EFFECTIVE APPLICATION OF GRASP

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Abstract. A greedy randomized adaptive search procedure (GRASP) is an iterative multistart metaheuristic for difficult combinatorial optimization. Each GRASP iteration consists of two phases: a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. Since 1989, GRASP has been applied to a wide range of combinatorial optimization problems, ranging from scheduling and routing to drawing and turbine balancing. In this paper, we cover the literature where GRASP is applied to scheduling, routing, logic, partitioning, location, graph theory, assignment, manufacturing, transportation, telecommunications, biology and related fields, automatic drawing, power systems, and VLSI design.

1. Introduction

Many combinatorial optimization problems especially those found in industry and government are either computationally intractable by their nature, or sufficiently large so as to preclude the use of exact algorithms. In such cases, heuristic methods are usually employed to find good, but not necessarily guaranteed optimal, solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization, avoid entrapment at local optima, and exploit the basic structure of the problem. Building on these notions, various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. The most promising of such techniques include simulated annealing [Kirkpatrick, 1984], tabu search [Glover, 1989, 1990, Glover and Laguna, 1997], genetic algorithms [Goldberg, 1989], variable neighborhood search [Hansen and Mladenović, 1998], and GRASP, or Greedy Randomized Adaptive Search Procedures, [Feo and Resende, 1989, 1995].

A GRASP is a multi-start or iterative process [Lin and Kernighan, 1973], in which each GRASP iteration consists of two phases, a construction phase, in which a feasible solution is produced, and a local search phase, in which a local optimum in the neighborhood of the constructed solution is sought. The best overall solution is kept as the result. An especially appealing characteristic of GRASP is the ease with which it can be implemented. Few parameters need to be set and tuned, and therefore development can focus on implementing efficient data structures to assure quick GRASP iterations.

In this chapter, we briefly review the literature of operations research and computer science applications of GRASP as well as industrial applications. We cover

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applications in scheduling, routing and transportation, logic and partitioning, graph theory, location and assignment, manufacturing, telecommunications and power systems. The chapter concludes with a section on applications in biology and related fields.

2. Scheduling

GRASP has been applied to a wide variety of scheduling problems. One of the pioneering works illustrating a GRASP for scheduling is the paper of Bard and Feo [Bard and Feo, 1989] appeared in 1989. This paper describes a method for efficiently sequencing cutting operations associated with the manufacture of discrete parts. The problem is first modeled as an integer program and then relaxed via Lagrangian relaxation into a min-cut problem on a bipartite network. To obtain lower bounds, a max-flow algorithm is applied and the corresponding solution is input to a GRASP. In [Feo et al., 1996] Feo et al. proposed a GRASP for single machine scheduling with sequence dependent setup costs and linear delay penalties. Here, the greedy function of the GRASP construction phase proposed is made up of two components: the switch over cost and the opportunity cost associated with not inserting a specific job in the next position and instead, inserting it after half of the unscheduled jobs have been scheduled. This greedy function tends to lead to a balance between the natural order and nearest neighbor approaches. The local search uses 2-exchange, insertion exchange, and a combination of the two. Another interesting application of GRASP for solving real-world scheduling problems is due to Xu and Chiu Xu and Chiu [2001], who proposed effective heuristic procedure for a field technician scheduling problem, where the objective is to assign a set of jobs at different locations with time windows to technicians with different job skills. Several heuristics, including a GRASP, are designed and tested for solving the problem. The greedy choice is to select jobs with the highest unit weight. The local search implements four different moves, among them the 2-exchange and a swap that exchanges an assigned job with another job unassigned under the candidate schedule.

Several hybrid approaches have been also proposed that combine GRASP with other techniques, including tabu search for the school timetabling problem Souza et al. [2001], path-relinking for the job shop scheduling Aiex et al. [2003] and Branch & Bound for the job shop scheduling Fernandes and Lourenço [2007] and a scheduling problem with non-related parallel machines and sequence-dependent setup times Rocha et al. [2004], where at each GRASP iteration, the greedy criterion adopted to build a feasible solution consists in sorting the jobs in a non-decreasing order using the due date and then assigning each job to the machine that can finish it first. In 2008, two further interesting hybrid techniques have been proposed. In Alvarez-Valdes et al. [2008b], Alvarez-Valdes et al. describe a Reactive GRASP for a strip-packing problem, while in Alvarez-Valdes et al. [2008a] Alvarez-Valdes et al. propose several heuristic algorithms based on GRASP and path relinking for project scheduling under partially renewable resources. In this case, the greedy selection is based on a priority rule and the improvement phase consists of two steps. First, it identifies the activities whose completion times must be reduced in order to have a new solution with the shortest makespan and labelling these activities as critical. Second, it moves critical activities in such a way that the resulting sequence is feasible according to precedence and resource constraints. In particular, the authors have designed two types of moves: 1) in a simple move, only a critical activity is moved, leaving the remaining activities unchanged; 2) in a double move, non-critical activities are moved to make the move of a critical activity possible.

3. Routing and Transportation

GRASP has been applied to several vehicle, aircraft, telecommunications, and inventory routing problems. In 1995, Kontoravdis and Bard Kontoravdis and Bard [1995] proposed a GRASP for minimizing the fleet size of temporarily constrained vehicle routing problems with two types of service. The greedy function of the construction phase takes into account both the overall minimum insertion cost and the penalty cost. Local search is applied to the best solution found every five iterations of the first phase, rather than to each feasible solution. In 1998, a modified and improved version of this GRASP has been extended for the inventory routing problem with satellite facilities by Bard et al. Bard et al. [1998], who proposed a methodology that decomposes the problem over the planning horizon, and then solves daily rather than multi-day vehicle routing problems.

In 2002, Carreto and Baker Carreto and Baker [2002] have investigated the incorporation of interactive tools into heuristic algorithms and proposed a GRASP interactive approach to the vehicle routing problem with backhauls. A GRASP has been used in the routes construction and improvement phase. The construction phase is implemented in a clustering heuristic that constructs the routes by clustering the remaining customers according to the vehicles defined by seeds while applying the 3-opt heuristic to reduce the total distance traveled by each vehicle. The greedy function takes into account routes with smallest insertion cost and costumers with biggest difference between the smallest and the second smallest insertion costs and smallest number of routes they can traverse.

In Corberán et al. [2002] Corberán et al. proposed a pure GRASP for the mixed Chinese postman problem, while a hybrid GRASP with tabu search has been proposed by Li et al. Li et al. [2004] for vehicle routing with both time window and limited number of vehicles. To efficiently solve the rural postman problem, de la Peña de la Peña [2004] published in 2004 an interesting case study to compare several metaheuristics approaches, including a simulated annealing, a GRASP, and a genetic algorithm.

GRASP has been used also to find approximate solutions of problems in air, rail, and intermodal transportation. One among the pioneering paper illustrating these applications appeared in 1989 due to Feo and Bard Feo and Bard [1989a], who studied the problem of locating maintenance stations and developing flight schedules that better meet the cyclical demand for maintenance. The problem has been formulated as large-scale mixed integer program, i.e. a minimum cost, multi-commodity flow network with integral constraints, where each airplane represents a separate commodity and each arc has an upper and lower capacity of flow. Since obtaining feasible solutions from the relative LP relaxation is difficult, the authors proposed a GRASP. Another interesting paper is due to Argüello et al. Argüello et al. [1997], who described a GRASP to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. The objective is to minimize the cost of reassigning aircraft to flights taking into account available resources and other system constraints. The proposed heuristic is a neighborhood search technique that takes as input an initial feasible solution, so that the construction phase is omitted. Two types of partial route exchange operations are described. The first type is the exchange of flight sequences with identical endpoints. In the second type, the sequence of flights being exchanged must have the same origination airport, but the termination airports are swapped.

4. Logic, Partitioning, and Graph Theoretic Applications

GRASP has been applied to problems in logic, including SAT, MAX-SAT, and logical clause inference. In 1996, Resende and Feo Resende and Feo [1996] proposed a GRASP for the satisfiability problem that can be applied to both the weighted and unweighted versions of the maximum satisfiability problem. The adaptive greedy function is a hybrid combination of two functions that seek to maximize the number of yet-unsatisfied clauses that become satisfied after the assignment of each construction iteration, and maximize the number of yet-unassigned literals in yet-unsatisfied clauses that become satisfied if opposite assignments were to be made. The local search flips the assignment of each variable, one at a time, checking if the new truth assignment increases the number of satisfied clauses. In 2006, Festa et al. Festa et al. [2006] proposed a hybrid GRASP with path relinking for the weighted maximum satisfiability problem (MAX-SAT). The authors designed accurate experiments to determine the effect of path relinking on the convergence of the GRASP.

Partitioning problems have been also treated with GRASP. These include number partitioning Argüello et al. [1996] and software/hardware partitioning Pu et al. [2006]. In particular, in Argüello et al. [1996] randomized methodologies are described for partitioning a finite set of integers into two disjoint subsets such that the difference of the sums of the elements in the subsets is minimized. Here, the greedy criterion consists in considering only large elements for differencing. In Pu et al. [2006], the authors propose a hybrid GRASP with tabu search as local search for software/hardware partitioning within timed automata.

Perhaps the largest class of problems for which GRASP has been applied is graph theory. The wide applicability of GRASP to these problems includes among others the maximum independent set problem, the maximum covering problem, the Steiner tree problem, the max cut problem, the feedback vertex set and arc set problems, graph coloring, and the multi-criteria minimum spanning tree problem.

For the maximum independent set, in 1994 Feo et al. Feo et al. [1994] proposed a GRASP whose greedy function orders admissible vertices with respect to the minimum admissible vertex degree, i.e. a vertex with the minimum degree and that is not adjacent to any vertex in the current independent set. The neighborhood definition used in the local search is $(2, 1)$-exchange, where two nonadjacent vertices can be added to the current solution if a single vertex from the solution is removed. In Resende [1998], Resende describes a GRASP for the maximum covering problem. Here, the greedy function is the total weight of the yet-uncovered demand points that become covered after the selection and the local search procedures uses a 2-exchange neighborhood structure.

In Martins et al. [1999], Martins et al. proposed a GRASP for approximately solving general instances of the Steiner problem in graphs. The construction phase is based on the distance network heuristic. A distance network corresponding to the original graph is built and associated with each edge of the distance network is a weight that takes into account the shortest distances in the original graph. With respect to the new weight distribution, Kruskal's algorithm is used to solve the minimum spanning tree problem and the edges in the MST so computed are replaced by the edges in the corresponding shortest paths in the original graph. The local search is based on insertions and eliminations of nodes to and from the current solution. A parallel version of a GRASP for the Steiner tree problem has been described in Martins et al. [2000], where the authors propose a local search that uses a combination of key-path based local search and node based local search.

For the max cut problem, in 2002 Festa et al. Festa et al. [2002] designed and tested a GRASP, a variable neighborhood search (VNS), a path relinking heuristic, and new hybrid heuristics that combine GRASP, VNS as GRASP local search, and path relinking.

Pardalos et al. Pardalos et al. [1999] designed a GRASP for finding approximate solutions to the feedback vertex set problem on a digraph. Several greedy functions have tested, all of them favoring vertices with high degree, while the local search procedure tries at each iteration to eliminate redundant vertices. In Festa et al. [2001] Festa et al. described a set of ANSI standard Fortran 77 subroutines to find approximate solutions of both the feedback vertex set problem and the feedback arc set problem. The GRASP of Pardalos et al. is used to produce the approximate solutions of the feedback set problem, while feedback arc set problems are converted into feedback vertex set problems and then solved.

For graph coloring, in 2001 Laguna and Martí Laguna and Martí [2001] proposed a GRASP, whose construction phase greedy function chooses the vertex having the maximum degree among the uncolored vertices adjacent to at least one colored vertex. The local search combines the two smallest cardinality color classes into one and tries to find a valid color for each violating vertex.

Very recently, for the multi-criteria minimum spanning tree problem in 2008 Arroyo et al. Arroyo et al. [2008] proposed a GRASP, whose local search tries to improve the current solution by defining a drop-and-add neighborhood, where the spanning trees are represented by Prufer numbers.

5. Location and Assignment Problems

GRASP has been successfully used to find approximate solutions to location and assignment problems.

Already in 1992, Klincewicz Klincewicz [1992] proposed two heuristics based on tabu search and GRASP for the p-hub location problem, whose local search procedure is based on the 2-exchange neighborhood. Later, in 2005 Pérez et al. Pérez et al. [2005] described a hybrid GRASP with path-relinking algorithm for the capacitated p-hub median problem. In this proposal, the greedy evaluation function has two phases: a location phase and an allocation phase. In the location phase, one element is added at each time to the set of hubs, while the allocation phase consists of an allocation to the nearest hub. As local search, the authors proposed a greedy procedure for location and an allocation to the nearest hub. For the p-median and other location problems, efficient multistart hybrid heuristics that combine elements of several traditional metaheuristics including GRASP have been recently proposed by Resende and Werneck Resende and Werneck [2004, 2007]. Here, the greedy criterion consists in choosing at each iteration one among the most profitable facilities, while the local search procedure is based on swapping facilities.

Starting from the pioneering work of Li et al. Li et al. [1994], GRASP has been applied to several hard assignment problems, including quadratic, biquadratic, and multidimensional assignment. For the quadratic assignment problem (QAP), in Li et al. [1994] the authors describe a GRASP, whose construction mechanism defines a greedy function on the assignment interaction cost and whose local search procedure is a 2-assignment exchange. The construction phase of the GRASP proposed by Li et al. has been later applied by Ahuja et al. Ahuja et al. [2000] to generate the initial population of a genetic algorithm for the QAP. In 1998, Mavridou et al. Mavridou et al. [1998] proposed a GRASP for finding approximate solutions of the biquadratic assignment problem. As in the case of GRASP for the QAP, the construction phase has two stages. The first stage simultaneously makes four assignments, selecting the pairs corresponding to the smallest interaction costs, while the second stage makes the remaining assignments, one at time. The greedy function in the second stage selects the assignment corresponding to the minimum interaction cost with respect to the already-made assignments. In the local search phase, 2-exchange is applied to the permutation constructed in the first phase. An interesting study has been conducted in 1999 by Fleurent and Glover Fleurent and Glover [1999]. In their paper, they showed that the GRASP for QAP of Li et al. Li et al. [1994] can be improved upon by using memory strategies, such as learning, intensification, candidate list strategies, and POP. For the multidimensional assignment problem, Robertson Robertson [2001] in 2001 proposed four GRASP implementations by combining two construction methods (randomized greedy and randomized max regret) and two local search methods (2-exchange and variable depth exchange). The greedy function of the randomized max regret construction method is a measure of the competition between the two leading cost candidates, while the maximum regret value corresponds to the candidate assignment that has the largest winning margin between itself and its next highest competitor. The variable depth exchange is an extension of the 2-exchange method that allows a more extensive search of the surrounding neighborhood space.

6. Manufacturing

GRASP has been applied to solve several optimization problems arising in manufacturing. The first paper example of this kind of application is due to Feo and Bard Feo and Bard [1989b], who in 1989 proposed a method for minimizing the sum of tool setup and volume removal times associated with metal cutting operations on a flexible machine. The problem was modeled as an integer program and relaxed into a min-cut problem on a simple network. After obtaining a tentative solution, the authors used a GRASP to identify good feasible points corresponding to alternative process plans. The same authors in 1991 Bard and Feo [1991] designed an algorithm for the manufacturing equipment selection problem, where the objective is to determine how many of each machine type to purchase and what fraction of the time each piece of equipment will be configured for a particular type of operation. The problem has converted into a MILP and a depth-first B&B has been used, employing the greedy randomized set covering heuristic of Feo and Resende Feo and Resende [1989]. Viewing the contribution that any machine makes to satisfy the demand of any process as the unit benefit associated with that machine. To derive a feasible solution, the heuristic iteratively selects machines among those with largest benefits.

More recently, for constrained two-dimensional non-guillotine cutting problems in 2005 Alvarez-Valdes et al. Alvarez-Valdes et al. [2005] proposed a GRASP, whose construction mechanism takes the smallest rectangle breaking the ties by the nearest distance to a corner of the stock rectangle. Then, two criteria have been considered to select the piece: 1) the first piece in a list ordered, giving priority to pieces which must be cut; 2) the piece producing the largest increase in the objective function. In 2008, Monkman et al. Monkman et al. [2008] described a production scheduling heuristic for an electronics manufacturer with sequence-dependent setup costs. The problem here studied involves assignment, sequencing, and time scheduling steps and has been modeled as a traveling salesman subset-tour problem. For the sequencing step, the authors proposed a GRASP, whose construction phase uses a cardinality based RCL and the greedy function takes into account the cost associated with the arcs of the underlying graph. The local search uses two different neighborhoods: a node elimination and a node swap neighborhood.

7. Telecommunications

Telecommunications and network design are two fields of quite a large number of applications of GRASP. In the following, only some recently appeared papers will be cited.

Liu et al. Liu et al. [2000] in 2000 proposed a hybrid GRASP for frequency assignment in mobile radio networks, whose local search is a simulated annealing and whose construction phase uses two greedy functions. The first chooses a vertex from the set of unselected vertices with high saturation degrees, while the second function is used to assign a frequency to the selected vertex. Prais and Ribeiro Prais and Ribeiro [2000] designed a GRASP for a matrix decomposition problem arising in the context of traffic assignment in communication satellites. The local search phase of the GRASP proposed is based on a new neighborhood, defined by constructive and destructive moves. In 2003, Amaldi et al. Amaldi et al. [2003] proposed a GRASP and a tabu search to efficiently solve the downlink base station (BS) location problem. Here, during the local search the following moves are considered: removing a BS, installing a new BS, removing an existing BS, and installing a new one (swap). The output GRASP solution is used as initial solution for a tabu search algorithm. In 2008, Commander at al. Commander et al. [2008] have designed and implemented a GRASP for the cooperative communication problem on ad hoc networks, where the problem consists in maximizing the amount of connectivity among a set of users, subject to constraints on the maximum distance traveled, as well as restrictions on what types of movement can be performed. The greedy function value of each candidate element is a measure of additional connections created by its insertion in the partial solution under construction. The local search procedure is based on a perturbation function consisting of selecting a wireless agent and rerouting.

8. Applications in Biology

It is only in the past few years that it has been shown that a large number of molecular biology problems can be formulated as combinatorial optimization problems, usually computationally intractable so as to employ heuristic methods to find good solutions in a reasonable amount of running time. Brown et al. Brown et al. [2000] proposed a GRASP for selecting a population subset for use in a high-density genetic mapping project. At each iteration of the construction phase, the authors add to the partial solution one among the r unchosen population members which most improve the objective function value. The authors investigated very small sized RCL (i.e. $r = 3$ and $r = 5$) and the local search they implemented removes from the current solution some members and greedily includes other members, until no further improving exchange can be done.

For the phylogeny problem (consisting in finding a phylogeny with the minimum number of evolutionary steps - the so-called parsimony criterion), Andreatta and Ribeiro Andreatta and Ribeiro [2002] and later Ribeiro and Vianna Ribeiro and Vianna [2005] proposed a GRASP and a variable neighborhood search and a GRASP that uses variable neighborhood descent for local search, respectively.

Phylogenetic footprints are short pieces of no-coding DNA sequence in genes that are conserved between evolutionary distant species. Fried et al. Fried et al. [2004] showed that solving the footprint sorting problem requires the solution of a minimum weight vertex feedback set problem and for solving the latter, they used the GRASP provided in Festa et al. Festa et al. [2001].

In 2007, Festa Festa [2007] proposed a GRASP to find improved solutions for the far from most string problem, where the objective is to determine a sequence far from most of the sequences in a given input set. In this case, it is intuitive to relate the greedy function to the occurrence of each character in a given position, while to realize the local search phase the 2-exchange algorithm has been used. A solution for the far from most string problem can help for example to identify a sequence fragment that distinguishes the pathogens from the host, so the potential exists to create a drug that harms several but not all pathogens.

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