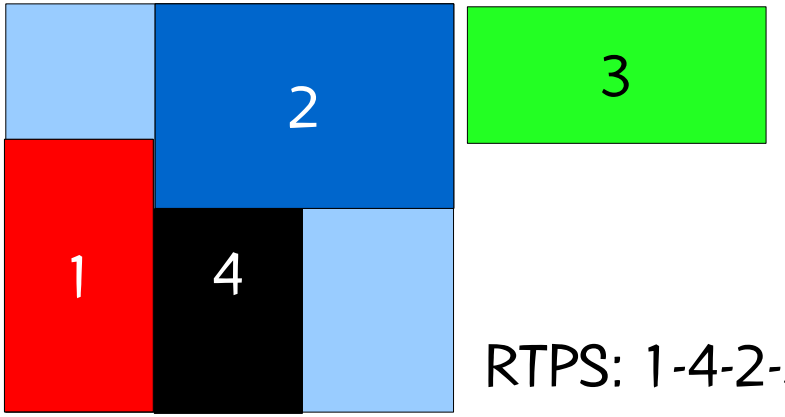


RTPS: 1-2-3-4

Similar infeasibilities are observed if 2, 3, or 4 is the first rectangle in the RTPS.



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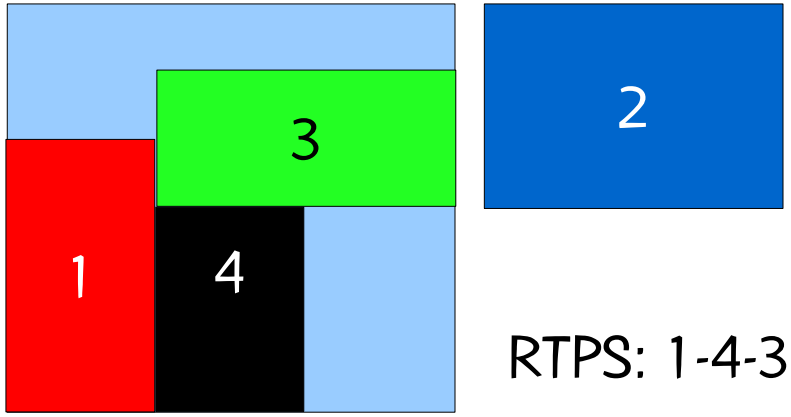

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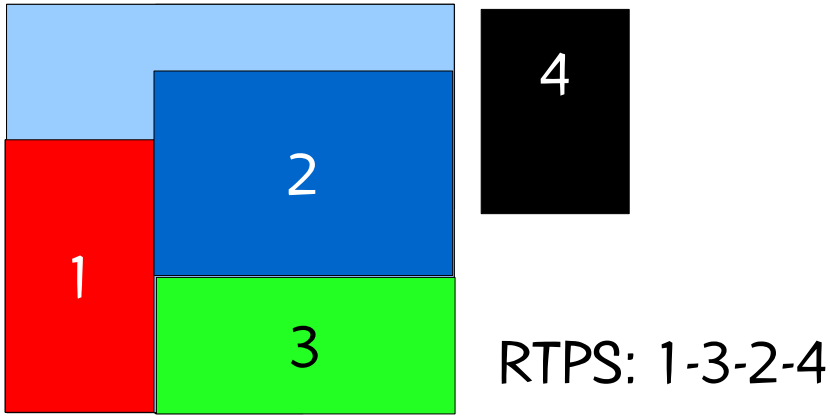
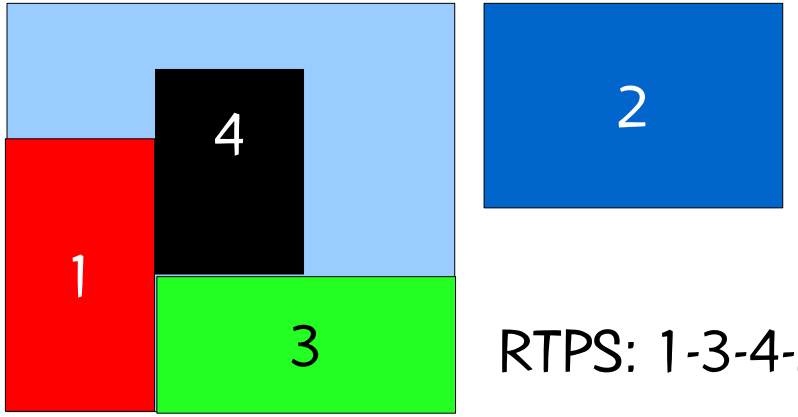


RTPS: 1-3-2-4



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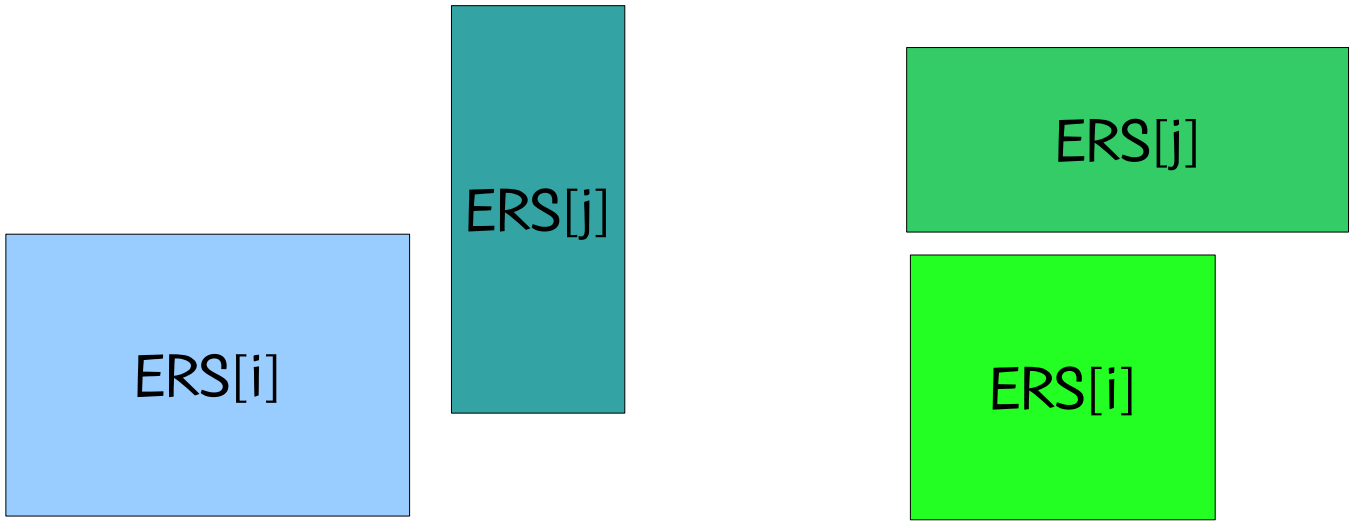
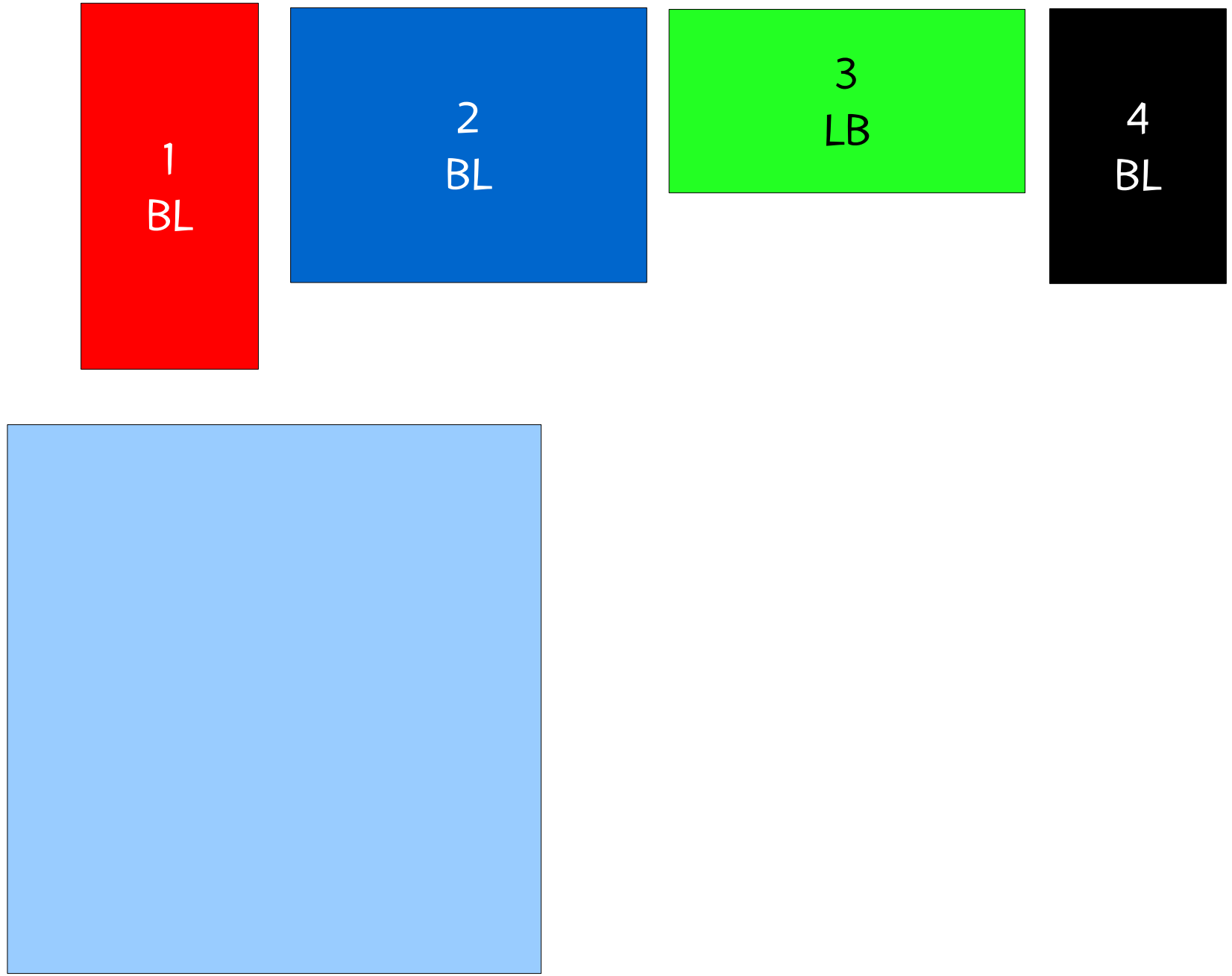
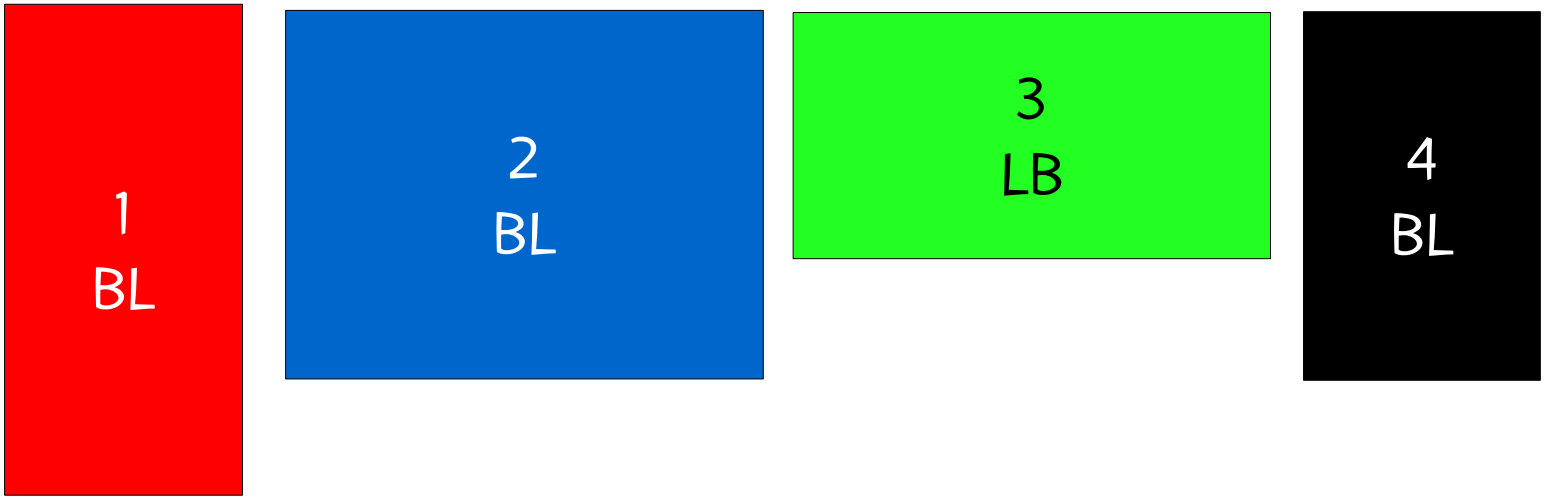
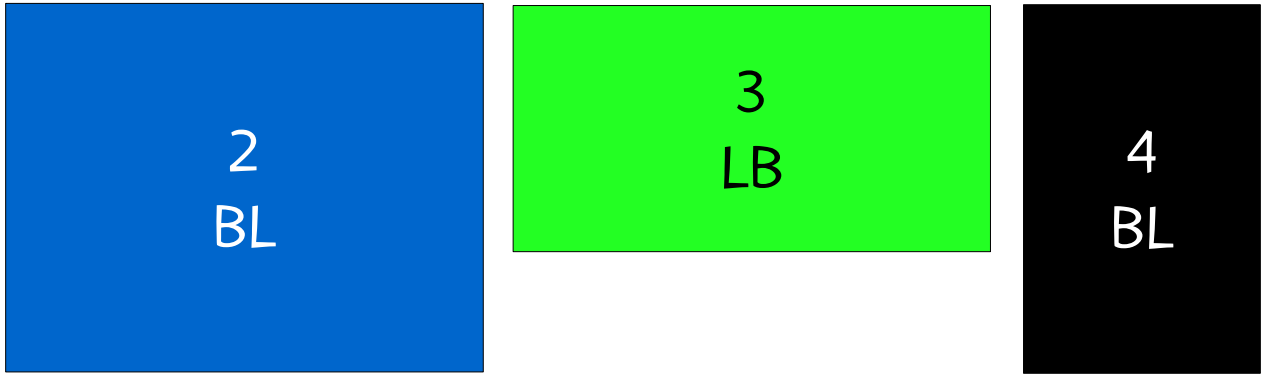
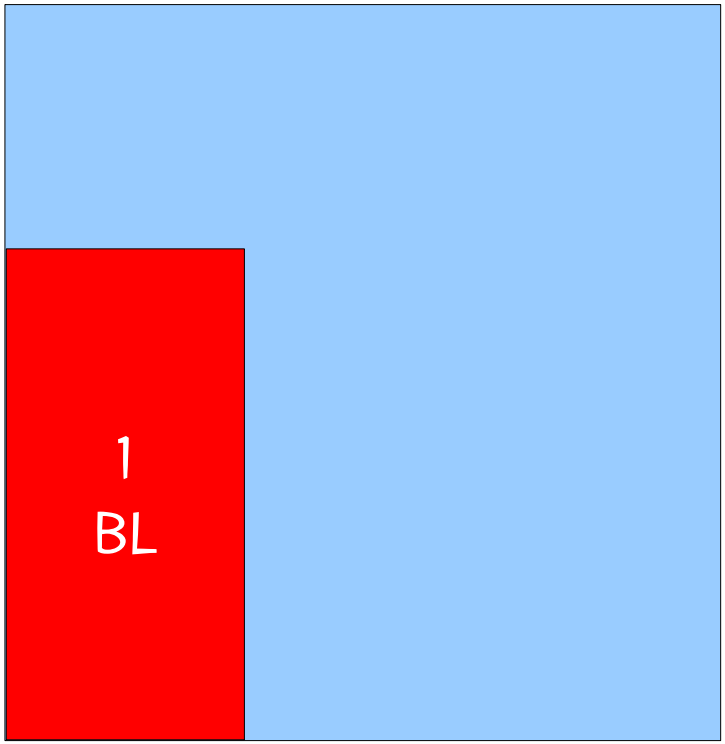


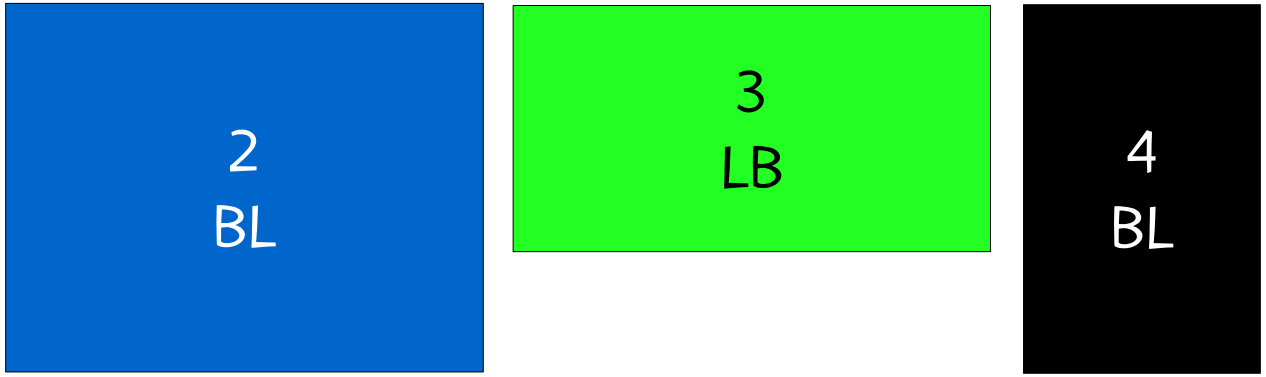
ERS[i] < ERS[j]

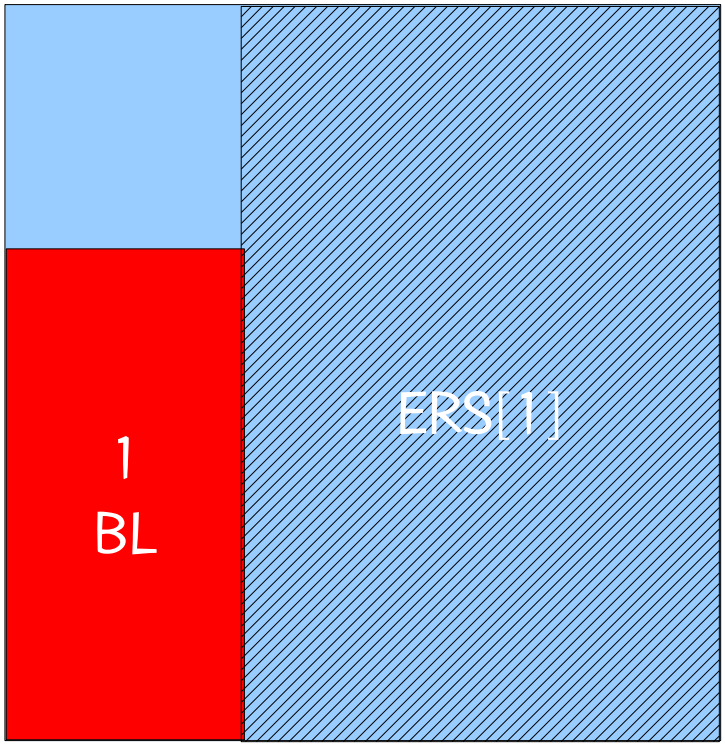

1
BL
2
BL
3
LB
4
BL

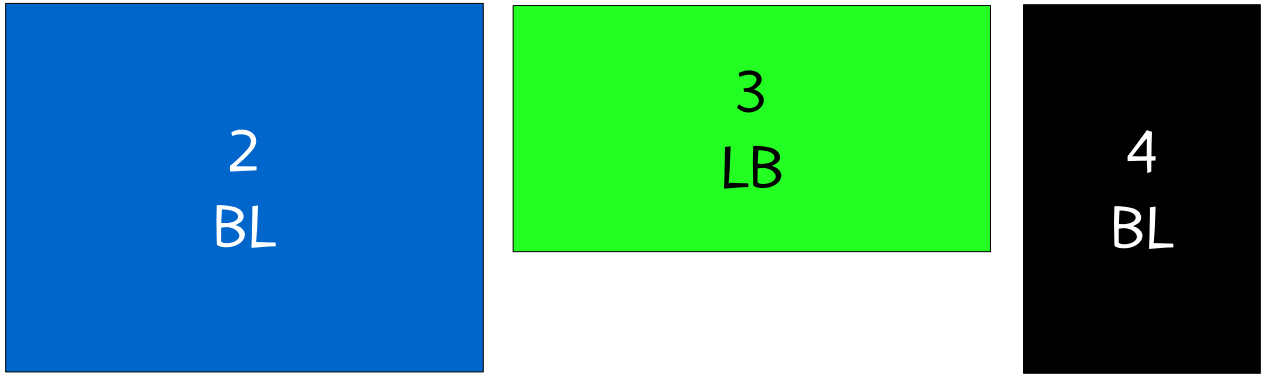


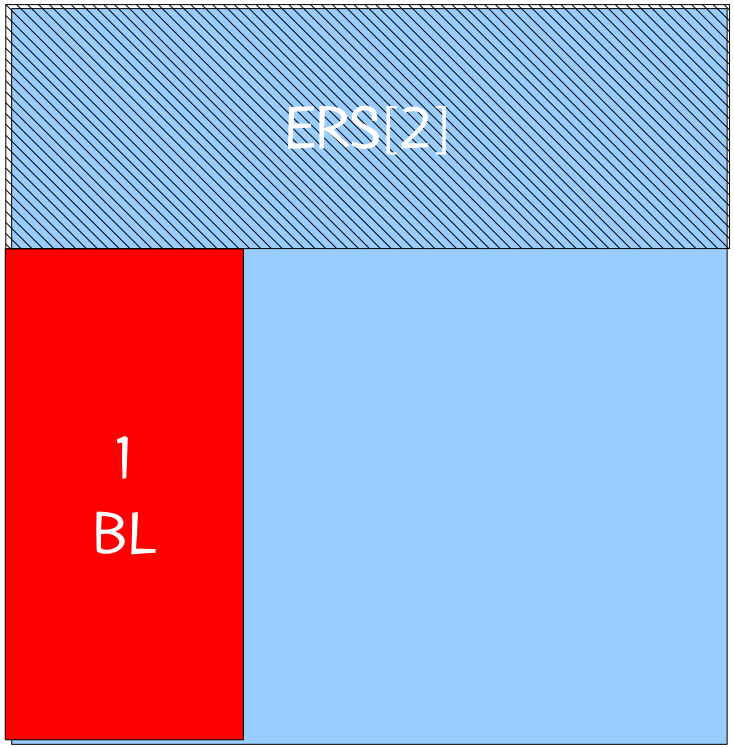


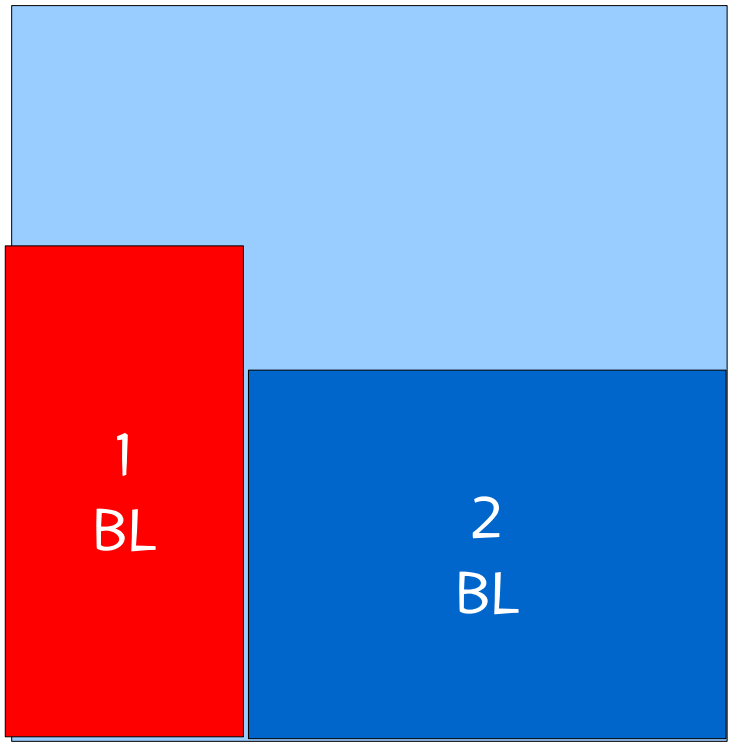






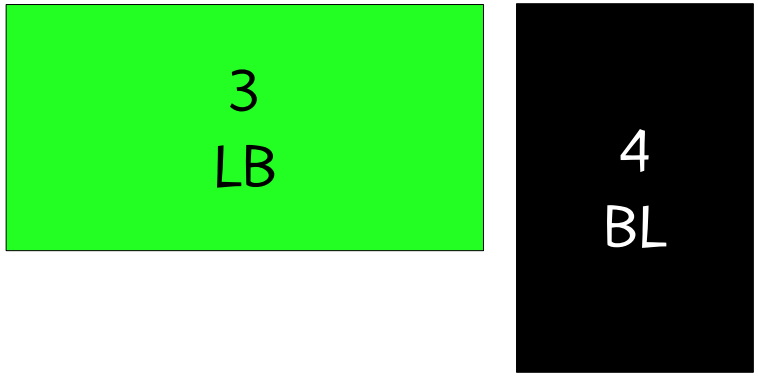


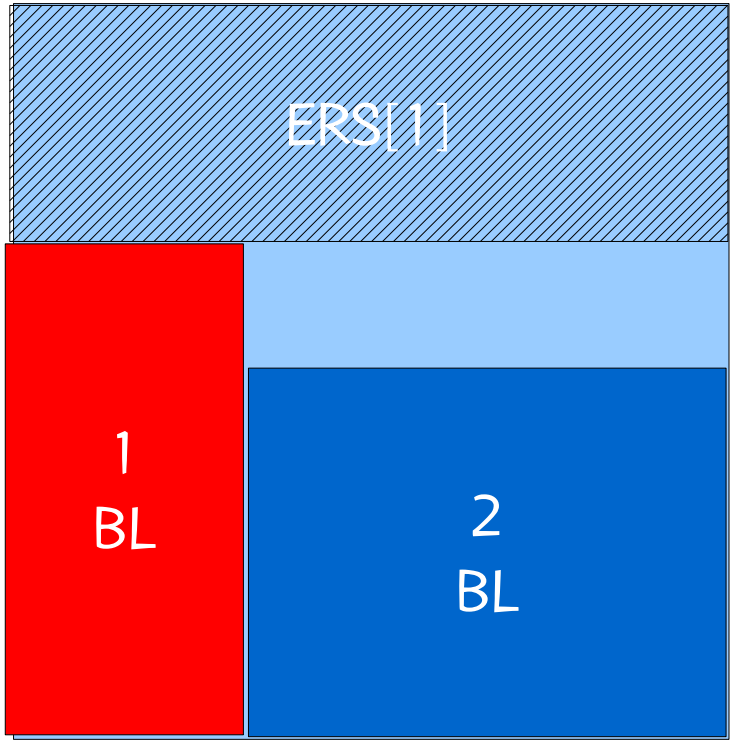


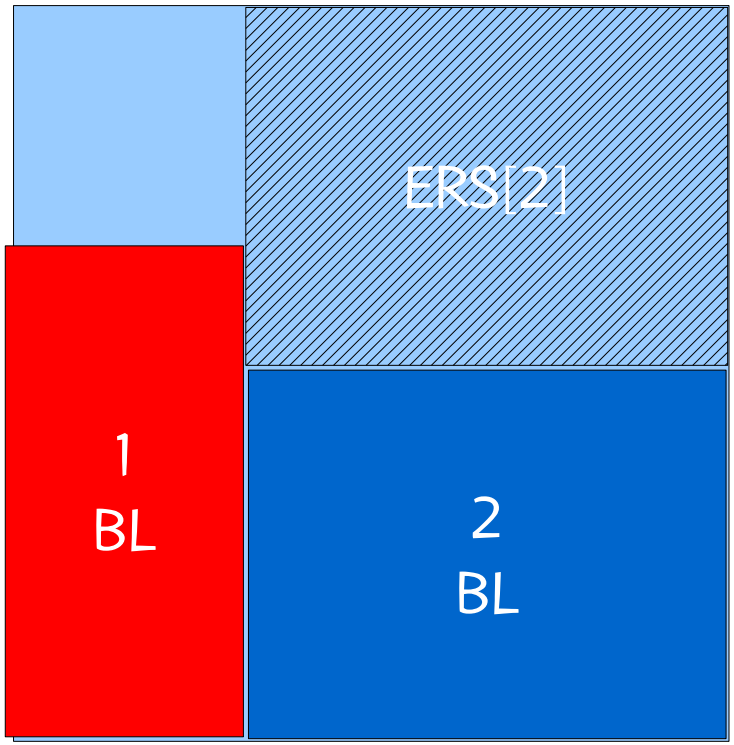


3
LB

4
BL







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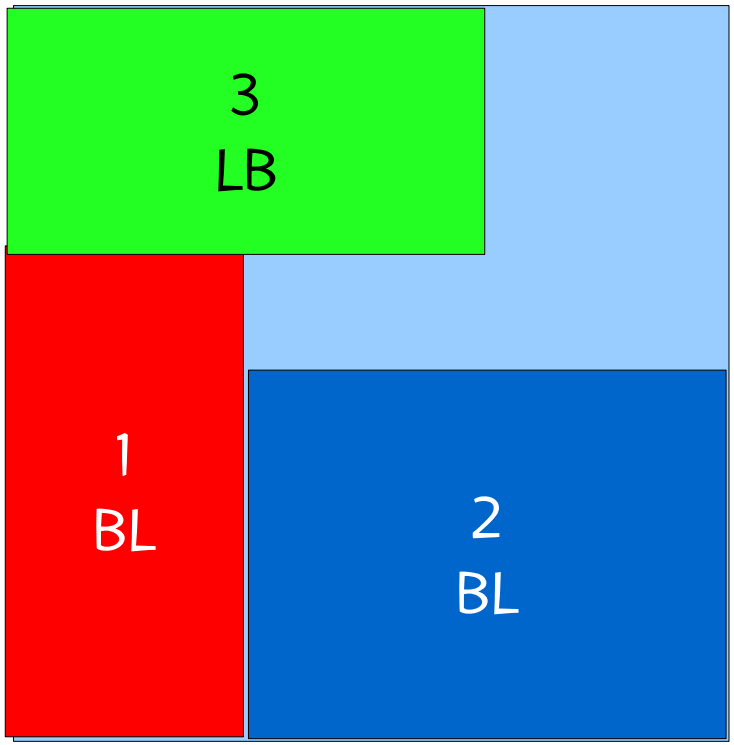

4
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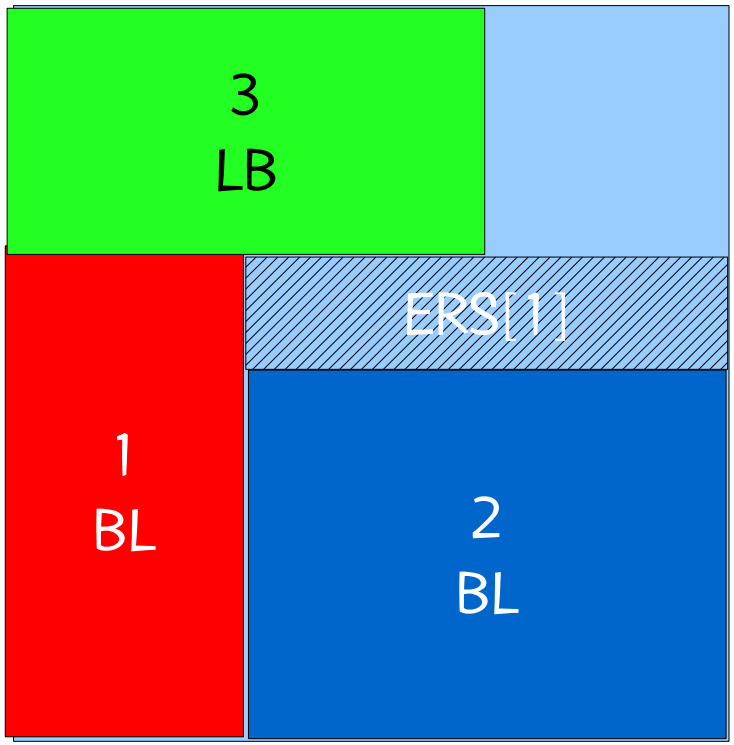
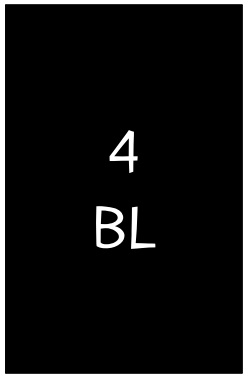
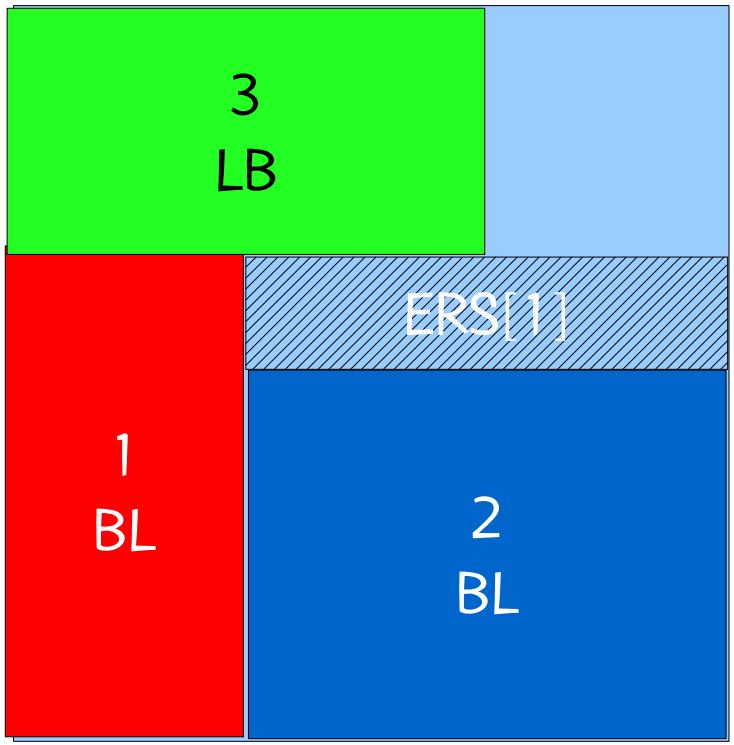
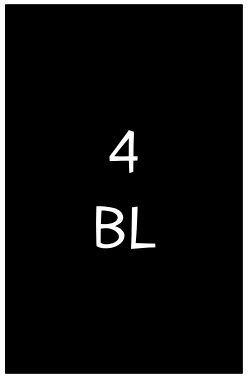


4
BL

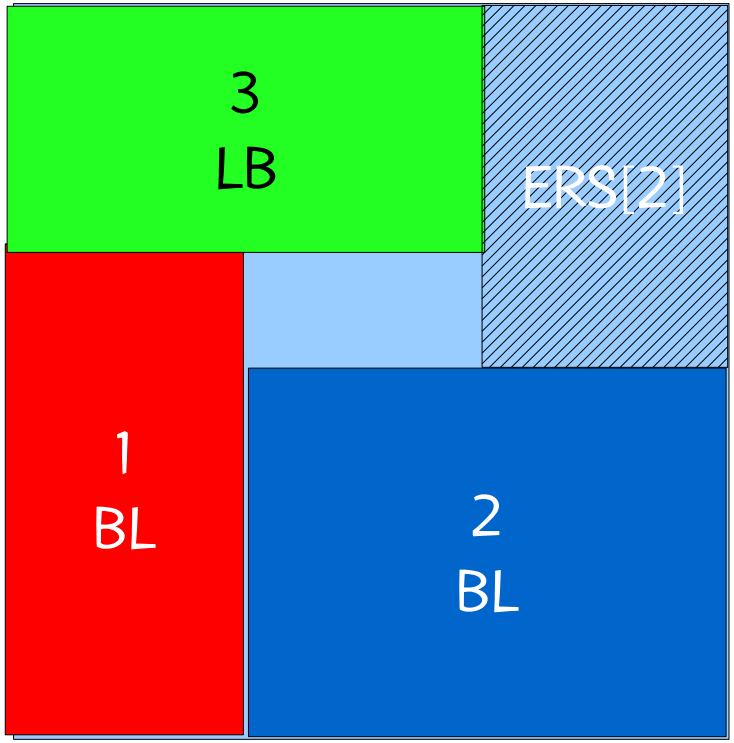


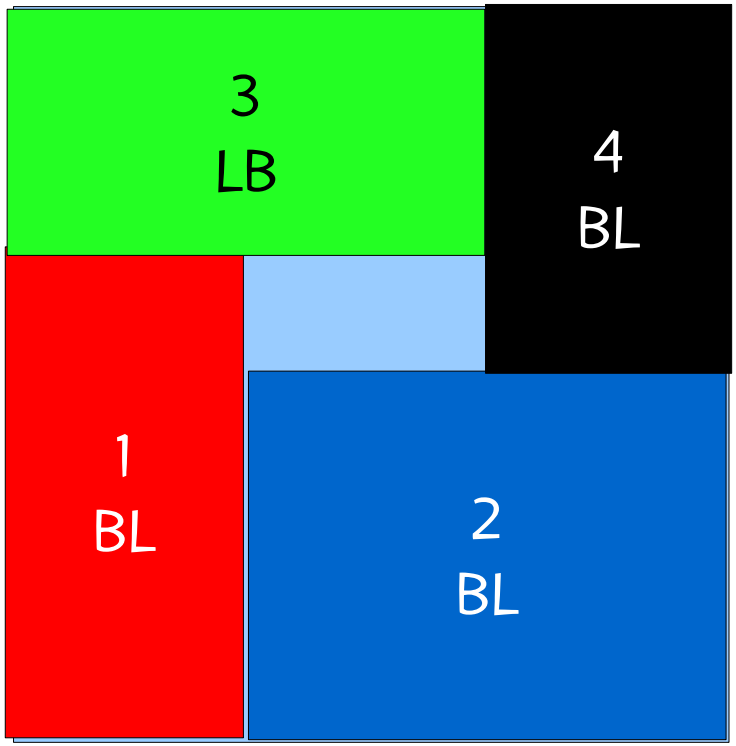
4 does not fit in ERS[1].





4 does fit
in ERS[2].





Optimal solution!

Experimental results

Design

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  - GRASP: greedy randomized adaptive search procedure of Alvarez-Valdes et al. (2005)
  - TABU: tabu search of Alvarez-Valdes et al. (2007)

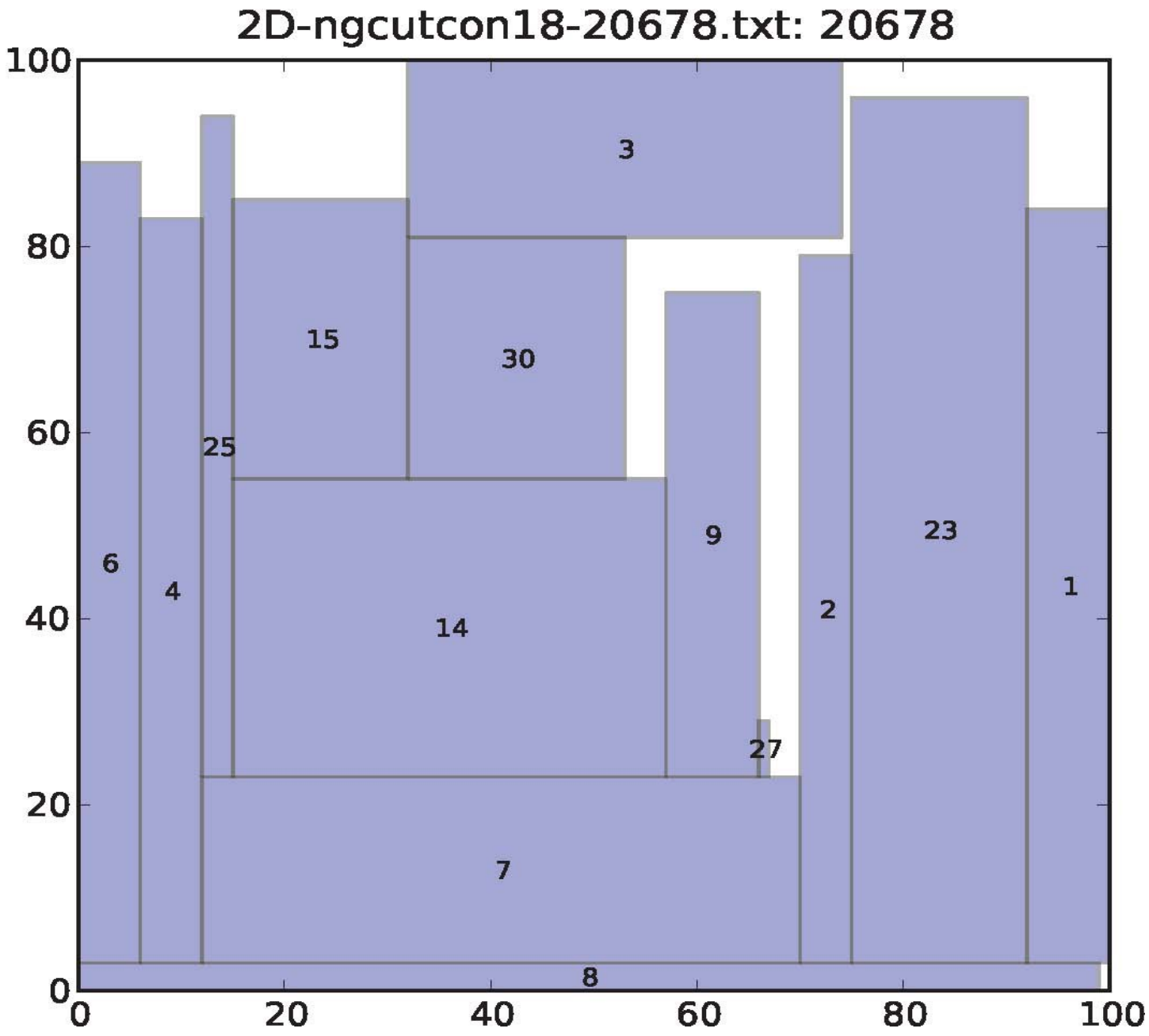
Number of best solutions / total instances

Problem	PH	GA	GRASP	TABU	BRKGA BL-LB-L-4NR
From literature (optimal)	13/21	**21/21**	18/21	**21/21**	**21/21**
Large random*	0/21	0/21	5/21	8/21	**20/21**
Zero-waste			5/31	17/31	**30/31**
Doubly constrained	11/21		12/21	17/21	**19/21**

* For large random: number of best average solutions / total instance classes

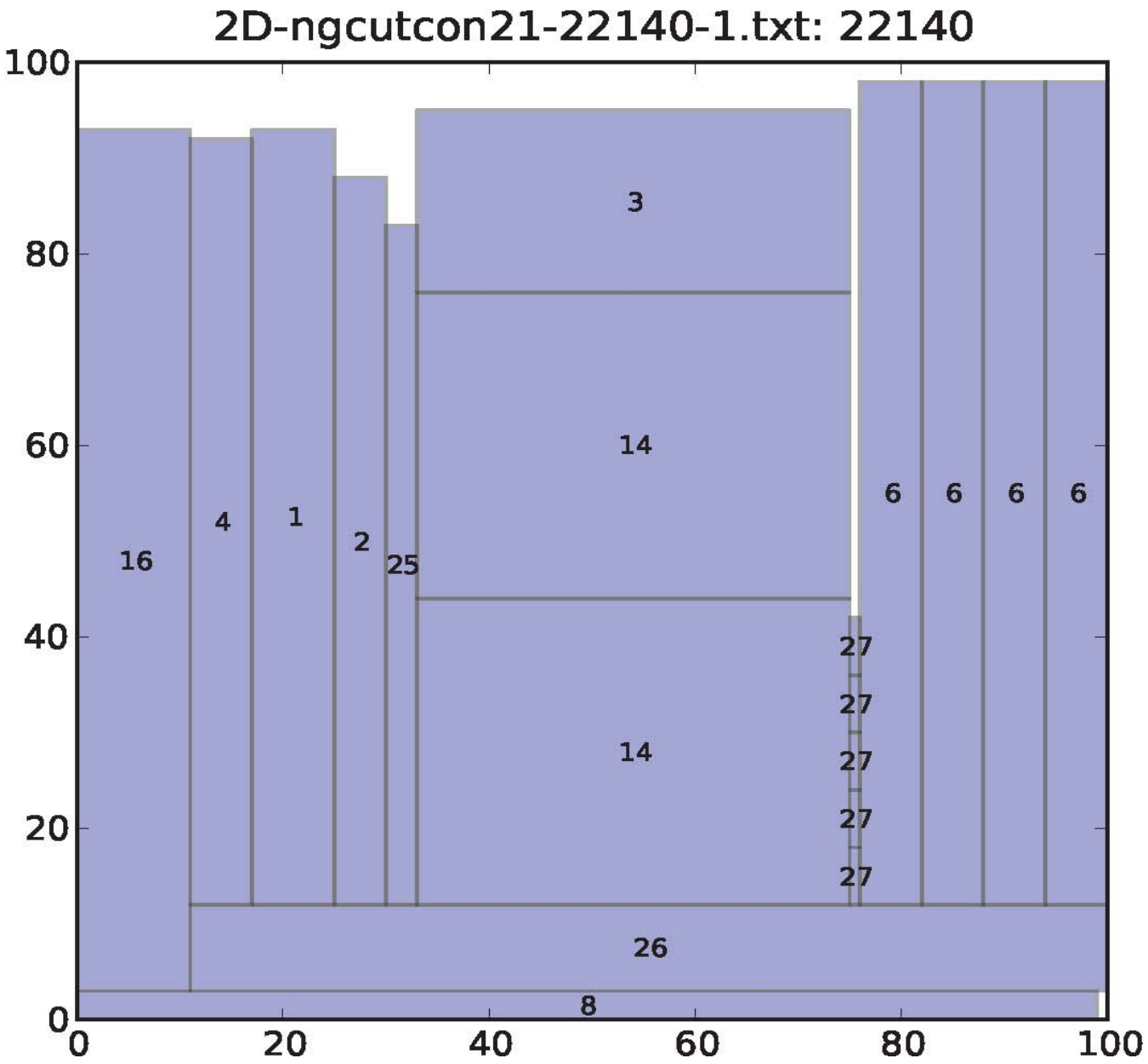
Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

Problem	Min solution time (secs)	Avg solution time (secs)	Max solution time (secs)
From literature (optimal)	0.00	0.05	0.55
Large random	1.78	23.85	72.70
Zero-waste	0.01	82.21	808.03
Doubly constrained	0.00	1.16	16.87



New BKS for a 100 x100 doubly constrained instance of Fekete & Schepers (1997) of value **20678.** Previous best was **19657** by tabu search of Alvarez-Valdes et al., (2007).

30 types
30 rectangles



New BKS for a 100 x 100 doubly constrained instance Fekete & Schepers (1997) of value **22140**.

Previous BKS was **22011** by tabu search of Alvarez-Valdes et al. (2007).

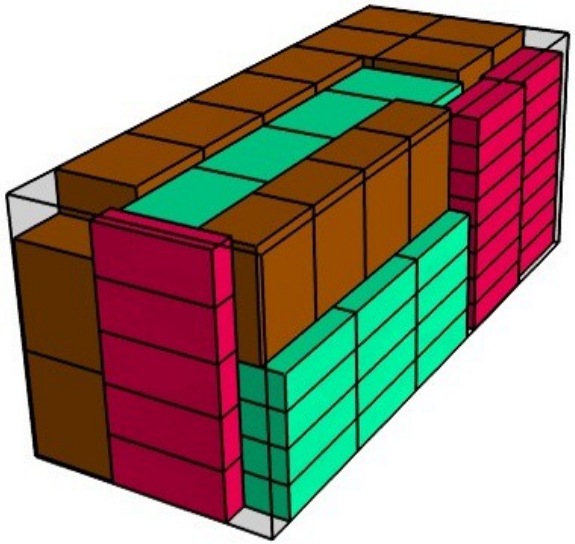
29 types
97 rectangles

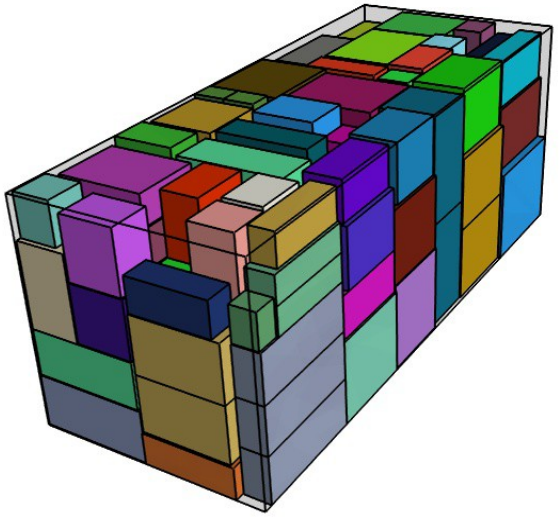
Some remarks

We have extended this to 3D packing:

J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers & Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: http://www.research.att.com/~mgcr/doc/brkga-pack3d.pdf





Packing with a BRKGA

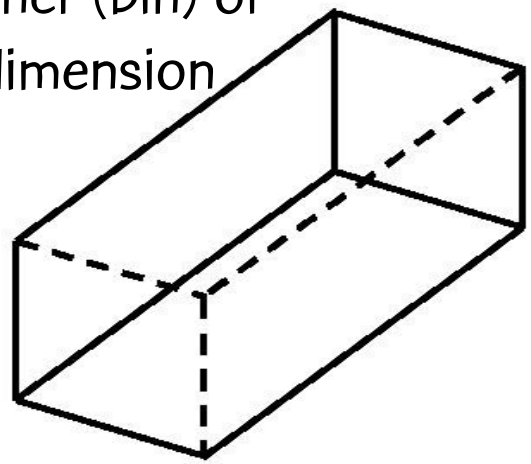
3D bin packing

J.F. Gonçalves and M.G.C.R., "A biased random-key genetic algorithm for a 2D and 3D bin packing problem," AT&T Labs Research Technical Report, 2012 (submitted).

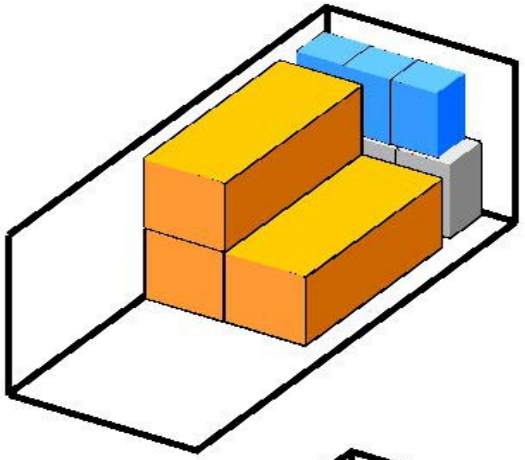
http://www.research.att.com/~mgcr/doc/brkga-binpacking.pdf

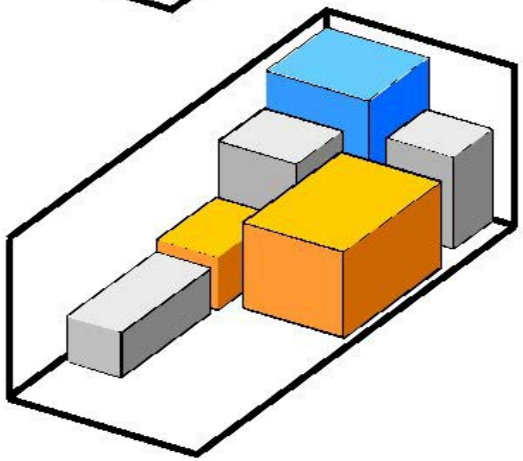
3D bin packing problem

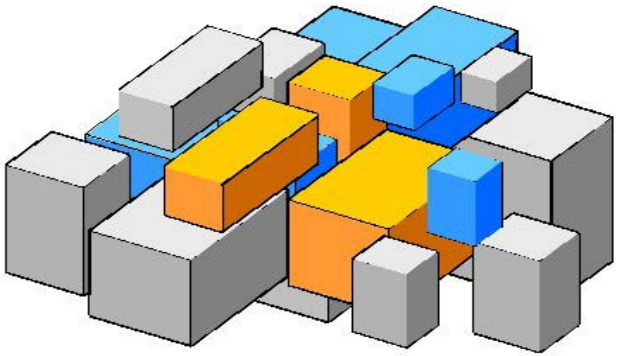
Container (bin) of fixed dimension



Minimize number of containers (bins) needed to pack all boxes





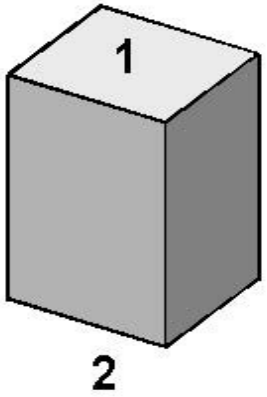


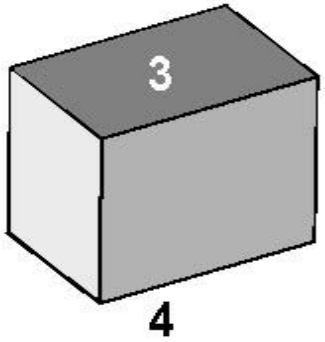
Boxes of different dimensions

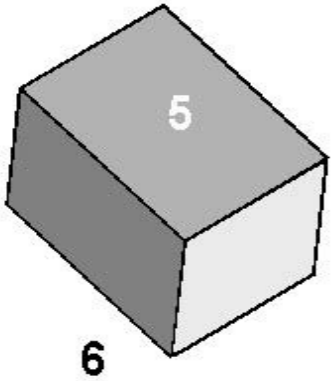
3D bin packing constraints

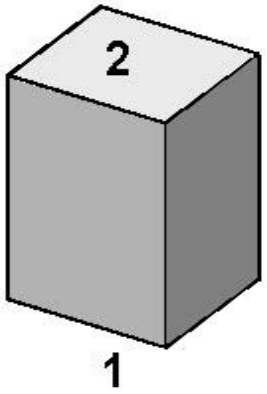
- Each box is placed completely within container
- Boxes do not overlap with each other
- Each box is placed parallel to the side walls of bin
- In some instances, only certain box orientations are allowed (there are at most six possible orientations)

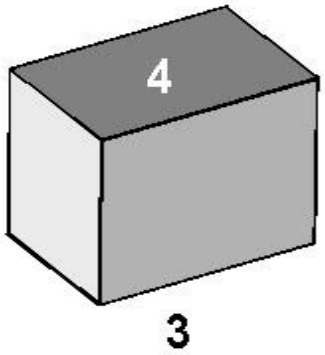
Six possible orientations for each box

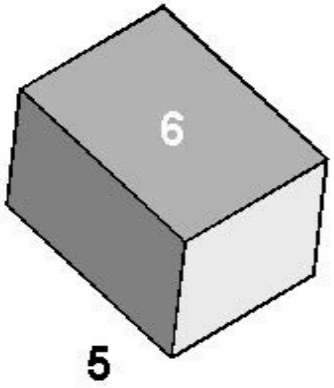






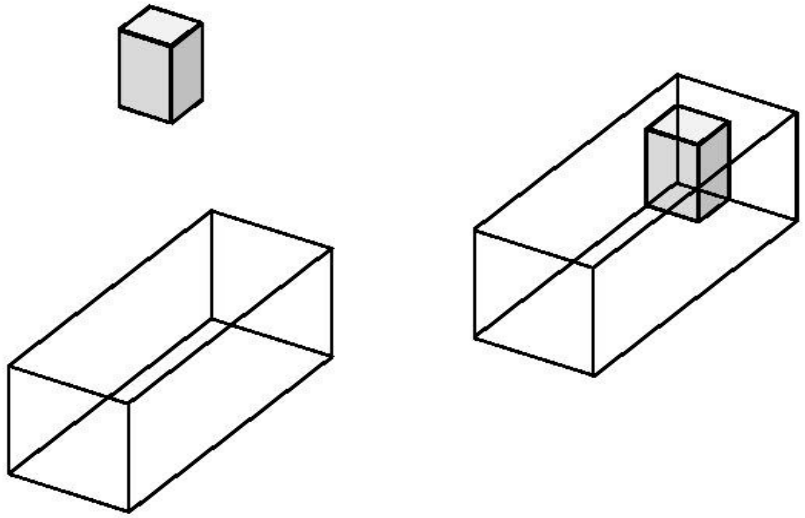


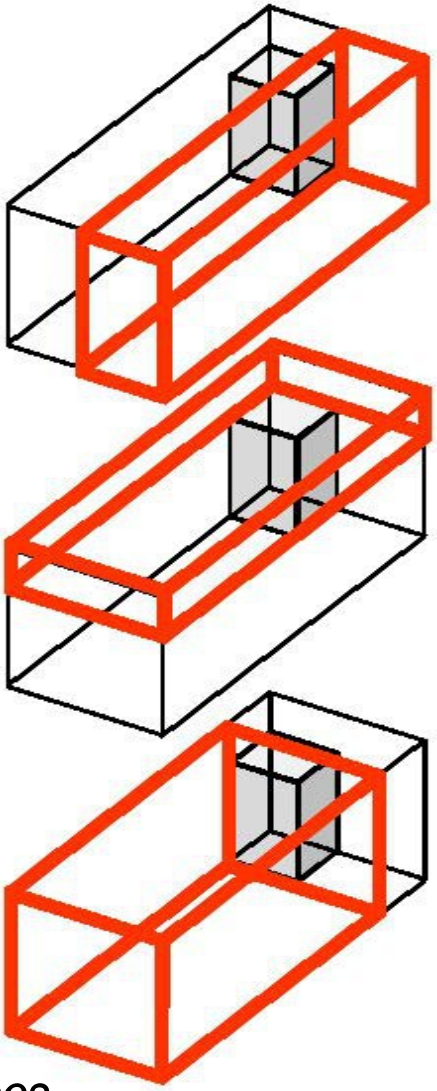




Difference process - DP

(Lai & Chan, 1997)





When box is placed in container ...

use DP to keep track of maximal free spaces

Encoding

Solutions are encoded as vectors of 3n random keys, where n is the number of boxed to be packed.

$$X = (x_1 , x_2 , \ldots, x_n , x_{n+1}, x_{n+2}, \ldots, x_{2n}, x_{2n+1}, x_{2n+2}, \ldots, x_{3n})$$

Box packing sequence | Placement heuristic | Box orientation

Decoding

1) Sort first n keys of X to produce sequence boxes will be packed;
2) Use second n keys of X to determine which placement heuristic to use (back-bottom-left or back-left-bottom):
   - if $x_{n+i} < ½$ then use back-bottom-left to pack i-th box
   - if $x_{n+i} \geq ½$ then use back-left-bottom to pack i-th box
3) Use third n keys of X to determine which of six orientations to use when packing box:
   - $x_{2n+i} \in [0,1/6)$: orientation 1;
   - $x_{2n+i} \in [1/6,2/6)$: orientation 2; ...
   - $x_{2n+i} \in [5/6,1]$: orientation 6.

Decoding

For each box

- scan containers in order they were opened
- use placement heuristic to place box in first container in which box fits with its specified orientation
- if box does not fit in any open container, open new container and place box using placement heuristic with its specified orientation

Fitness function

Instead of using as fitness measure the number of bins (NB)

- use adjusted fitness: aNB
- aNB = NB + (LeastLoad / BinVolume), where
  - LeastLoad is load on least loaded bin
  - BinVolume is volume of bin: H x W x L

Experiment

- Parameters:
  - population size: $p = 30n$
  - size of elite partition: $p_e = .10p$
  - number of of mutans: $p_m = .15p$
  - crossover probability: 0.7
  - stopping criterion: 300 generations

Experiment

- Instances:
  - 320 instances of Martello et al. (2000)
  - generator is available at http://www.diku.dk/~pisinger/codes/html
  - 8 classes
  - 40 instances per class
  - 10 instances for each value of n ∈ {50, 100, 150, 200)

Experiment

- We compare BRKGA with:
  - TS3, the tabu search of Lodi et al. (2002)
  - GLS, the guided local search of Faroe et al. (2003)
  - TS2PACK, the tabu search of Crainic et al. (2009)
  - GRASP, the greedy randomized adaptive search procedure of Parreno et al. (2010)

Class 1 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.5	**13.4**	**13.4**	**13.4**	**13.4**	**13.4**
100	25.1	**26.6**	**26.6**	**26.6**	26.7	26.7
150	34.7	**36.4**	**36.4**	36.7	37.0	37.0
200	48.4	**50.9**	**50.9**	51.2	51.1	51.2
Sum:	120.7	**127.3**	**127.3**	127.9	128.2	128.3

Class 2 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.7	**13.8**	**13.8**	**13.8**	Did not run	
100	24.1	**25.6**	25.7	25.7		
150	35.1	**36.7**	36.9	37.2		
200	47.5	**49.4**	**49.4**	50.1		
Sum:	119.4	**125.5**	125.8	126.8		

Class 3 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	12.3	**13.3**	**13.3**	**13.3**	Did not run	
100	24.7	**25.9**	26.0	26.0		
150	36.0	**37.5**	37.6	37.7		
200	47.8	**49.8**	50.0	50.5		
Sum:	120.8	**126.5**	126.9	127.5		

Class 4 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	28.7	**29.4**	**29.4**	**29.4**	**29.4**	**29.4**
100	57.6	59.0	59.0	59.0	**58.9**	59.0
150	85.2	**86.8**	**86.8**	**86.8**	**86.8**	**86.8**
200	116.3	**118.8**	**118.8**	**118.8**	**118.8**	119.0
Sum:	287.8	294.0	294.0	294.0	**293.9**	294.2

Class 5 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	7.3	**8.3**	**8.3**	8.4	**8.3**	**8.3**
100	12.9	**15.0**	**15.0**	**15.0**	15.2	15.1
150	17.4	**20.0**	20.1	20.4	20.1	20.2
200	24.4	**27.1**	**27.1**	27.6	27.4	27.2
Sum:	62.0	**70.4**	70.5	71.4	71.0	70.8

Class 6 - Bin size: 10 x 10 x 10

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	8.7	**9.8**	**9.8**	9.9	**9.8**	**9.8**
100	17.5	**18.8**	19.0	19.1	19.1	19.1
150	26.9	**29.2**	**29.2**	29.4	**29.2**	29.4
200	35.0	**37.2**	37.4	37.7	37.7	37.7
Sum:	88.1	**95.0**	95.4	96.1	95.8	96.0

Class 7 - Bin size: 40 x 40 x 40

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	6.3	**7.4**	**7.4**	7.5	**7.4**	**7.4**
100	10.9	**12.2**	12.5	12.5	12.3	12.3
150	13.7	**15.2**	16.0	16.1	15.8	15.8
200	21.0	**23.4**	23.5	23.9	23.5	23.5
Sum:	51.9	**58.2**	59.4	60.0	59.0	59.0

Class 8 - Bin size: 100 x 100 x 100

Boxes	LB	BRKGA	GRASP	TS3	TS2PACK	GLS
50	0.8	**9.2**	**9.2**	9.3	**9.2**	**9.2**
100	17.5	18.9	18.9	18.9	**18.8**	18.9
150	21.3	**23.5**	24.1	24.1	23.9	23.9
200	26.7	**29.3**	29.8	30.3	30.0	29.9
Sum:	66.3	**80.9**	82.0	82.6	81.9	81.9

Summary

Class	Bin size	BRKGA	GRASP	TS3	TS2PACK	GLS
1	100^3	**127.3**	**127.3**	127.9	128.2	128.3
2	100^3	**125.5**	125.8	126.8		
3	100^3	**126.5**	126.9	127.5		
4	100^3	294.0	294.0	294.0	**293.9**	294.2
5	100^3	**70.4**	70.5	71.4	71.0	70.8
6	10^3	**95.0**	95.4	96.1	95.8	96.0
7	40^3	**58.2**	59.4	60.0	59.0	59.0
8	100^3	**80.9**	82.0	82.6	81.9	81.9
Sum(rows 1, 4-8):		**725.8**	728.6	732.0	729.8	730.2
Sum(rows 1-8):		**977.8**	981.3	986.3		

Concluding remarks

- Reviewed BRKGA framework
- Applied framework to
  - 2D/3D packing to maximize value packed
  - 2D/3D bin packing to minimize number of bins
- All decoders were simple heuristics
- BRKGA "learned" how to "operate" the heuristics
- In all cases, several new best known solutions were produced

Thanks!

These slides and all of the papers cited in this lecture can be downloaded from my homepage:

http://www.research.att.com/~mgcr