A GRASP FOR PBX TELEPHONE MIGRATION SCHEDULING

DIOGO V. ANDRADE AND MAURICIO G.C. RESENDE

ABSTRACT. A PBX, or *private branch exchange*, is a private telephone network used within an enterprise. The PBX telephone migration problem arises when an enterprise acquires a new PBX to replace an existing one. Phone numbers need to migrate from the old system to the new system over a time horizon. A penalty, assigned to the each pair of phones, is incurred if the pair is migrated in different time periods. The objective is to assign phones to time periods such that no more than a given number of phones is assigned to any period and the total penalty is minimized. We present a GRASP (greedy randomized adaptive search procedure) for this problem.

1. INTRODUCTION

A PBX, or *private branch exchange*, is a private telephone network used within an enterprise. Besides allowing users to share a number of outside lines, PBXes have many features, such as call forwarding, call recording, call transfer, and voice messaging.

Some PBX features require groups of phone numbers to be defined. These include, for example, multi-line hunt (MLH), call pickup (CPU), intercom (ICOM), series completion (SC), and shared telephone number (STN) groups. An MLH group consists of a cycle of phone numbers. When a call is made to a phone in the cycle and the call is not answered, it is transfered to the next phone in the cycle. This is repeated until someone picks up. A CPU group is a set of phone numbers where any phone in the group can pickup a call made to any other phone in group. Any phone in an ICOM group can speed dial to any other group member. A SC group is an ordered list of phone numbers. If a call made to the first phone is not answered, it is transfered to the next. This is repeated until someone picks up. If the last phone in the list does not pick up, voice mail answers the call. An STN group is a set of phone numbers for which calls made to them are answered by a single phone (e.g. an assistant). In an enterprise there may exist several MLH, CPU, ICOM, SC, and STN groups and a single phone number may be a member of more than one group.

We consider a problem that arises when an enterprise acquires a new PBX to replace an existing one. Phone numbers need to migrate from the old system to the new system over a time horizon of T time periods. Each group g has a penalty p_g associated with it. Let $\varphi(u) \in \{1,2,\dots,T\}$ be the time period in which phone number u migrates. If two phones u and v in group g migrate in time periods $\varphi(u)$ and $\varphi(v)$, respectively, then a penalty $p_g|\varphi(u) - \varphi(v)|$ is incurred. Let the telephone numbers to be migrated in the planning horizon be $1,2,\dots,P$. We further require that during each time period at most ρ phones are allowed to migrate and assume that $T \times \rho \geq P$, i.e. that there exists a feasible schedule.

The objective is to schedule the migration plan so that the total migration penalty is minimized. This involves assigning phone numbers to time periods such that no more

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than ρ phones are assigned to a single period. This problem is called the *PBX telephone migration scheduling problem*.

2. GRASP FOR PBX TELEPHONE MIGRATION SCHEDULING

In this section, we describe a GRASP, or greedy randomized adaptive search procedure, for finding good PBX telephone migration schedules. GRASP [1, 2, 5] is a metaheuristic for combinatorial optimization that has been applied to a wide range of optimization problems [3]. A GRASP is a multi-start procedure, where each iteration consists of a construction phase followed by a local search phase. The construction phase produces a starting solution for the local search phase. This solution is constructed using a semi-greedy algorithm [4], i.e. a randomized greedy algorithm. Whereas in a greedy algorithm the next element to be added to the solution is the best choice if one were to ignore the effects of adding future elements, in a semi-greedy algorithm a set of good choices is determined and an element is selected at random from this set. A neighborhood of a feasible solution is the set of solutions obtained by minimally perturbing the solution. This perturbation, also called a *move*, depends on the structure of the solution. Given a feasible solution, the local search procedure examines the neighborhood of this solution seeking a better-quality solution. If one if found, local search is reapplied, starting from the new improved solution. Local search ends when there is no improving solution in the neighborhood of the current solution, which is said to be a local optimum.

A solution of the migration problem is an assignment of phone numbers to time periods such that each time period has no more than ρ telephone numbers assigned to it. We next describe a construction procedure, three local search neighborhoods, and the local search procedure that uses these neighborhoods.

The construction procedure sequences the phone numbers and assigns them evenly to each time period. Let $\pi(u)$ be the position of phone number u is the sequence. The first $\lfloor P/T \rfloor$ numbers in the sequence are assigned to time period 1, the second $\lfloor P/T \rfloor$ numbers are assigned to time period 2, and so on. The last time period may have less than $\lfloor P/T \rfloor$ numbers assigned to it. To describe the construction procedure, consider a graph $G = (V,E)$, where V is the set of phone numbers and $(u,v) \in E$ if and only if there is a penalty associated with migrating phone numbers u and v in different time periods. Since u and v may jointly belong to more than one group, there may be more than one penalty associated with moving them in different time periods. Let $w_{u,v}$ be the sum of the penalties associated with moving phone numbers u and v in different time periods. The goal of the construction procedure is to generate a diverse set of good-quality solutions. We use as an approximation of solution cost the function $\sum_{(u,v)\in E} w_{u,v} \mid \pi(u) - \pi(v) \mid$. Solutions with small function cost will tend to have pairs of numbers with high penalties sequenced close to each other.

Suppose that $k-1$ vertices have been already sequenced and we wish to select the next (k-th) vertex to sequence from the set Γ of vertices yet to be sequenced. For $u \in \Gamma$, let $f(u)$ be the penalty-weighted degree of u with respect to all $v \in \Gamma \setminus \{u\}$, i.e. $f(u) = \sum_{v\in\Gamma\setminus\{u\}} w_{u,v}$. Likewise, for $u \in \Gamma$, let $b(u)$ be the penalty-weighted degree of u with respect to all $v \in V \setminus \Gamma \setminus \{u\}$, i.e. $b(u) = \sum_{v\in V\setminus\Gamma\setminus\{u\}} w_{u,v}$. A greedy choice for the next vertex is the one which minimizes $f(u) - b(u)$. This criterion can be explained by the following observation. Let S be the sum of weights ($\sum_{(u,v)\in E} w_{u,v}$ for $u \in V \setminus \Gamma$ and $v \in \Gamma$). Notice that by removing u from Γ, $\Delta S = f(u) - b(u)$. Therefore, by choosing the smallest value of $f(u) - b(u)$ we minimize the remaining sum of weights. The greedy algorithm goes as follows. Initially, the set Γ of yet to be sequenced vertices is set to V. For all $v \in \Gamma$,

$b(v) = 0$ and $f(v) = \sum_{u\in\Gamma\setminus\{v\}} w_{v,u}$. The first vertex in the sequence, v_1, is chosen as the vertex v having the smallest $f(v)$ value. It is removed from set Γ, i.e. $\Gamma = \Gamma \setminus \{v_1\}$. Suppose that $k-1$ vertices have been already sequenced and we now select the next vertex, v_k, to sequence. First, the values $b(v)$ and $f(v)$ for all $v \in \Gamma$ need to be updated to take into account the selection of vertex v_{k-1} in the previous iteration. For all $v \in \Gamma$, if $(v, v_{k-1}) \in E$ then $b(v) = b(v) + w_{v,v_{k-1}}$ and $f(v) = f(v) - w_{v,v_{k-1}}$. Then $v_k = \text{argmin}_{v\in\Gamma}\{f(v) - b(v)\}$ is chosen and removed from Γ, i.e. $\Gamma = \Gamma \setminus \{v_k\}$. This procedure is repeated until $n-1$ vertices are sequenced and only a single vertex remains in set Γ. The remaining vertex is the last vertex in the sequence. The GRASP construction uses a randomized version of the greedy construction described above. At each step of the construction, instead of selecting the vertex $v \in \Gamma$ with the smallest $f(v) - b(v)$ value, a restricted candidate list made up of vertices $v \in \Gamma$ with small $f(v) - b(v)$ values is set up and a vertex is chosen at random from this set.

Once a feasible solution is constructed, local search is applied to this solution. The local search uses three neighborhoods: swap phones, move phone, and swap periods. The *swap phone* neighborhood of assignment φ is the set of all assignments φ' in which phones u and v are in periods i and j, respectively, in φ and in periods j and i, respectively, in φ' and all other phones have the same assignment in φ and φ'. A move in this neighborhood does not alter the size of the time periods. The *move phone* neighborhood of assignment φ is the set of all assignments φ' in which phone u was in period i in φ and is in period j ($j \neq i$) is φ' and all other phones have the same assignment in φ and φ' as long as φ' has no more than ρ phone numbers. A move in this neighborhood shrinks time period i and increases time period j. Finally, the *swap periods* neighborhood of assignment φ is the set of all assignments φ' in which all phones assigned to period i in φ are assigned to time period j ($j \neq i$) in φ' and all phones assigned to period j in φ' are assigned to time period i ($i \neq j$) in φ.

The local search looks for an improving assignment in the swap phones neighborhood. If one is found, the move is made and the local search restarts. If no improving move in the swap phones neighborhood is found, the local search proceeds to look for an improving assignment in the move phone neighborhood. If one is found, the move is made and the local search restarts. If no improving move in the move phone neighborhood is found, the local search proceeds to look for an improving assignment in the swap periods neighborhood. If one is found, the move is made and the local search restarts. If no improving move is found in all three neighborhoods, local search terminates with a locally optimal solution.

3. An example

We consider here an example of a real PBX telephone migration scheduling problem where an organization has eight weeks to migrate 2855 phones. These phones make up 223 groups (with as few as one phone and at most 804 phones per group). At most 375 phones can be migrated in one week. The penalties for groups MLH, CPU, ICOM, SC, and STN are 10, 4, 3, 2, and 1, respectively.

Figure 1 compares the solution produced with GRASP with a migration plan proposed by planners (Plan). To facilitate the visual comparison of the two solutions, the penalty for GRASP is shown above the axis while the penalty for the planner's solution is shown below the axis. The figure shows how penalties accumulate from time period 1 to time period 7. The GRASP reduced the total penalty by 28%. The period sizes for the best solution found were 357, 367, 354, 374, 374, 374, 371, and 284.

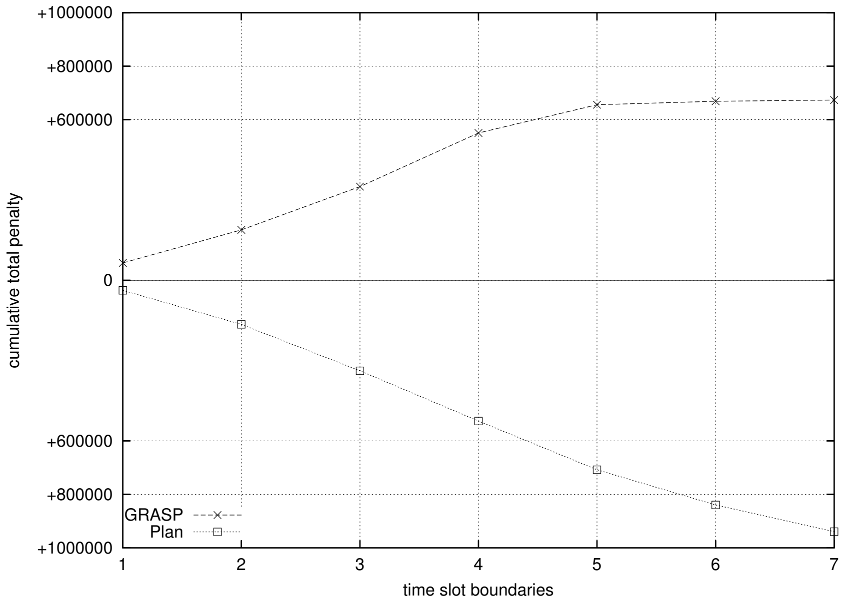

FIGURE 1. *GRASP versus original plan.*

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(D.V. Andrade) RUTCOR, RUTGERS UNIVERSITY, PISCATAWAY, NJ, USA
E-mail address, D.V. Andrade: `dandrade@rutcor.rutgers.edu`

(M.G.C. Resende) AT&T LABS RESEARCH, FLORHAM PARK, NJ, USA.
E-mail address, M.G.C. Resende: `mgcr@research.att.com`