# GRASP WITH EVOLUTIONARY PATH-RELINKING

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ABSTRACT. We propose GRASP with evolutionary path-relinking, a metaheuristic resulting from the hybridization of GRASP, path-relinking, and evolutionary path-relinking. This metaheuristic is applied to a network migration problem. Experiments show that a GRASP with evolutionary path-relinking heuristic finds solutions faster than a heuristic based on GRASP with path-relinking as well as one based on pure GRASP.

## 1. GRASP

GRASP, or greedy randomized adaptive search procedure [4, 5], is a metaheuristic for combinatorial optimization. It consists of multiple applications of local search, each starting from a different solution. Starting solutions are generated using some type of greedy randomized construction procedure, such as the semi-greedy algorithm of Hart and Shogan [7], the sample greedy algorithm of Resende and Werneck [10], or a perturbation scheme like in Canuto, Resende, and Ribeiro [2].

## 2. PATH-RELINKING

Path-relinking (PR) is a search strategy that explores trajectories connecting two solutions [6]. Given two solutions, their common elements are kept constant and the space of solutions spanned by these elements is searched with the objective of finding a better solution. The size of the solution space grows exponentially with the difference between the two solutions and PR explores only a small portion of the space by moving between the two solutions in a greedy way. One way of carrying out PR is to make one of the solutions the *initial* solution and the other the *target* and move from the initial solution to the target. Ribeiro and Rosseti [11] introduced *mixed* PR, where the roles of guiding and target solutions are interchanged at each step of the procedure. Faria Jr. et al. [3] introduced greedy randomized adaptive PR, where instead of moving between the two solutions in a greedy way, the moves are done in a greedy randomized fashion.

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procedure GRASP(i_max, EvPR_freq)
    Initialize pool P ← ∅
    for i = 1,...,i_max do
        x ← GreedyRandomizedConstruction()
        x ← LocalSearch(x)
        x ← PathRelinking(x,P)
        Test x for membership in pool P;
        if mod(i,EvPR_freq) = 0 then
            P ← EvPathRelinking(P);
        end if
    end for
    return(x* ← argmin(P));
end GRASP;
```

FIGURE 1. Pseudo-code for GRASP with EvPR.

## 3. GRASP WITH PATH-RELINKING

Laguna and Martí [8] proposed integrating GRASP with path-relinking. A pool of elite solutions found in the search is maintained. Membership in the pool is based on quality and diversity, i.e. not only do pool solutions have to be of good quality but they also must be sufficiently different from one another. At the end of each GRASP iteration, PR is applied between the GRASP iterate and a solution chosen at random from the pool. The solution resulting from PR is tested for membership in the elite set. A survey of GRASP with PR is given in Resende and Ribeiro [9].

## 4. EVOLUTIONARY PATH-RELINKING

Resende and Werneck [10], introduced evolutionary path-relinking (EvPR) as a post-processing phase for GRASP with PR. In EvPR, the solutions in the pool are evolved as a series of populations $P_1, P_2, \dots$ of equal size. The initial population ($P_0$) is the pool of elite solutions produced by GRASP with PR. In iteration $k$ of EvPR, path-relinking is applied between a set of pairs of solutions in population $P_k$ and, with the same rules used to test for membership in the pool of elite solutions, each resulting solution is tested for membership in population $P_{k+1}$. This evolutionary process is repeated until no improvement is seen from one population to the next.

## 5. GRASP WITH EVOLUTIONARY PATH-RELINKING

Figure 1 shows pseudo-code for GRASP with evolutionary path-relinking. GRASP with EvPR is GRASP with PR, where at fixed intervals an intensification of the elite pool is done by way of EvPR. The pool resulting from the EvPR phase is used as the elite pool when GRASP with PR resumes.

We highlight a few important implementation issues for the design of effective EvPR procedures. Since path-relinking between two lower-quality elite solutions is often unfruitful, we limit each iteration of EvPR to path-relinking between the $\alpha$ best elite solutions and the $\beta$ best elite solutions, where $\alpha \leq \beta$. Since EvPR can be repeatedly applied on the same pool (in consecutive GRASP with PR iterations), we make use of greedy randomized adaptive mixed PR with the objective of exploring different trajectories during different applications of EvPR. Lower-quality solutions that have been in the pool for a large number of GRASP iterations should be forced out of the elite pool. To implement this we associate with each elite solution an age. When the solution enters the elite set, its age is zero. At each call of the procedure `EvPathRelinking` in line 8 of the pseudo-code in Figure 1 the ages of each pool member are incremented by one. If an elite solution is among the $\gamma$ worst elite solutions and its age is above a given threshold, then it is removed from the elite set.

## 6. Experimental results

We illustrate GRASP with EvPR on a network migration scheduling problem, where inter-nodal traffic from an outdated telecommunications network is to be migrated to a new network. Nodes in the old network are deloaded in sequence. All traffic originating, terminating, or passing through the node in the old network is moved to a specific node in the new network. The amount of new capacity needed to achieve the migration depends on the sequence that nodes in the old network are deloaded.

Using real and artificially generated instances, we compare pure GRASP, GRASP with PR, and several variants of GRASP with EvPR. Time-to-target (TTT) plots [1] show that GRASP with PR outperforms pure GRASP and that GRASP with EvPR outperforms GRASP with PR.

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