Biased random-key genetic algorithms

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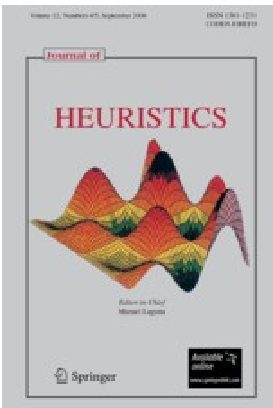
Reference



J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Tech report version:

http://www.research.att.com/~mgcr/doc/srkga.pdf

Summary

- Metaheuristics and basic concepts of genetic algorithms
- Random-key genetic algorithm of Bean (1994)
- Biased random-key genetic algorithms (BRKGA)
  - Encoding / Decoding
  - Initial population
  - Evolutionary mechanisms
  - Problem independent / problem dependent components
  - Multi-start strategy
  - Specifying a BRKGA
  - Application programming interface (API) for BRKGA
- Example of a BRKGA for 2-dim packing
- Brief overview of literature

Metaheuristics

Metaheuristics are heuristics to devise heuristics.

Metaheuristics

Metaheuristics are high level procedures that coordinate simple heuristics, such as local search, to find solutions that are of better quality than those found by the simple heuristics alone.

Metaheuristics

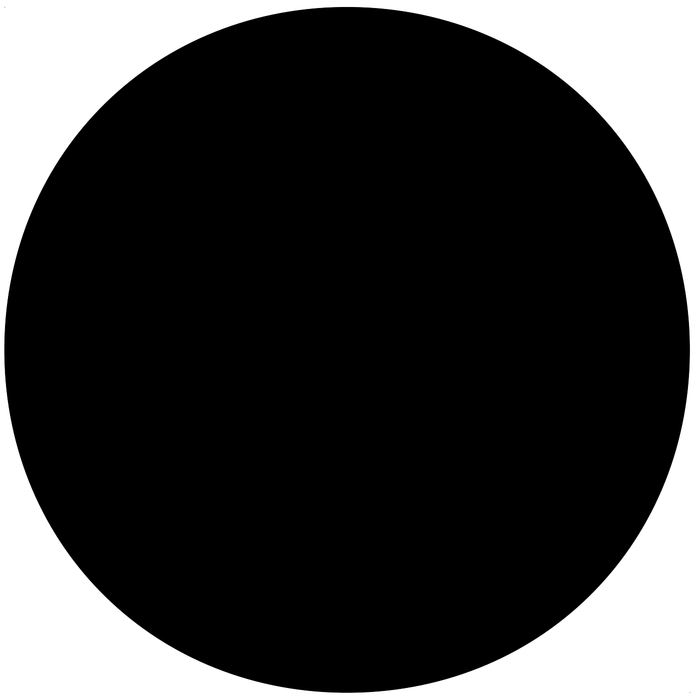
Metaheuristics are high level procedures that coordinate simple heuristics, such as local search, to find solutions that are of better quality than those found by the simple heuristics alone.

Examples: GRASP and C-GRASP, simulated annealing, genetic algorithms, tabu search, scatter search, ant colony optimization, variable neighborhood search, and biased random-key genetic algorithms (BRKGA).

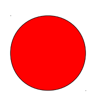
Genetic algorithms

Genetic algorithms

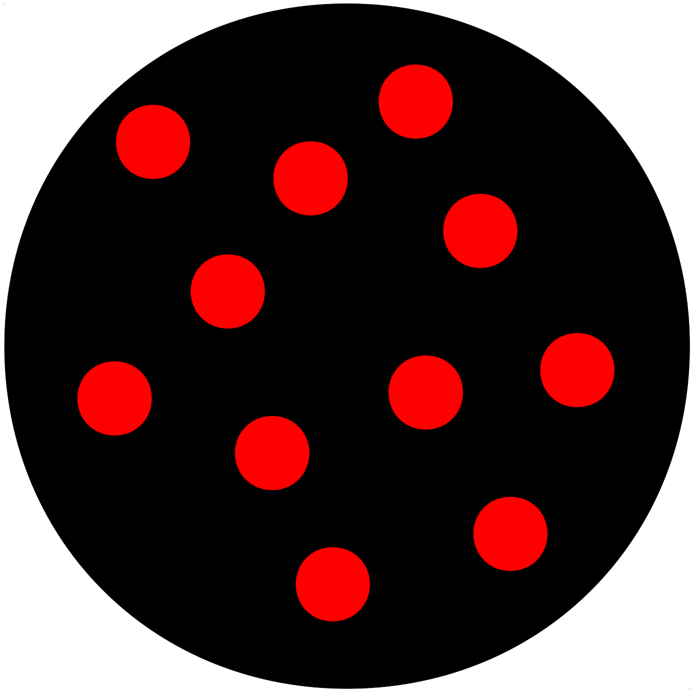
Holland (1975)



Adaptive methods that are used to solve search and optimization problems.

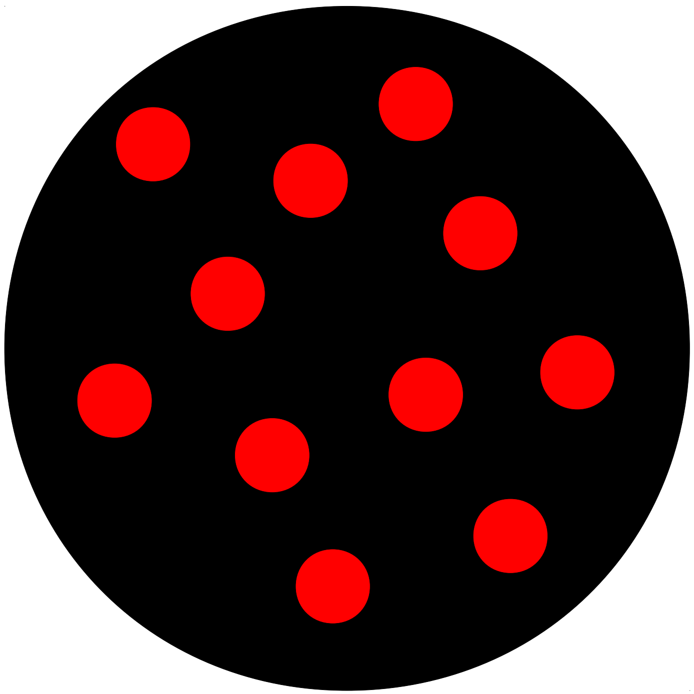
Individual: solution 

Genetic algorithms



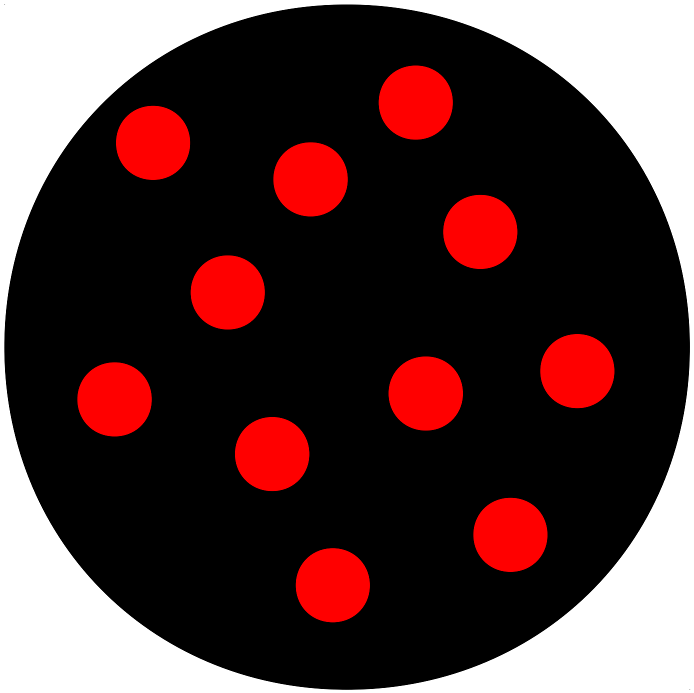
Individual: solution (chromosome = string of genes)
Population: set of fixed number of individuals

Genetic algorithms



Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

Genetic algorithms



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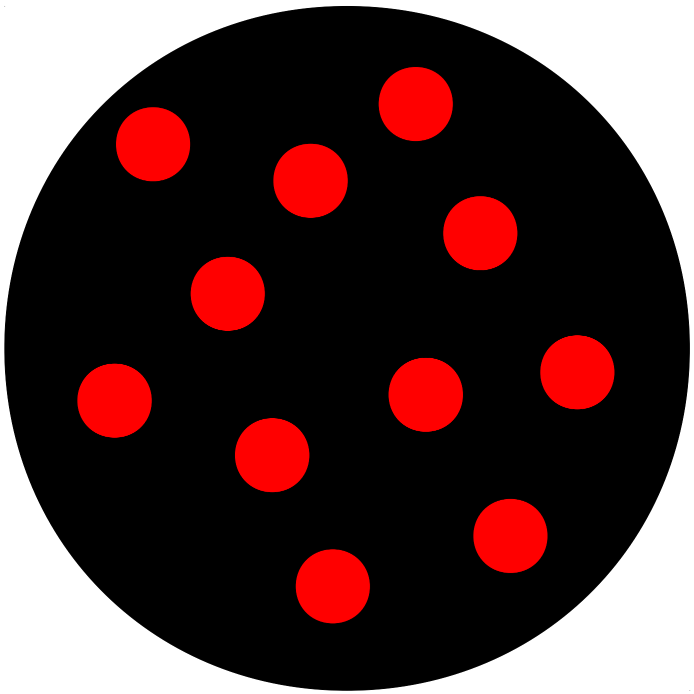
A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

Genetic algorithms

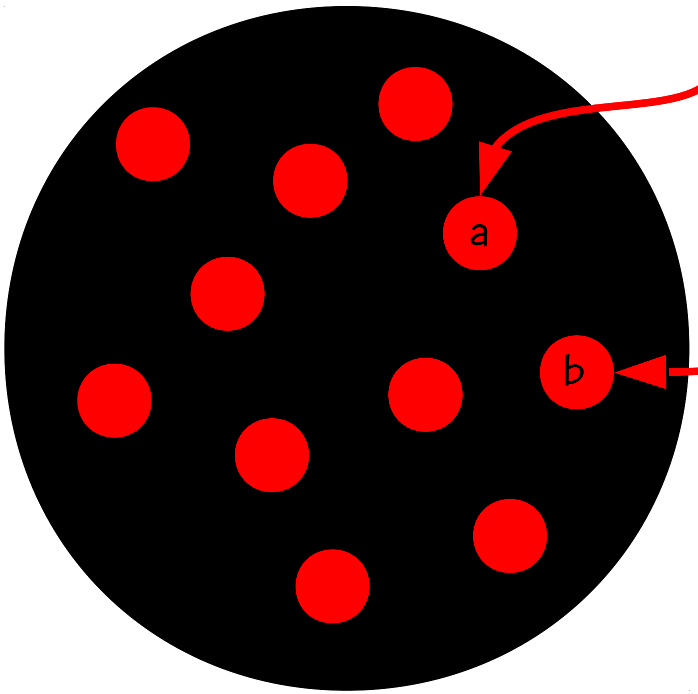


Genetic algorithms evolve population applying Darwin's principle of survival of the fittest.

A series of generations are produced by the algorithm. The most fit individual of the last generation is the solution.

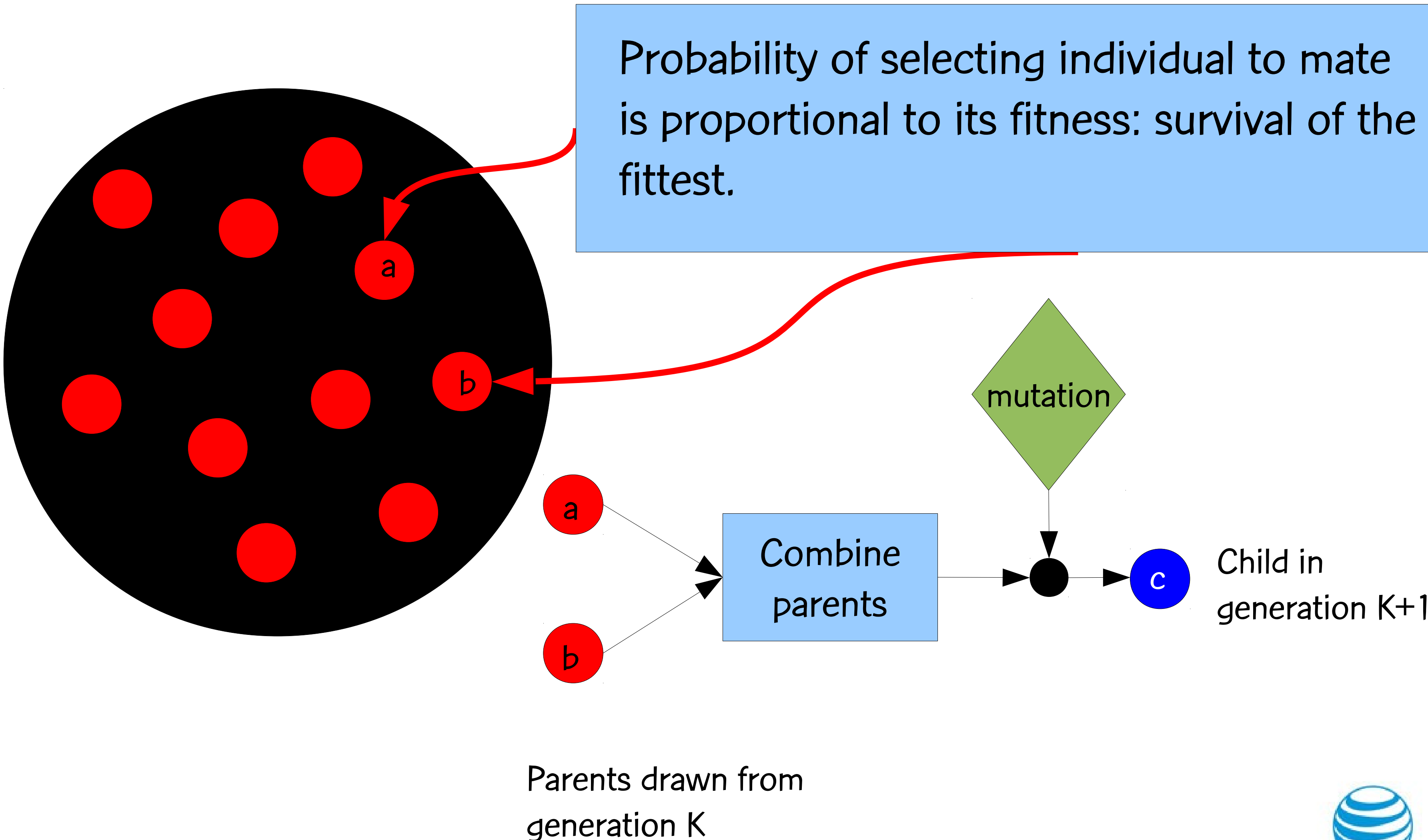
Individuals from one generation are combined to produce offspring that make up next generation.

Genetic algorithms

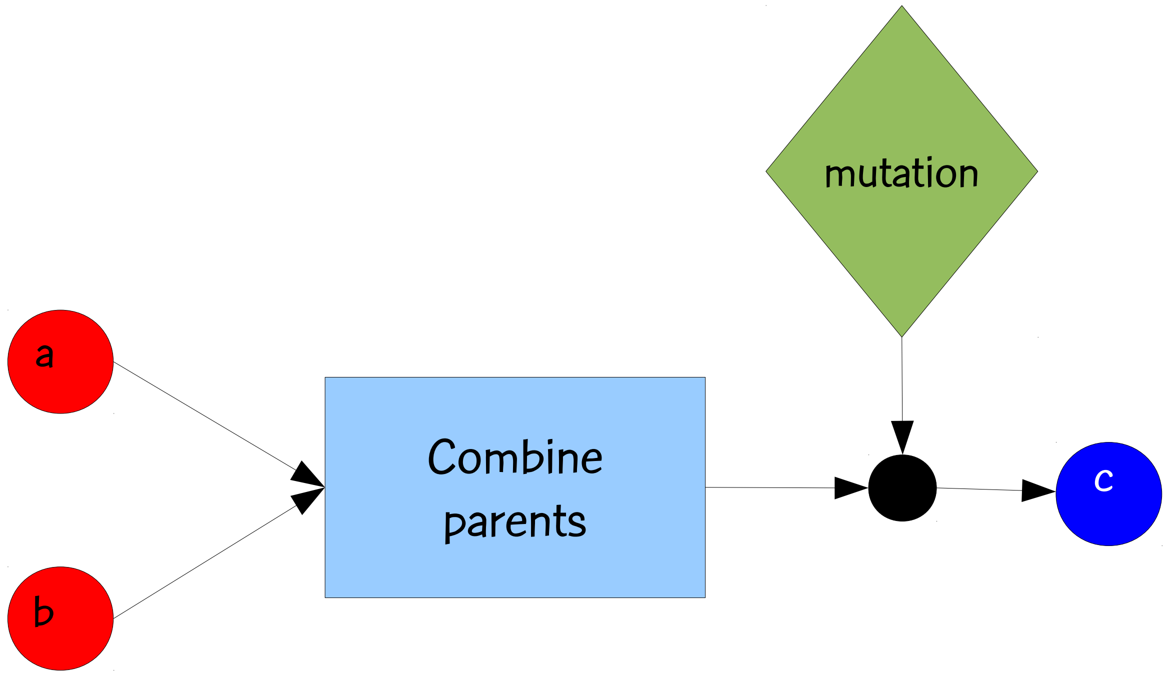


Probability of selecting individual to mate is proportional to its fitness: survival of the fittest.

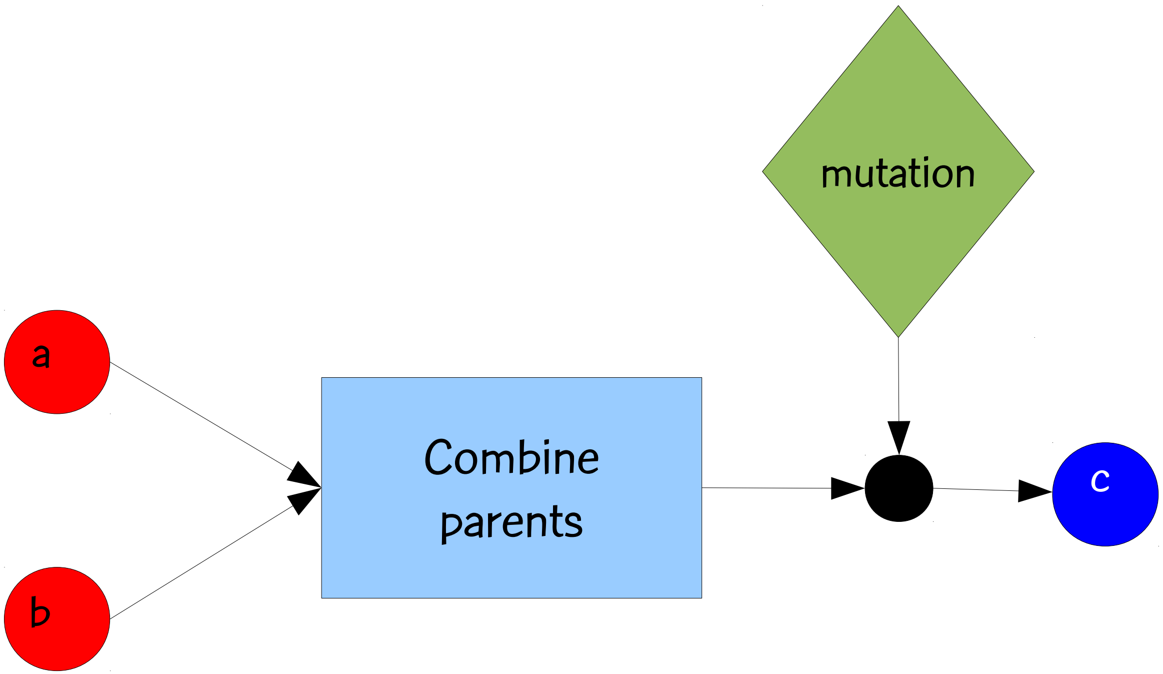
Genetic algorithms



Crossover and mutation



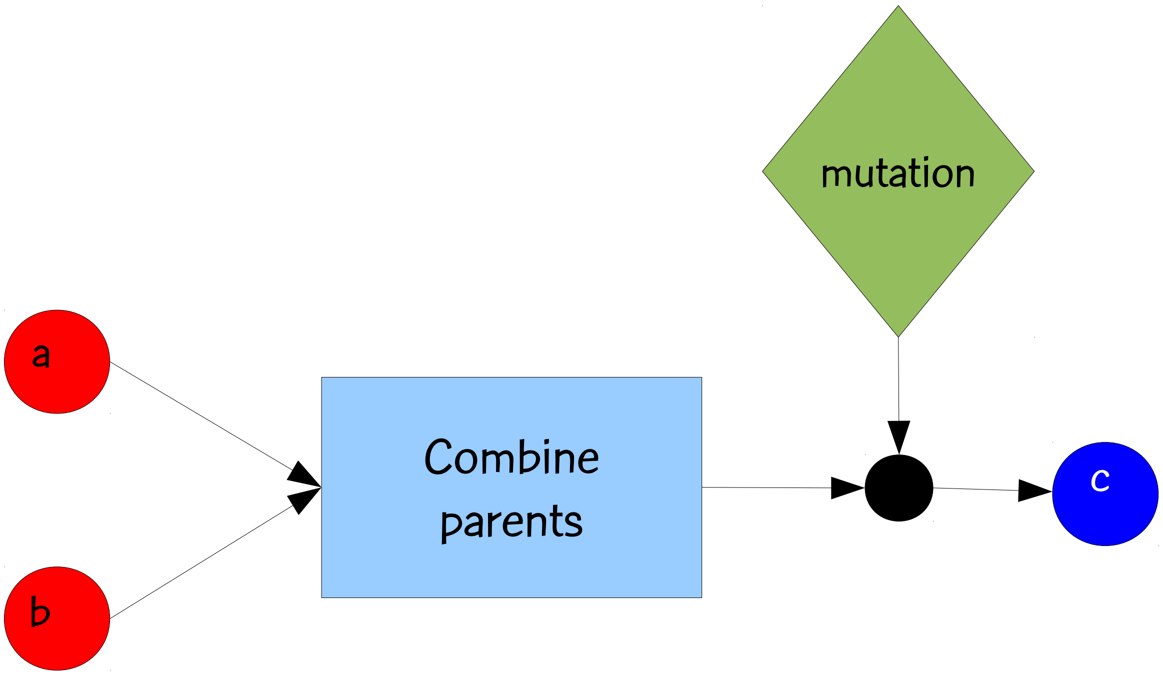
Crossover and mutation



Crossover: Combines parents ... passing along to offspring characteristics of each parent ...

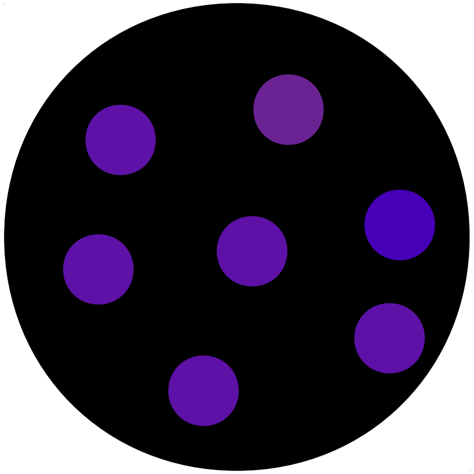
Intensification of search

Crossover and mutation

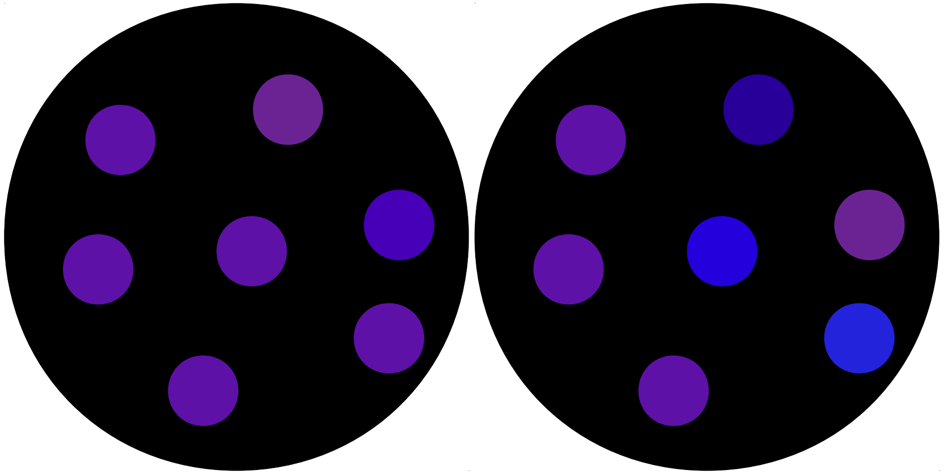


Mutation: Randomly changes chromosome of offspring ...
Driver of evolutionary process ...
Diversification of search

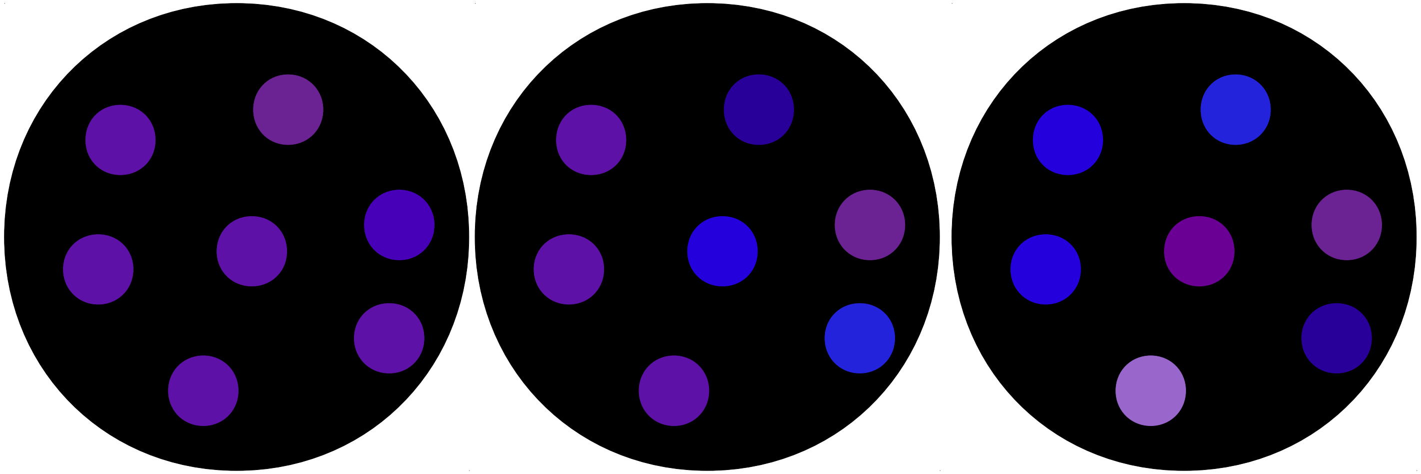
Evolution of solutions



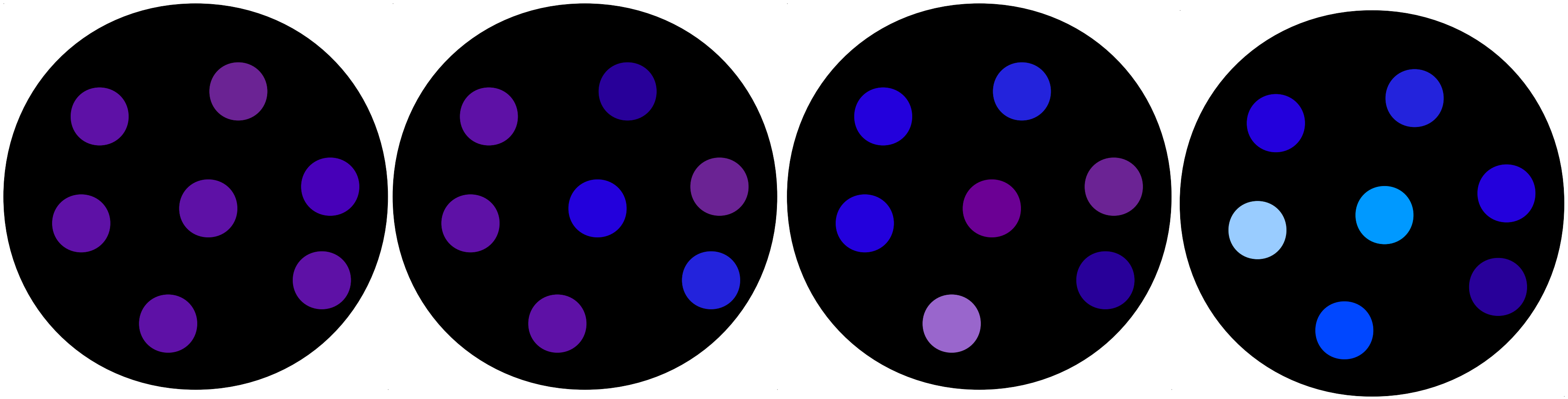
Evolution of solutions



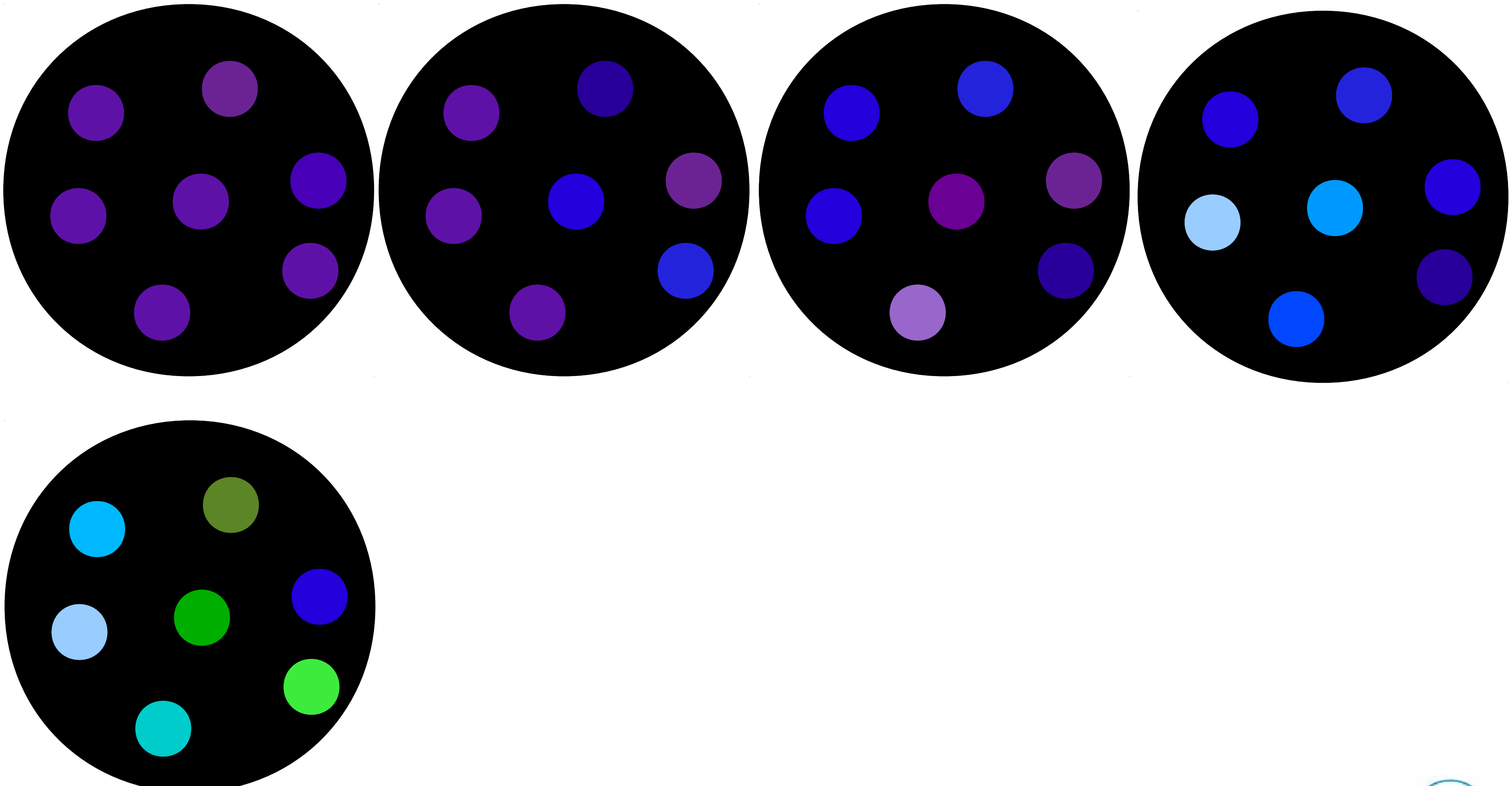
Evolution of solutions



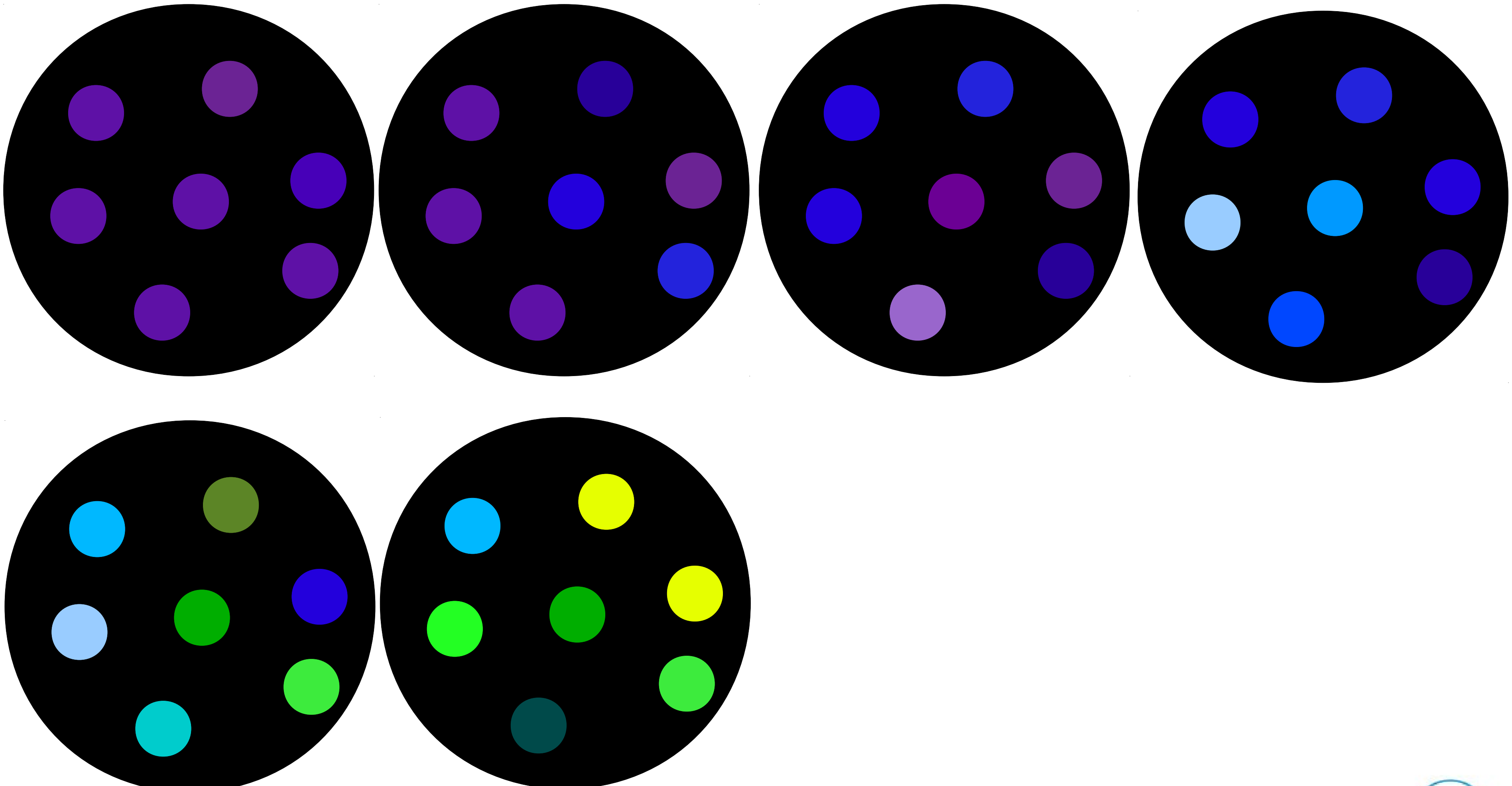
Evolution of solutions



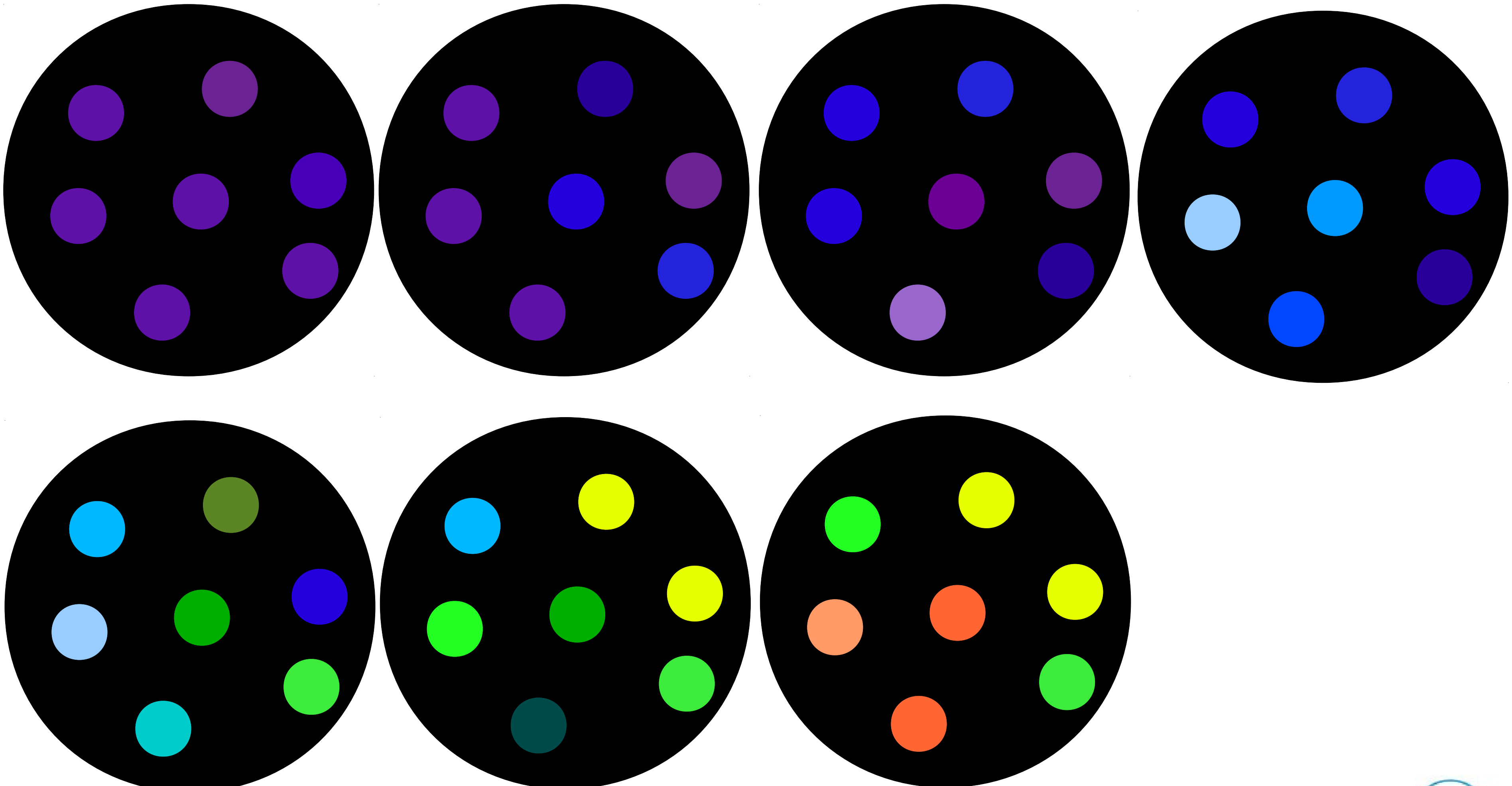
Evolution of solutions



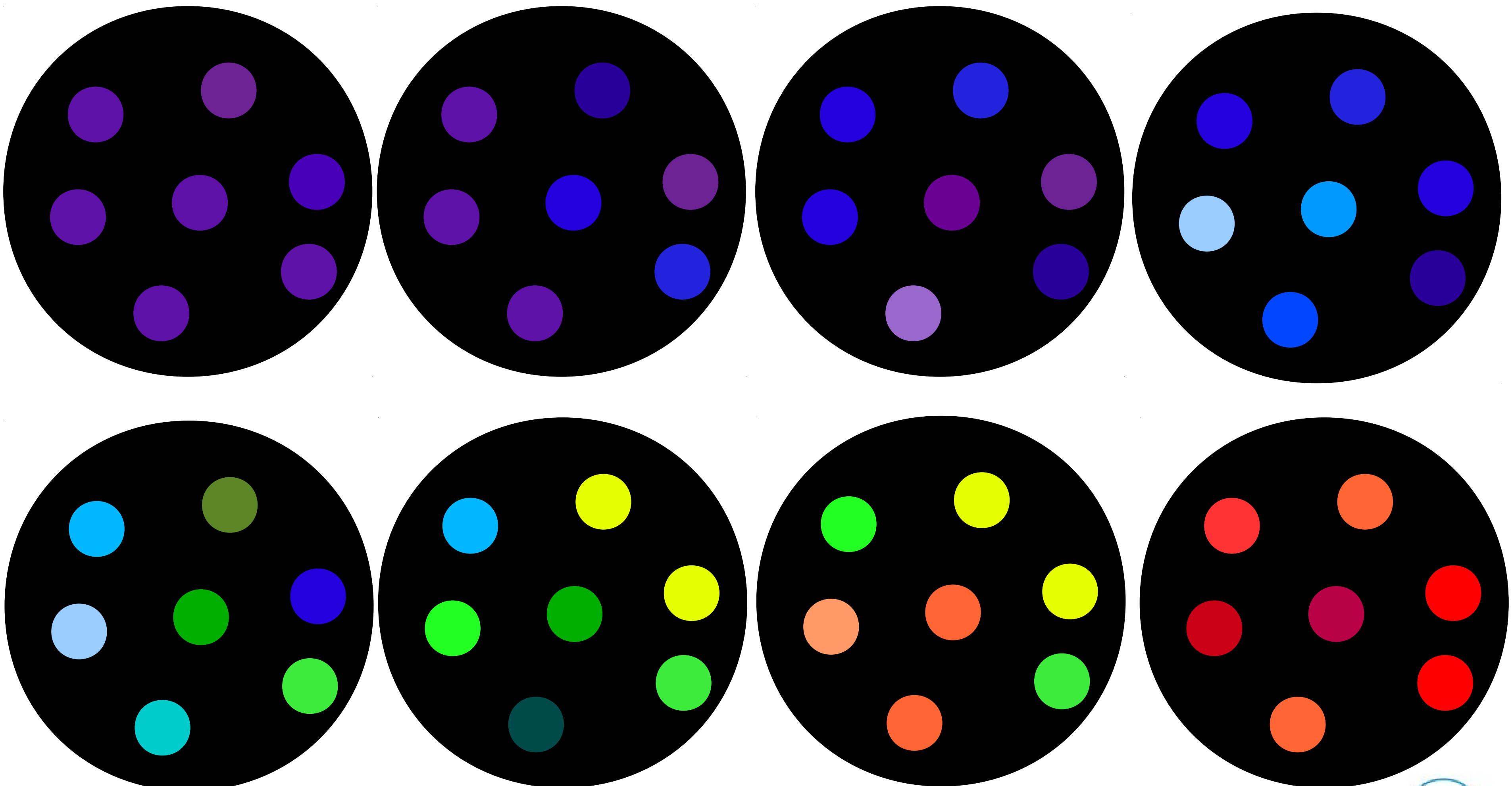
Evolution of solutions



Evolution of solutions



Evolution of solutions



Encoding solutions with random keys

Encoding with random keys

- A random key is a real random number in the continuous interval [0,1).

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- Solutions of optimization problems can be encoded by random keys.

Encoding with random keys

- A random key is a real random number in the continuous interval [0,1).
- A vector X of random keys, or simply random keys, is an array of n random keys.
- Solutions of optimization problems can be encoded by random keys.
- A decoder is a deterministic algorithm that takes a vector of random keys as input and outputs a feasible solution of the optimization problem.

Encoding with random keys: Sequencing

Encoding

[1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658]

Encoding with random keys: Sequencing

Encoding

[1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658]

Decode by sorting vector of random keys

[1, 2, 4, 5, 3]

X = [0.099, 0.216, 0.368, 0.658, 0.802]

Encoding with random keys: Sequencing

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658]

encodes the sequence: 1 – 2 – 4 – 5 – 3

Encoding with random keys: Subset selection (select 3 of 5 elements)

Encoding

$$[\quad 1, \quad 2, \quad 3, \quad 4, \quad 5\ ]$$

$$X = [\ 0.099,\ 0.216,\ 0.802,\ 0.368,\ 0.658\ ]$$

Encoding with random keys: Subset selection (select 3 of 5 elements)

Encoding

[1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658]

Decode by sorting vector of random keys

[1, 2, 4, 5, 3]

X = [0.099, 0.216, 0.368, 0.658, 0.802]

Encoding with random keys: Subset selection (select 3 of 5 elements)

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658]

encodes the subset: {1, 2, 4 }

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Encoding with random keys: Assigning integer weights ∈ [0,10] to a subset of 3 of 5 elements

Encoding

$$[\quad 1,\quad 2,\quad 3,\quad 4,\quad 5 \mid 1,\quad 2,\quad 3,\quad 4,\quad 5\ ]$$

$$X = [\ 0.099, 0.216, 0.802, 0.368, 0.658 \mid 0.4634, 0.5611, 0.2752, 0.4874, 0.0348\ ]$$

Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

Encoding

[1, 2, 3, 4, 5 | 1, 2, 3, 4, 5]

X = [0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]

Decode by sorting the first 5 keys and assign as the weight the value $W_i =$ **floor** $[\, 10\, X_{5+i} \,] + 1$ to the 3 elements with smallest keys X_i, for $i = 1,\ldots,5$.

Encoding with random keys: Assigning integer weights $\in [0,10]$ to a subset of 3 of 5 elements

Therefore, the vector of random keys:

X = [0.099, 0.216, 0.802, 0.368, 0.658 | 0.4634, 0.5611, 0.2752, 0.4874, 0.0348]

encodes the weight vector W = (5,6,–,5,–)

Genetic algorithms and random keys

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.

GAs and random keys

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- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).

S = (0.25, 0.19, 0.67, 0.05, 0.89)
s(1) s(2) s(3) s(4) s(5)

GAs and random keys

- Introduced by Bean (1994) for sequencing problems.
- Individuals are strings of real-valued numbers (random keys) in the interval [0,1).
- Sorting random keys results in a sequencing order.

$$S = (\ 0.25,\ 0.19,\ 0.67,\ 0.05,\ 0.89\ )$$
$$\quad s(1)\quad s(2)\quad s(3)\quad s(4)\quad s(5)$$

$$S' = (\ 0.05,\ 0.19,\ 0.25,\ 0.67,\ 0.89\ )$$
$$\quad s(4)\quad s(2)\quad s(1)\quad s(3)\quad s(5)$$

Sequence: 4 – 2 – 1 – 3 – 5

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)

a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)

GAs and random keys

- Mating is done using parametrized uniform crossover (Spears & DeJong , 1990)
- For each gene, flip a biased coin to choose which parent passes the allele (key, or value of gene) to the child.

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c = ( )

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c = (0.25)

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a = (0.25, 0.19, 0.67, 0.05, 0.89)
b = (0.63, 0.90, 0.76, 0.93, 0.08)
c = (0.25, 0.90)

GAs and random keys

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a = (0.25, 0.19, 0.67, 0.05, 0.89)
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c = (0.25, 0.90, 0.76)

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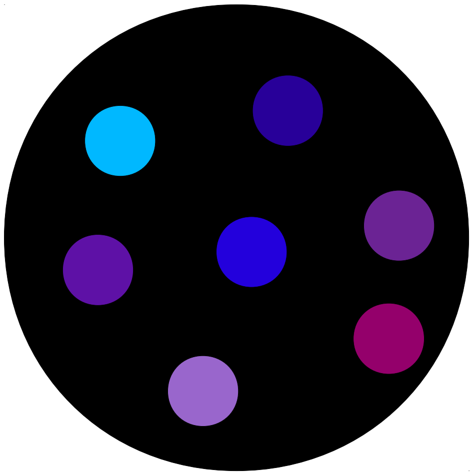
b = (0.63, 0.90, 0.76, 0.93, 0.08)
c = (0.25, 0.90, 0.76, 0.05, 0.89)

If every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

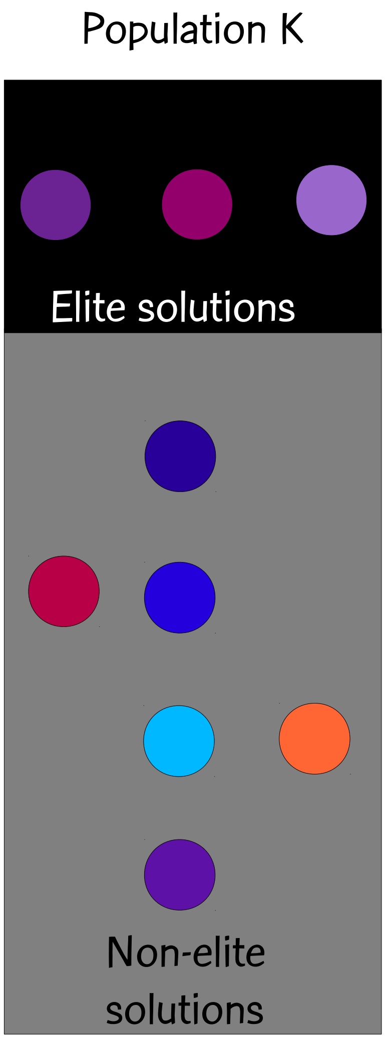
GAs and random keys

Initial population is made up of P random-key vectors, each with N keys, each having a value generated uniformly at random in the interval [0,1).



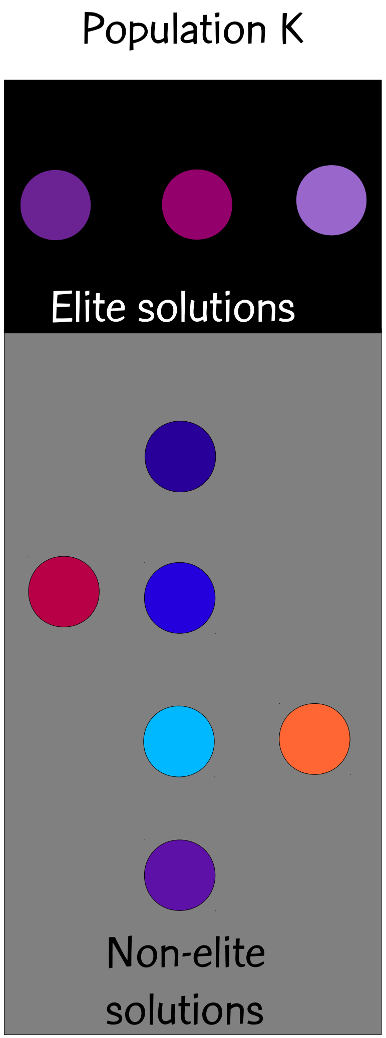
GAs and random keys

At the K-th generation, compute the cost of each solution ...



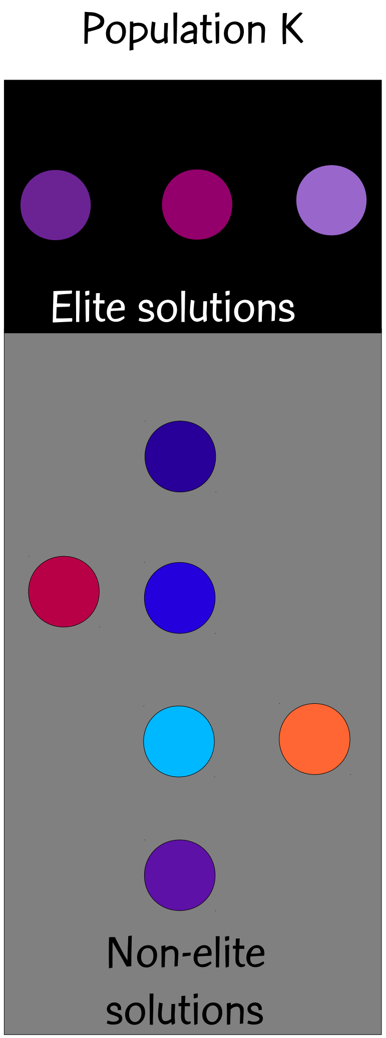
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets:



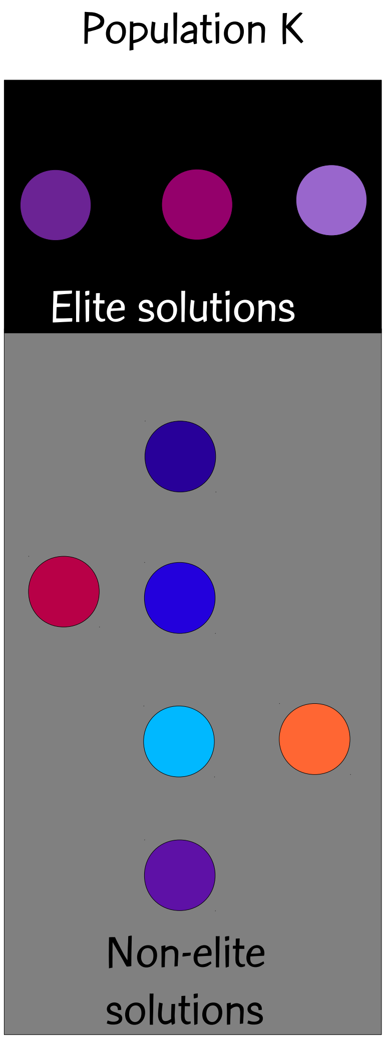
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions.



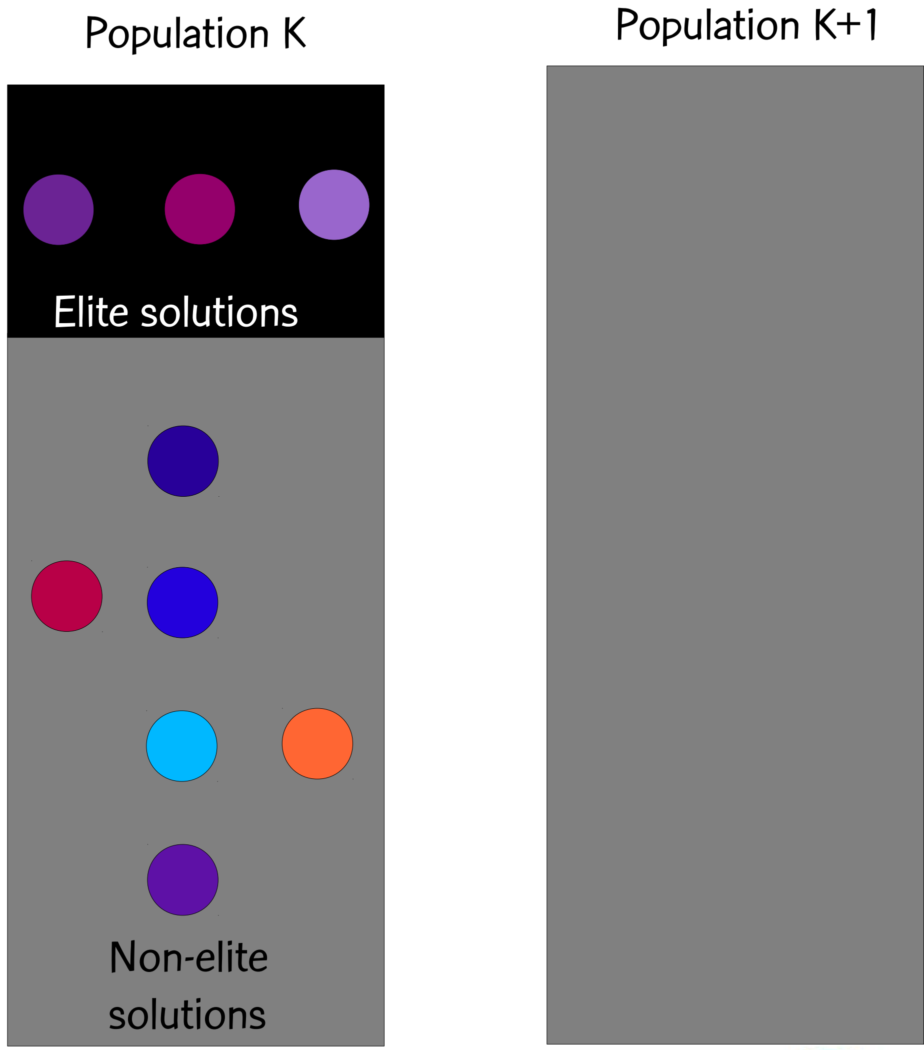
GAs and random keys

At the K-th generation, compute the cost of each solution and partition the solutions into two sets: elite solutions and non-elite solutions. Elite set should be smaller of the two sets and contain best solutions.



GAs and random keys

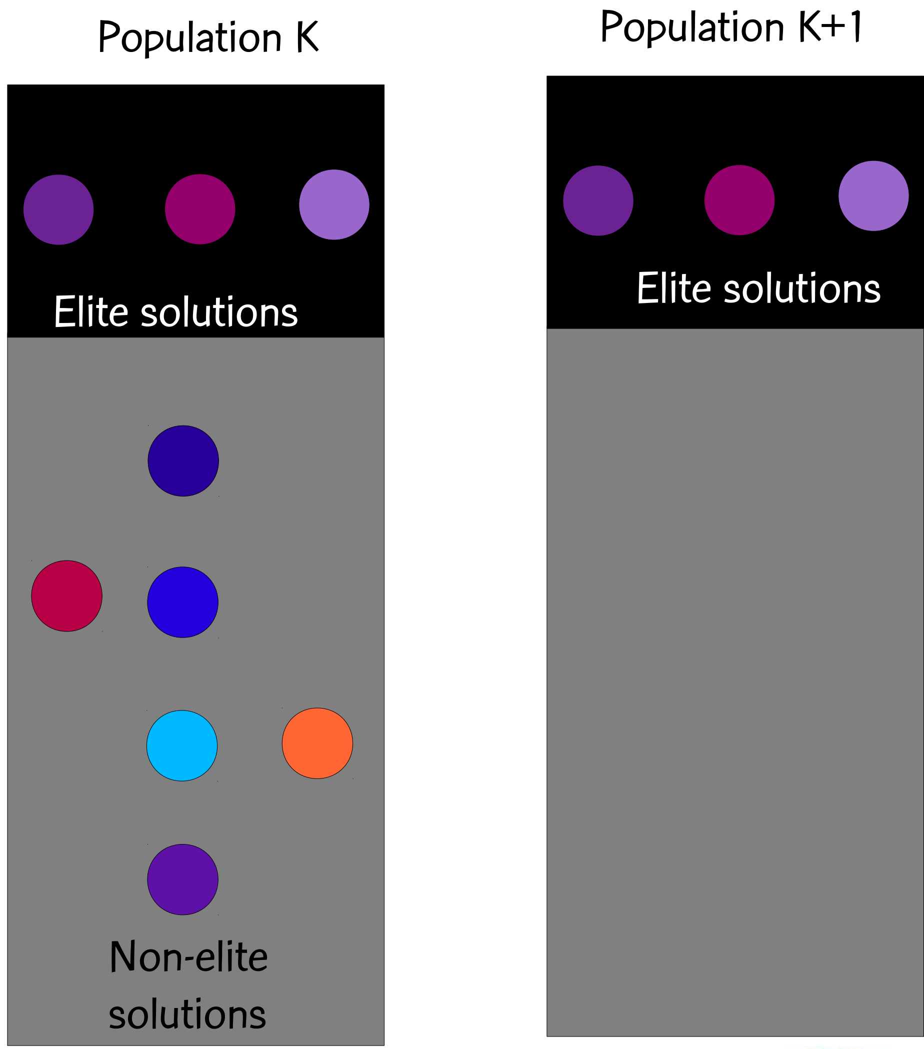
Evolutionary dynamics



GAs and random keys

Evolutionary dynamics

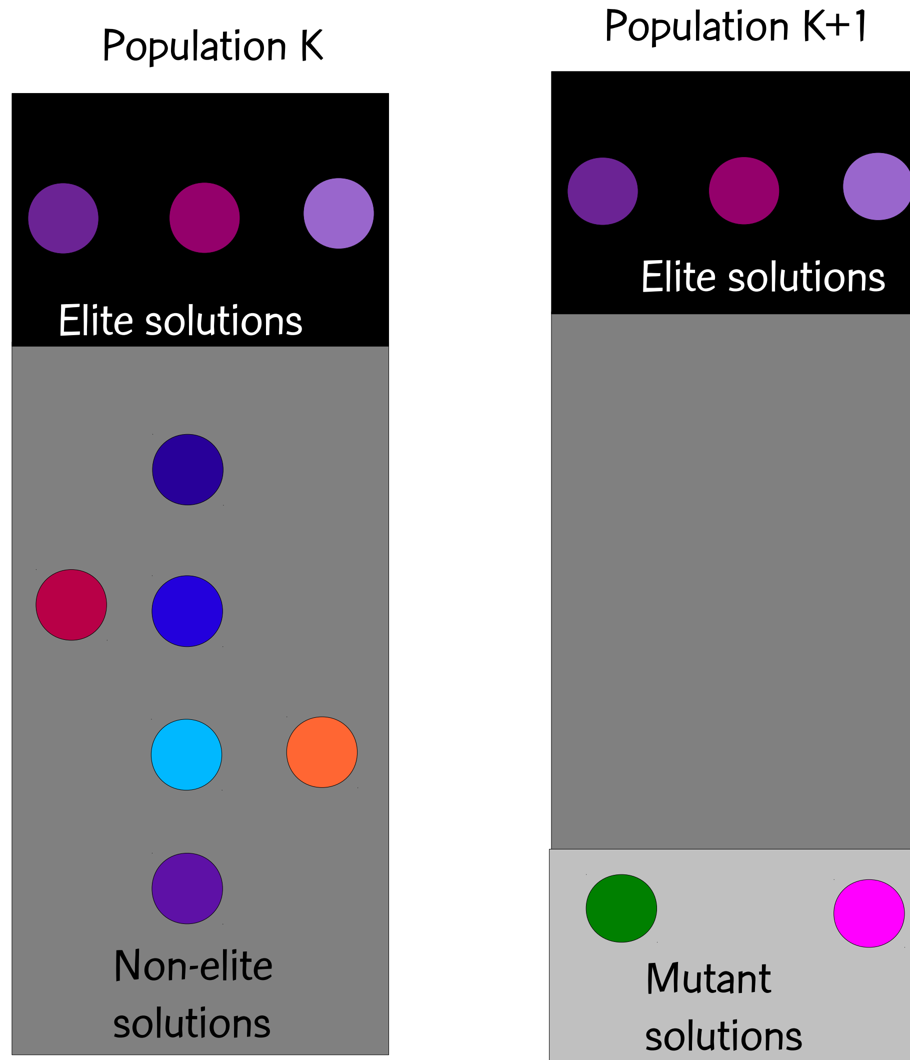
– Copy elite solutions from population K to population K+1



GAs and random keys

Evolutionary dynamics

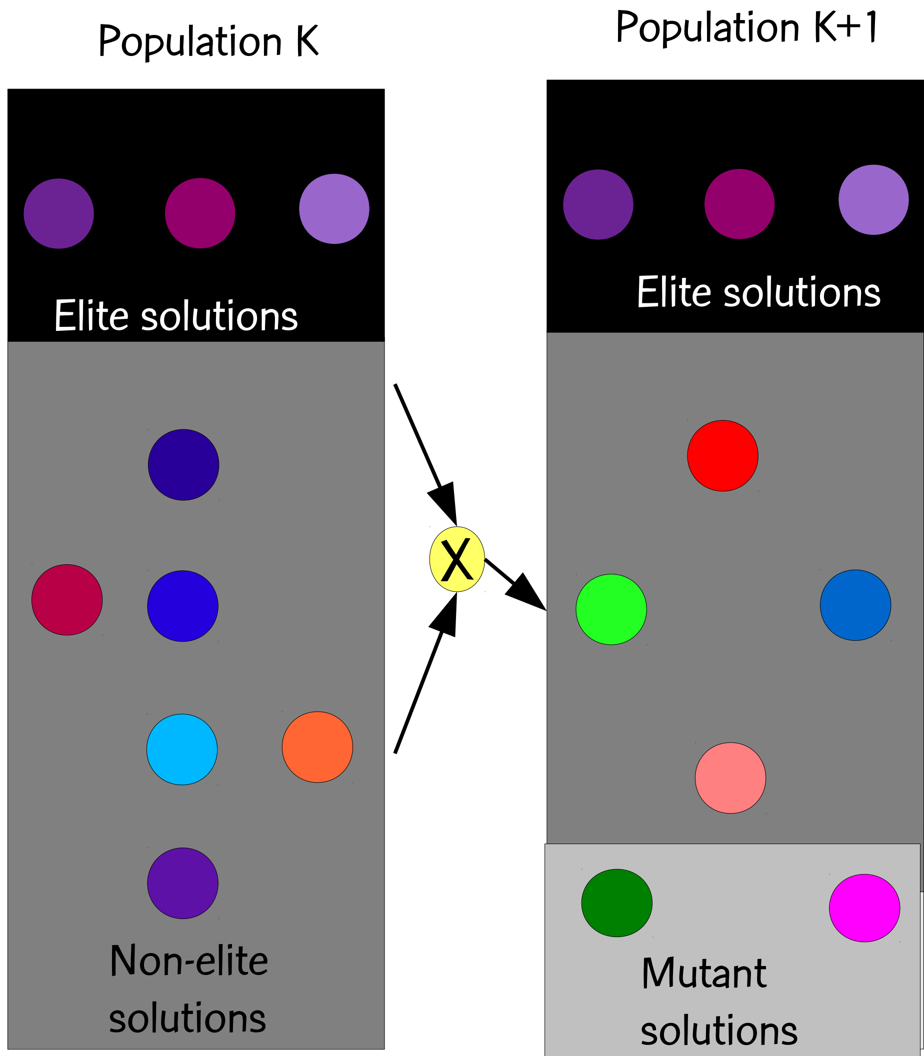
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1



GAs and random keys

Evolutionary dynamics

- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.



Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).

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Biased random key genetic algorithm

- A biased random key genetic algorithm (BRKGA) is a random key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parametrized uniform crossover is applied.

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

BRKGA

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

How RKGA & BRKGA differ

RKGA

both parents chosen at random from entire population

either parent can be parent A in parametrized uniform crossover

BRKGA

both parents chosen at random but one parent chosen from population of elite solutions

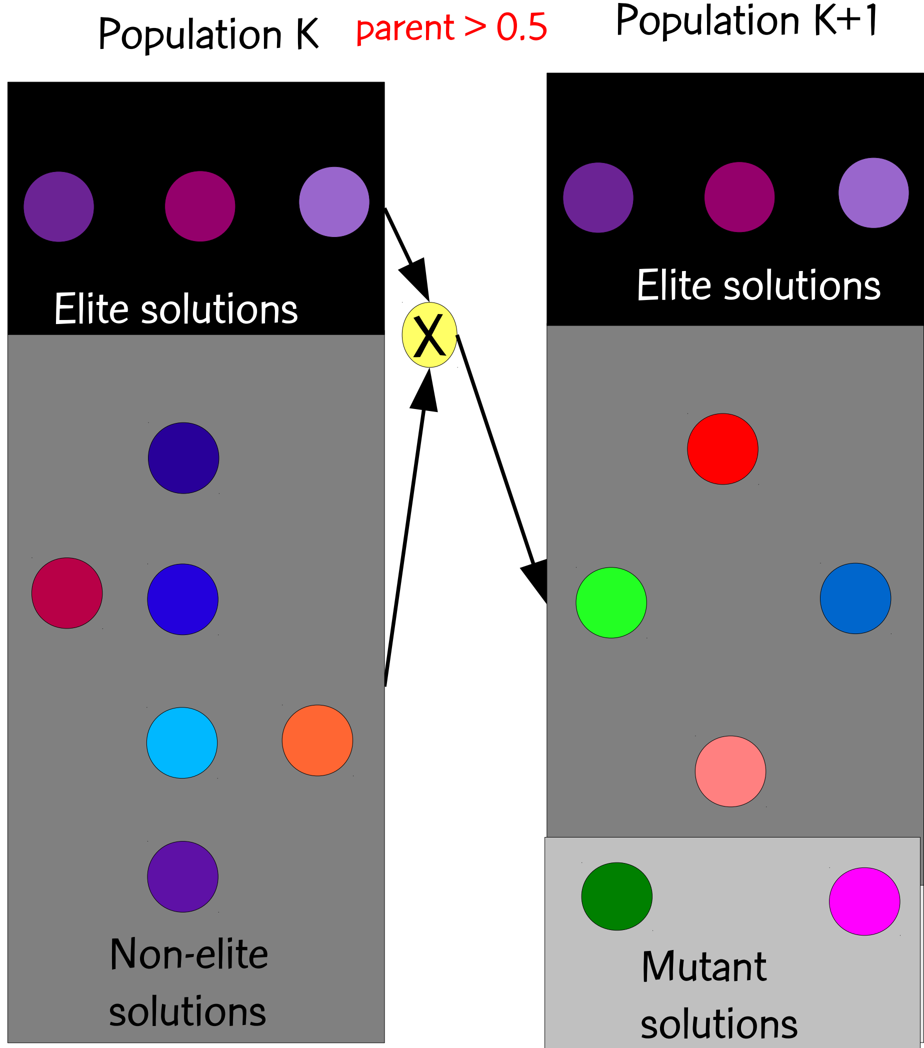
best fit parent is parent A in parametrized uniform crossover

Biased random key GA

Evolutionary dynamics

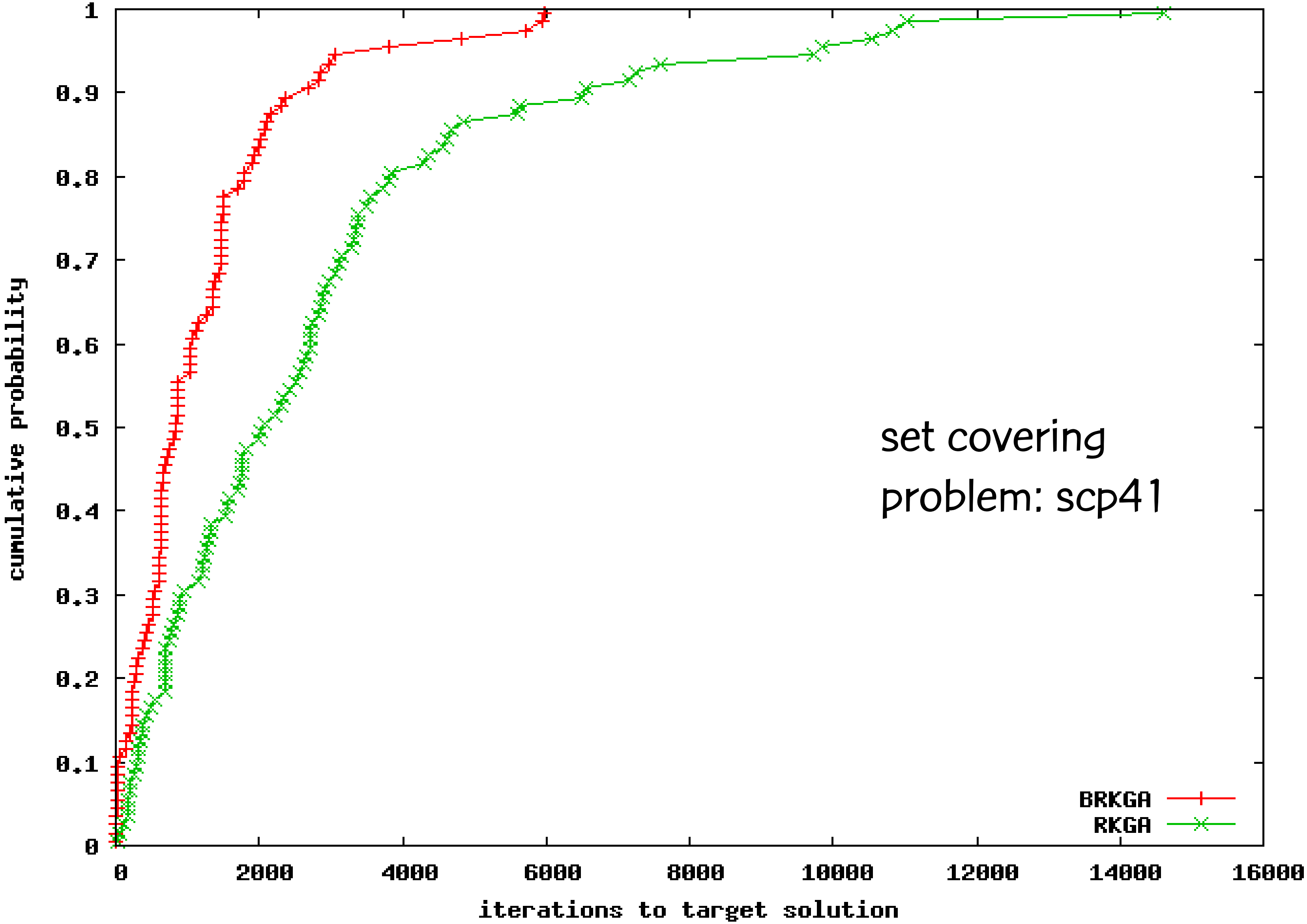
- Copy elite solutions from population K to population K+1
- Add R random solutions (mutants) to population K+1
- While K+1-th population < P
  - RANDOM-KEY GA: Use any two solutions in population K to produce child in population K+1. Mates are chosen at random.
  - BIASED RANDOM-KEY GA: Mate elite solution with other solution of population K to produce child in population K+1. Mates are chosen at random.

BRKGA: Probability child inherits key of elite parent > 0.5

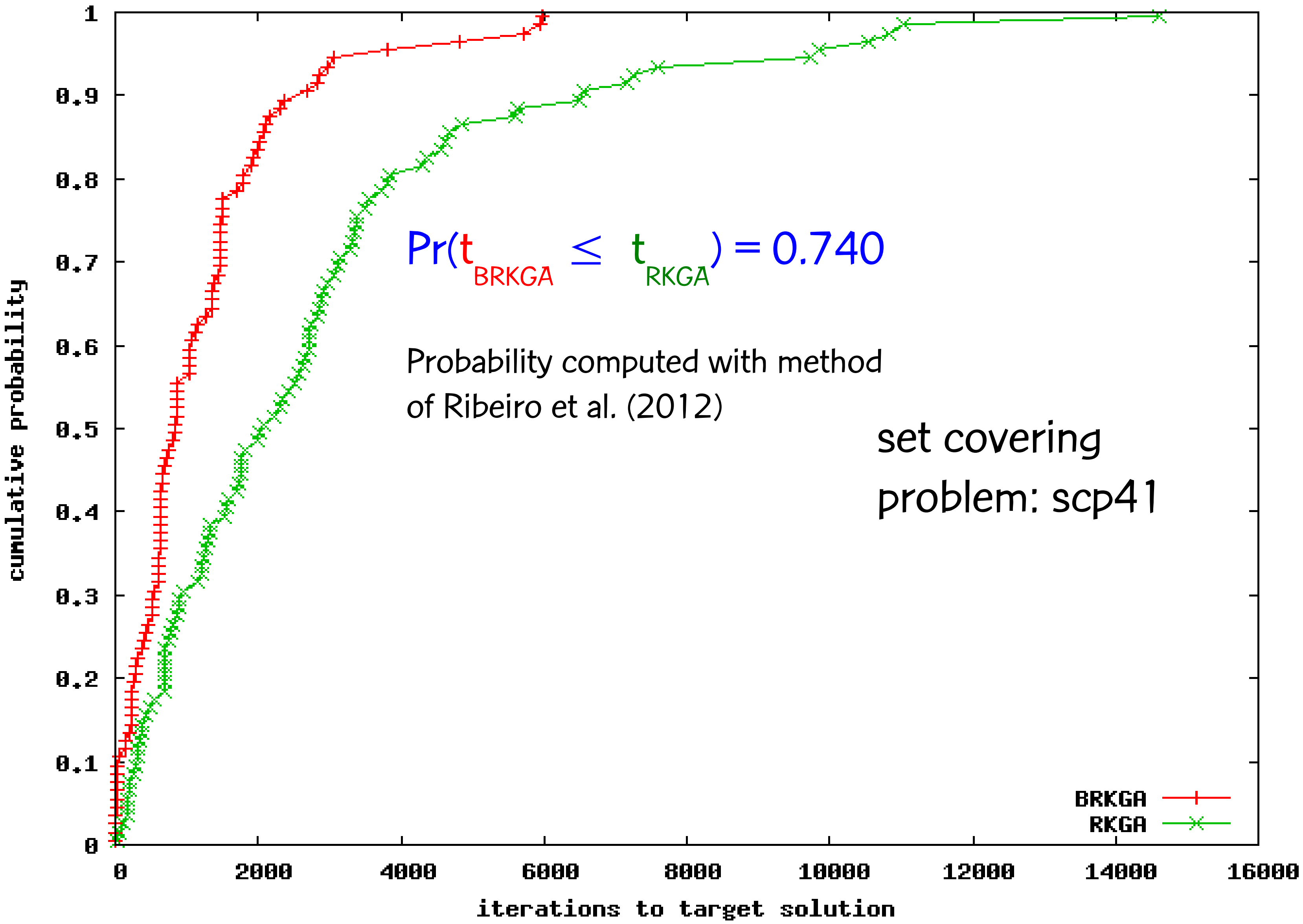


Paper comparing BRKGA and Bean's Method

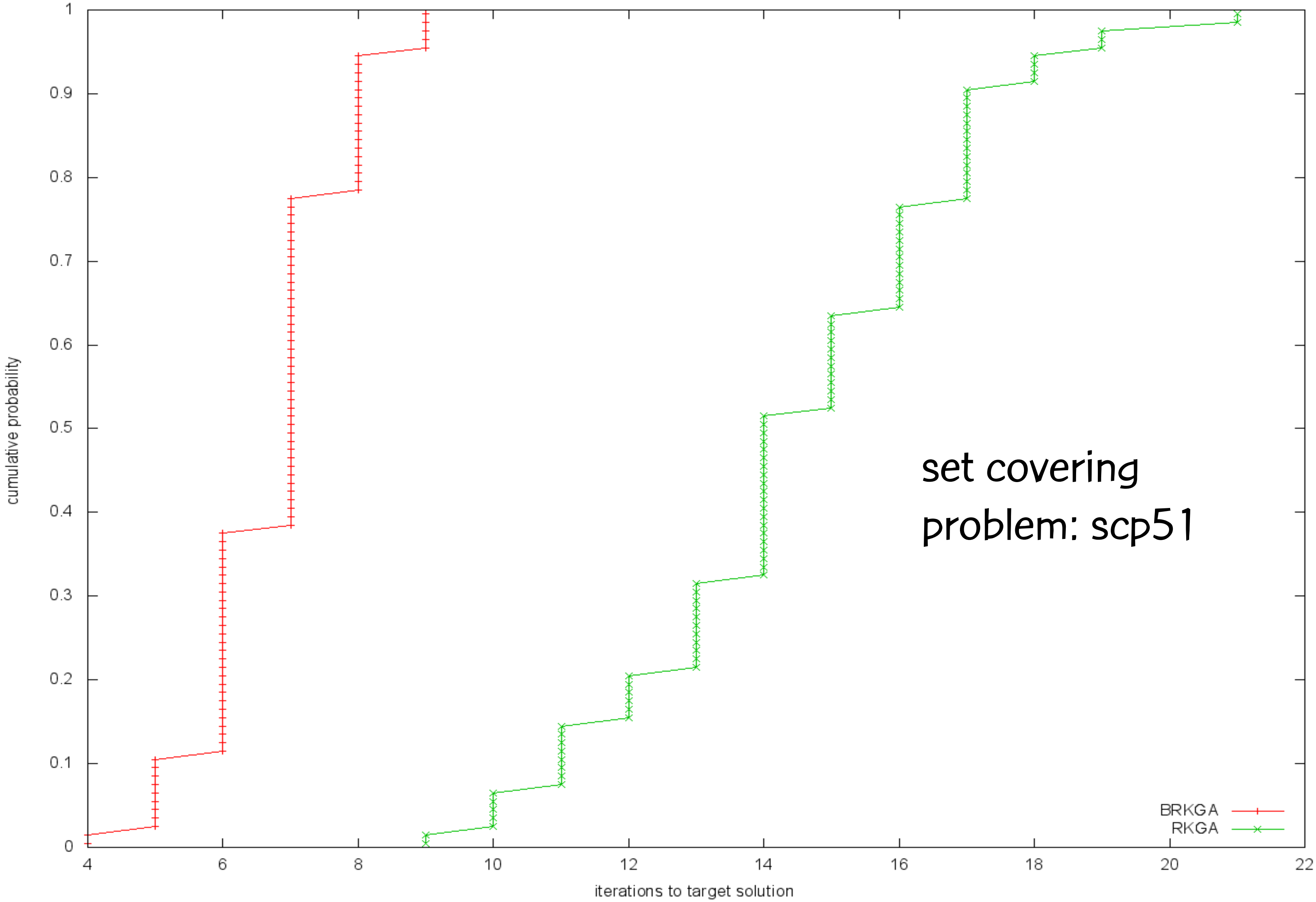
Gonçalves, R., and Toso, "Biased and unbiased random-key genetic algorithms: An experimental analysis", AT&T Labs Research Technical Report, Florham Park, December 2012.



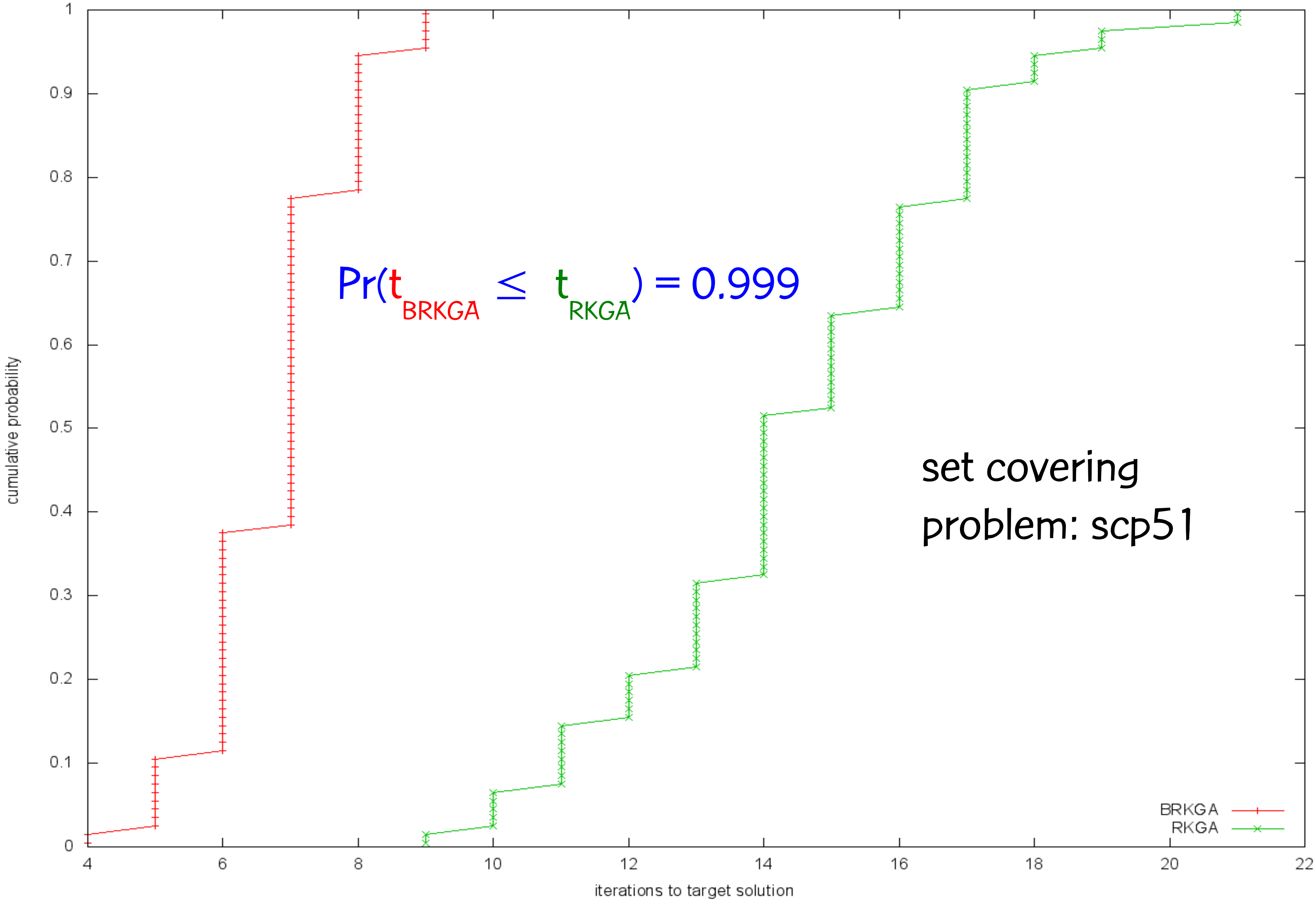
set covering
problem: scp41
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2000
4000
6000
8000
10000
12000
14000
16000



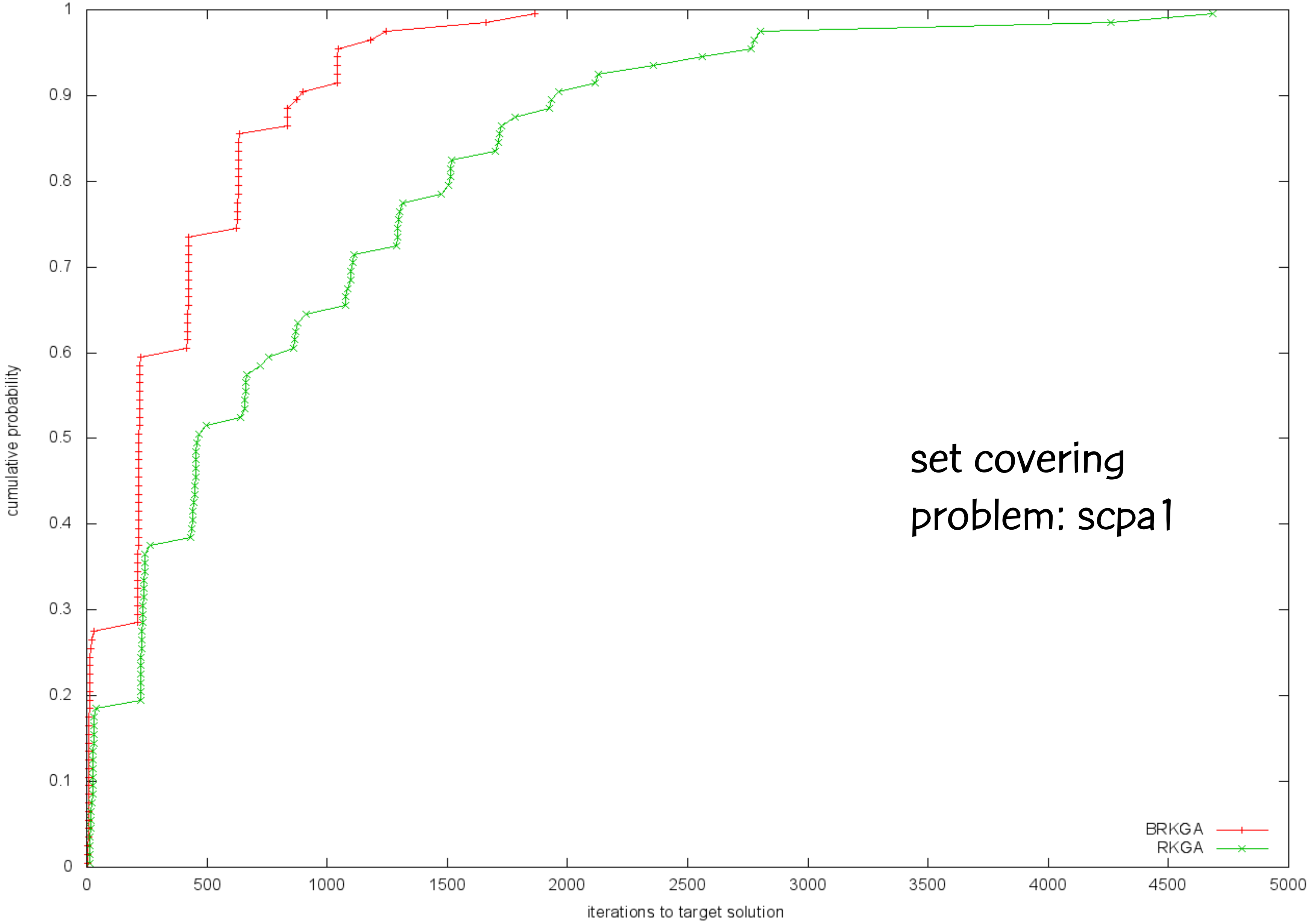
cumulative probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2000
4000
6000
8000
10000
12000
14000
16000
iterations to target solution
$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.740$
Probability computed with method of Ribeiro et al. (2012)
set covering problem: scp41
BRKGA
RKGA



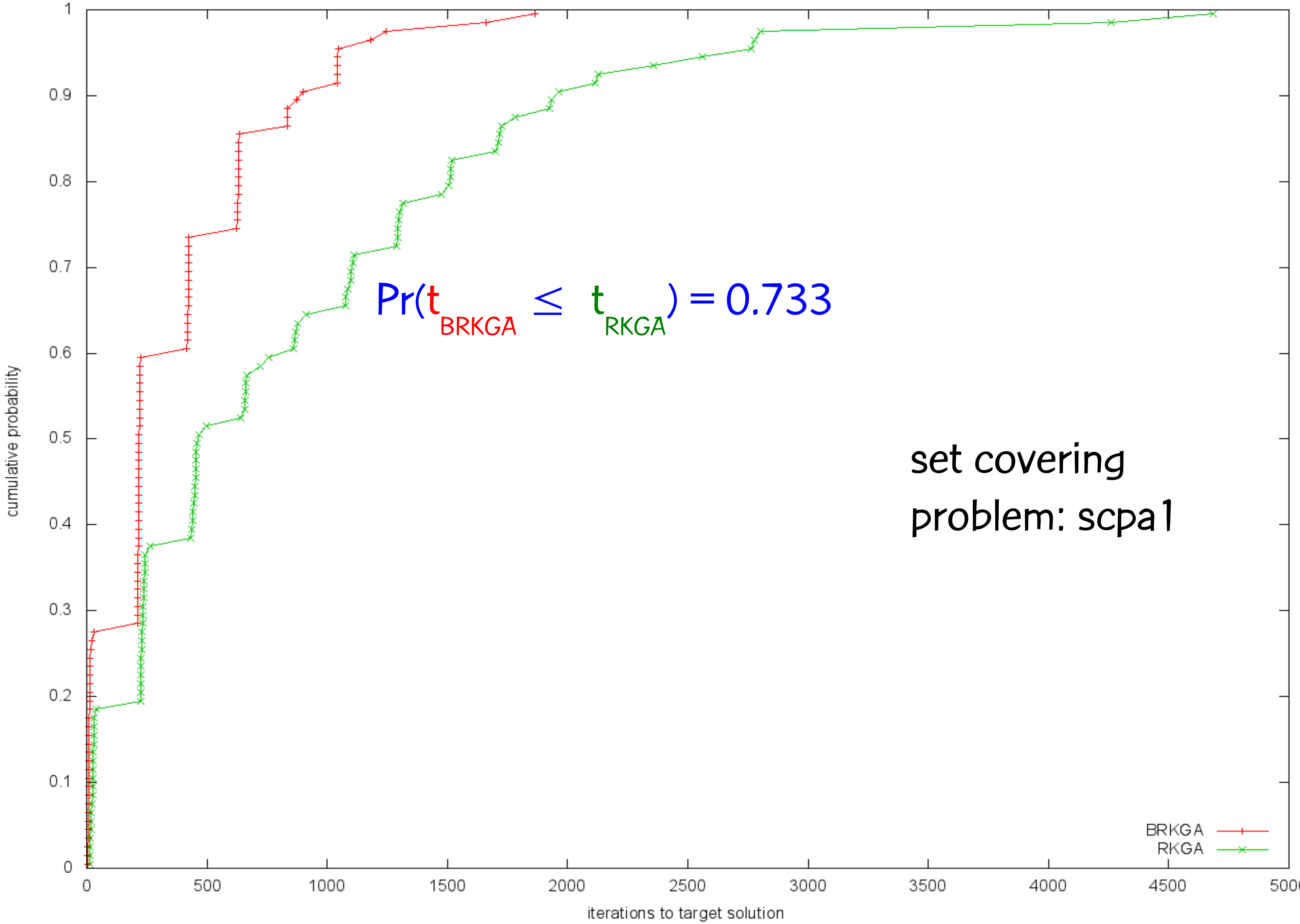
cumulative probability
iterations to target solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
4
6
8
10
12
14
16
18
20
22
BRKGA
RKGA
set covering
problem: scp51



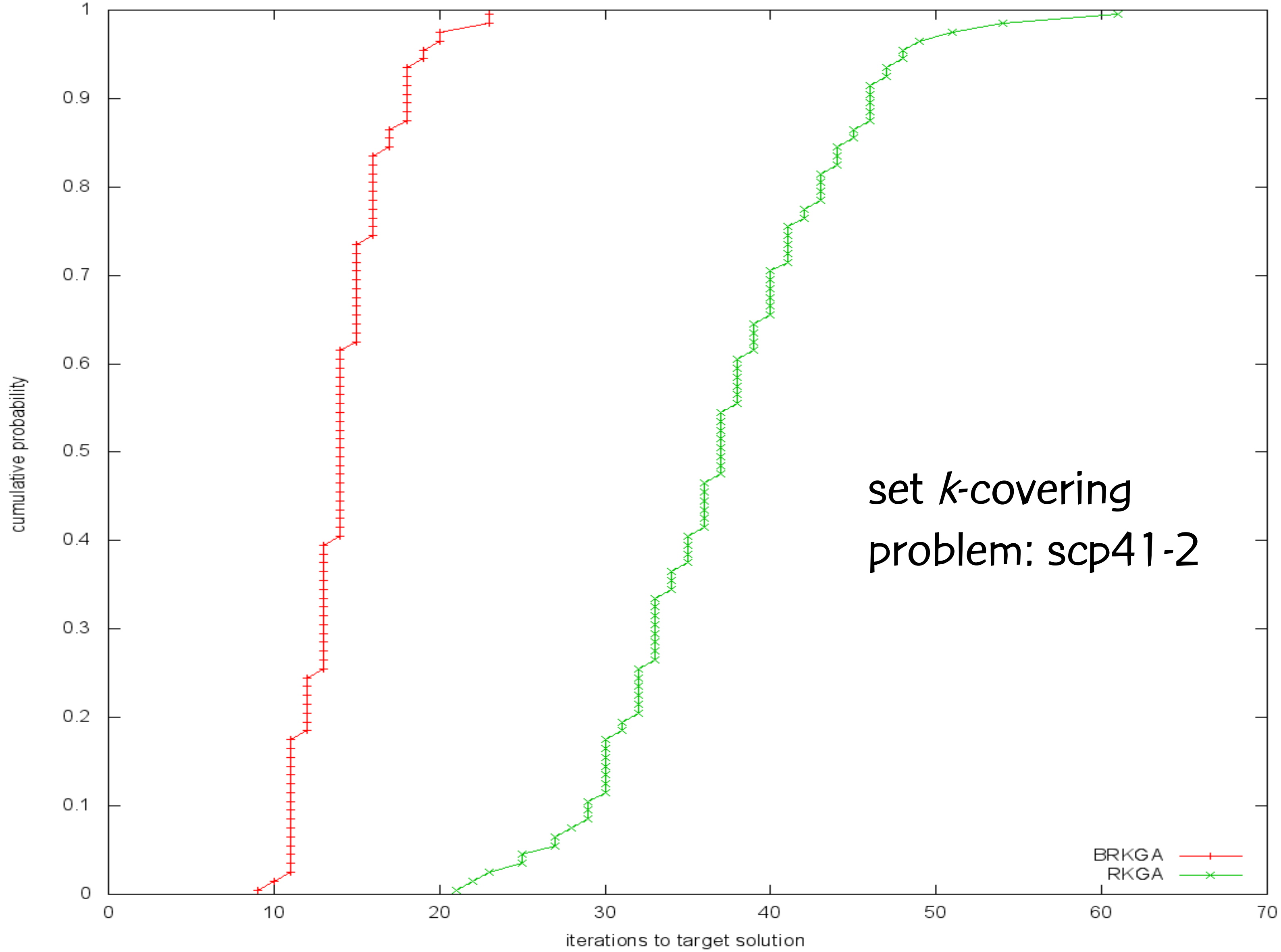
$\Pr(t_{BRKGA} \leq t_{RKGA}) = 0.999$
set covering problem: scp51
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
4
6
8
10
12
14
16
18
20
22

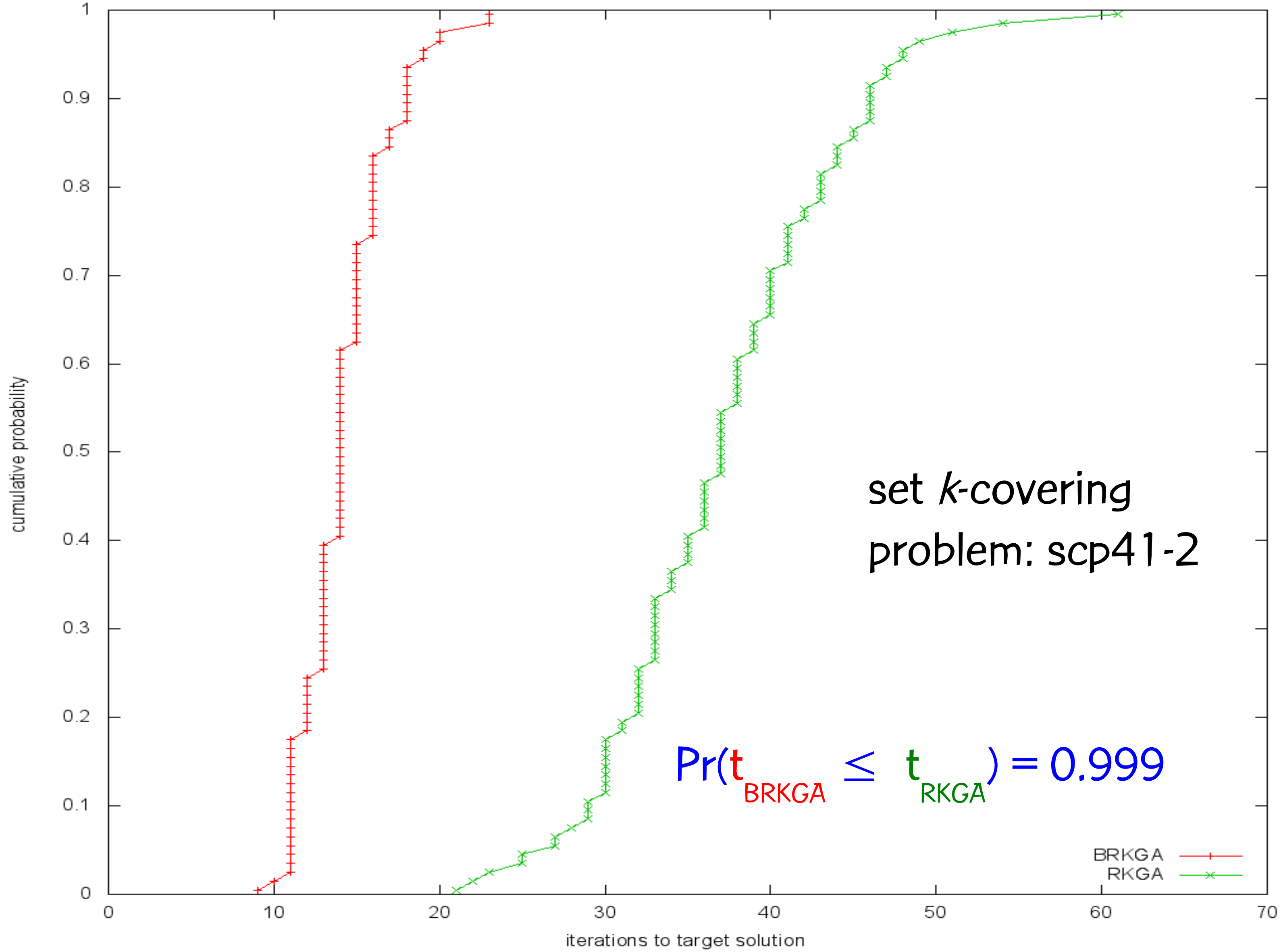


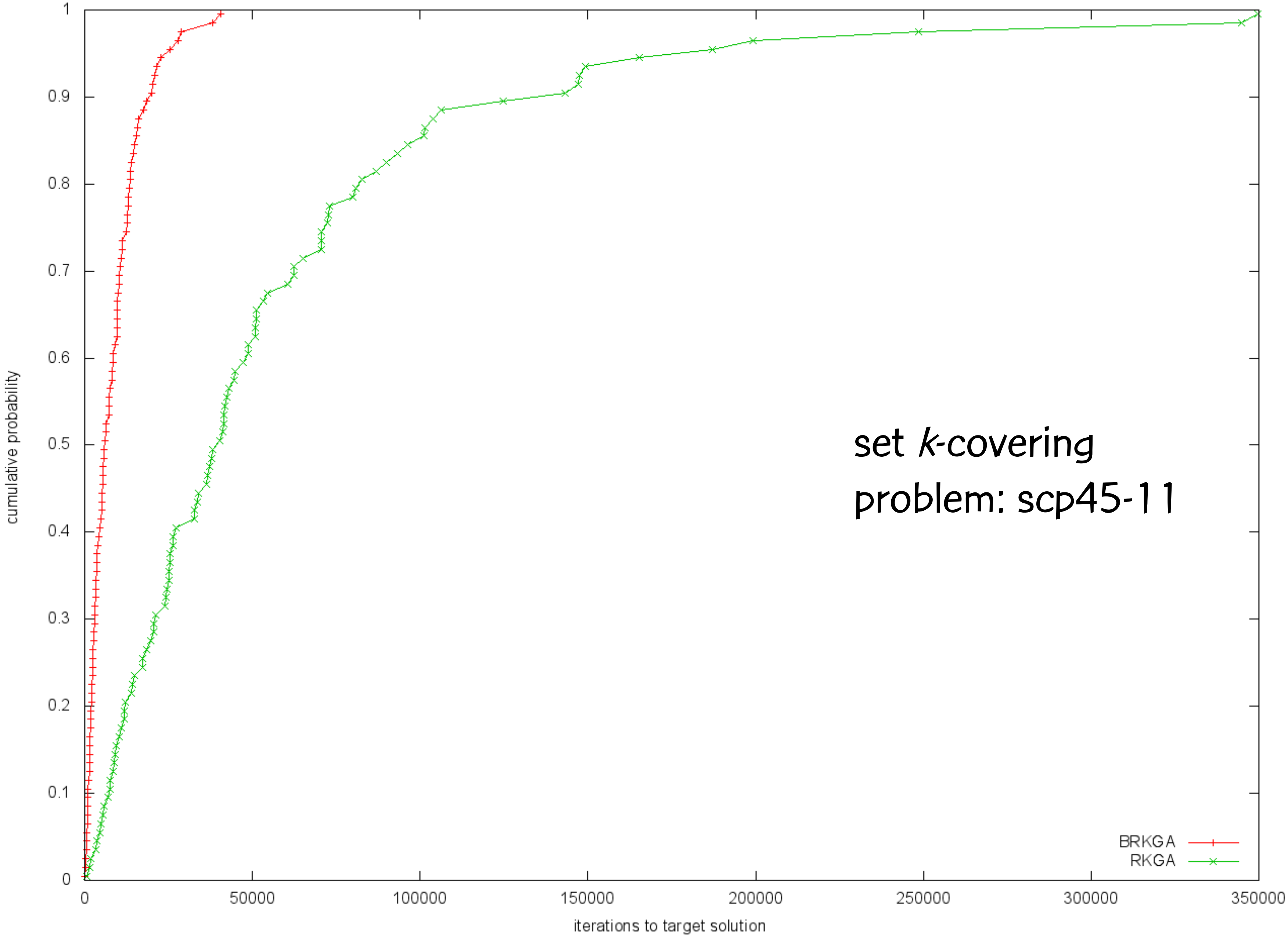
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
cumulative probability
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
iterations to target solution
set covering
problem: scpa1
BRKGA
RKGA



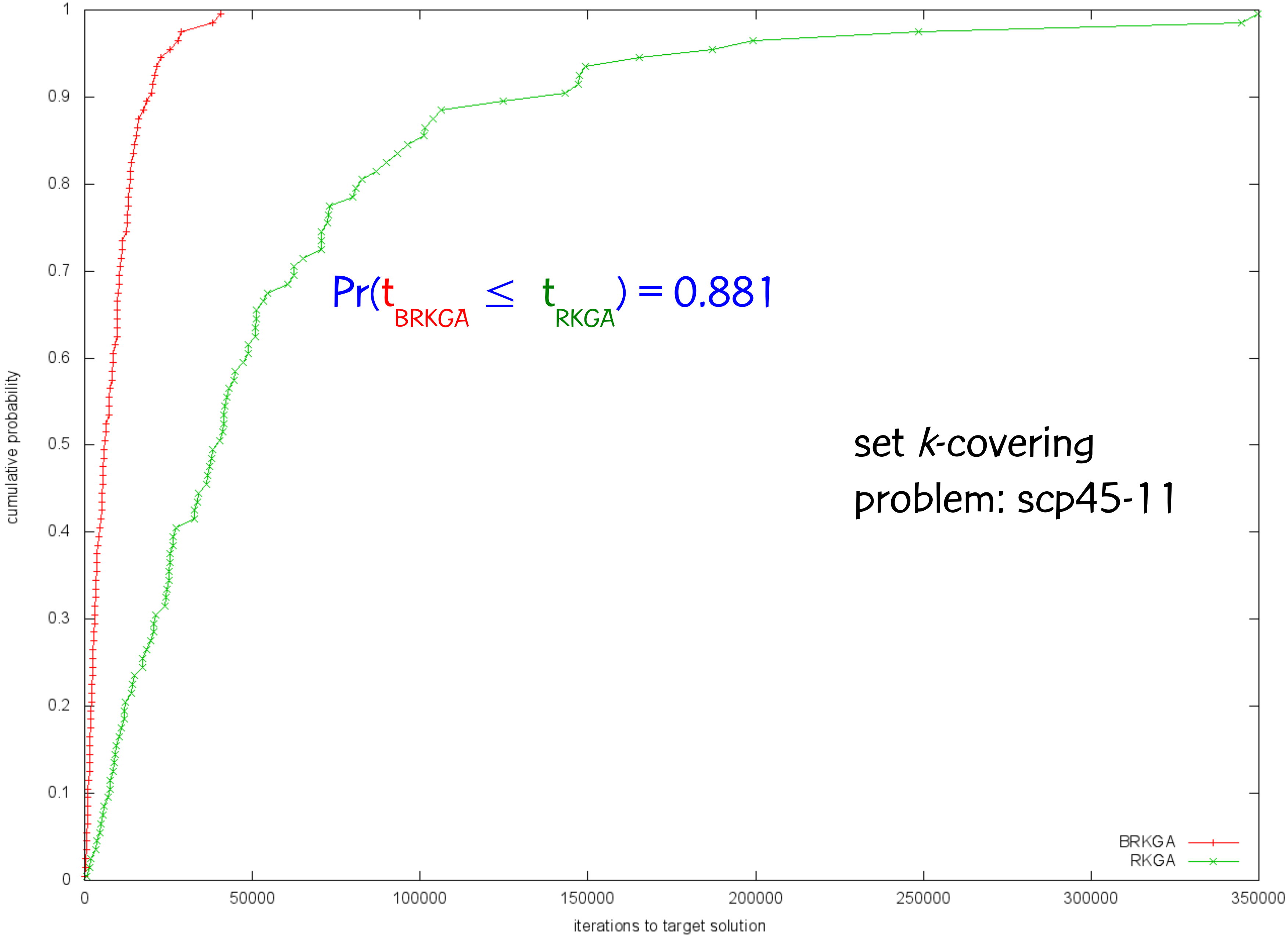
$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.733$
set covering
problem: scpa1
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000


cumulative probability
iterations to target solution
set k-covering
problem: scp41-2
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
20
30
40
50
60
70

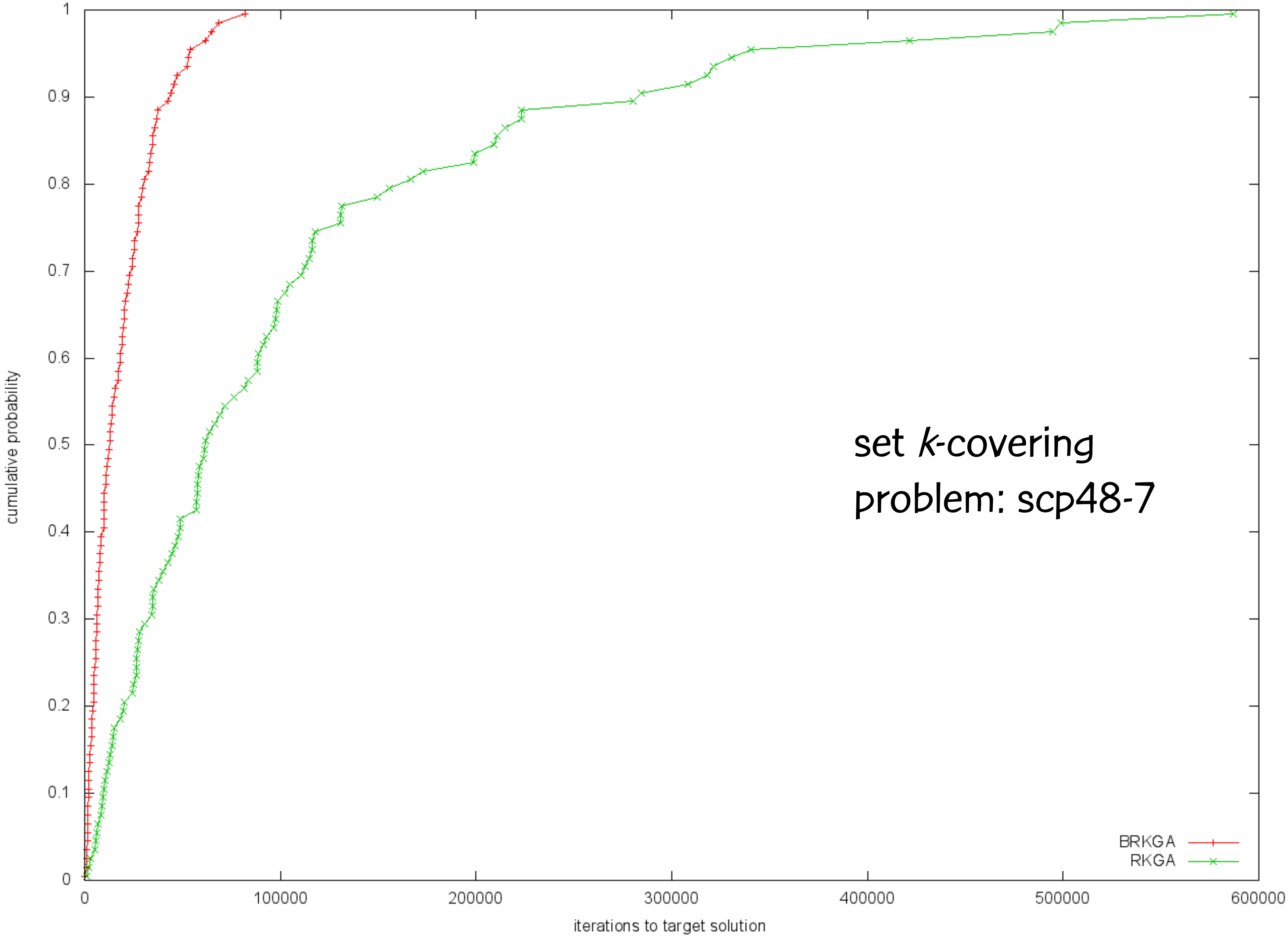

cumulative probability
iterations to target solution
set k-covering problem: scp41-2
Pr(t BRKGA ≤ t RKGA) = 0.999
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70

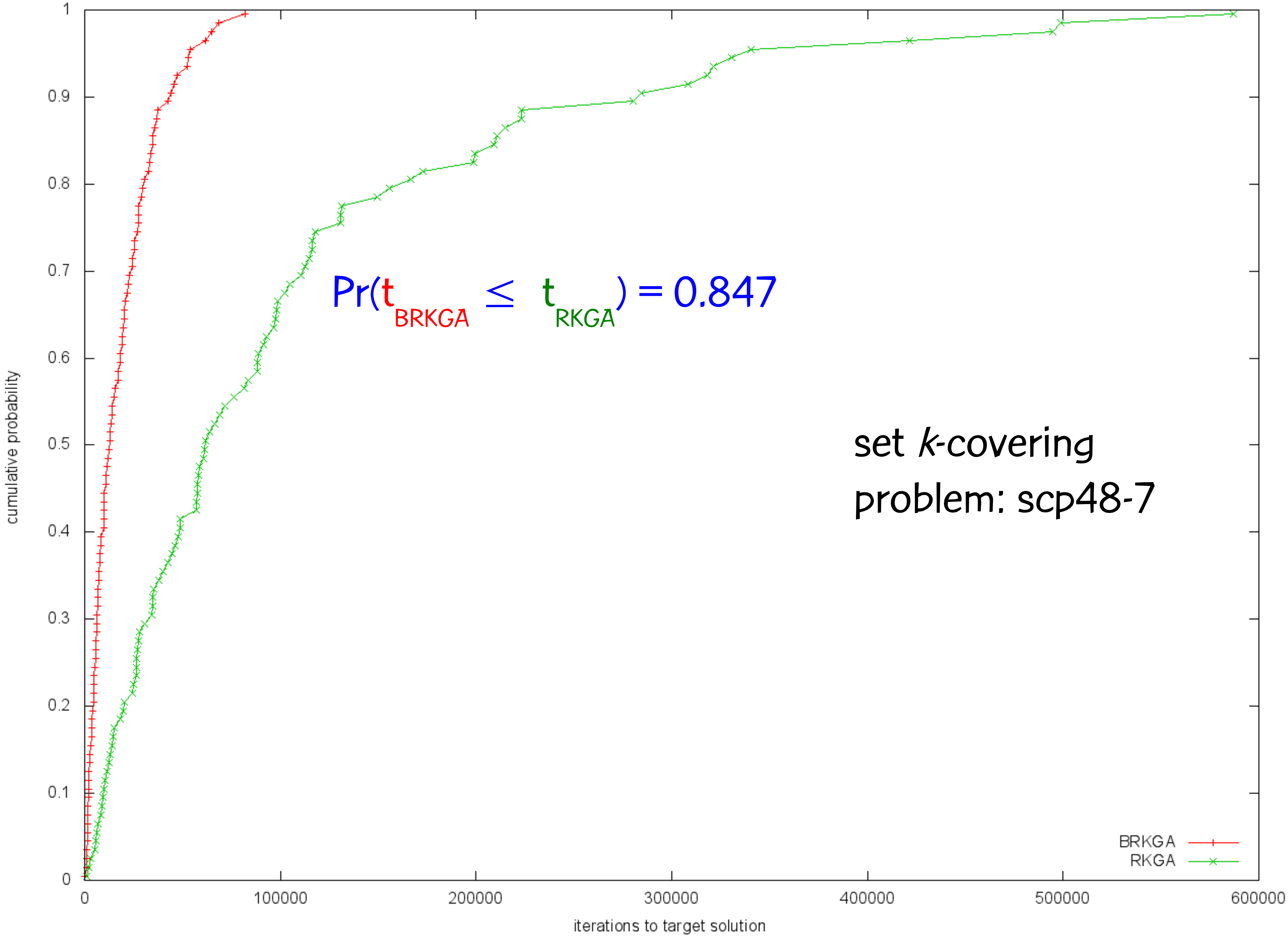


cumulative probability
iterations to target solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50000
100000
150000
200000
250000
300000
350000
BRKGA
RKGA
set *k*-covering problem: scp45-11



$Pr(t_{BRKGA} \leq t_{RKGA}) = 0.881$
set *k*-covering
problem: scp45-11
cumulative probability
iterations to target solution
BRKGA
RKGA
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50000
100000
150000
200000
250000
300000
350000


cumulative probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100000
200000
300000
400000
500000
600000
iterations to target solution
BRKGA
RKGA
set *k*-covering problem: scp48-7



$\Pr(t_{BRKGA} \leq t_{RKGA}) = 0.847$

set *k*-covering problem: scp48-7

Observations

- Random method: keys are randomly generated so solutions are always vectors of random keys

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- Elitist strategy: best solutions are passed without change from one generation to the next (incumbent is kept)

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- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent > 0.5 Not so in the RKGA of Bean.

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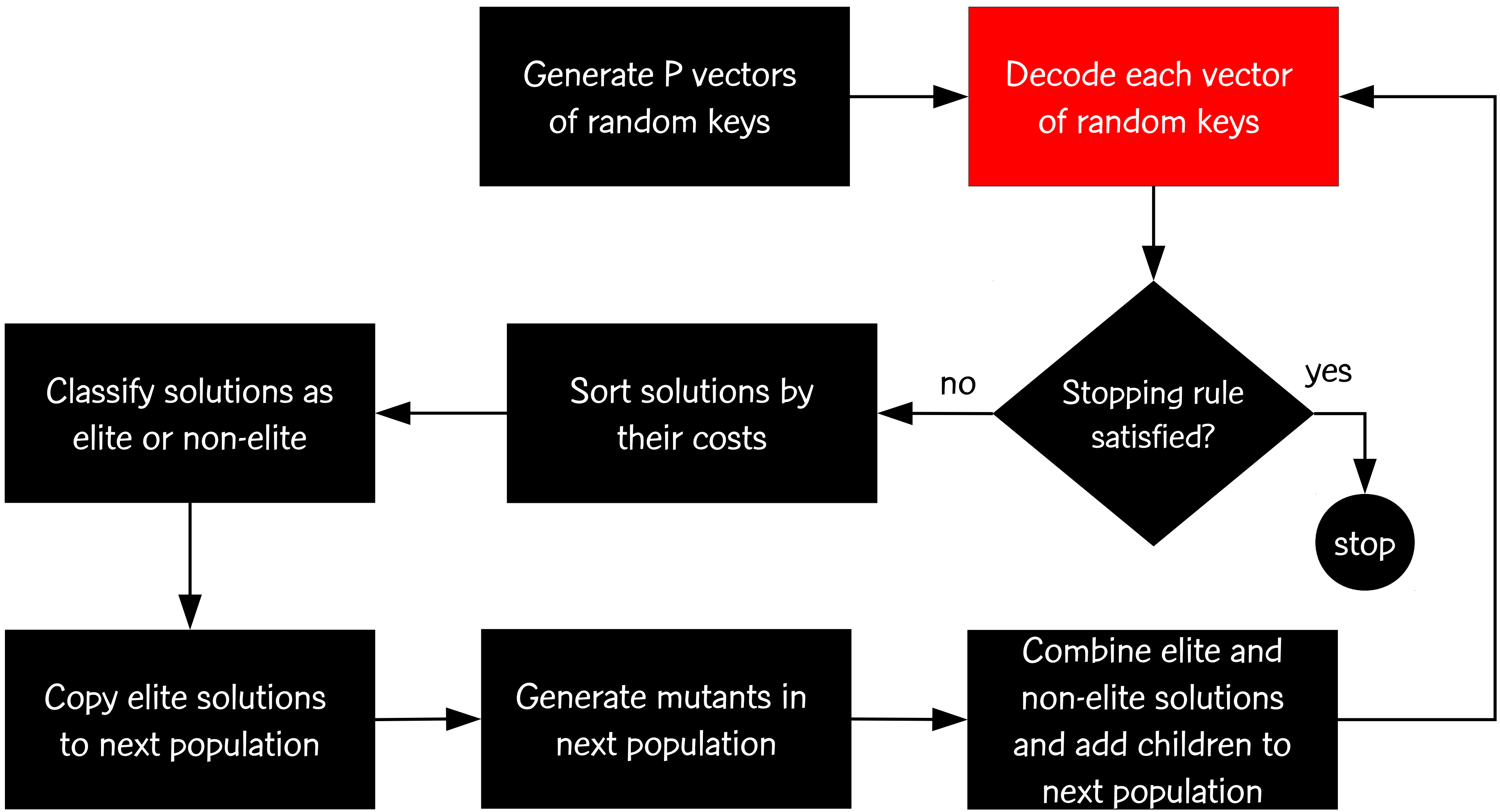
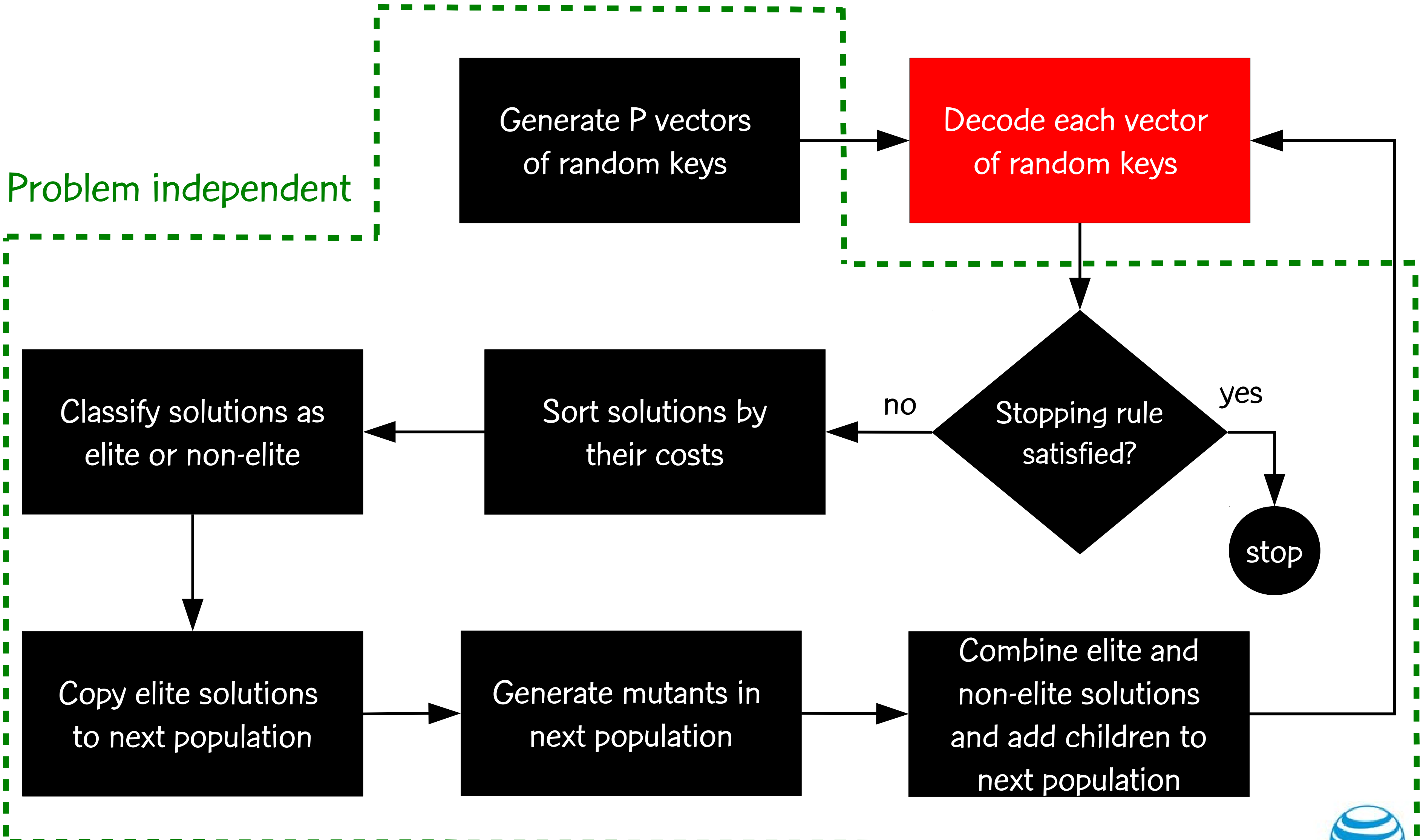
- Child inherits more characteristics of elite parent: one parent is always selected (with replacement) from the small elite set and probability that child inherits key of elite parent > 0.5 Not so in the RKGA of Bean.
- No mutation in crossover: mutants are used instead (they play same role as mutation in GAs … help escape local optima)

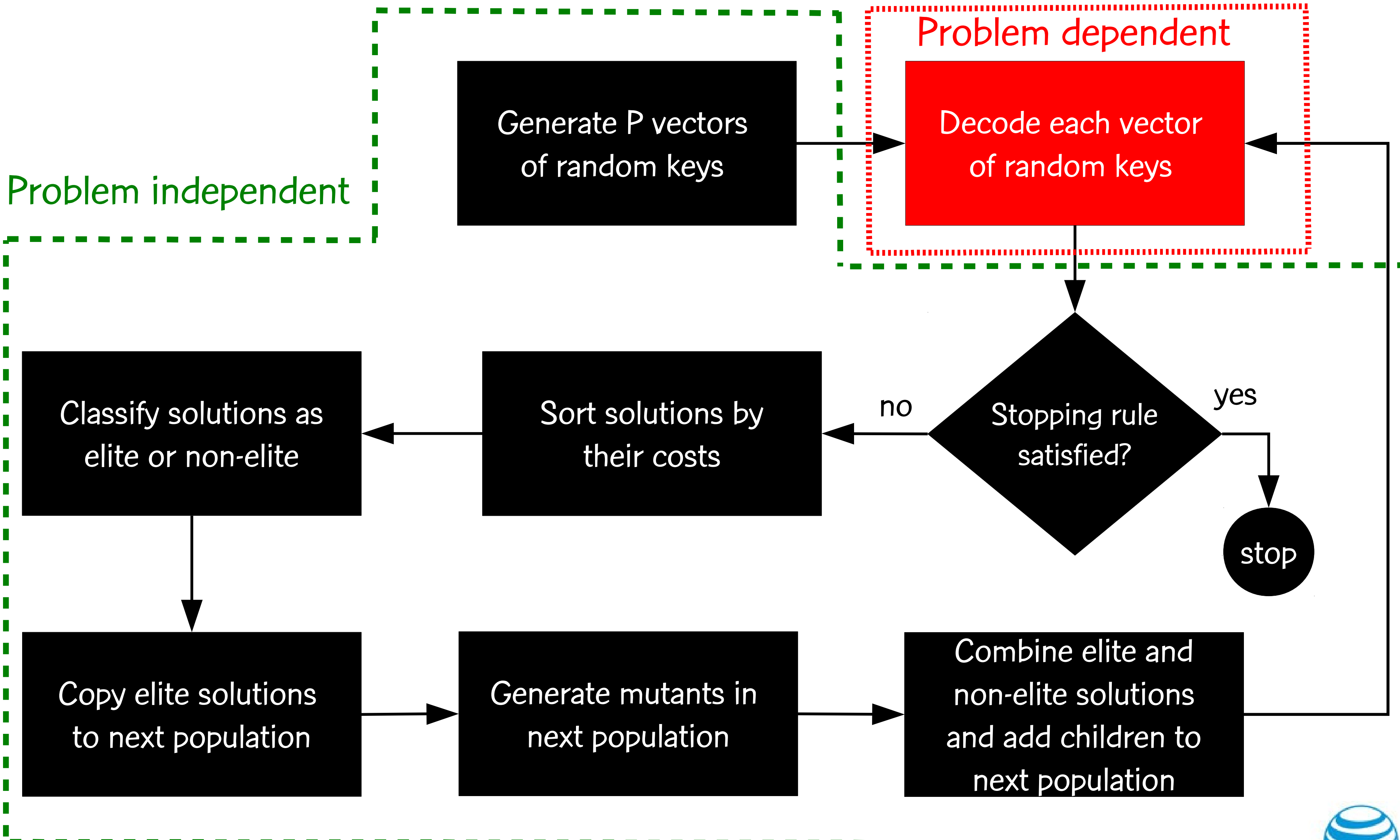
Framework for biased random-key genetic algorithms



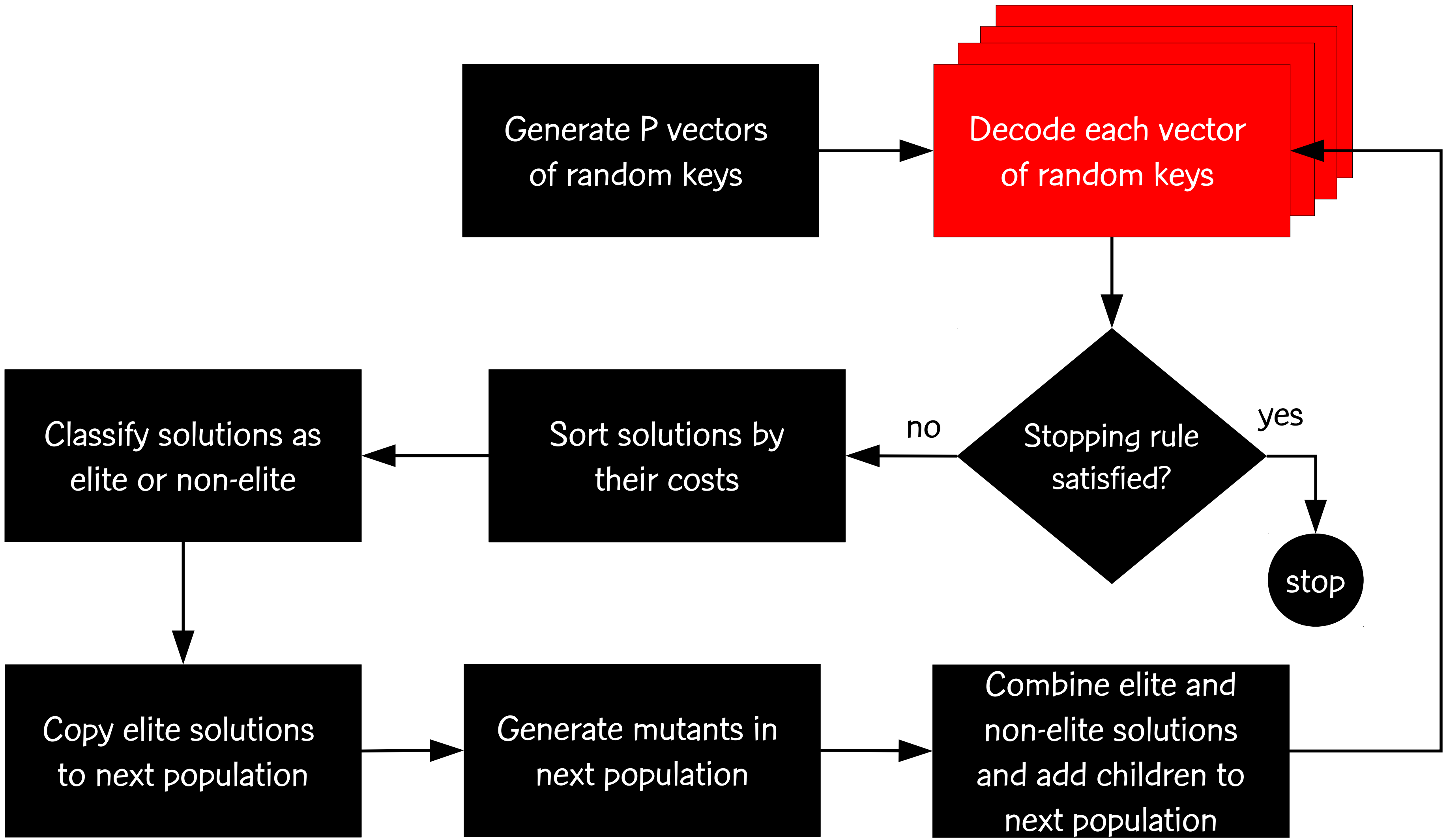
Framework for biased random-key genetic algorithms



Framework for biased random-key genetic algorithms



Decoding of random key vectors can be done in parallel

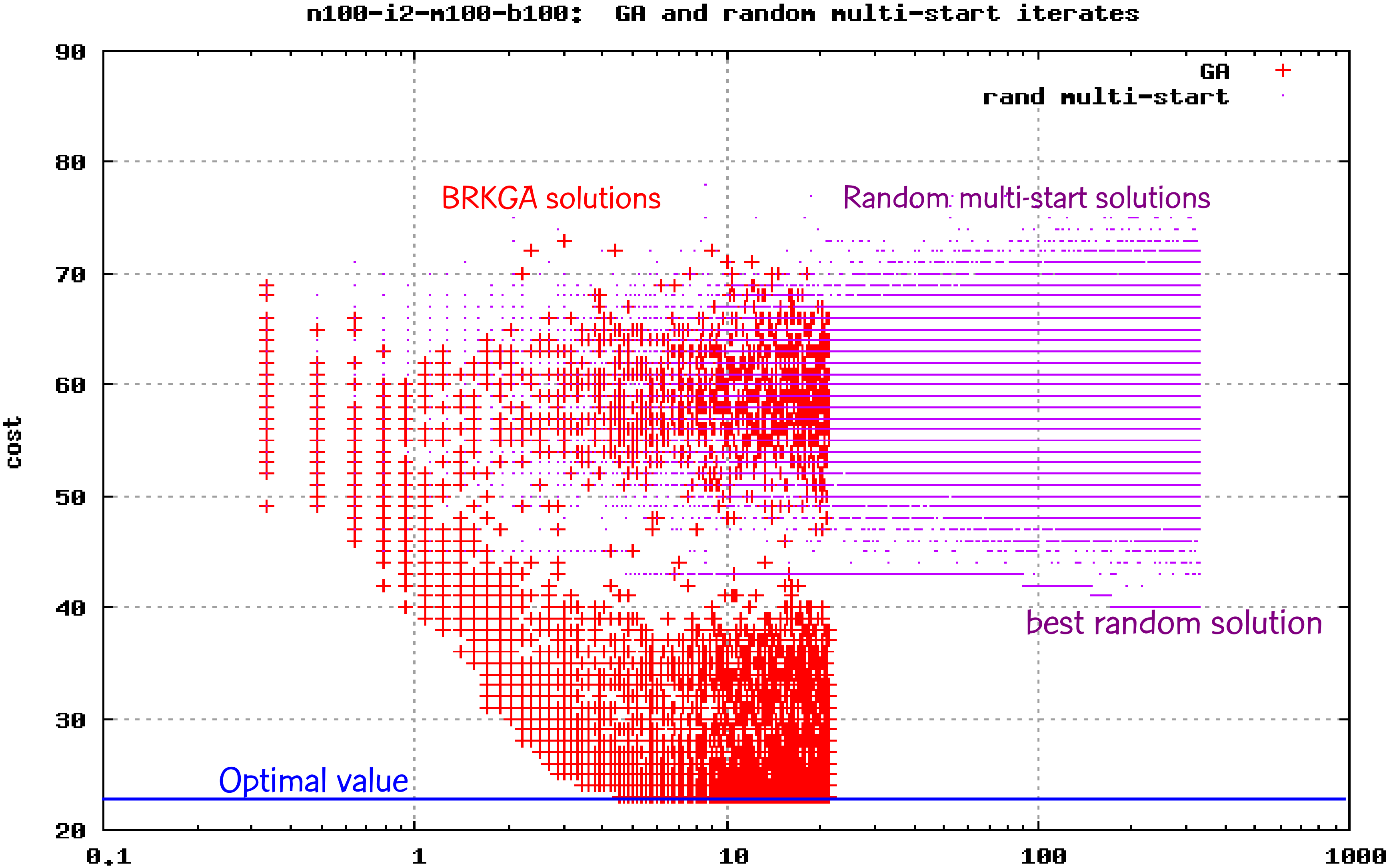


Is a BRKGA any different from applying the decoder to random keys?

- Simulate a random multi-start decoding method with a BRKGA by setting size of elite partition to 1 and number of mutants to P–1
- Each iteration, best solution is maintained in elite set and P–1 random key vectors are generated as mutants … no mating is done since population already has P individuals

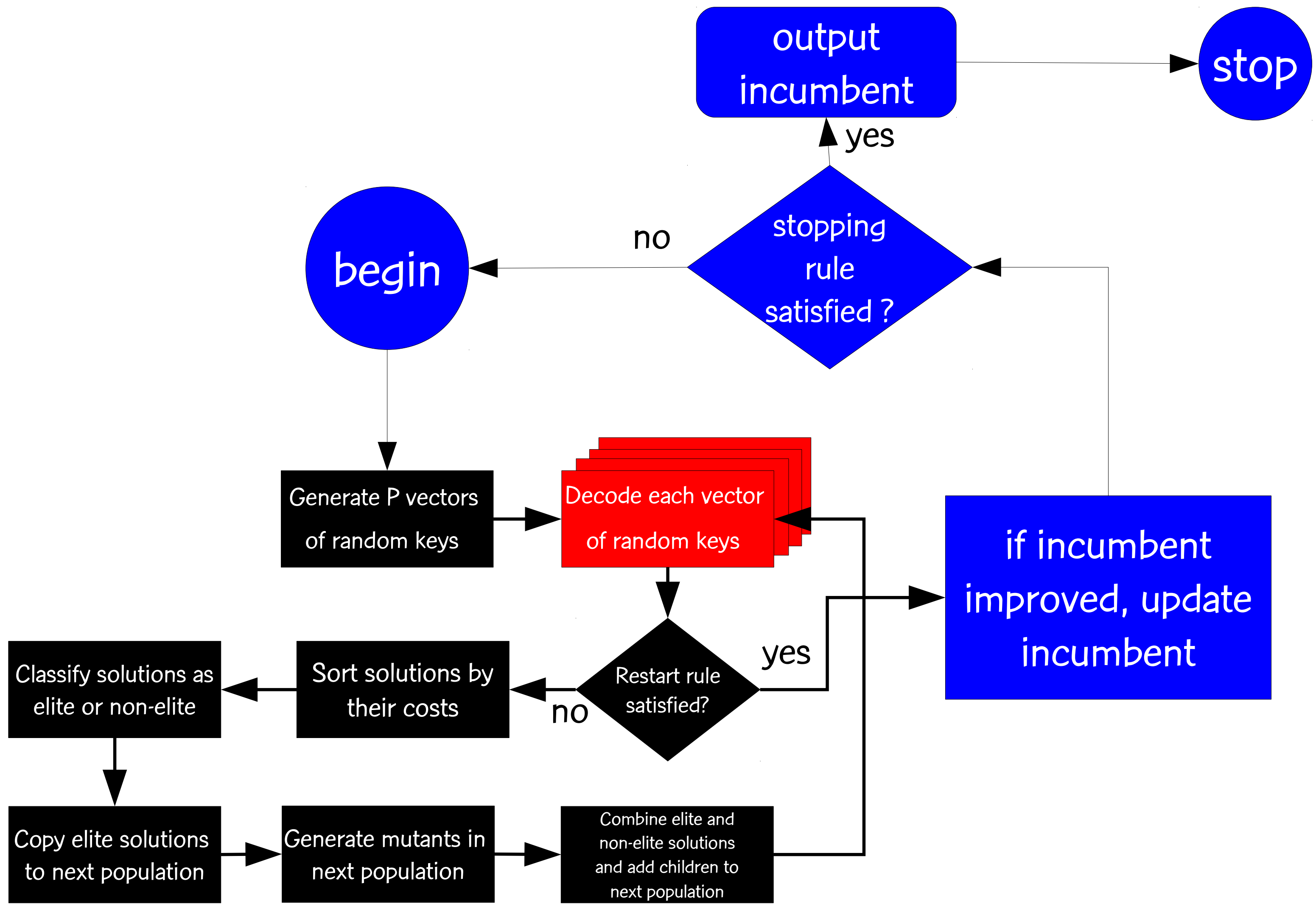
Network monitor location problem (opt = 23)

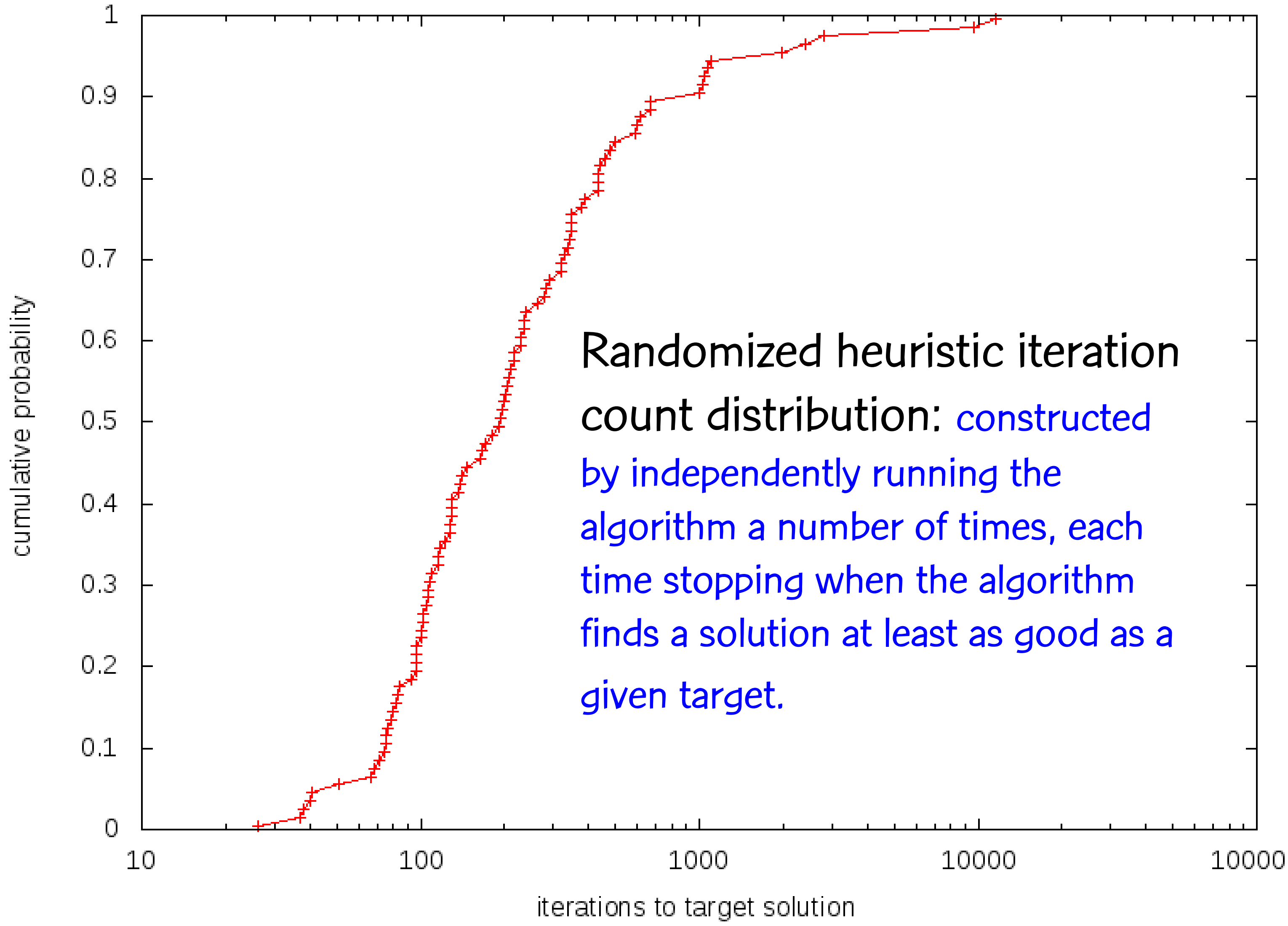
solution



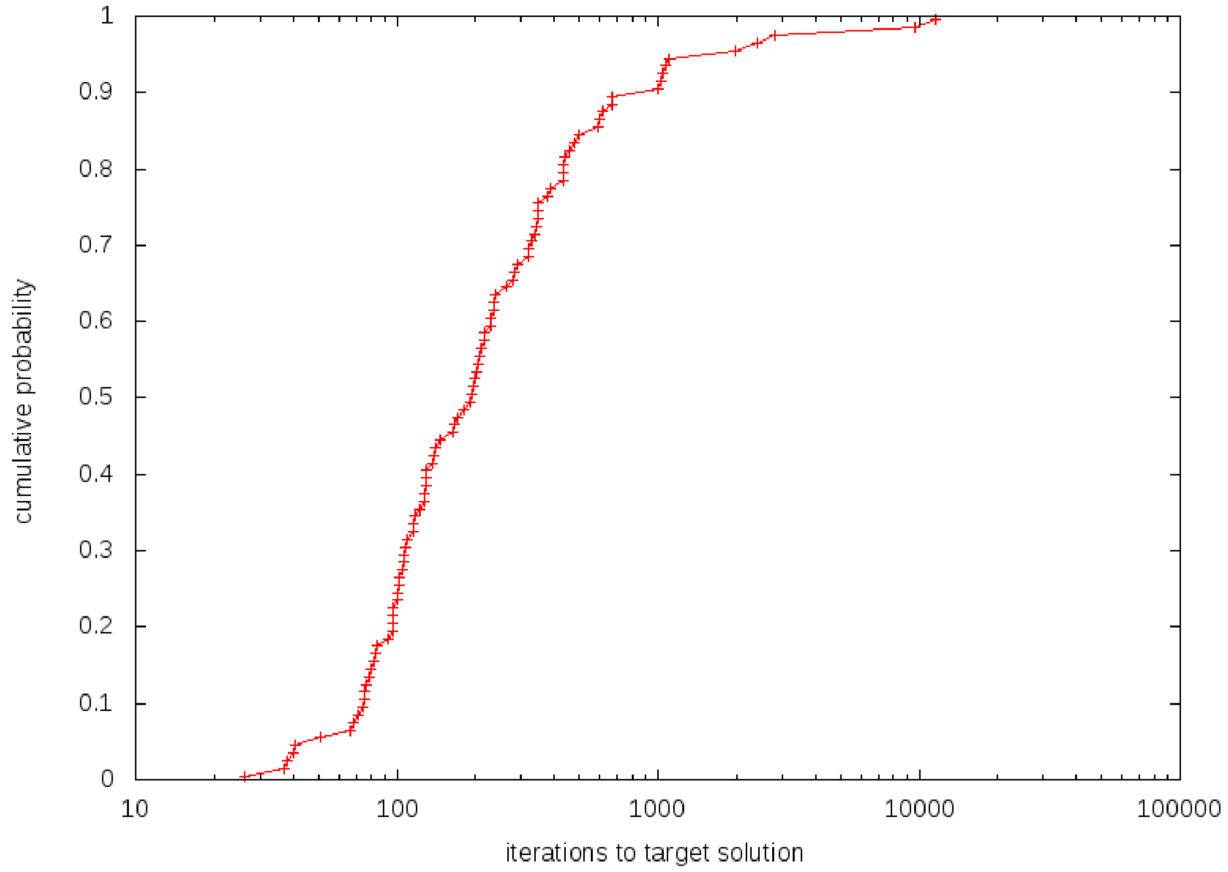
Time (ibm t41 secs)

BRKGA in multi-start strategy

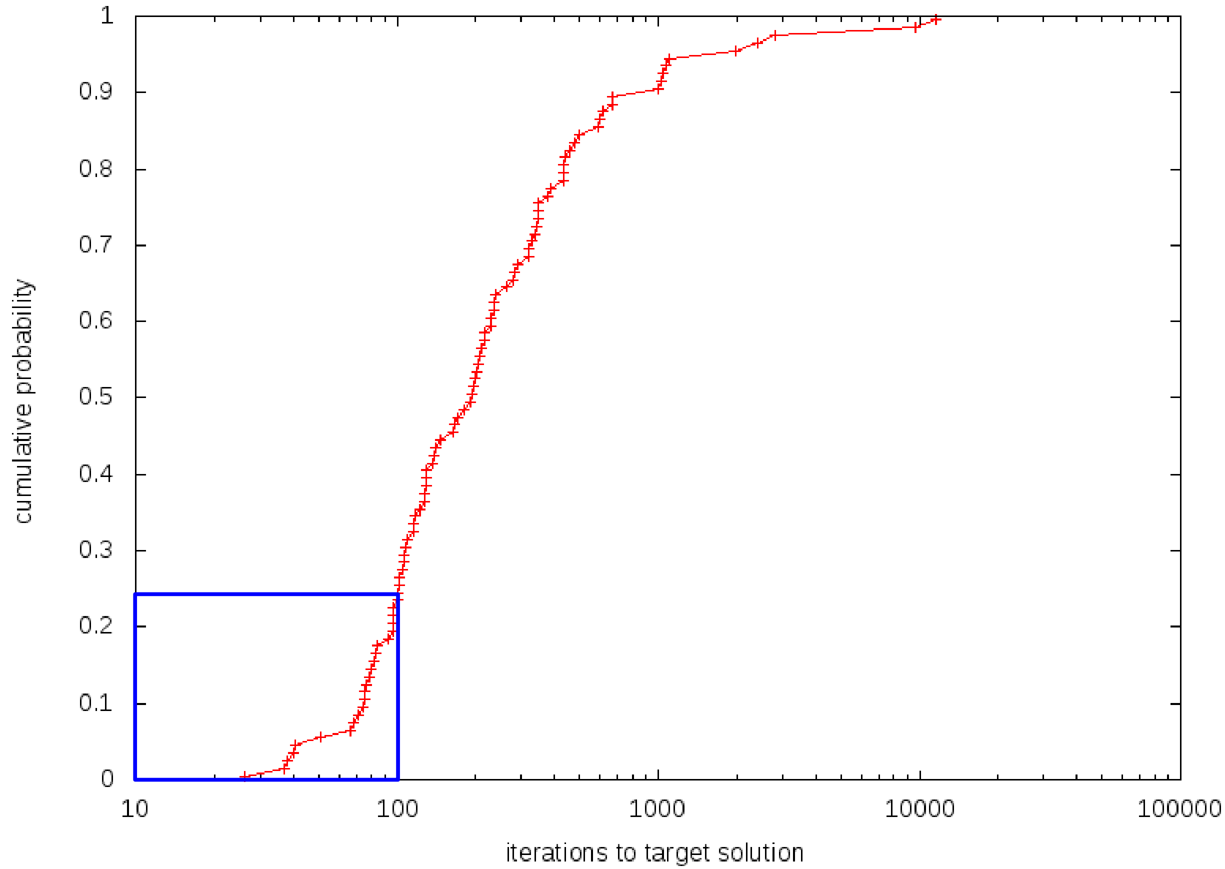




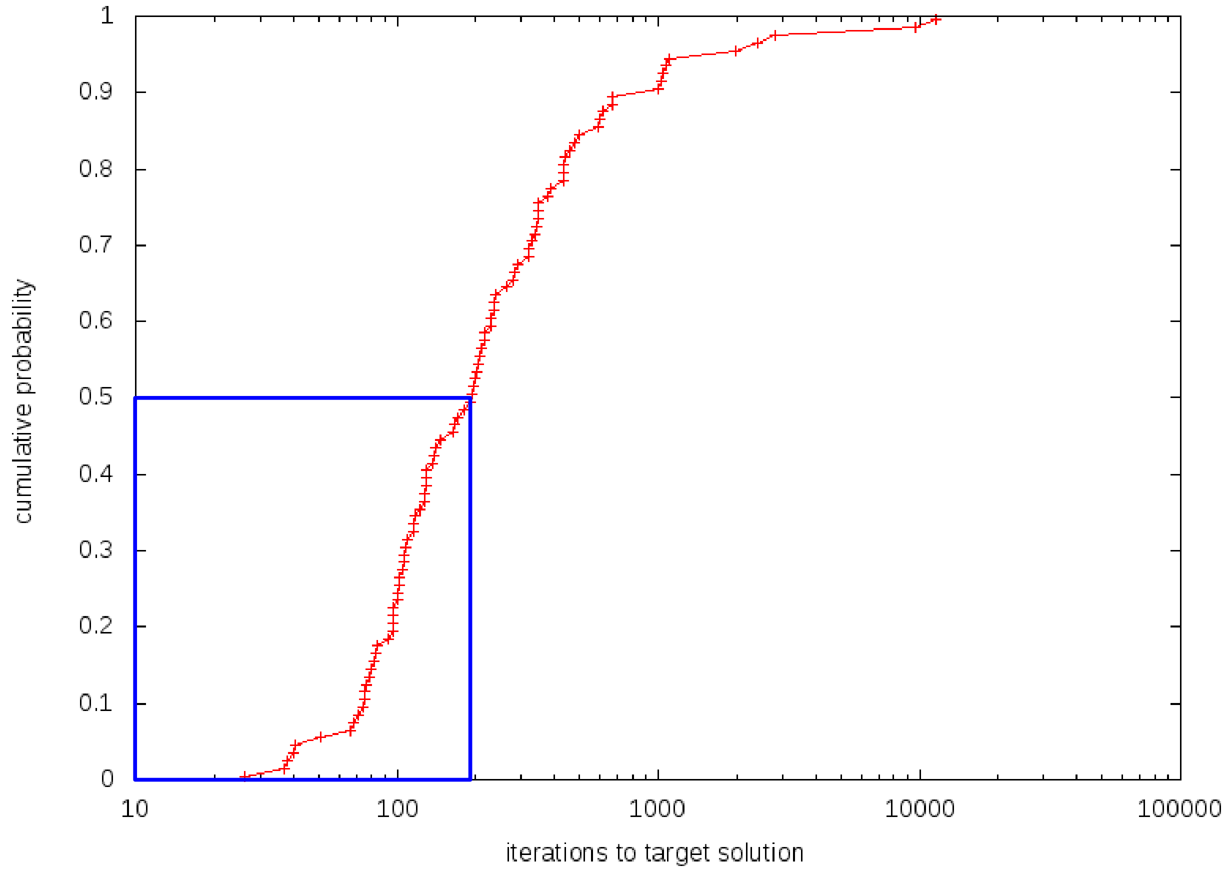
Randomized heuristic iteration count distribution: constructed by independently running the algorithm a number of times, each time stopping when the algorithm finds a solution at least as good as a given target.



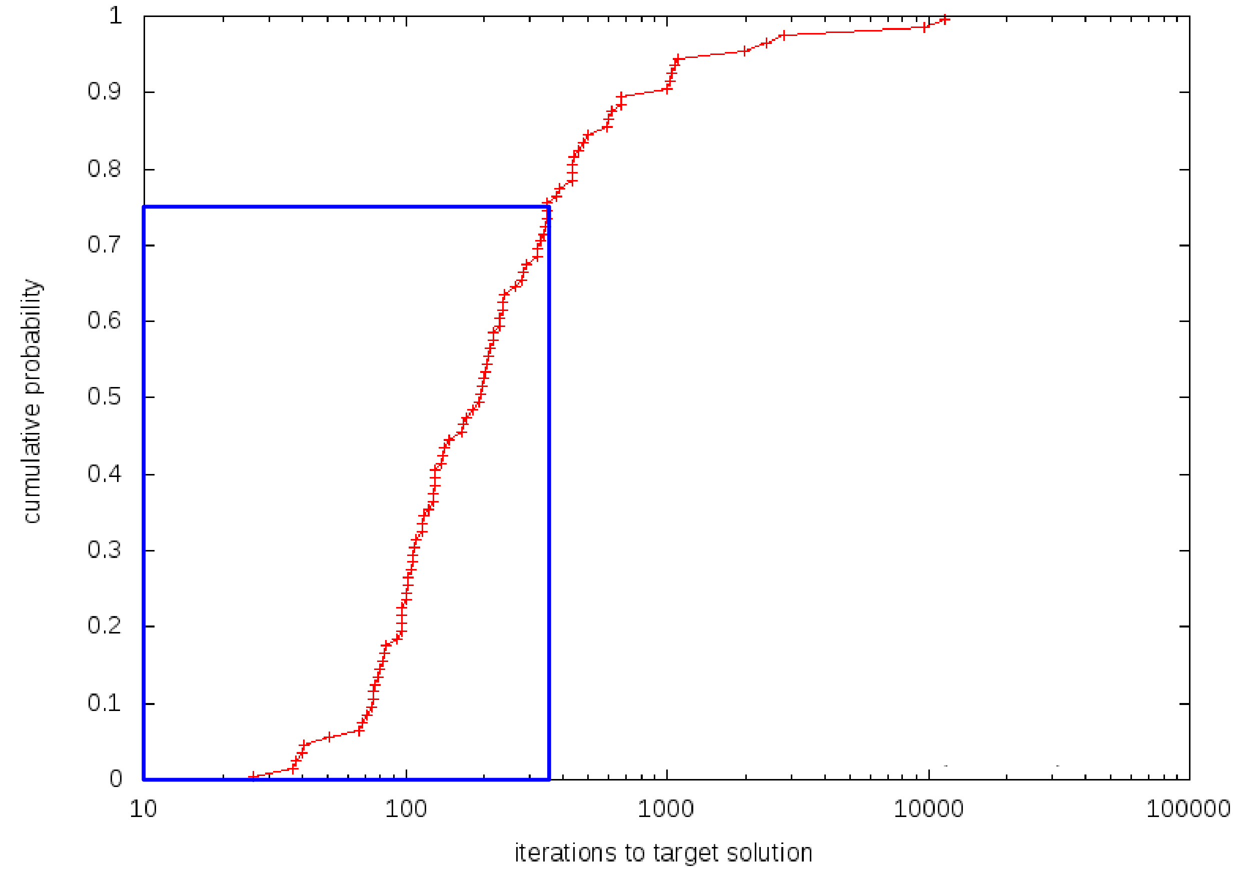
In most of the independent runs, the algorithm finds the target solution in relatively few iterations:



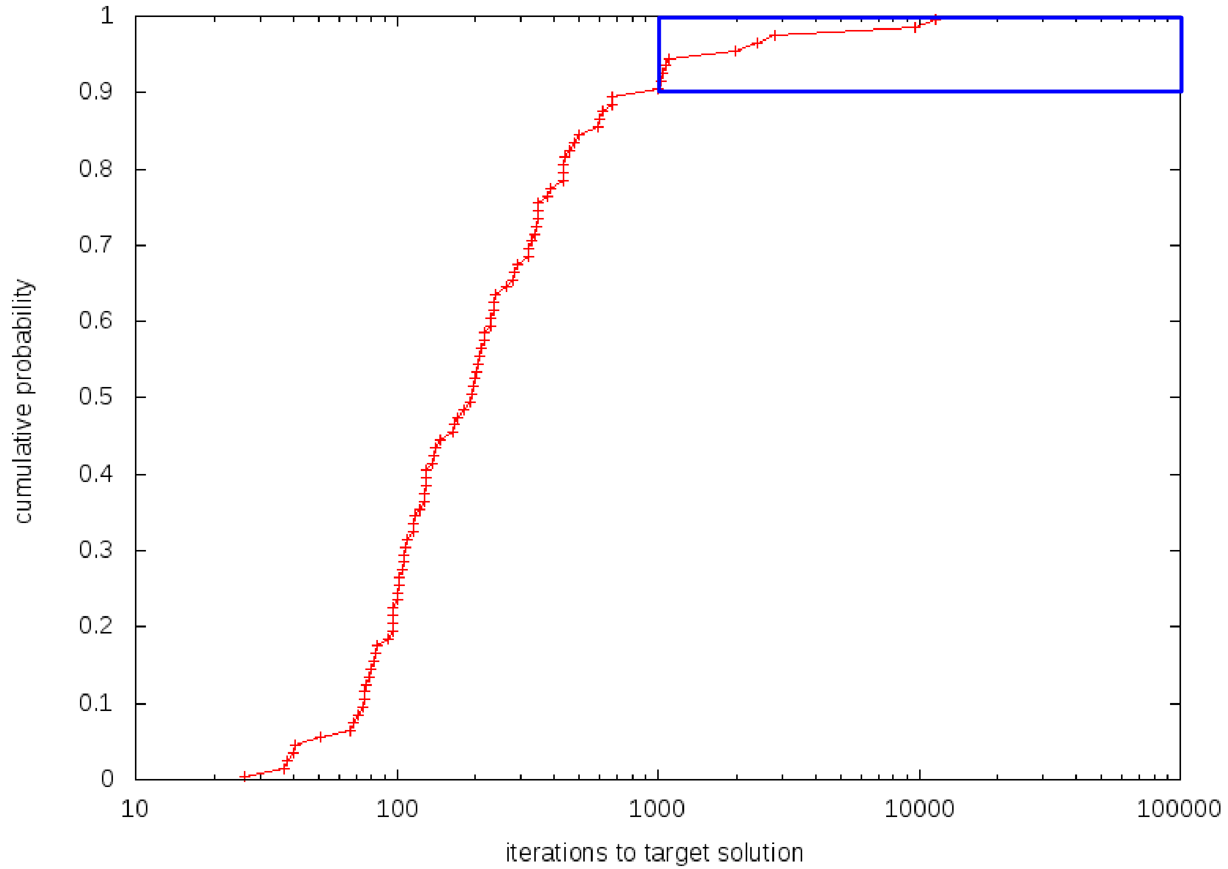
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 25% of the runs take fewer than 101 iterations



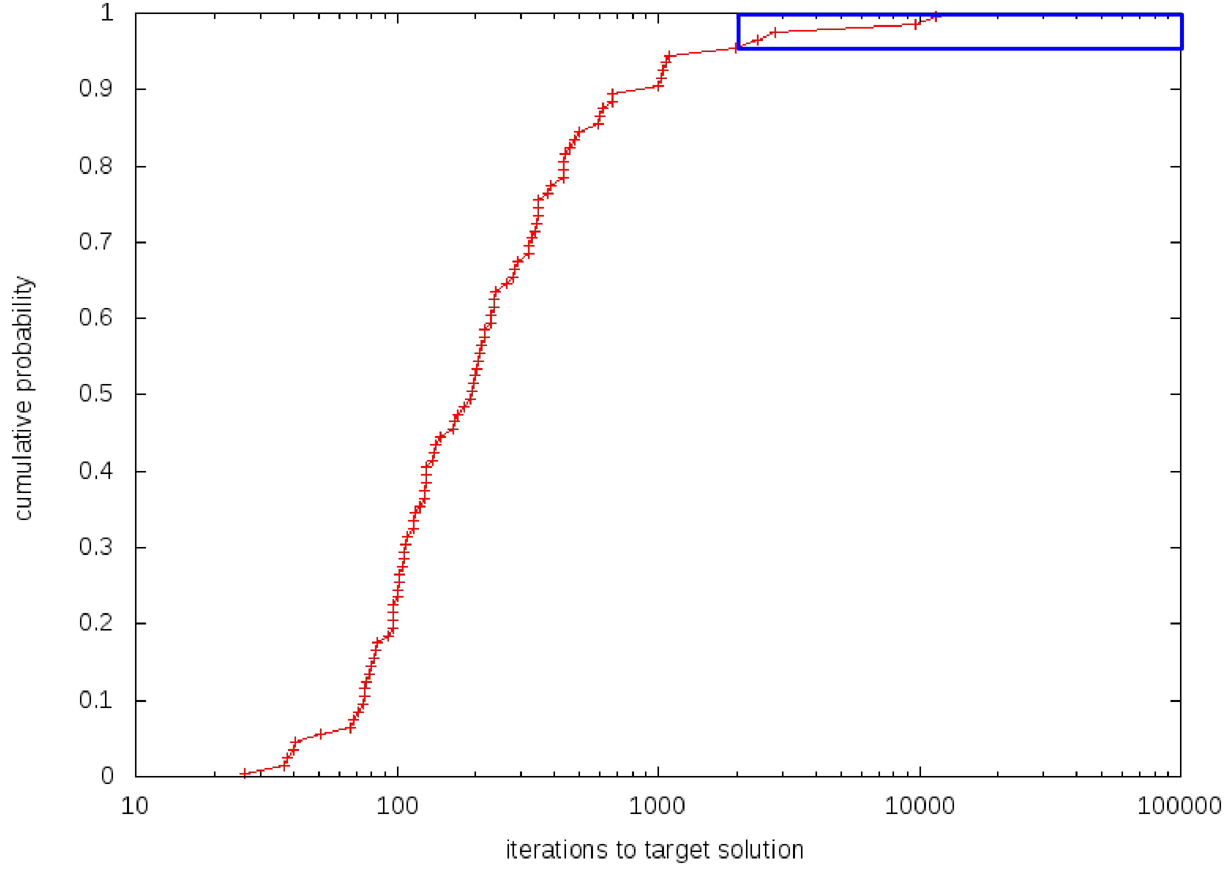
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 50% of the runs take fewer than 192 iterations



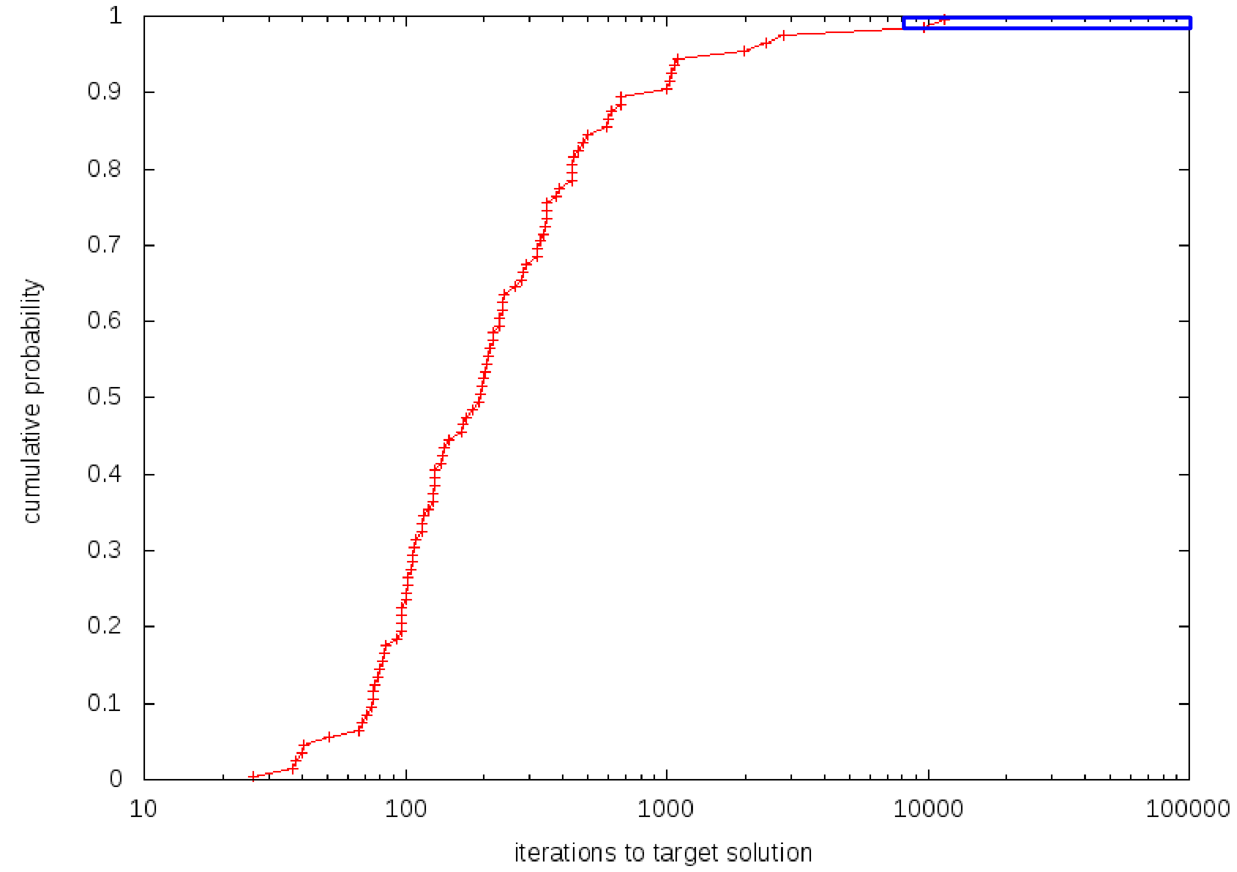
In most of the independent runs, the algorithm finds the target solution in relatively few iterations: 75% of the runs take fewer than 345 iterations



However, some runs take much longer: 10% of the runs take over 1000 iterations



However, some runs take much longer: 5% of the runs take over 2000 iterations



However, some runs take much longer: 2% of the runs take over 9715 iterations

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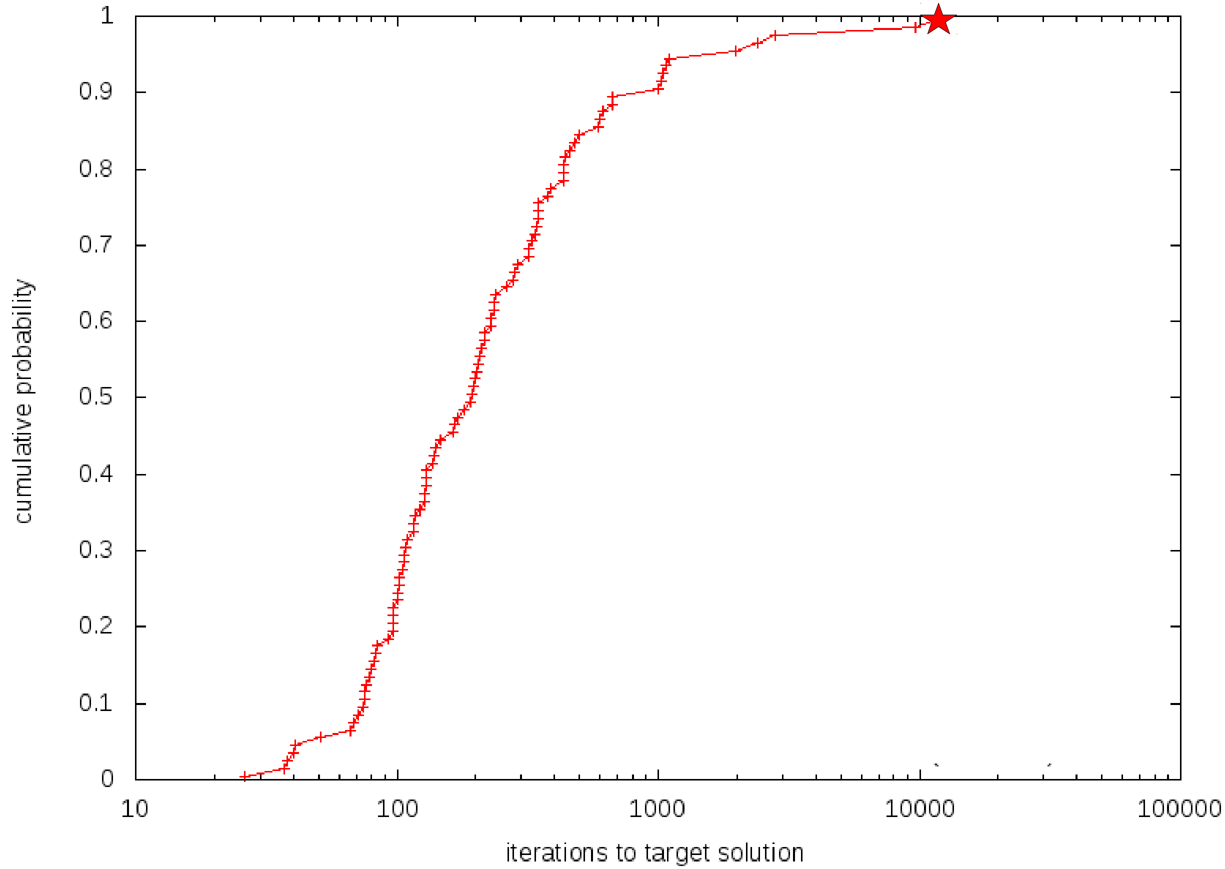

However, some runs take much longer: the longest run took 11607 iterations

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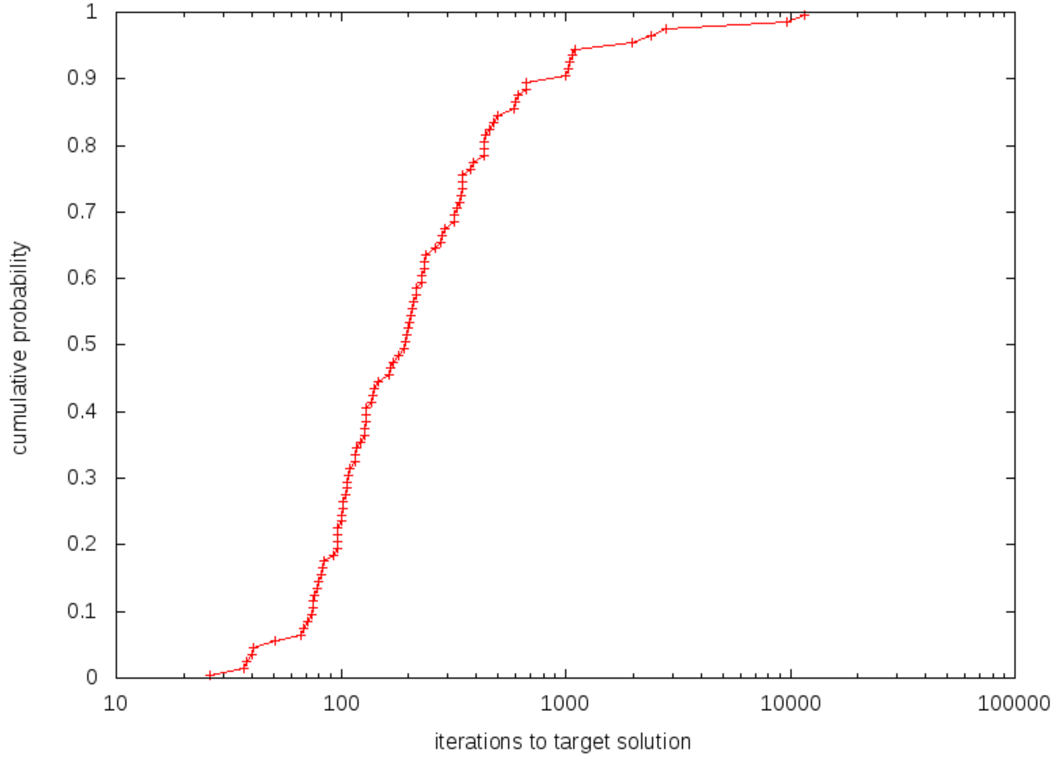
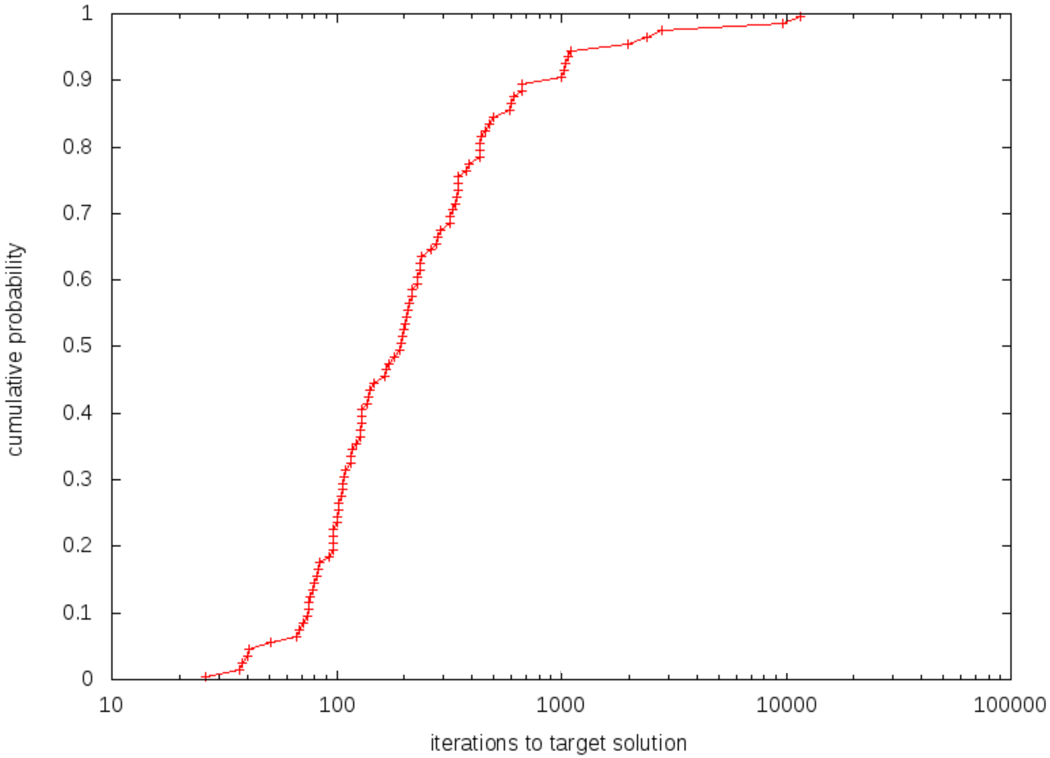


Probability that algorithm will take over 345 iterations: 25% = 1/4



Probability that algorithm will take over 345 iterations: 25% = 1/4

By restarting algorithm after 345 iterations, probability that new run will take over 690 iterations: 25% = 1/4

Probability that algorithm with restart will take over 690 iterations: probability of taking over 345 X probability of taking over 690 iterations given it took over 345 = ¼ x ¼ = $1/4^2$

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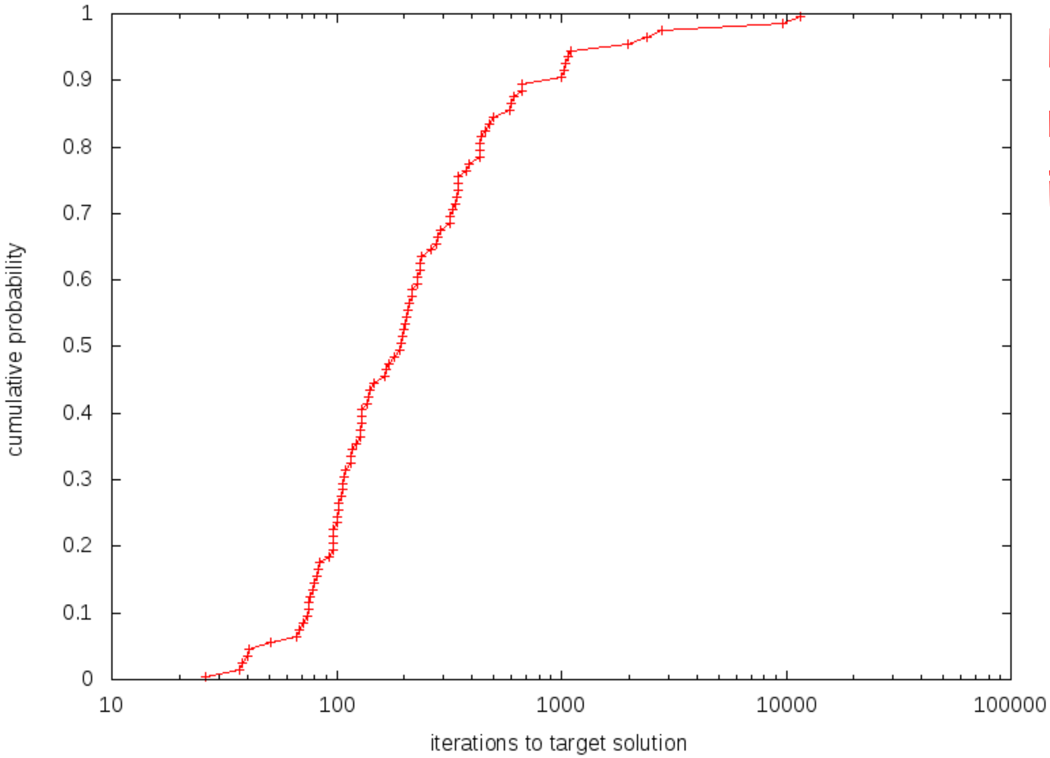
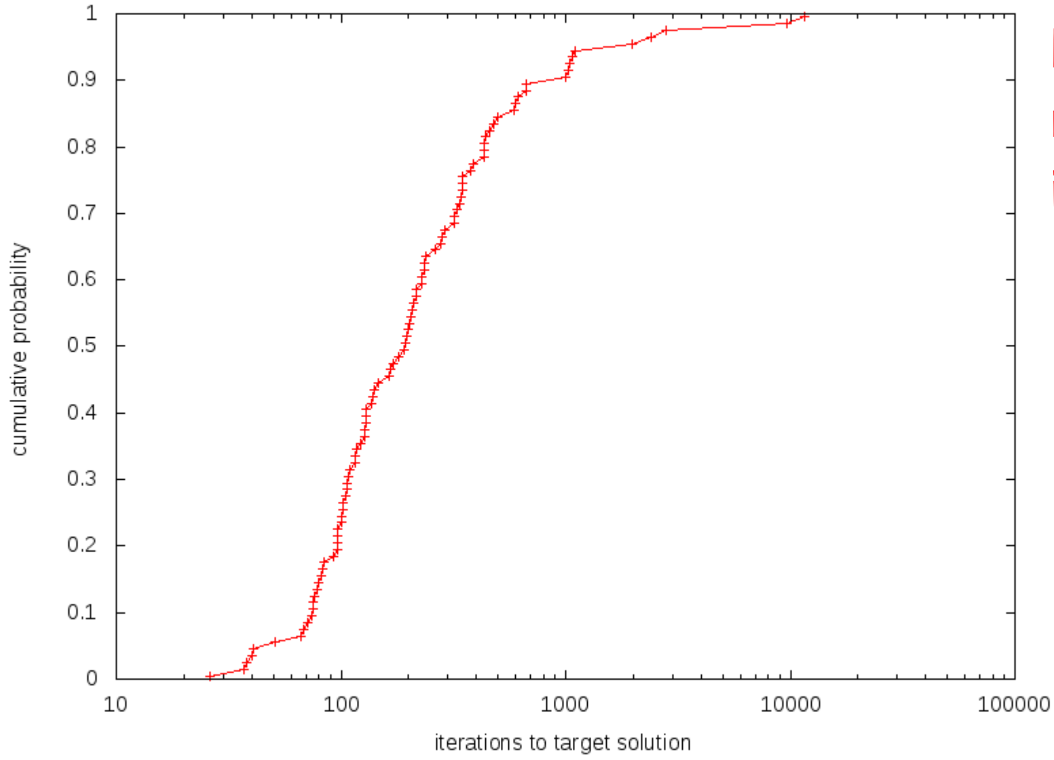
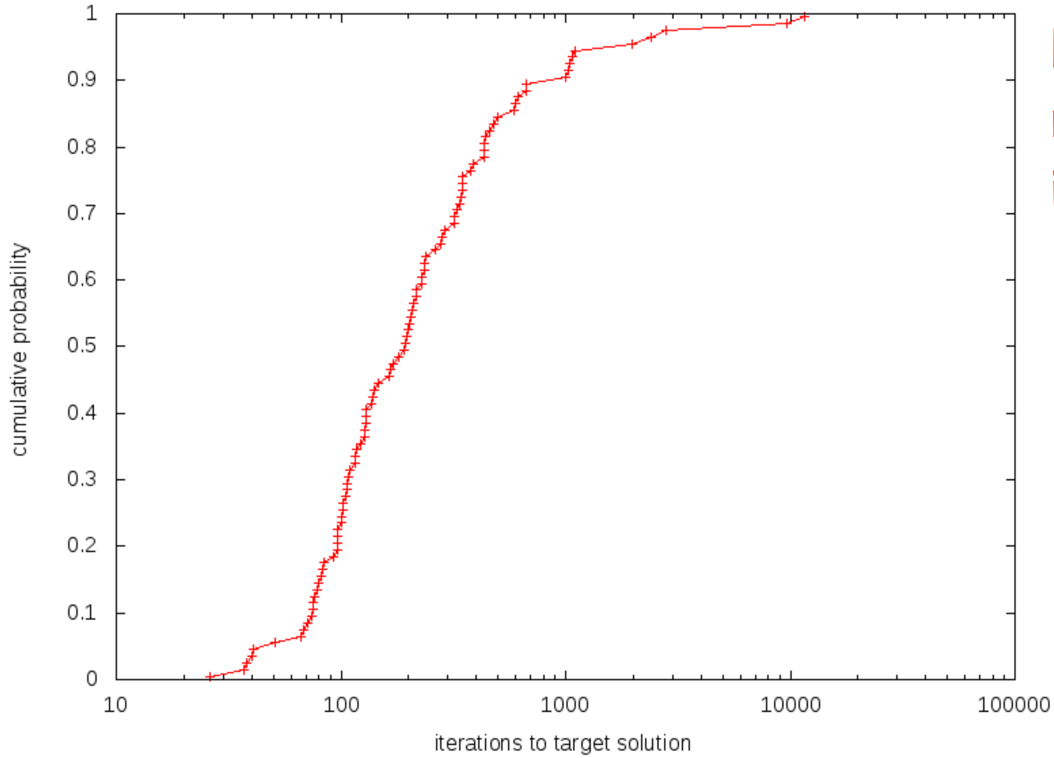


Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$



Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$



Probability that algorithm will still be running after K periods of 345 iterations: $1/4^K$

For example, probability that algorithm with restart will still be running after 1725 iterations (5 periods of 345 iterations): $1/4^5 \cong 0.0977\%$

This is much less than the 5% probability that the algorithm without restart will take over 2000 iterations.

Restart strategies

- First proposed by Luby et al. (1993)
- They define a restart strategy as a finite sequence of time intervals $S = \{\tau_1, \tau_2, \tau_3, \ldots\}$ which define epochs τ_1, $\tau_1+\tau_2$, $\tau_1+\tau_2+\tau_3$, ... when the algorithm is restarted from scratch.
- Luby et al. (1993) prove that the optimal restart strategy uses $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$, where τ^* is a constant.

Restart strategies

- Luby et al. (1993)
- Kautz et al. (2002)
- Palubeckis (2004)
- Sergienko et al. (2004)
- Nowicki & Smutnicki (2005)
- D'Apuzzo et al. (2006)
- Shylo et al. (2011a)
- Shylo et al. (2011b)
- Resende & Ribeiro (2011)

Restart strategy for BRKGA

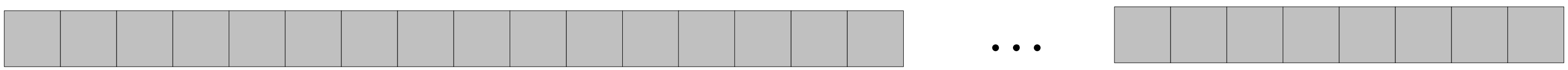
- Recall the restart strategy of Luby et al. where equal time intervals $\tau_1 = \tau_2 = \tau_3 = \cdots = \tau^*$ pass between restarts.
- Strategy requires τ^* as input.
- Since we have no prior information as to the runtime distribution of the heuristic, we run the risk of:
  - choosing τ^* too small: restart variant may take long to converge
  - choosing τ^* too big: restart variant may become like no-restart variant

Restart strategy for BRKGA

- We conjecture that number of iterations between improvement of the incumbent (best so far) solution varies less w.r.t. heuristic/ instance/ target than run times.
- We propose the following restart strategy: Keep track of the last generation when the incumbent improved and restart BRKGA if K generations have gone by without improvement.
- We call this strategy restart(K)

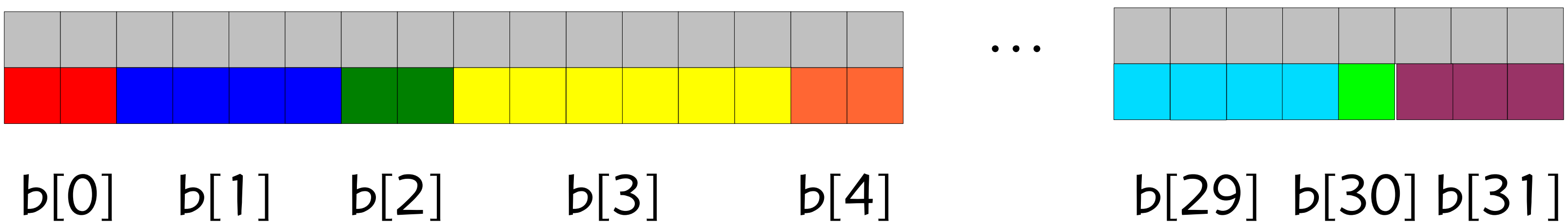
Example of restart strategy for BRKGA: Load balancing

Given an unordered sequence of 1024 integers p[0], p[1], ..., p[1023]



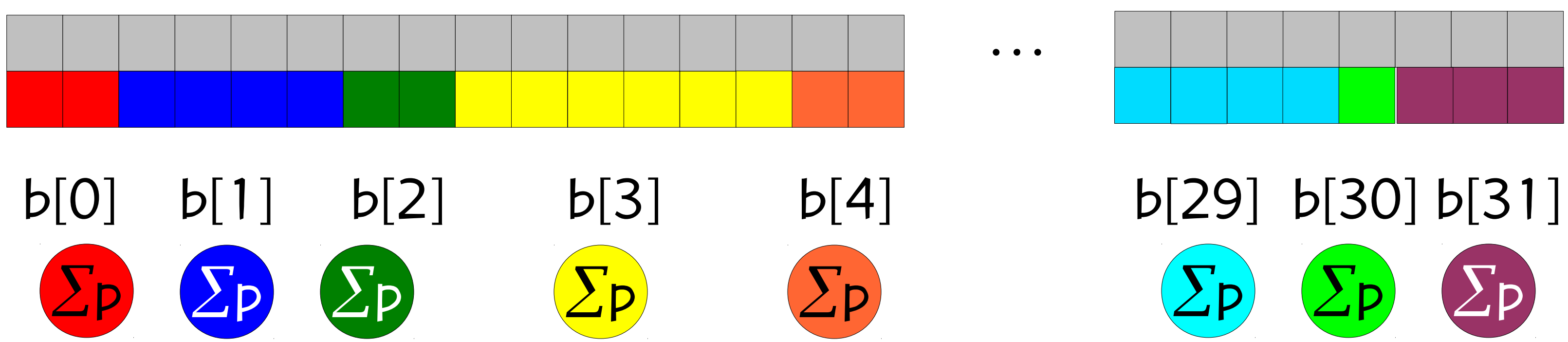
Example of restart strategy for BRKGA: Load balancing

Place consecutive numbers in 32 buckets b[0], b[1], ..., b[31]



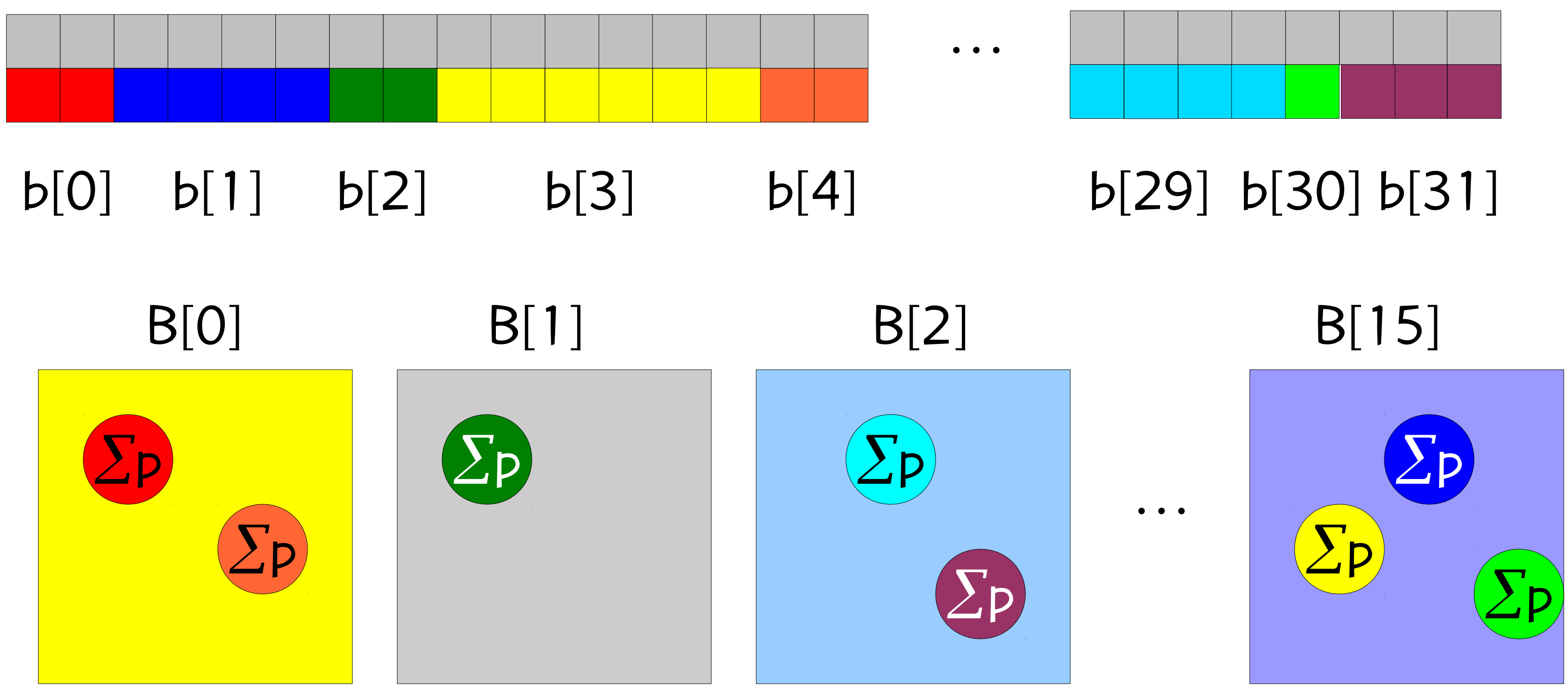
Example of restart strategy for BRKGA: Load balancing

Add the numbers in each bucket b[0], b[1], ..., b[31]



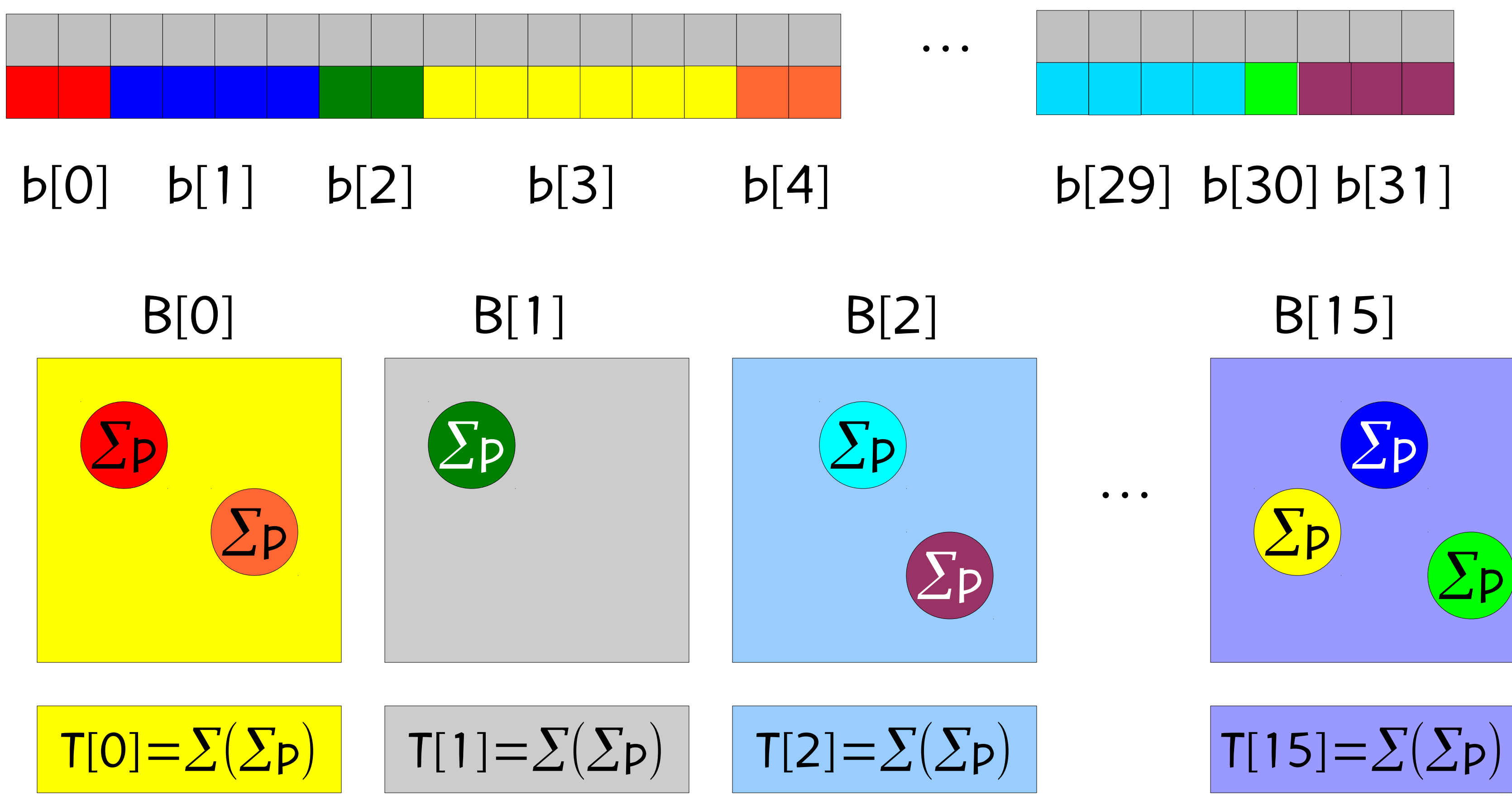
Example of restart strategy for BRKGA: Load balancing

Place the buckets in 16 bins B[0], B[1], ..., B[15]



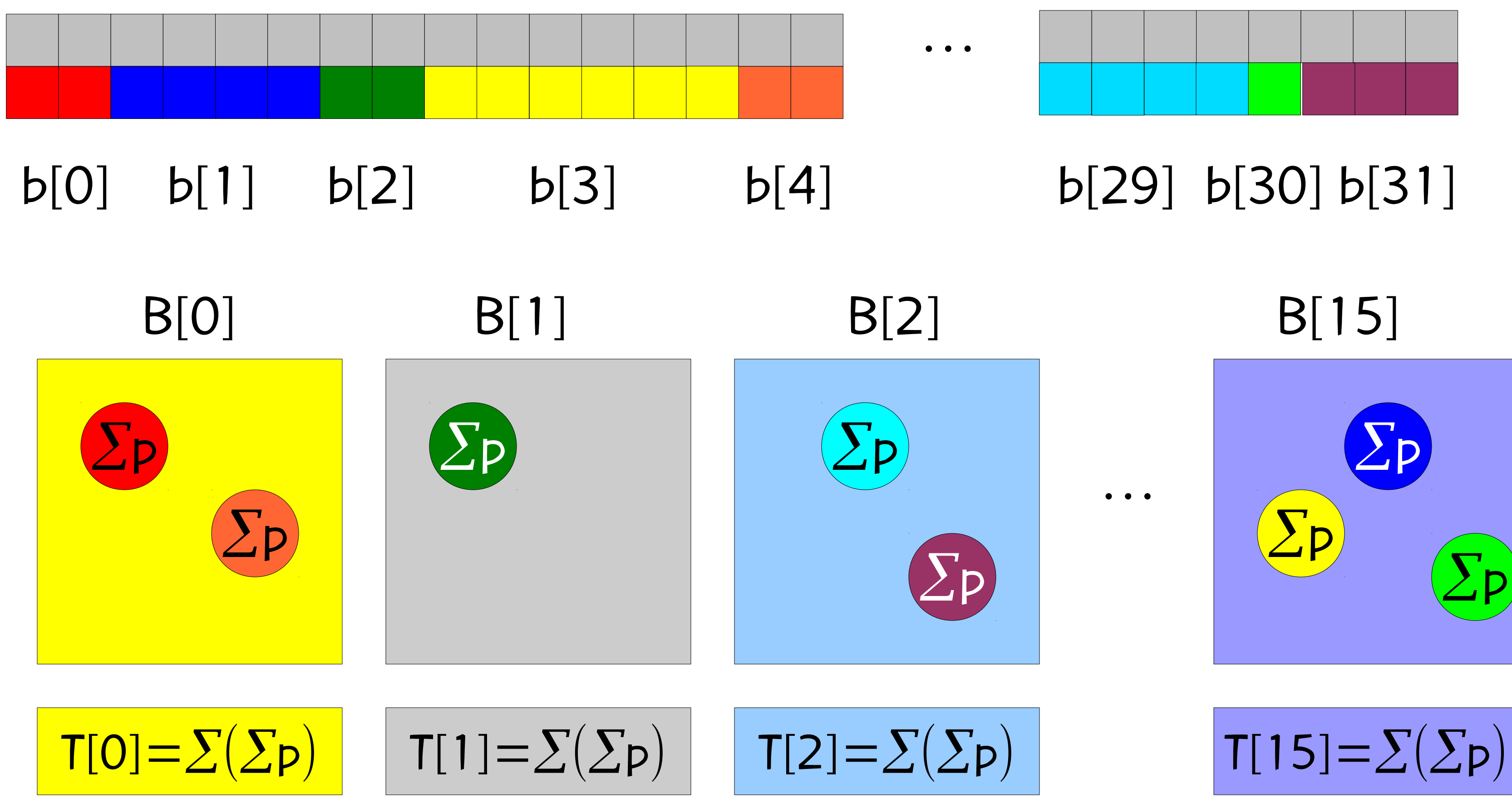
Example of restart strategy for BRKGA: Load balancing

Add up the numbers in each bin B[0], B[1], ..., B[15]



Example of restart strategy for BRKGA: Load balancing

OBJECTIVE: Minimize { Maximum (T[0], T[1], ..., T[15]) }



Example of restart strategy for BRKGA: Load balancing

Encoding

X = [x[1], x[2], ..., x[32] | x[32+1], x[32+2], ..., x[32+16]]

Decoding

x[1], x[2], ..., x[32] are used to define break points for buckets

x[32+1], x[32+2], ..., x[32+16] are used to determine to which bins the buckets are assigned

Example of restart strategy for BRKGA: Load balancing

Encoding

X = [x[1], x[2], ..., x[32] | x[32+1], x[32+2], ..., x[32+16]]

Decoding

x[1], x[2], ..., x[32] are used to define break points for buckets

Size of bucket i = floor (1024 × x[i]/(x[1]+x[2]+···+x[32])), i=1,...,15

Size of bucket 16 = 1024 – sum of sizes of first 15 buckets

Example of restart strategy for BRKGA: Load balancing

Encoding

X = [x[1], x[2], ..., x[32] | x[32+1], x[32+2], ..., x[32+16]]

Decoding

x[1], x[2], ..., x[32] are used to define break points for buckets

x[32+1], x[32+2], ..., x[32+16] are used to determine to which bins the buckets are assigned

Bin that bucket i is assigned to = floor (16 × x[32+i]) + 1

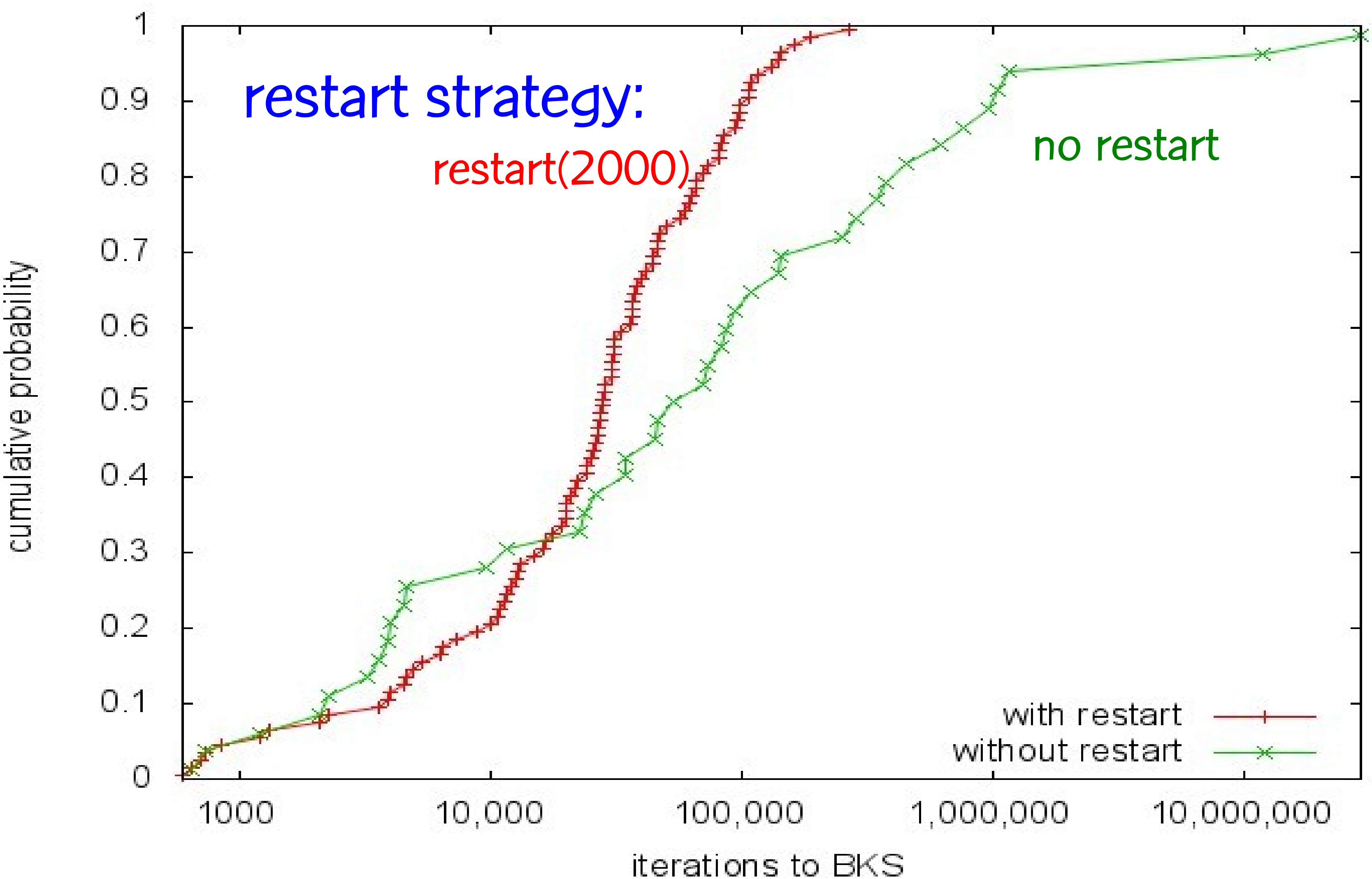
Example of restart strategy for BRKGA: Load balancing

Decoding (Local search phase)

- **while** (there exists a bucket in the most loaded bin that can be moved to another bin and not increase the maximum load) **then**
  - move that bucket to that bin
- **end while**

Make necessary chromosome adjustments to last 16 random keys of vector of random keys to reflect changes made in local search phase: Add or subtract an integer value from chromosome of bucket that moved to new bin.

Example of restart strategy for BRKGA: Load balancing



Specifying a BRKGA

Specifying a biased random-key GA

- Encoding is always done the same way, i.e. with a vector of N random-keys (parameter N must be specified)

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- Decoder that takes as input a vector of N random-keys and outputs the corresponding solution of the combinatorial optimization problem and its cost (this is usually a heuristic)
- Parameters

Specifying a biased random-key GA

Parameters:

- Size of population
- Parallel population parameters
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

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- Size of population: a function of N, say N or 2N
- Parallel population parameters
- Size of elite partition
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
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Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
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- Size of mutant set
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- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
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- Size of elite partition: 15-25% of population
- Size of mutant set
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

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Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , ν = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, $p = 3$, $v = 2$, and $x = 200$
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N, say 2N or 10N
- Stopping criterion

Specifying a biased random-key GA

Parameters:

- Size of population: a function of N, say N or 2N
- Parallel population parameters: say, p = 3 , v = 2, and x = 200
- Size of elite partition: 15-25% of population
- Size of mutant set: 5-15% of population
- Child inheritance probability: > 0.5, say 0.7
- Restart strategy parameter: a function of N, say 2N or 10N
- Stopping criterion: e.g. time, # generations, solution quality, # generations without improvement

brkgaAPI: A C++ API for BRKGA

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA.

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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
  - population management
  - evolutionary dynamics

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- Implemented in C++ and may benefit from shared-memory parallelism if available.

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- Cross-platform library handles large portion of problem independent modules that make up the framework, e.g.
  - population management
  - evolutionary dynamics
- Implemented in C++ and may benefit from shared-memory parallelism if available.
- User only needs to implement problem-dependent decoder.

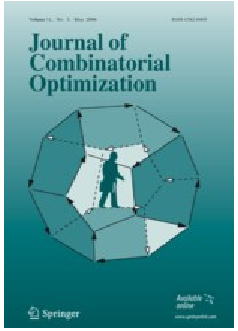
brkgaAPI: A C++ API for BRKGA

Paper: Rodrigo F. Toso and M.G.C.R., "A C++ Application Programming Interface for Biased Random-Key Genetic Algorithms," AT&T Labs Technical Report, Florham Park, August 2011.

Software: http://www.research.att.com/~mgcr/src/brkgaAPI

An example BRKGA: Packing weighted rectangles

Reference



J.F. Gonçalves and M.G.C.R., "A parallel multi-population genetic algorithm for a constrained two-dimensional orthogonal packing problem," Journal of Combinatorial Optimization, vol. 22, pp. 180-201, 2011.

Tech report:

http://www.research.att.com/~mgcr/doc/pack2d.pdf

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Constrained orthogonal packing

- Given a large planar stock rectangle (W, H) of width W and height H;

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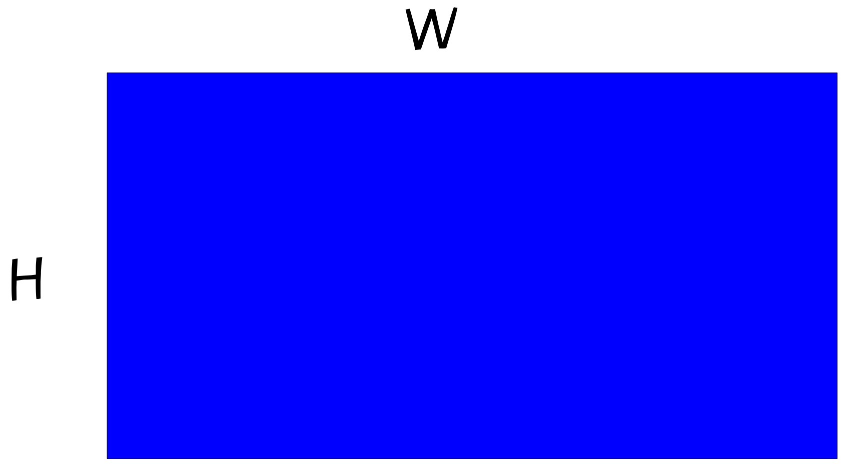
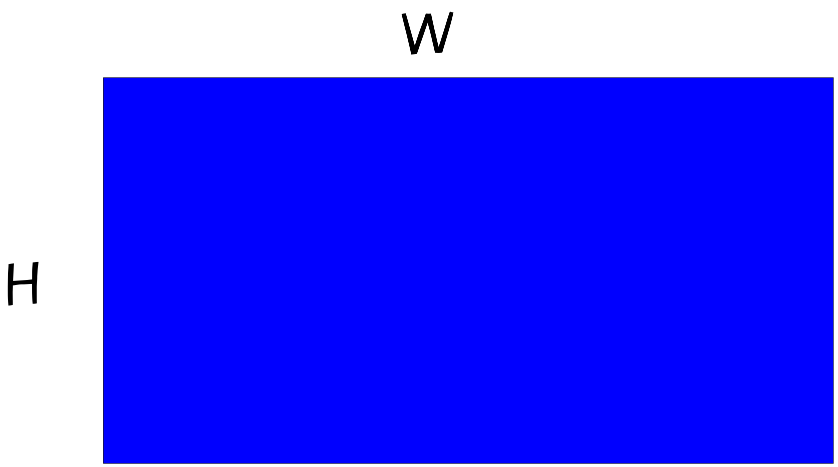


Constrained orthogonal packing

- Given a large planar stock rectangle (W, H) of width W and height H;
- Given N smaller rectangle types (w[i], h[i]), i = 1,...,N, each of width w[i], height h[i], and value v[i];



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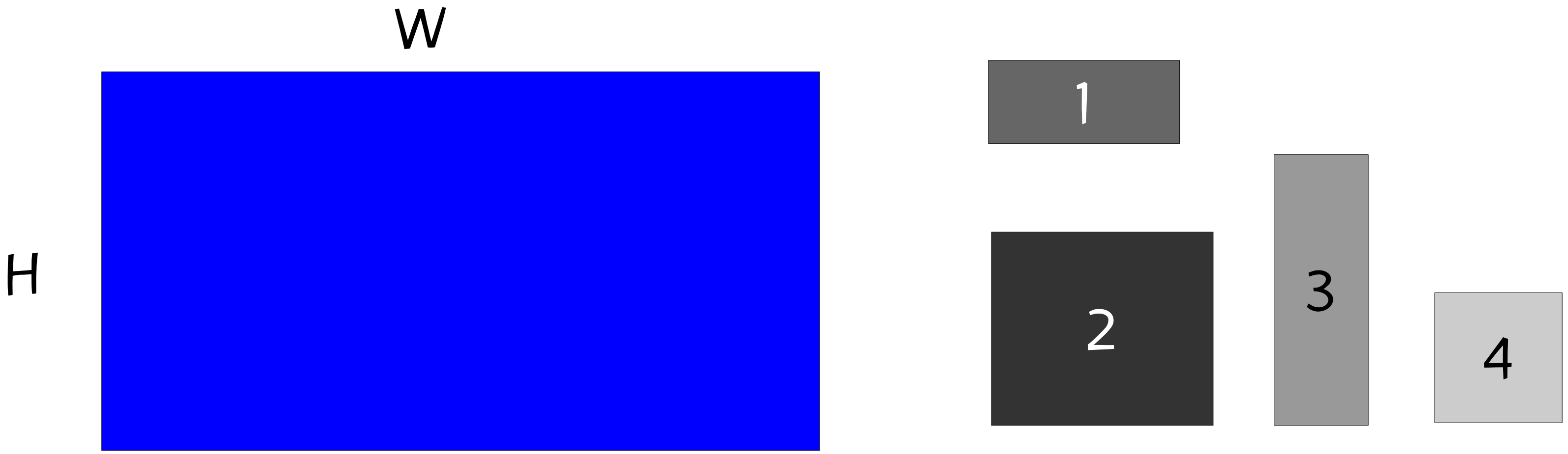
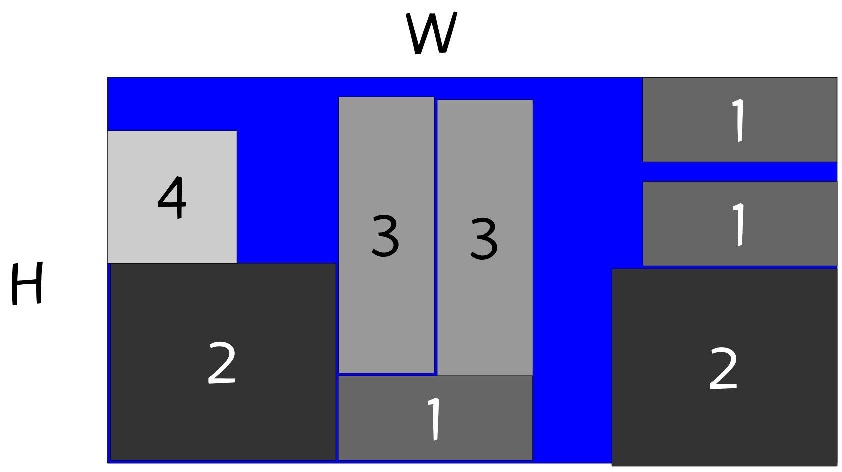


Constrained orthogonal packing

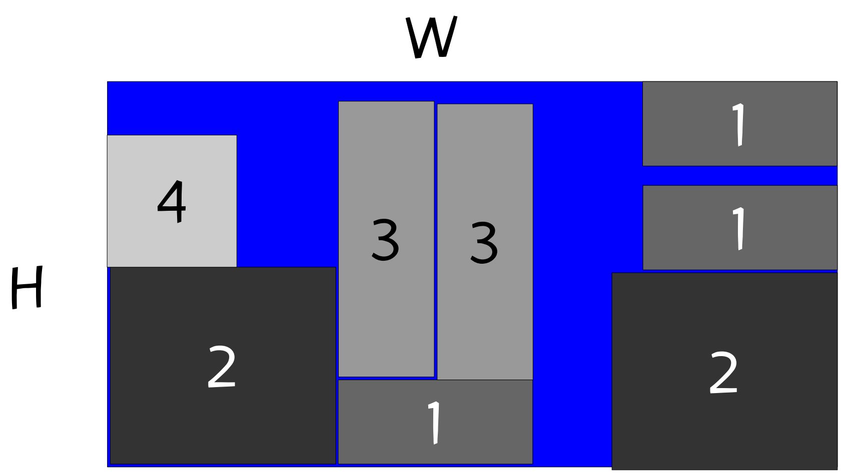
- r[i] rectangles of type i = 1, ..., N are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;



Constrained orthogonal packing

- $r[i]$ rectangles of type $i = 1, \ldots, N$ are to be packed in the large rectangle without overlap and such that their edges are parallel to the edges of the large rectangle;
- For $i = 1, \ldots, N$, we require that:

$$0 \leq P[i] \leq r[i] \leq Q[i]$$



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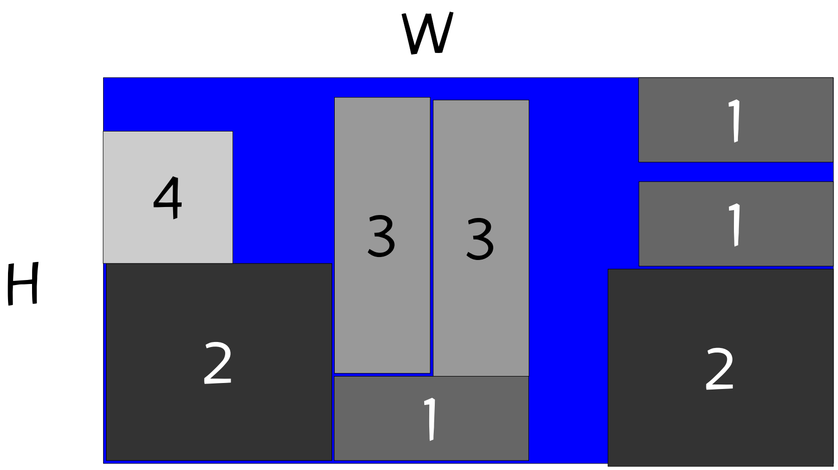

Suppose $5 \leq r[1] \leq 12$

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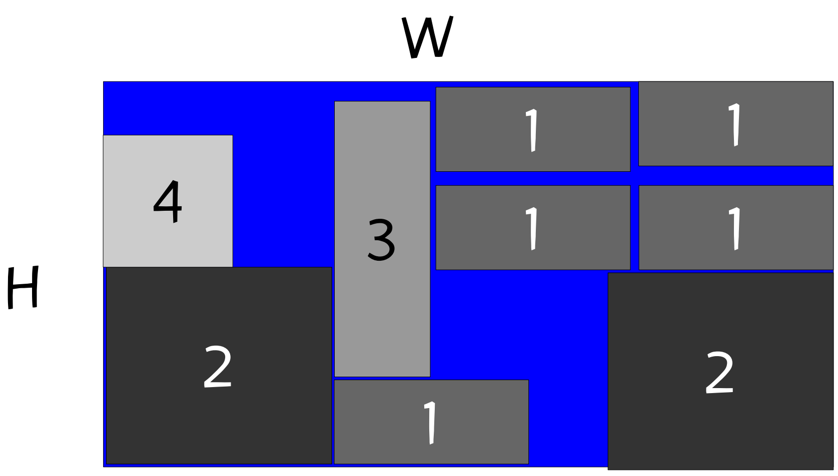
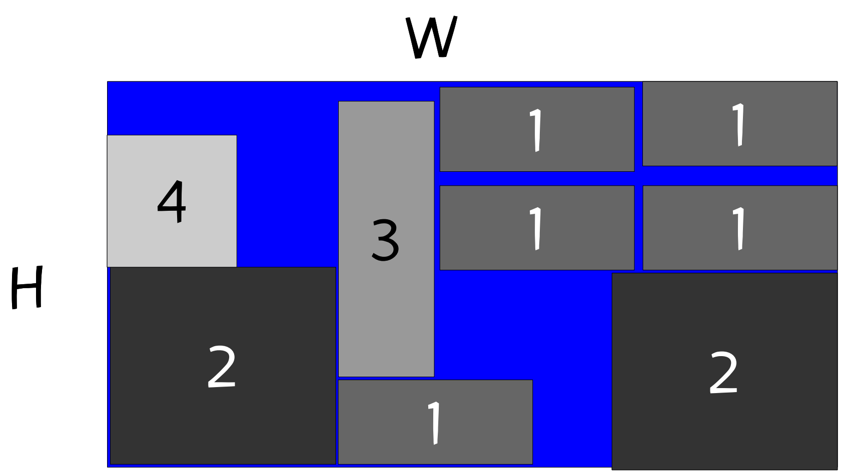


Suppose $5 \leq r[1] \leq 12$

Objective

Among the many feasible packings, we want to find one that maximizes total value of packed rectangles:

$$v[1]\, r[1] + v[2]\, r[2] + \cdots + v[N]\, r[N]$$



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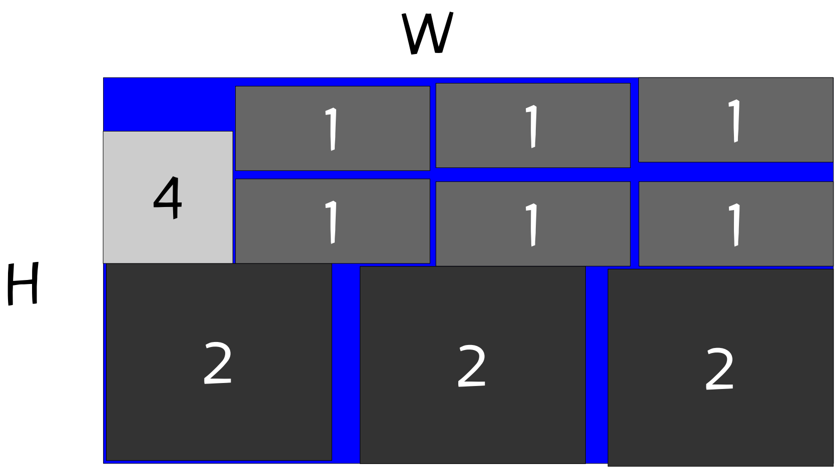

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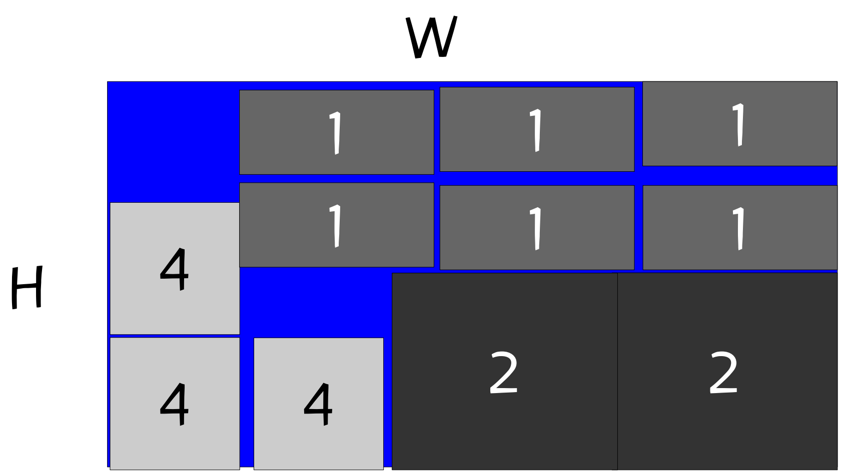

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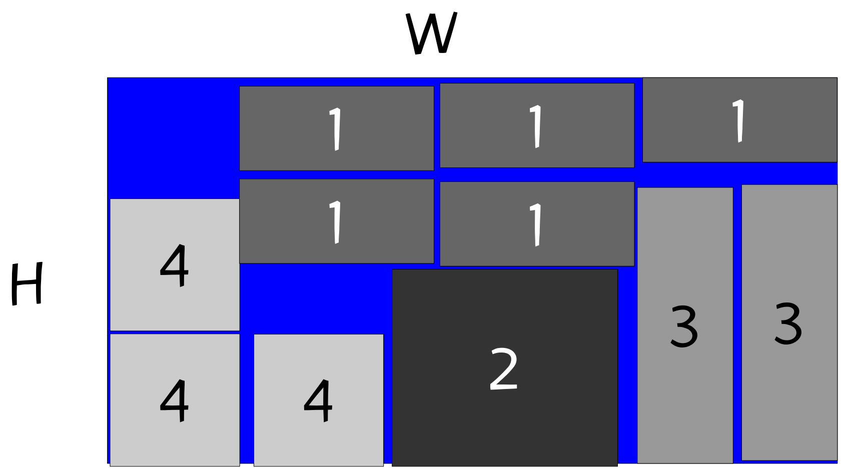
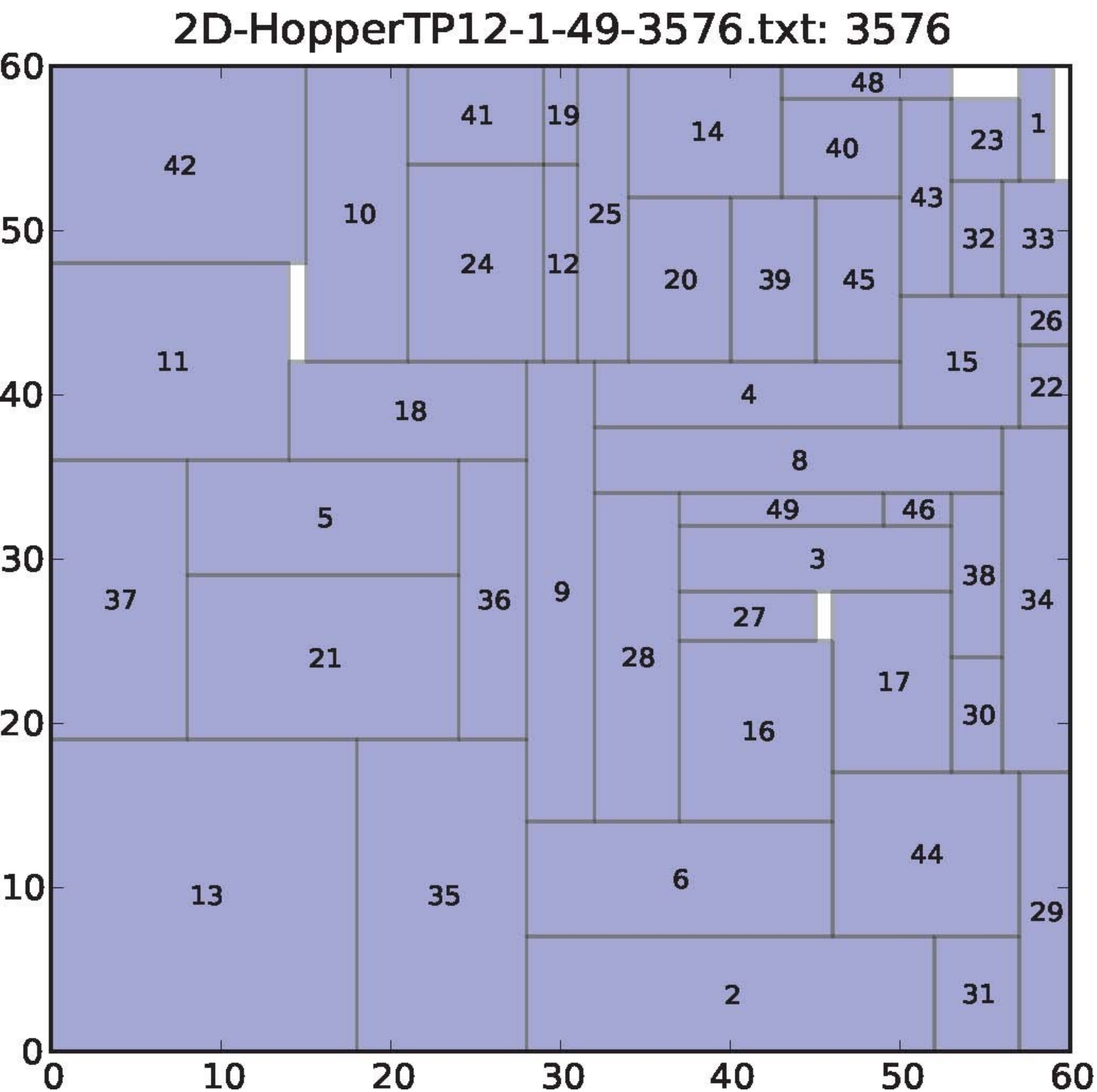


Applications

Problem arises in several production processes, e.g.

- Textile
- Glass
- Wood
- Paper

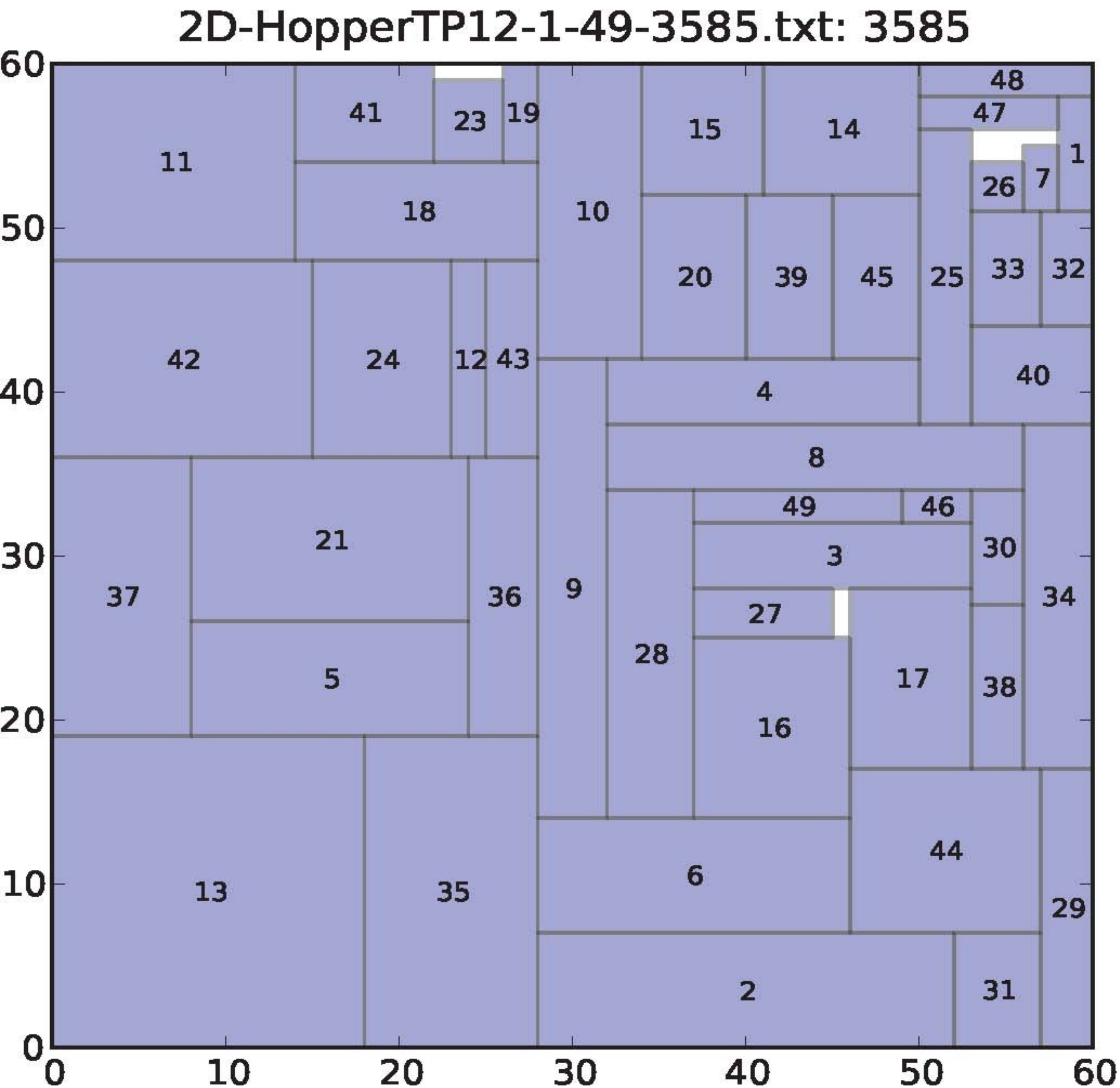
where rectangular figures are cut from large rectangular sheets of materials.



Hopper & Turton, 2001
Instance 4-1 60 x 60
Value: 3576

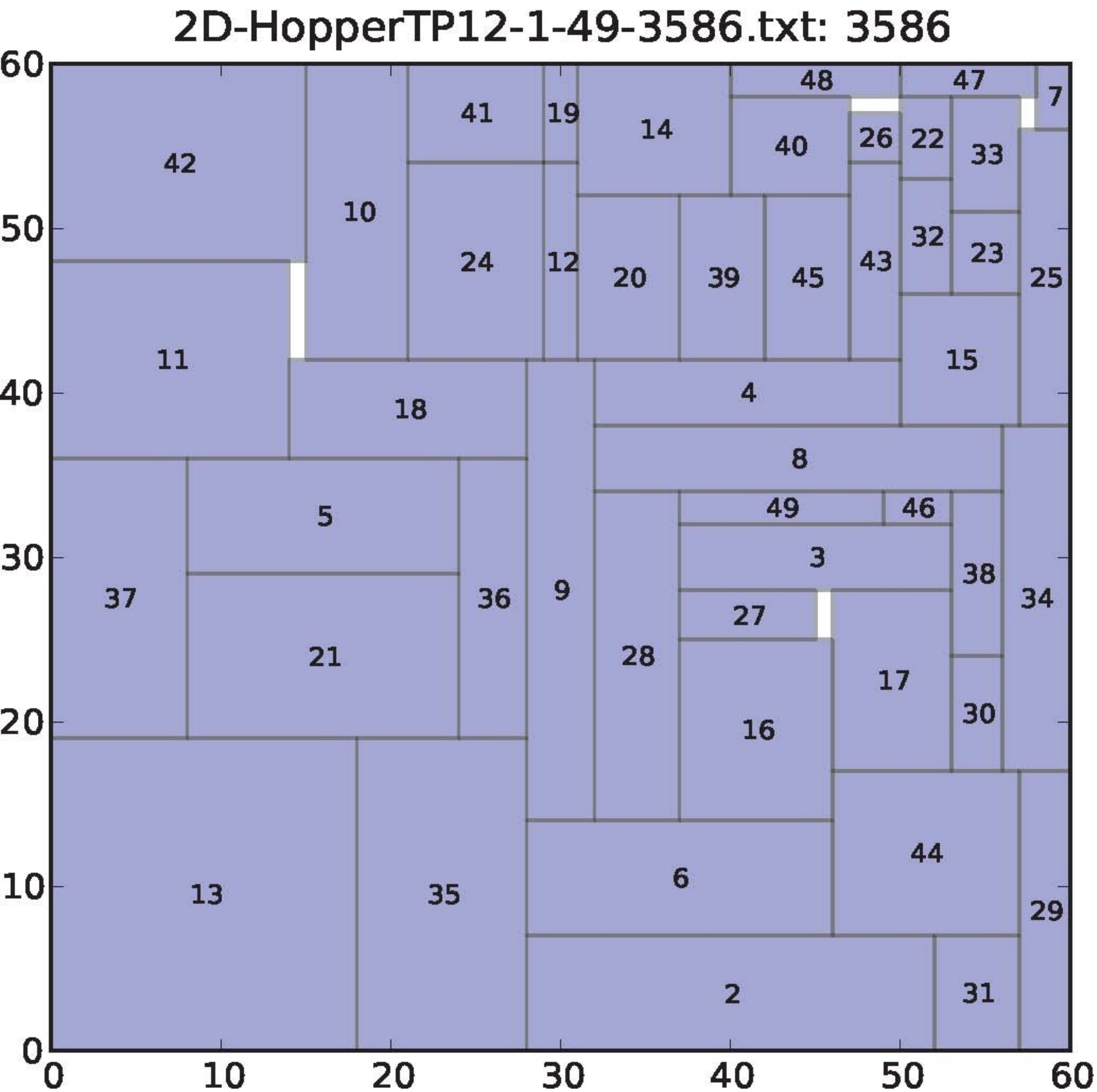
Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

BRKGA



Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3585

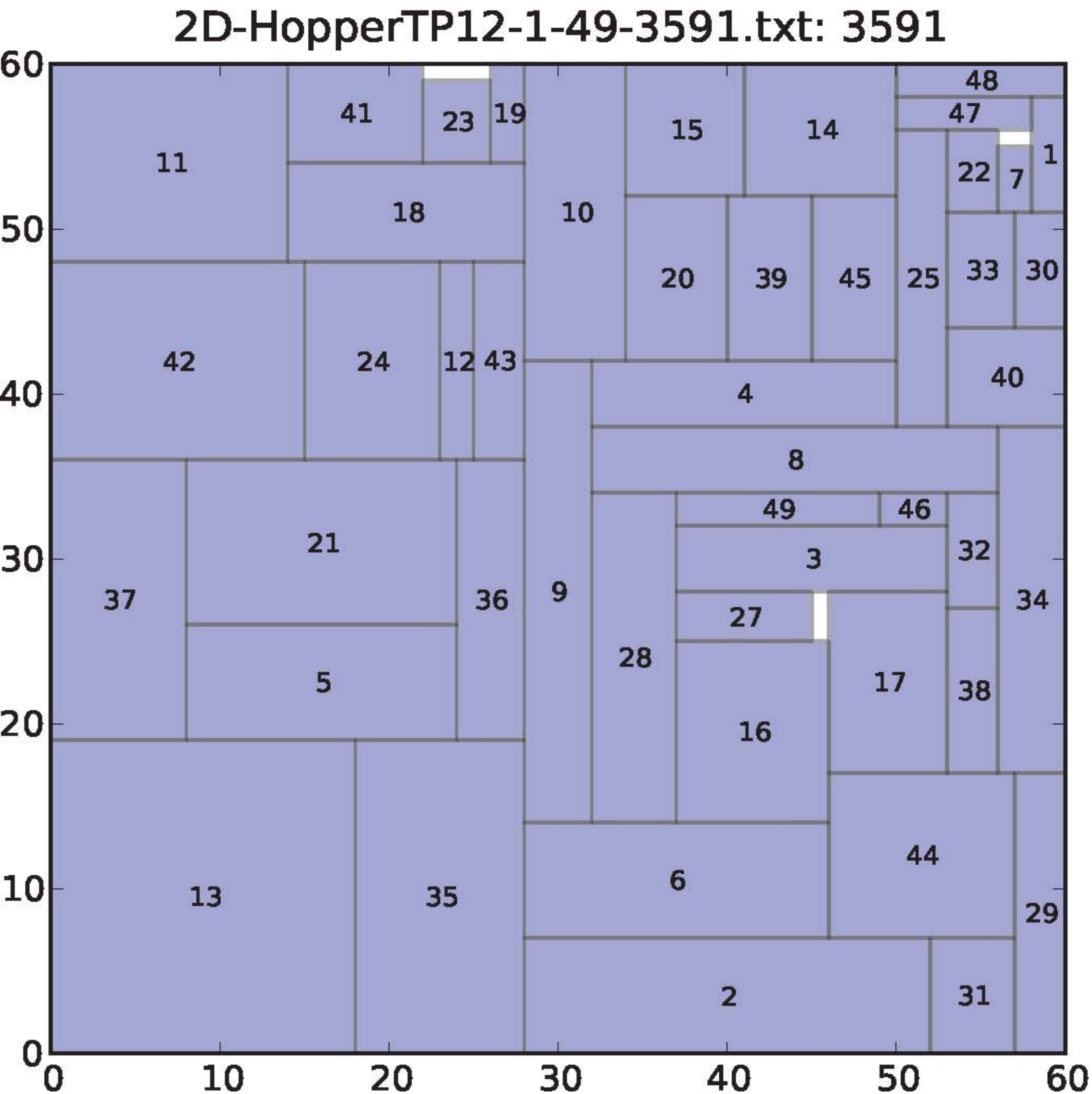
Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)



Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3586

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

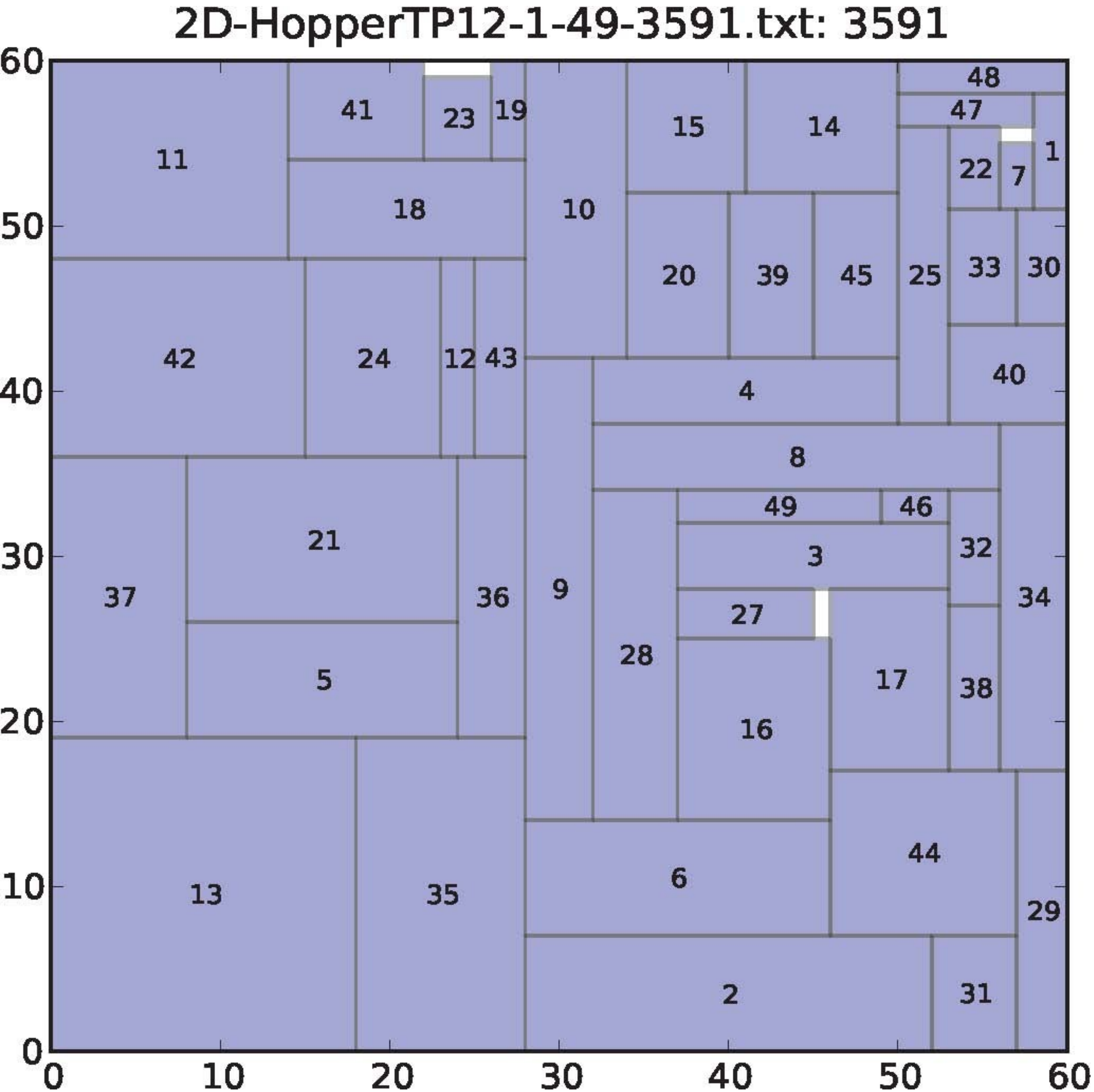
BRKGA



Hopper & Turton, 2001
Instance 4-2 60 x 60
Value: 3591

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

BRKGA



Hopper & Turton, 2001

Instance 4-2 60 x 60

Value: 3591

New best known solution!

Previous best: 3580 by a Tabu Search heuristic (Alvarez-Valdes et al., 2007)

BRKGA

BRKGA for constrained 2-dim orthogonal packing

Encoding

- Solutions are encoded as vectors K of

$$2N' = 2 \{ Q[1] + Q[2] + \cdots + Q[N] \}$$

random keys, where Q[i] is the maximum number of rectangles of type i (for i = 1, ..., N) that can be packed.

- K = (k[1], ..., k[N'], k[N'+1], ..., k[2N'])

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Rectangle type packing sequence (RTPS)

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random keys, where Q[i] is the maximum number of rectangles of type i (for i = 1, ..., N) that can be packed.
- $K = (\underbrace{k[1], \ldots, k[N'],}_{\text{Rectangle type packing sequence (RTPS)}} \quad \underbrace{k[N'+1], \ldots, k[2N']}_{\text{Vector of placement procedures (VPP)}})$

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Decoding

- Simple heuristic to pack rectangles:
  - Make Q[i] copies of rectangle i, for i = 1, ..., N.
  - Order the $N' = Q[1] + Q[2] + \cdots + Q[N]$ rectangles in some way.
  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order.

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  - Process the rectangles in the above order. Place the rectangle in the stock rectangle according to one of the following heuristics: bottom-left (BL) or left-bottom (LB). If rectangle cannot be positioned, discard it and go on to the next rectangle in the order. **Use the last N' keys to determine which heuristic to use. If k[N'+i] > 0.5 use LB, else use BL.**

Decoding

- A maximal empty rectangular space (ERS) is an empty rectangular space not contained in any other ERS.
- ERSs are generated and updated using the Difference Process of Lai and Chan (1997).
- When placing a rectangle, we limit ourselves only to maximal ERSs. We order all the maximal ERSs and place the rectangle in the first maximal ERS in which it fits.
- Let (x[i], y[i]) be the coordinates of the bottom left corner of the i-th ERS.

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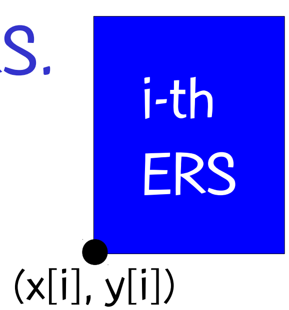

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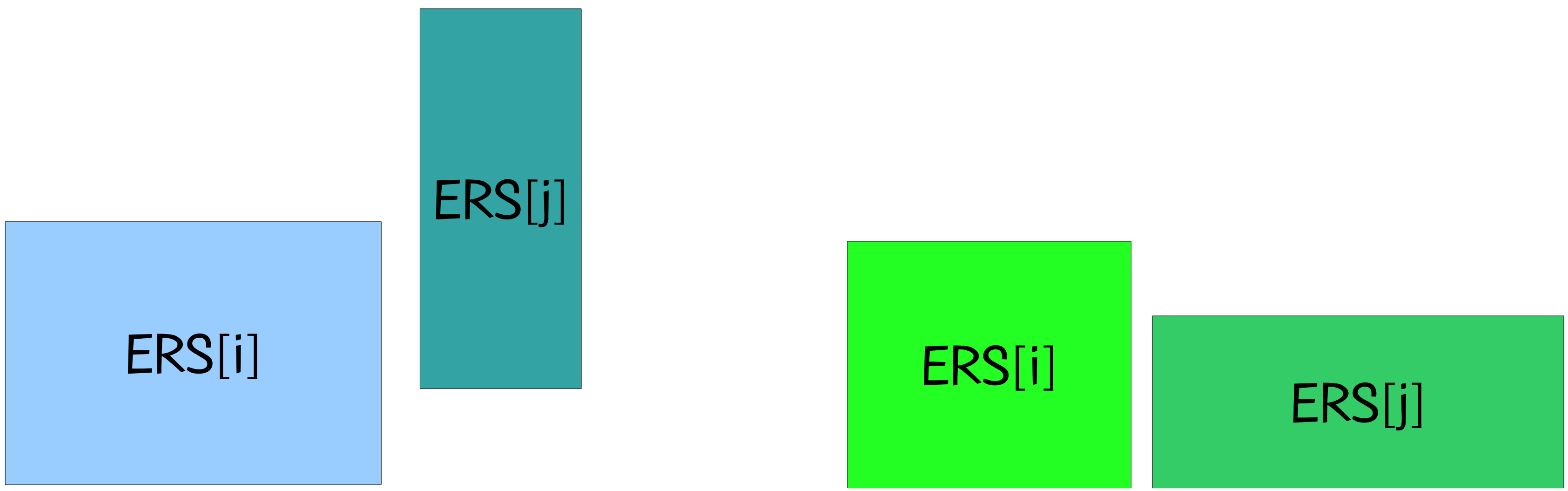
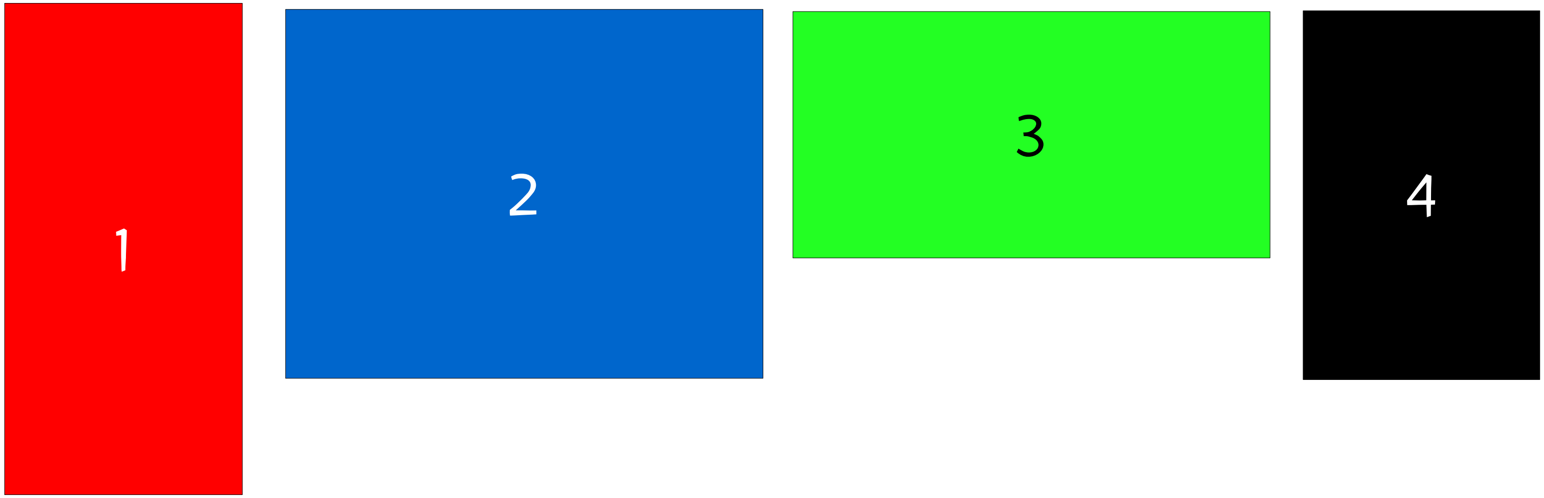
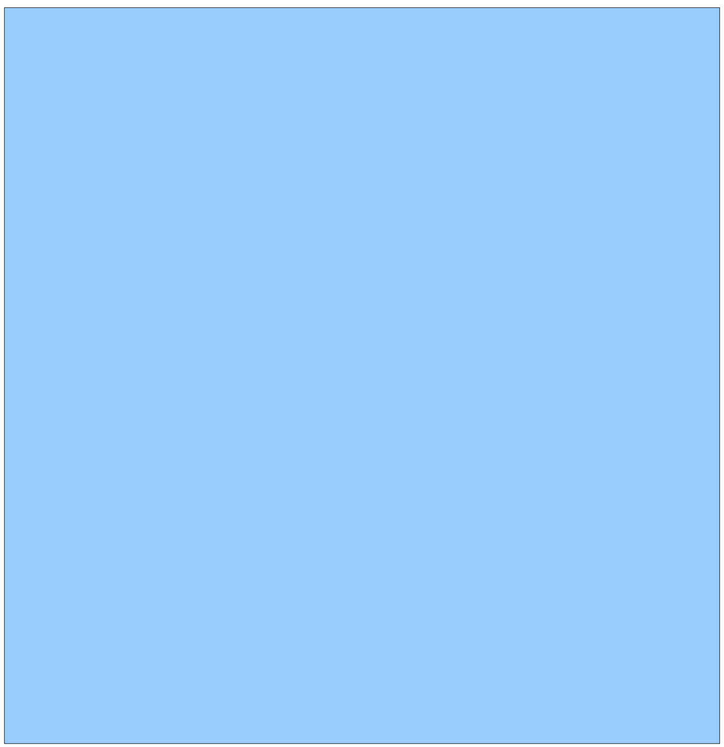


ERS[i] < ERS[j]

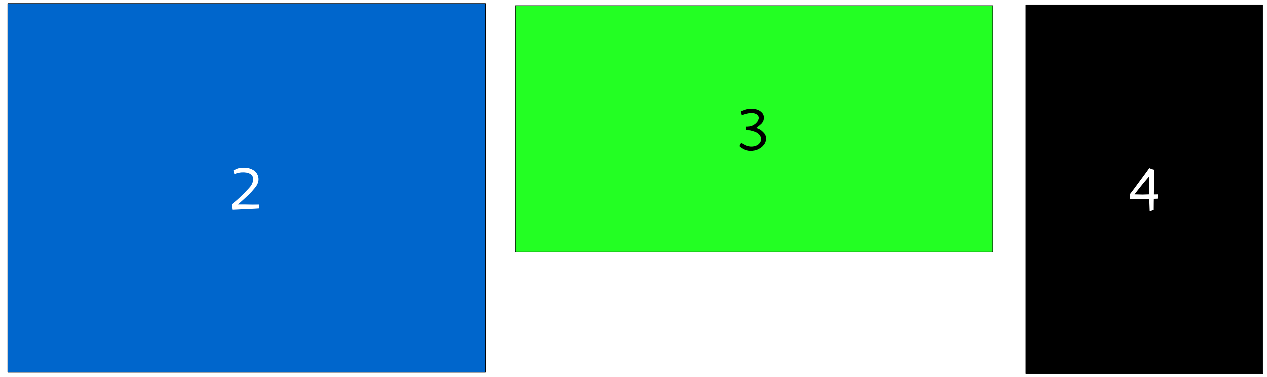


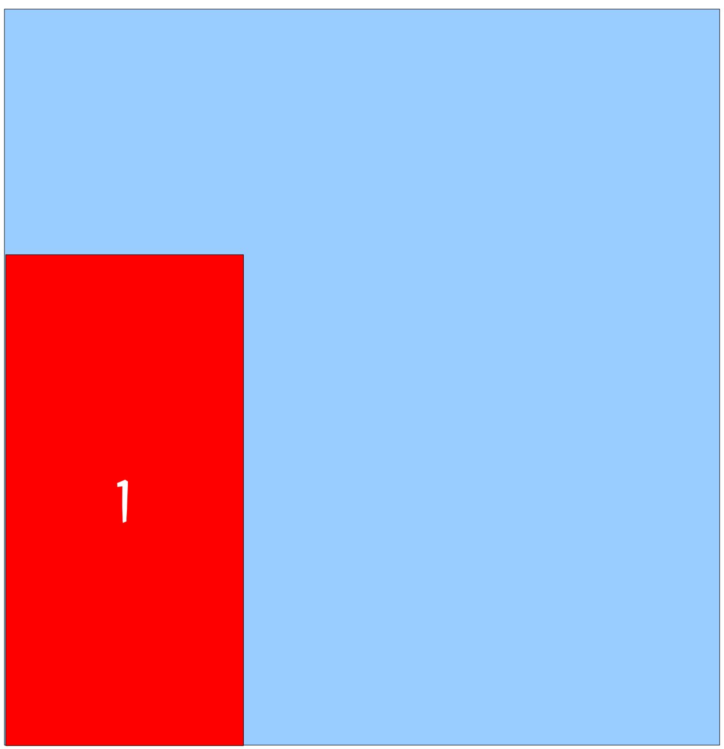


BL can run into problems even on small instances (Liu & Teng, 1999).

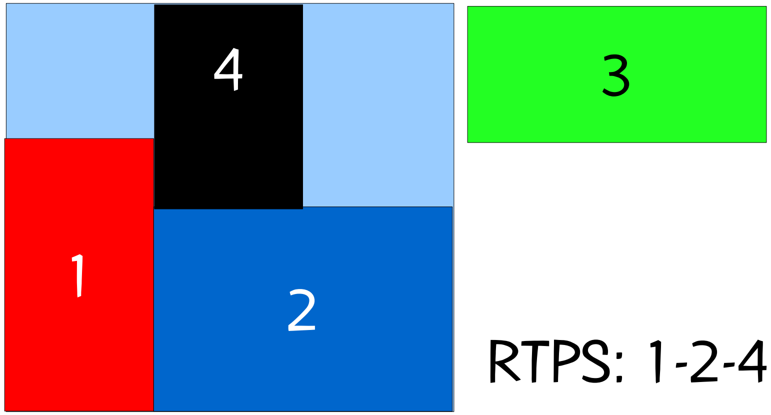
Consider this instance with 4 rectangles.

BL cannot find the optimal solution for any RTPS.

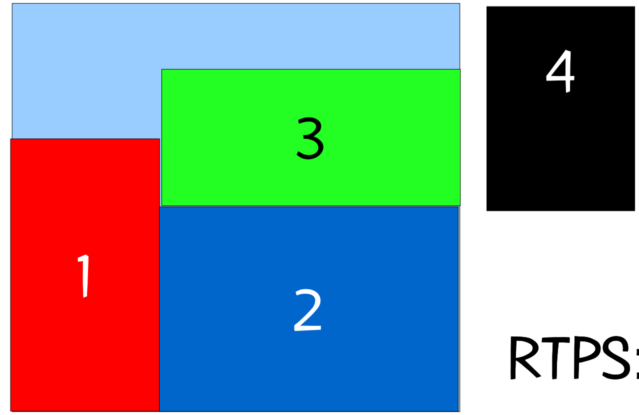




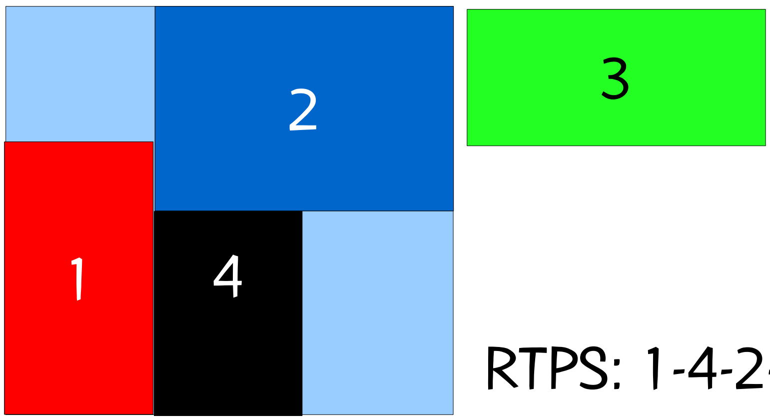
We show 6 rectangle type packing sequences (RTPS's) where we fix rectangle 1 in the first position.



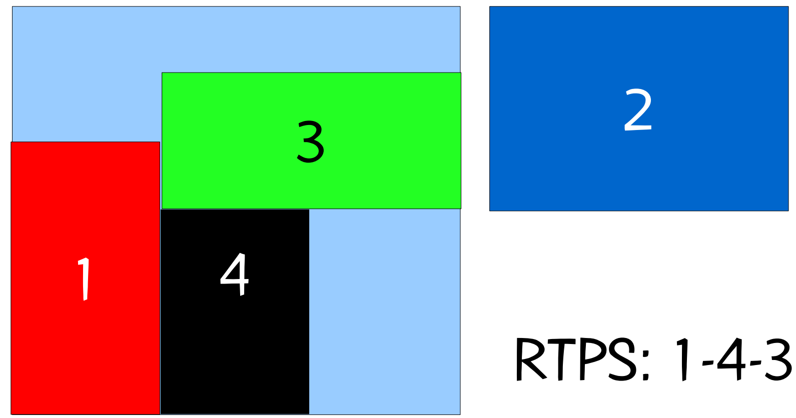
RTPS: 1-2-4-3



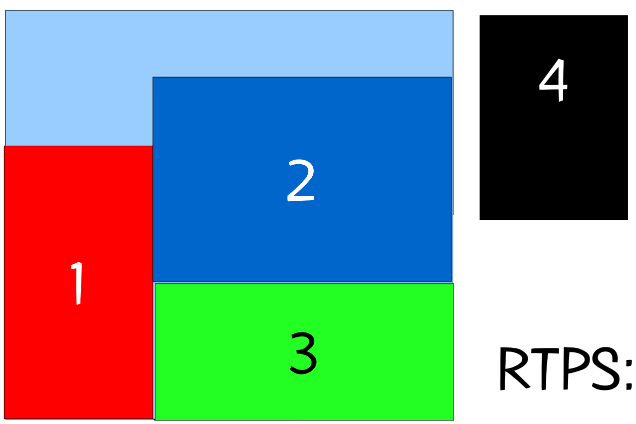
RTPS: 1-2-3-4



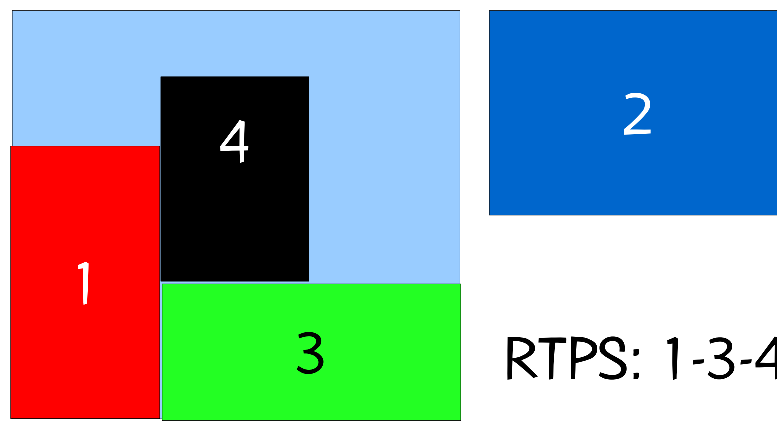
RTPS: 1-4-2-3



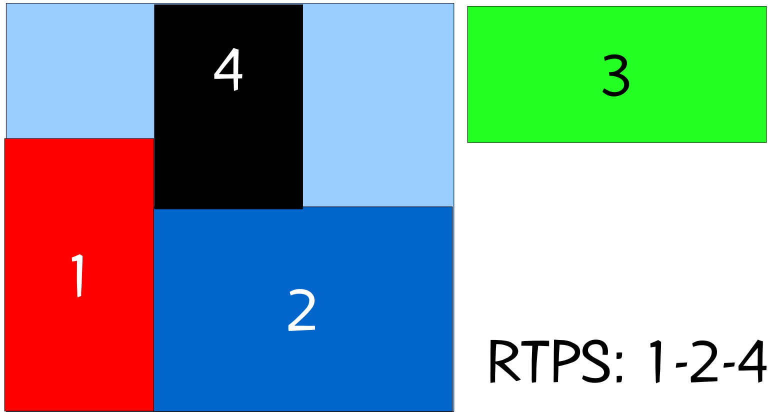
RTPS: 1-4-3-2



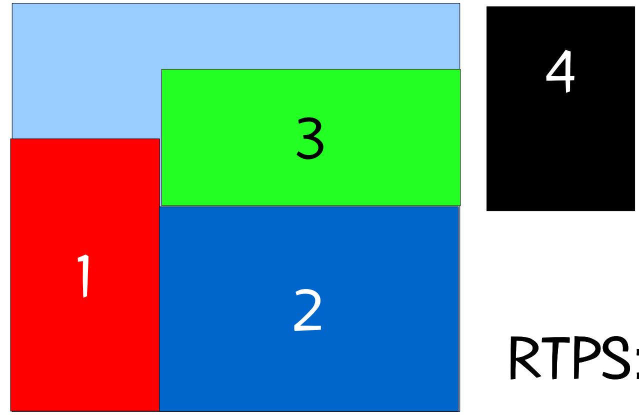
RTPS: 1-3-2-4



RTPS: 1-3-4-2

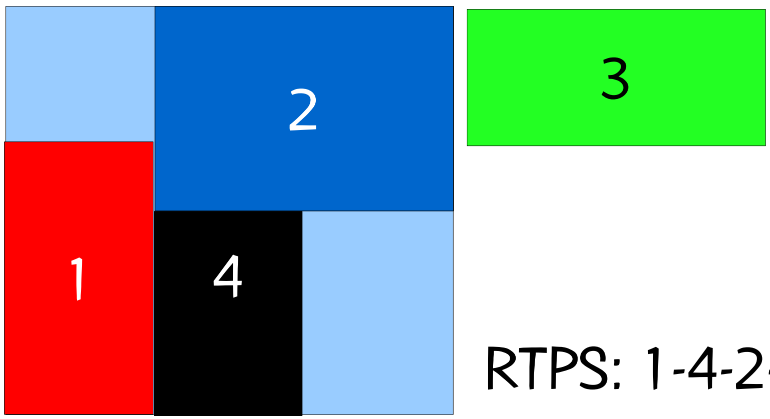


RTPS: 1-2-4-3

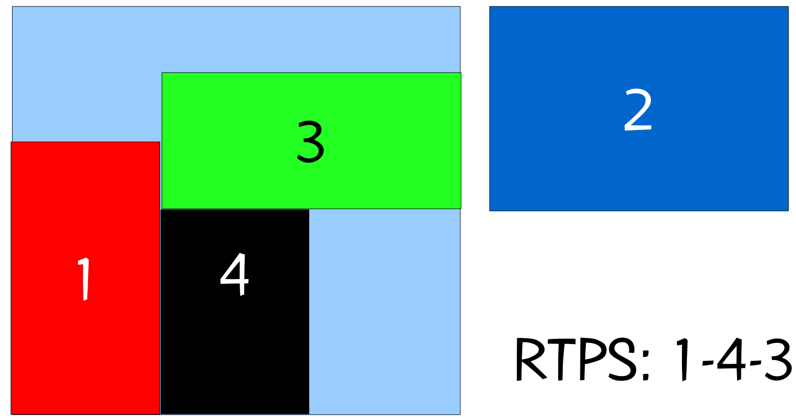


RTPS: 1-2-3-4

Similar infeasibilities are observed if 2, 3, or 4 is the first rectangle in the RTPS.



RTPS: 1-4-2-3



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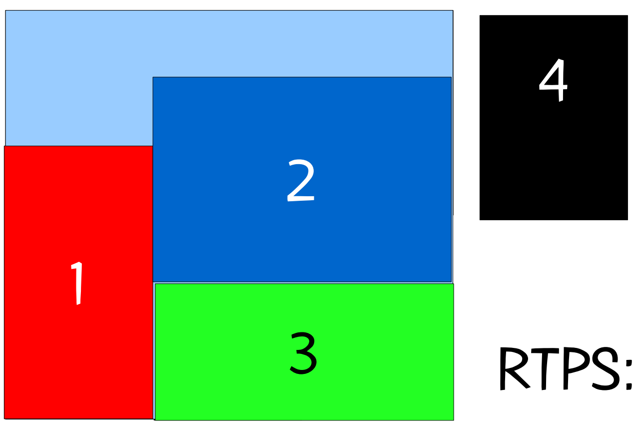

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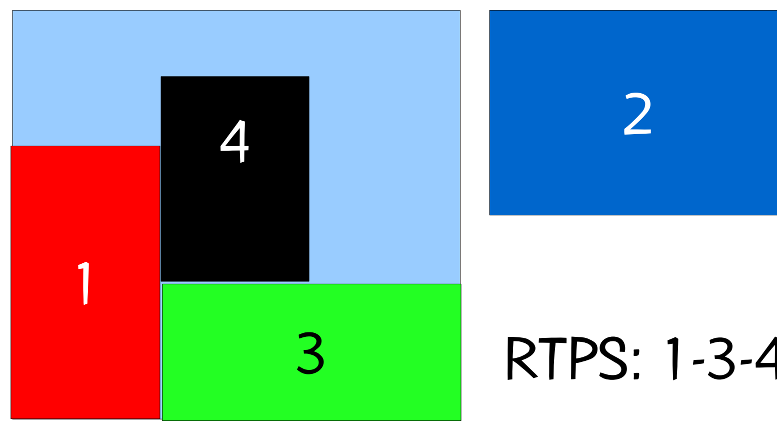

RTPS: 1-3-4-2

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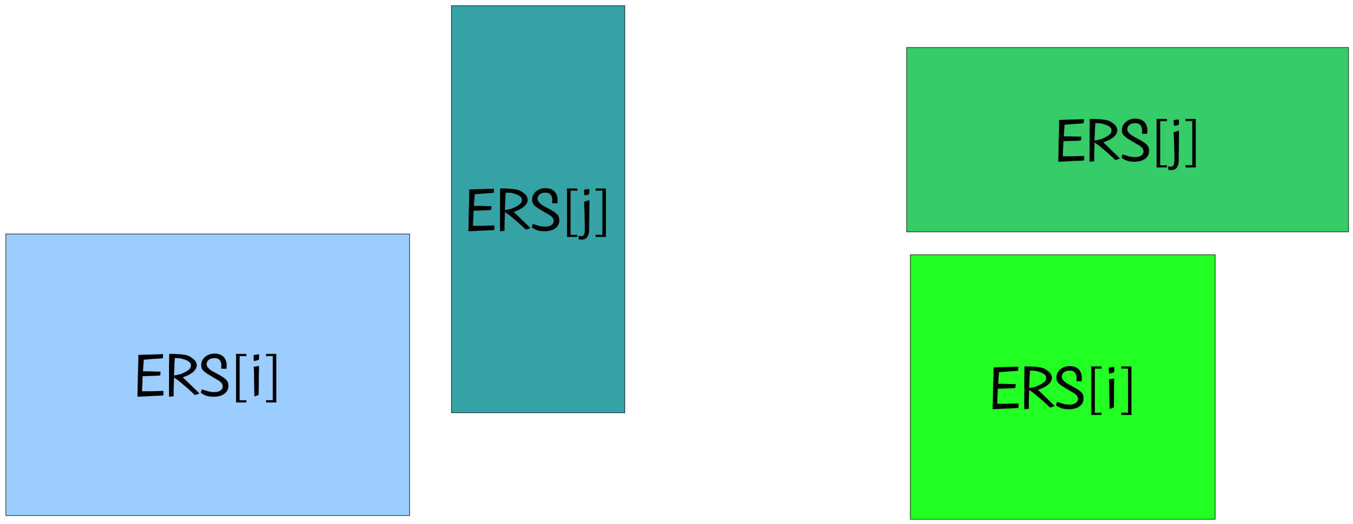


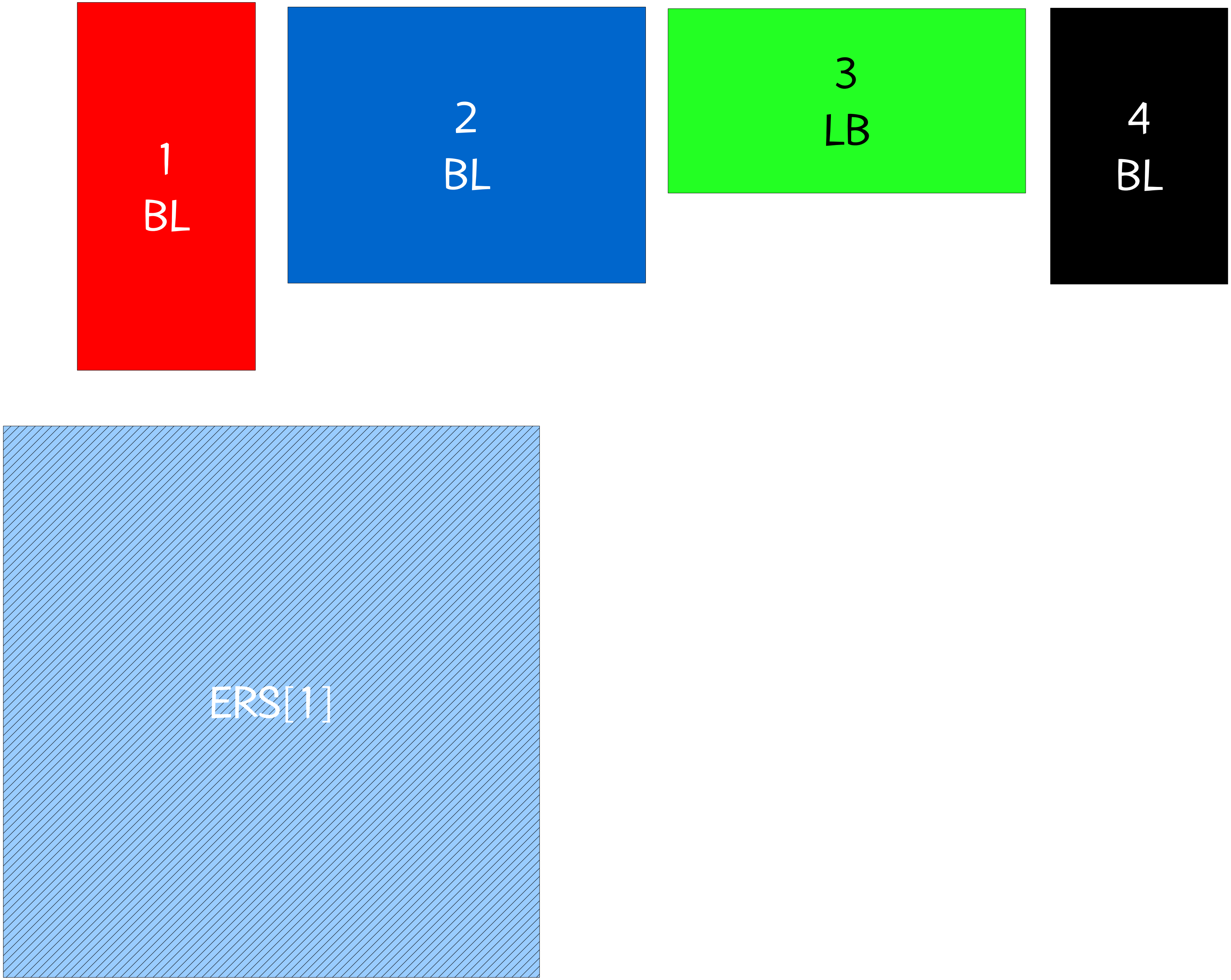
ERS[i] < ERS[j]

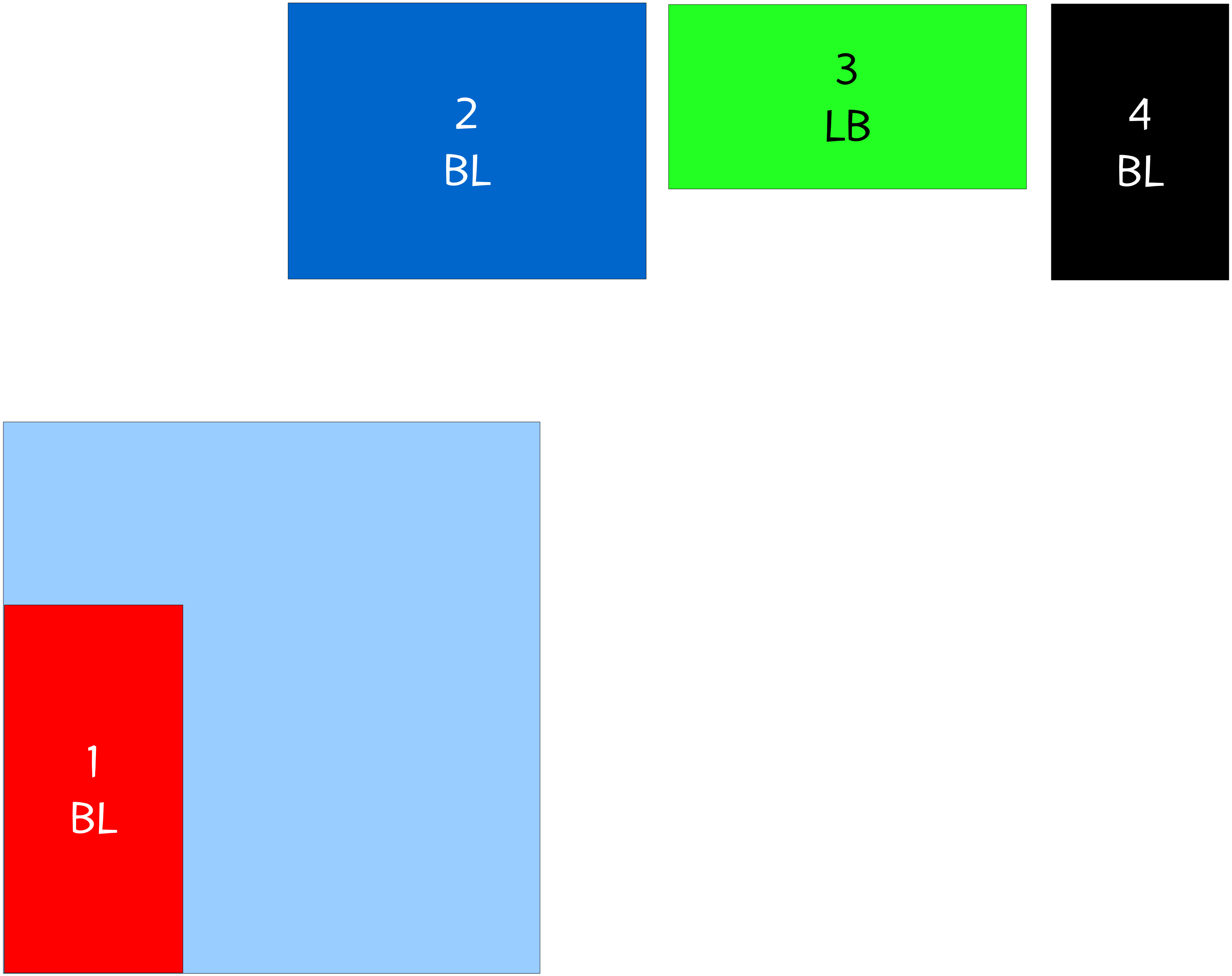
1
BL

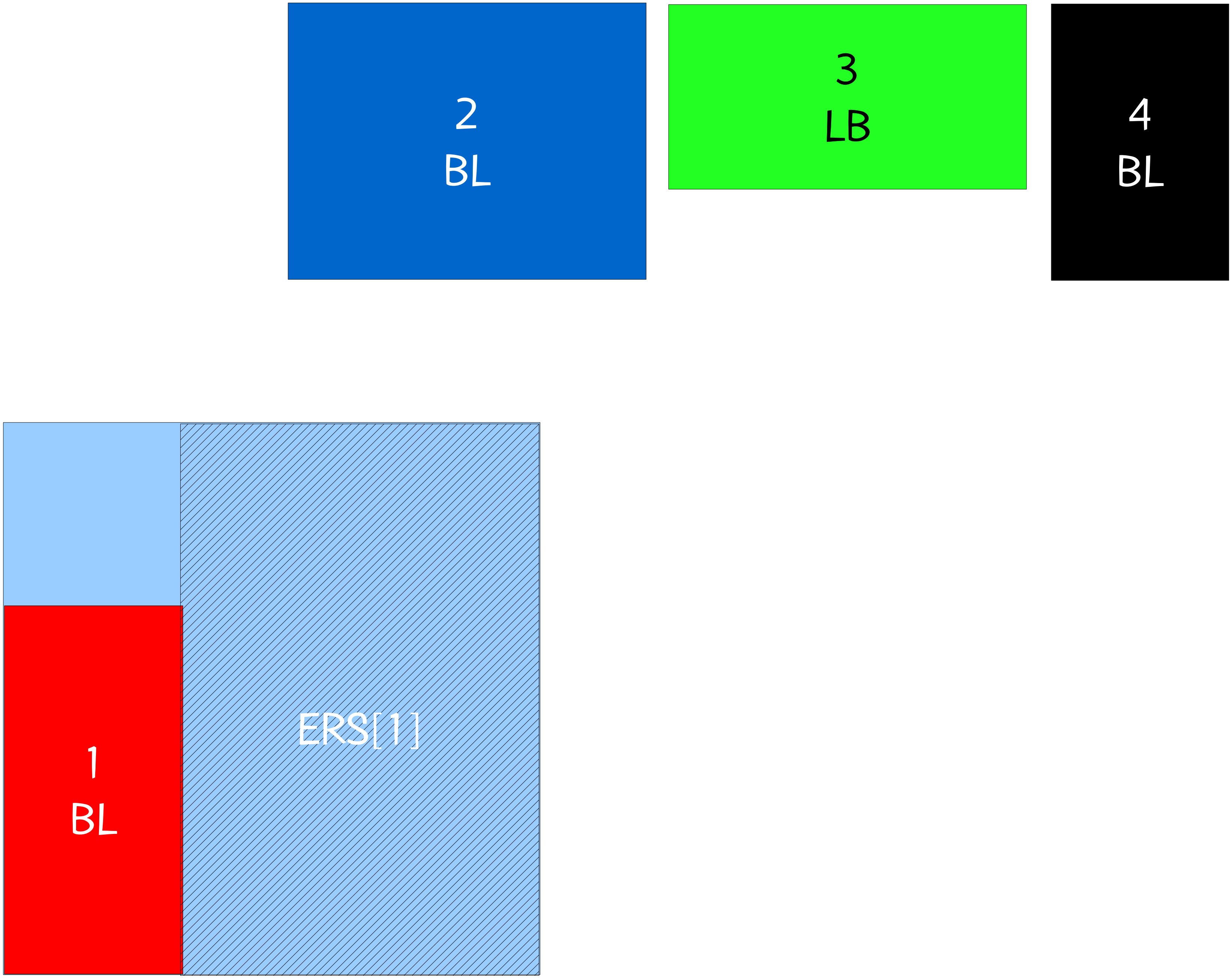
2
BL

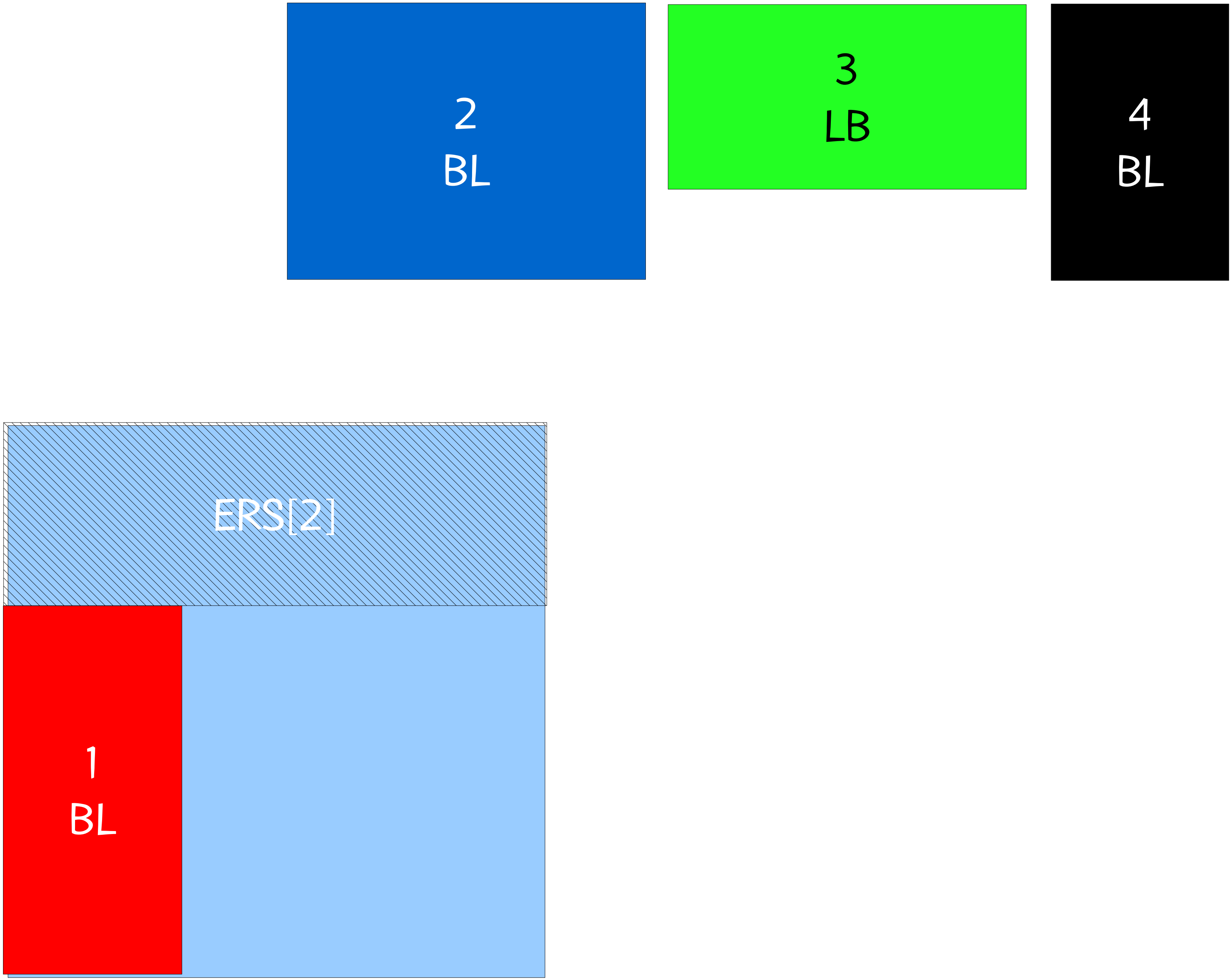
3
LB

4
BL

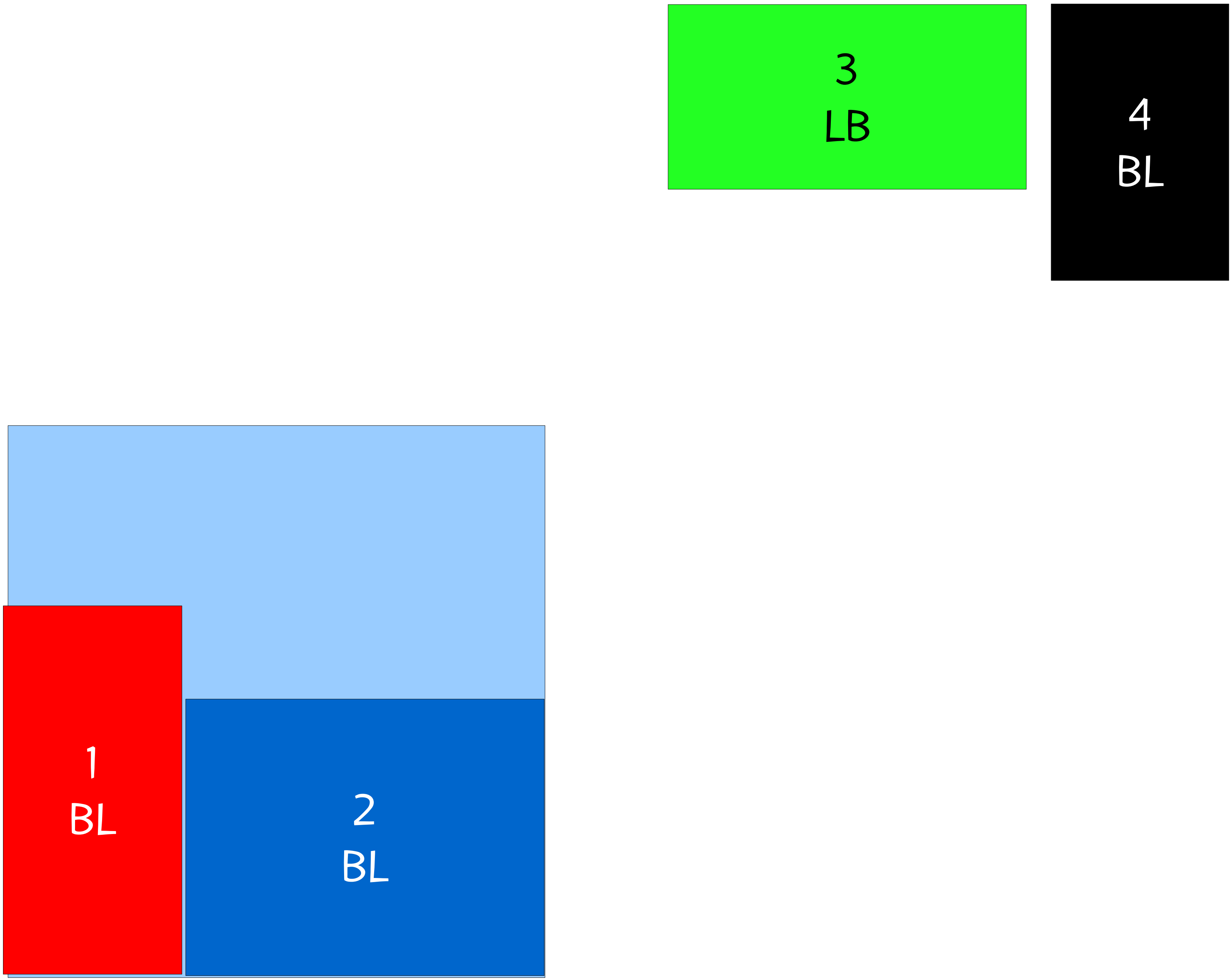

1
BL
2
BL
3
LB
4
BL
ERS[1]


2
BL
3
LB
4
BL
1
BL


2
BL
3
LB
4
BL
ERS[1]
1
BL



BRKGA


3
LB
4
BL
1
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2
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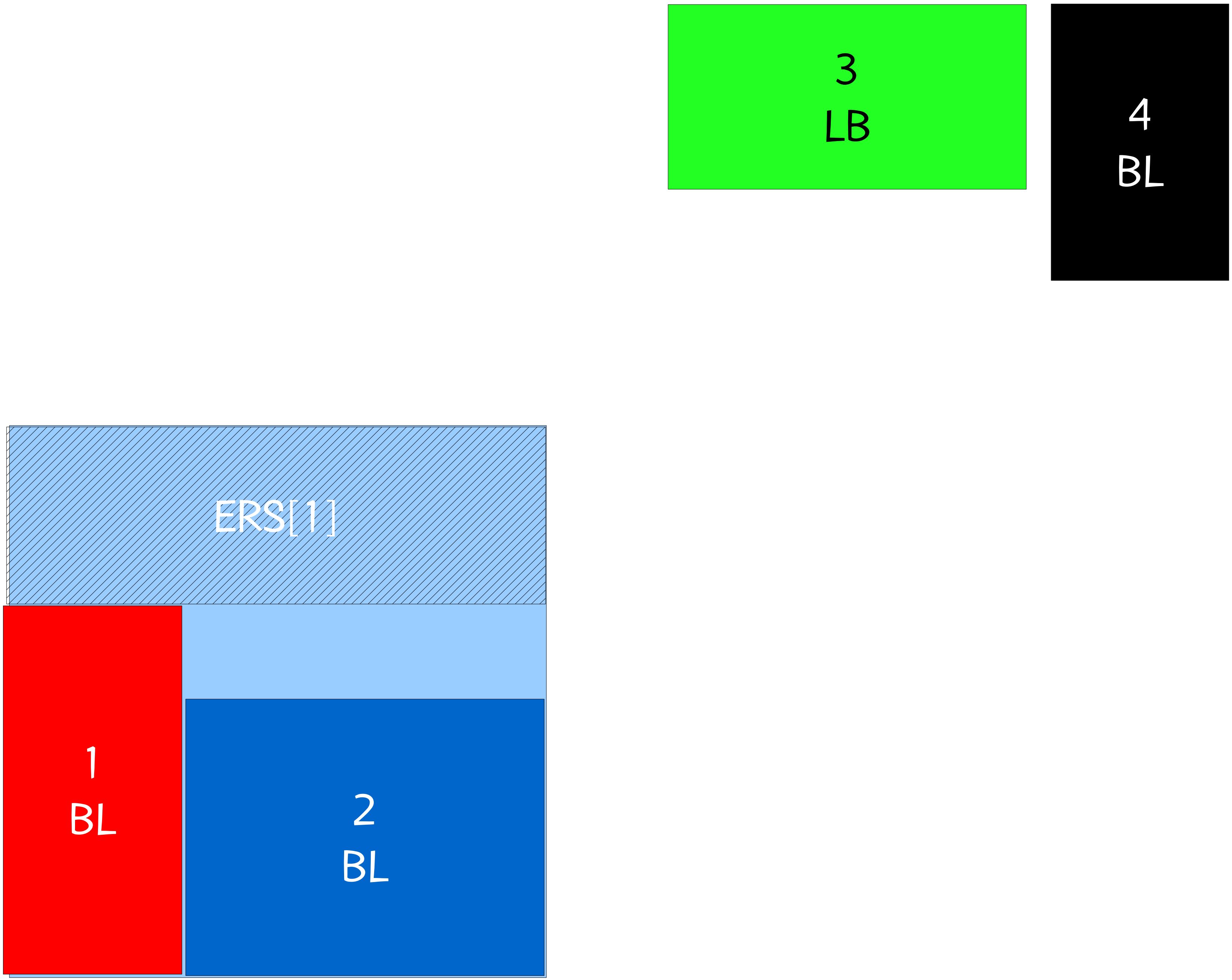
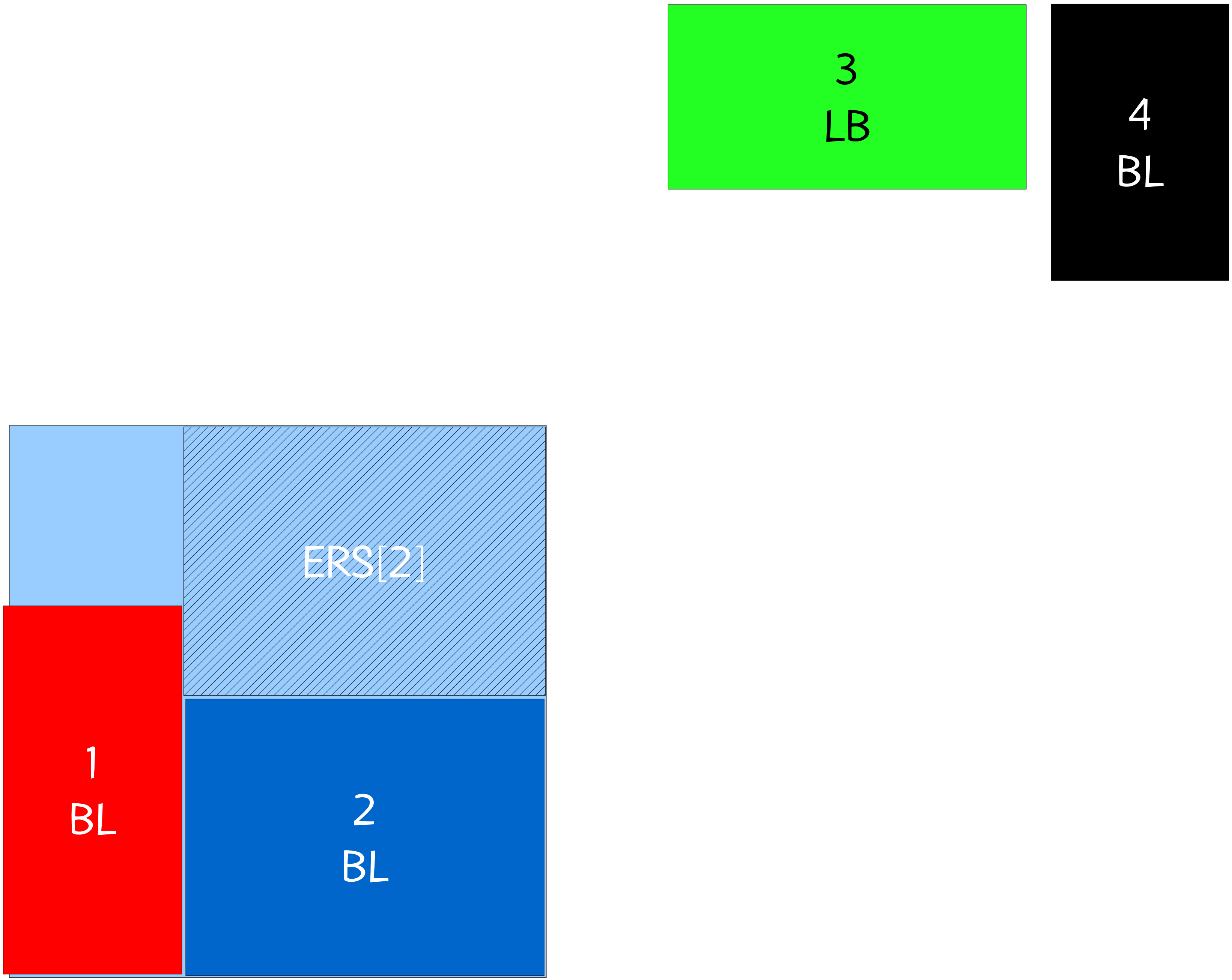
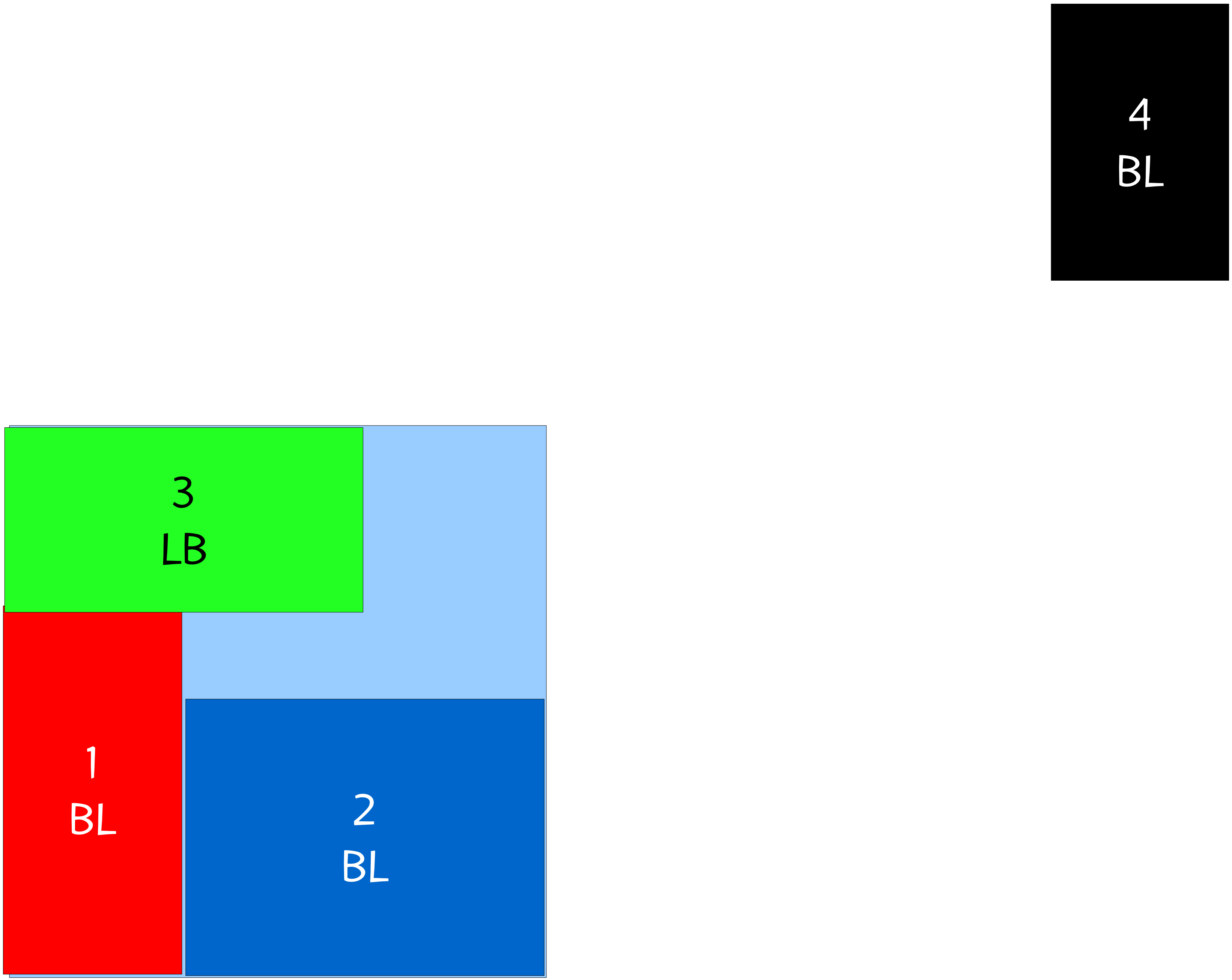

3
LB
4
BL
ERS[1]
1
BL
2
BL


3
LB
4
BL
ERS[2]
1
BL
2
BL


4
BL
3
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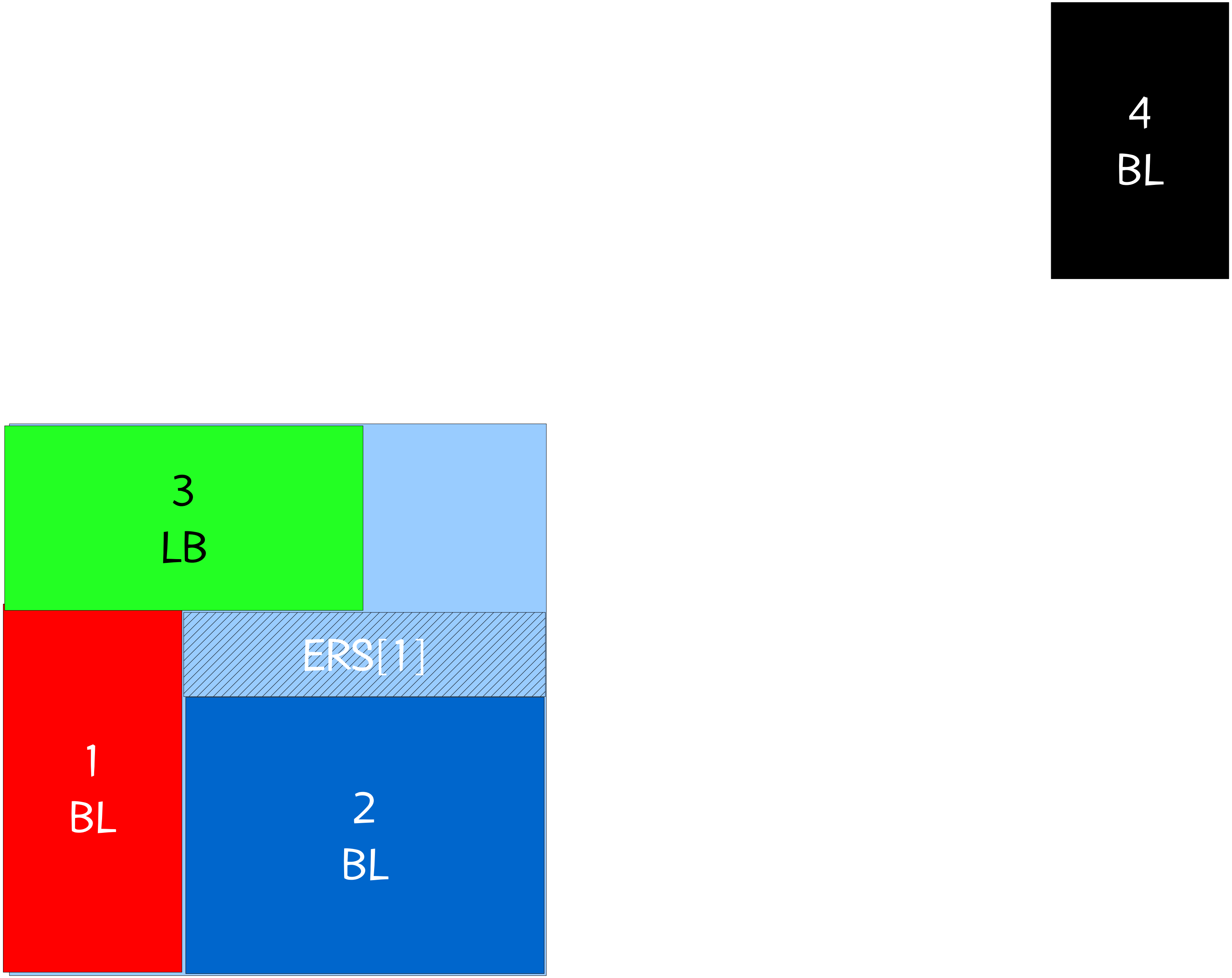
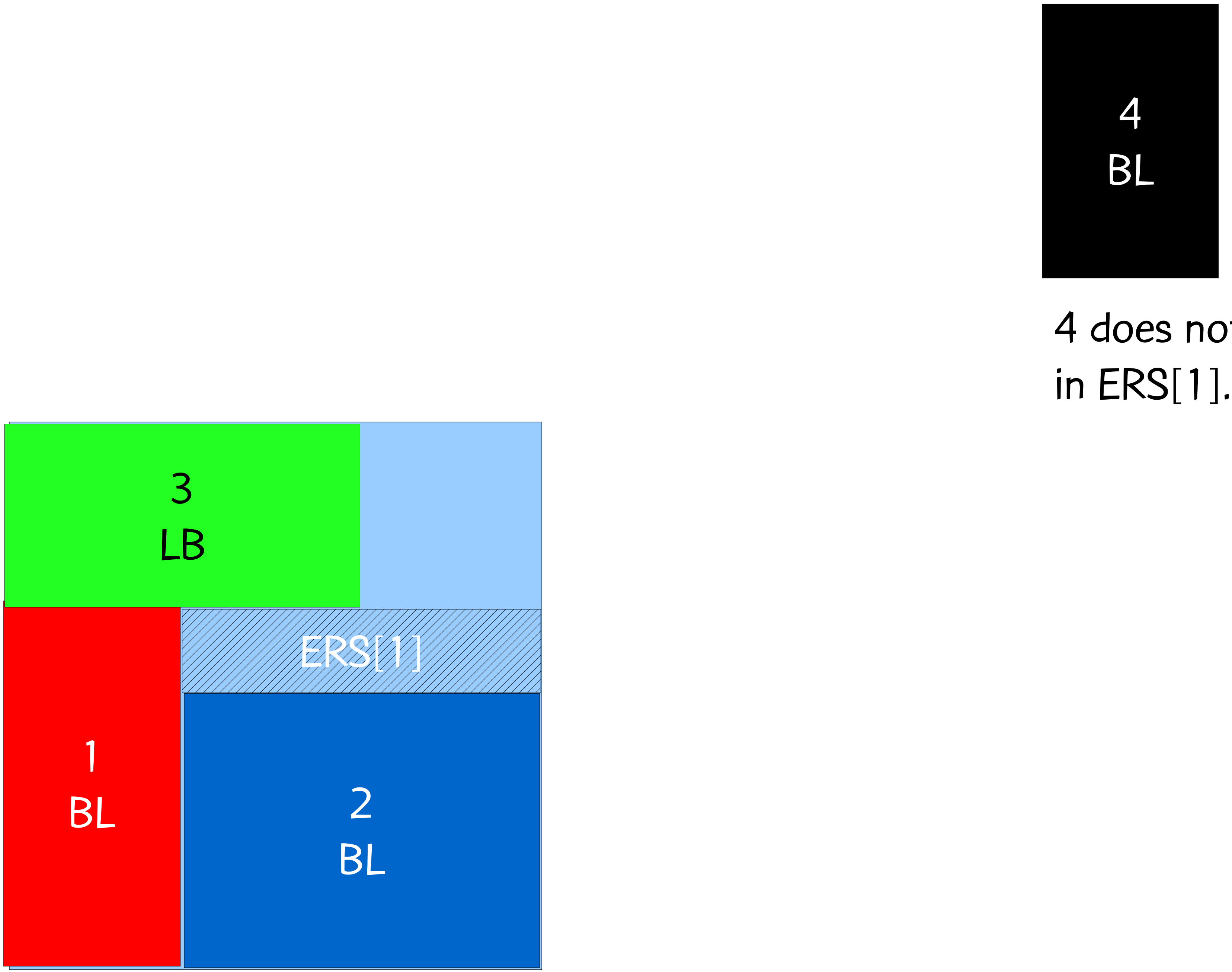
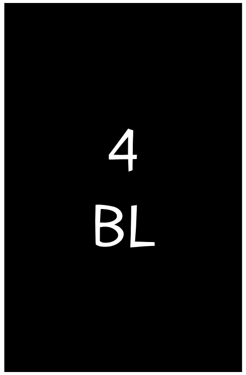
1
BL
2
BL

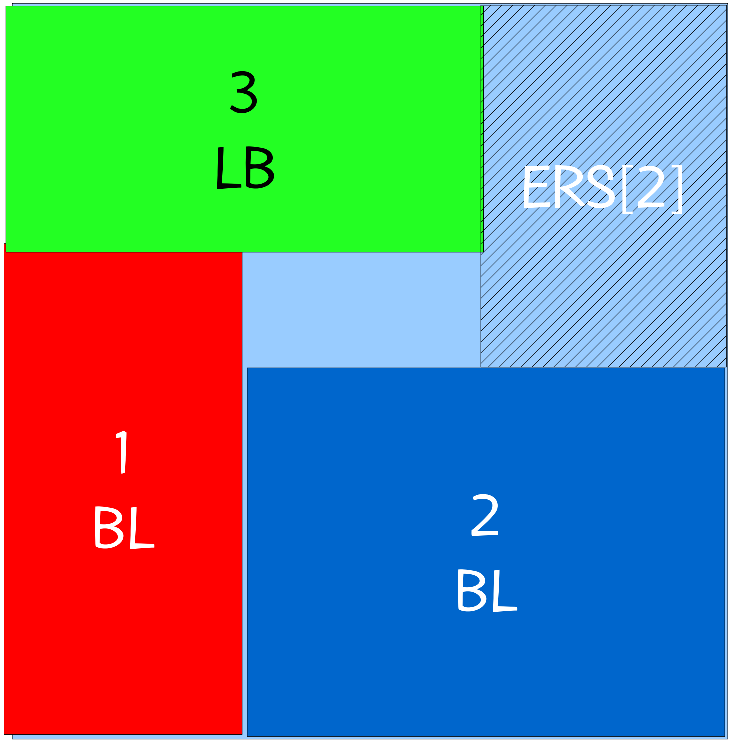

4
BL
3
LB
ERS[1]
1
BL
2
BL

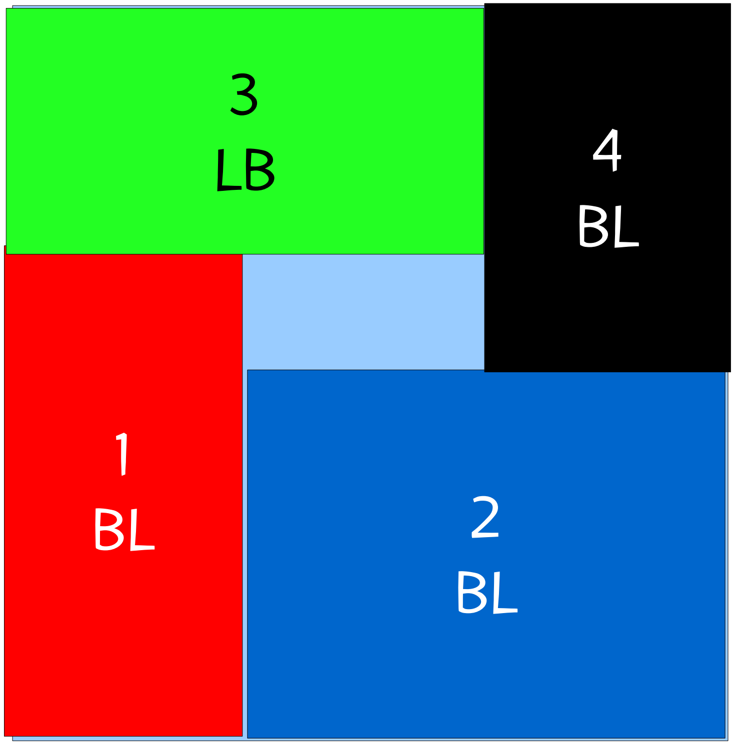


4 does not fit in ERS[1].



4 does fit in ERS[2].





Optimal solution!

Experimental results

Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:

Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:
  - PH: population-based heuristic of Beasley (2004)

Design

- We compare solution values obtained by the parallel multi-population BRKGA with solutions obtained by the heuristics that produced the best computational results to date:
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  - TABU: tabu search of Alvarez-Valdes et al. (2007)

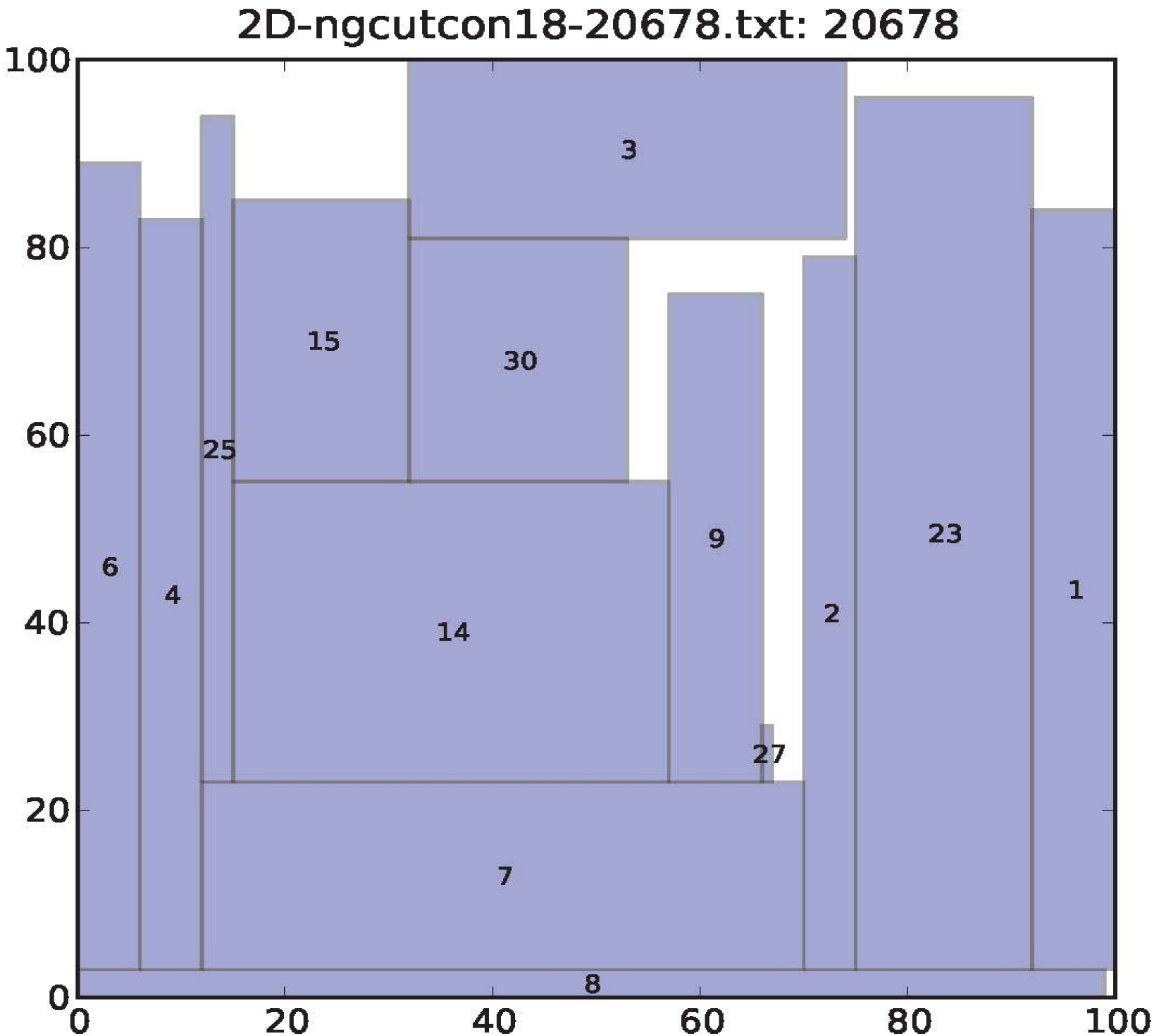
Number of best solutions / total instances

Problem	PH	GA	GRASP	TABU	BRKGA BL-LB-L-4NR
From literature (optimal)	13/21	**21/21**	18/21	**21/21**	**21/21**
Large random*	0/21	0/21	5/21	8/21	**20/21**
Zero-waste			5/31	17/31	**30/31**
Doubly constrained	11/21		12/21	17/21	**19/21**

* For large random: number of best average solutions / total instance classes

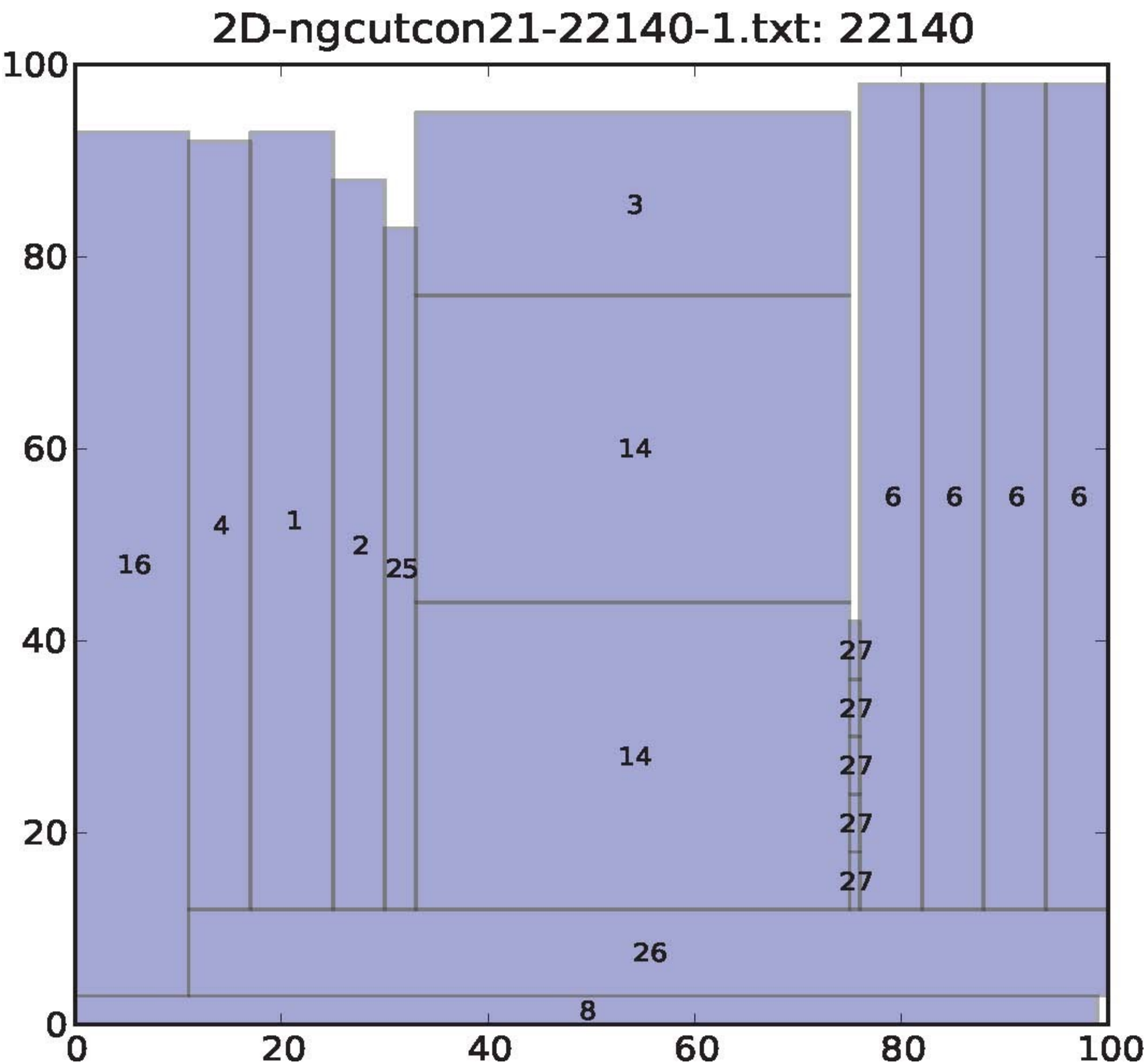
Minimum, average, and maximum solution times (secs) for BRKGA (BL-LB-L-4NR)

Problem	Min solution time (secs)	Avg solution time (secs)	Max solution time (secs)
From literature (optimal)	0.00	0.05	0.55
Large random	1.78	23.85	72.70
Zero-waste	0.01	82.21	808.03
Doubly constrained	0.00	1.16	16.87



New BKS for a 100 x100 doubly constrained instance of Fekete & Schepers (1997) of value **20678.** Previous best was **19657** by tabu search of Alvarez-Valdes et al., (2007).

30 types
30 rectangles



New BKS for a 100 x 100 doubly constrained instance Fekete & Schepers (1997) of value **22140**.

Previous BKS was **22011** by tabu search of Alvarez-Valdes et al. (2007).

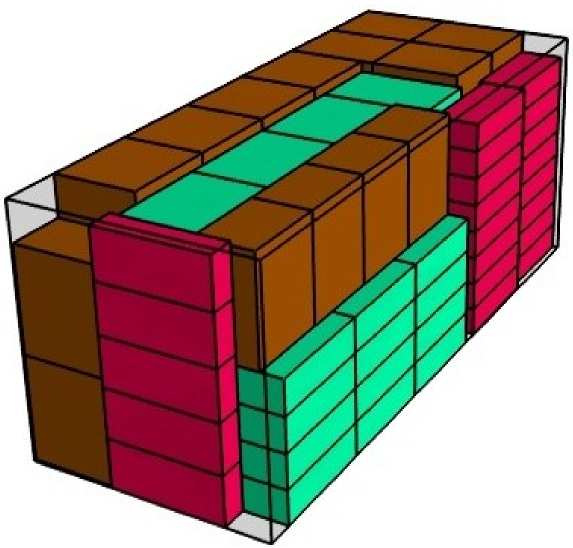
29 types
97 rectangles

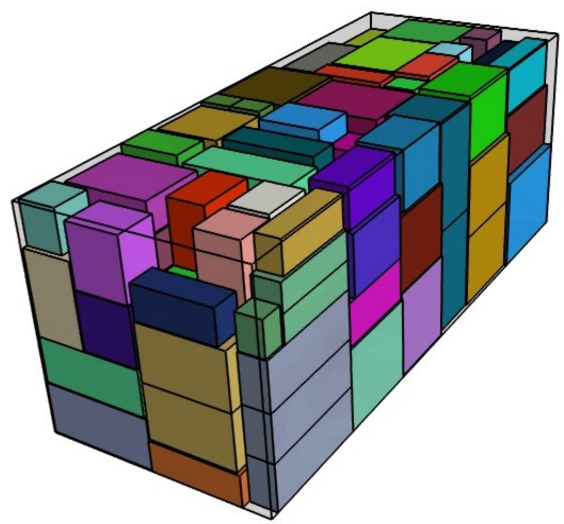
Some remarks

We have extended this to 3D packing:

J.F. Gonçalves and M.G.C.R., "A parallel multi-population biased random-key genetic algorithm for a container loading problem," Computers & Operations Research, vol. 29, pp. 179-190, 2012.

Tech report: http://www.research.att.com/~mgcr/doc/brkga-pack3d.pdf





BRKGA

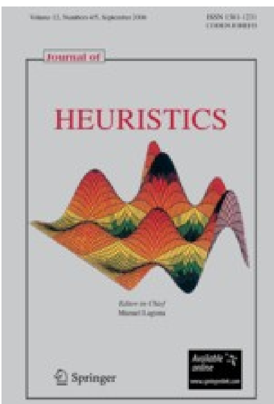
Literature survey

Literature

- BRKGAs have been applied in a wide range of areas.
- The following is a sampling of some papers that appeared in the literature applying BRKGAs.

Survey

- Survey: Gonçalves and R. (2011)



J.F. Gonçalves and M.G.C.R., "Biased random-key genetic algorithms for combinatorial optimization," J. of Heuristics, vol.17, pp. 487-525, 2011.

Telecommunications

- Routing: Ericsson, R., Pardalos (2002), Buriol et al. (2002, 2005), Reis et al. (2011), Noronha, R., Ribeiro (2007, 2008, 2011), Heckeler et al. (2011)
- Design: Andrade et al. (2006), Buriol, R., Thorup (2007)
- Network monitoring: Breslau et al. (2011)
- Regenerator location: Duarte et al. (2011)
- Fiber installation in optical networks: Goulart et al. (2011)
- Path-based recovery in flexgrid optical networks: Castro et al. (2012)

Telecommunications (cont'd)

- Handover minimization: Morán-Mirabal et al. (2012)
- Survivable IP/MPLS-over-WSON multi-layer network: Ruiz et al. (2011), Pedrola et al. (2011)
- Survey: R. (2012)

Scheduling

- Job-shop scheduling: Gonçalves, Mendes, R. (2005), Gonçalves and R. (2012)
- Single machine scheduling: Valente et al. (2006), Valente and Gonçalves (2008)
- Resource constrained project scheduling: Gonçalves, Mendes, R. (2008, 2009), Gonçalves, R., Mendes (2011)
- Selection and scheduling of observations on Earth observing satellites: Tangpattanakul, Josefowiez, Lopez (2012)

Production planning

- Assembly line balancing: Gonçalves and Almeida (2002)
- Manufacturing cell formation: Gonçalves and R. (2004)
- Single machine scheduling: Valente et al. (2006), Valente and Gonçalves (2008)
- Assembly line worker assignment and balancing: Moreira et al. (2010)
- Lot sizing and scheduling with capacity constraints and backorders: Gonçalves and Sousa (2011)

Network optimization

- Concave minimum cost flow: Fontes and Gonçalves (2007)
- Robust shortest path: Coco, Noronha, Santos (2012)
- Tree of hubs location: Pessoa, Santos, R. (2012)
- Hop-constrained trees in nonlinear cost flow networks: Fontes and Gonçalves (2012)
- Capacitated arc routing: Martinez, Loiseau, R. (2011)

Power systems

- Unit commitment: Roque, Fontes, Fontes (2010, 2011)
- Multi-objective unit commitment: Roque, Fontes, Fontes (2012)

Packing

- 2D orthogonal packing: Gonçalves and R. (2011)
- 3D container loading: Gonçalves and R. (2012a)
- 2D/3D bin packing: Gonçalves and R. (2012b)

Covering

- Steiner triple systems: R. et al. (2012)
- Covering by pairs: Breslau et al. (2011)

Transportation

- Tollbooth assignment: Buriol. et al. (2009, 2010)

Auctions

- Combinatorial auctions: Andrade et al. (2012)

Automatic parameter tuning

- GRASP with path-relinking: Festa et al. (2010)
- GRASP with evolutionary path-relinking: Morán-Mirabal, González-Velarde, R. (2012)

Continuous global optimization

- Bound-constrained optimization: Silva, Pardalos, R. (2012)

Software

- C++ API: Toso and R. (2012)

Thanks!

These slides and all of the papers cited in this lecture can be downloaded from my homepage:

http://www.research.att.com/~mgcr

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