A biased random-key genetic algorithm for the unequal area facility layout problem*

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This paper presents a biased random key genetic algorithm (*BRKGA*) for the unequal area facility layout problem (*UA-FLP*) where a set of rectangular facilities with given area requirements has to be placed, without overlapping, on a rectangular floor space. The objective is to find the location and the dimensions of the facilities such that the sum of the weighted distances between the centroids of the facilities is minimized. A hybrid approach combining a *BRKG*A, to determine the order of placement and the dimensions of each facility, a novel placement strategy, to position each facility, and a linear programming model, to fine-tune the solutions, is developed. The proposed approach is tested on 100 random datasets and 28 of benchmark datasets taken from the literature and compared against 21 other benchmark approaches. The quality of the approach was validated by the improvement of the best known solutions for 19 of the 28 extensively studied benchmark datasets.

Keywords: Facilities planning and design, facility layout, biased random-key genetic algorithms, random-keys.

1 Introduction

The facility layout design (*FLP*) problem is a challenging non-linear combinatorial optimization problem encountered in many service and manufacturing organizations. The problem has been extensively studied in the literature and good reviews can be found in Kusiak and Heragu (1987) and Meller and Gau (1996).

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The variant of the *FLP* we focus on in this paper was originally formulated by Armour and Buffa (1963) and involves determining the most cost-efficient arrangement of a given number of rectangular facilities with unequal area requirements within a given rectangular floor space. This problem is denoted as *UA-FLP*. The objective of the problem is to minimize the cost associated with the interactions between facilities (the cost of material-handling flows). This cost is commonly represented by the sum of the products (over all facility pairs) of the weighted rectilinear distance and the material-handling flow between the centroids. The constraints of the problem include facility area requirements and shape restrictions, as well as making sure that the facilities do not overlap and are located within the boundary of the space floor.

The *UA-FLP* is NP-hard since it is a generalization of the quadratic assignment problem (QAP), where the optimization is carried over a finite set of possible facility locations. The QAP was shown to be NP-hard by Sahni and Gonzalez (1976). Various methods and procedures have been proposed to solve the *FLP* and can be classified classified into:

- *Exact procedures* - Montreuil (1991) proposed one of the first MIP formulations of the *FLP* on the continuous plane. The model includes disjunctive constraints to prevent facility overlaps and bounded perimeter constraints to enforce specified facility area and shape requirements. The largest problem instance solved optimally by Montreuil's original formulation had six facilities. Meller et al. (1998) used valid inequalities to tighten Montreuil's formulation and were able to solve optimally problems with up to eight facilities. Sherali et al. (2003) used a polyhedral outer approximation to the facility area to improve the model of Meller et al. (1998) and solved a nine-facility problem optimally. Castillo and Westerlund (2005) developed an ε-accurate approximation. However, the largest problem solved had only nine facilities. By reformulating the *FLP*, using the sequence-pair representation of Murata et al. (1996), Meller et al. (2007) were able to solve problems with up to 11 facilities. Konak et al. (2006) modeled the facility area constraints exactly using a set of linear constraints derived from the structure of the flexible bay structure representation and reported solving problems having up to 14 facilities. Banerjee et al. (1992), Montreuil et al. (1993), and Banerjee et al. (1997) used design skeletons to reduce the complexity of MIP formulations for the *FLP*. Lacksonen (1997) fixed the orientations of obvious facility pairs using a pre-processing heuristic.

- *Heuristics and meta-heuristics* - Tam and Li (1991) proposed a hierarchical approach that employs a divide-and-conquer strategy consisting of three phases: (1) cluster analysis, (2) initial layout, and (3) layout refinement. The cluster analysis generates a hierarchical structure of the layout. The second phase produces an initial layout of each cluster which is subsequently refined in the third phase. Langevin et al. (1994) developed a heuristic approach based on Montreuil's MIP model (Montreuil, 1991) to solve spine layout problems. Kado (1996) developed six types of genetic algorithms using a slicing-tree structure representation. Garces-Perez et al. (1996) proposed a multiple purpose genetic programming kernel to generate slicing trees that are converted into candidate solutions. Schnecke and Vornberger (1997) introduced a genetic algorithm with a tree-structured genotype representation and hybrid problem-specific operators. Dunker et al. (2003) developed a coevolutionary approach that improved mutation and crossover operators and clustered the facilities into groups. Scholz et al. (2009) proposed a tabu search algorithm with a slicing-tree representation and incorporated a bounding curve for solving fixed and flexible facilities in *UA-FLPs*. Their tabu search incorporated four types of neighborhood moves to find better solutions. Komarudin and Wong (2010) proposed an Ant System to solve the *UA-FLP* using a slicing-tree representation and several types of local search to improve its search performance. Wong and Komarudin (2010) developed an Ant System algorithm for solving *UA-FLP*s using an improved flexible bay structure representation called modified-FBS (mFBS). McKendall Jr and Hakobyan (2010) introduced an approach which uses a boundary search construction technique that places facilities along the boundaries of already placed facilities. The solution is improved using a tabu search. Kulturel-Konak and Konak (2011b) use an ant colony optimization approach which uses a flexible bay structure representation. Kulturel-Konak and Konak (2011a) produced an hybrid particle swarm optimization and local search approach using a relaxed flexible bay structure (RFBS).

- *Matheuristics* - Gau and Meller (1999) proposed an algorithm that iterates between a genetic algorithm with a slicing-tree representation and a mixed-integer program with a subset of the binary variables set via the genetic algorithm. Montreuil et al. (2004) put together an algorithm based on ant colony optimisation (ACO) and the zone-based MIP (Montreuil et al., 2002) where the ACO-based heuristic searches for assignments of facilities to zones, and then the zone-based MIP is used to determine the detailed layout, including the input/output points of the facilities. Liu and Meller (2007) proposed a GA combining the sequence-pair representation (Murata et al., 1996) with the MIP model of Sherali et al. (2003). For a given sequence-pair, the corresponding layout is determined in this hybrid approach using the linear programming (LP) relaxation of the MIP model. Bozer and Wang (2012) introduced a hybrid approach based on a new representation called the graph-pair representation. The graph-pair representation encodes the relative locations of the facilities and the shape and a uses an LP to determine the exact location of each facility. A simulated annealing algorithm is used to search for new layouts based on the graph-pair representation. Recently, Kulturel-Konak and Konak (2013) proposed a hybrid genetic algorithm and linear programming approach to solve the *UA-FLP* which uses a new encoding scheme, called location/shape and which represents the relative facility positions based on the centroids and orientations of the facilities. After the relative facilities positions are set by the GA, the actual facility locations and shapes are determined by solving an LP problem.

Our contribution to solve the *UA-FLP* is a matheuristic which combines a *biased random-key genetic algorithm (BRKGA)*, a novel placement strategy, and a linear programming model to fine-tune the solutions.

The remainder of the paper is organized as follows. Section 2 presents a formal model of the *UA-FLP*. Section 3 introduces the new approach, describing in detail the *BRKGA*, the novel placement strategy, and the fitness function. Finally, in Section 4, we report on computational experiments, and in Section 5 make concluding remarks.

2 Problem Formulation

Let N be the number of rectangular facilities with unequal areas to be placed, without any overlap, on a rectangular floor space with dimensions (W, H) along the X- and Y-axis, respectively. Each facility $i = 1, ..., N$ is defined by its dimensions along the horizontal and vertical axis (w_i, h_i), its area $A_i = w_i \times h_i$, and the maximum aspect ratio, R_{max}, which due to practical reasons imposes the maximum permissible ratio between its longest and shortest dimensions, i.e., $R_{max} \geq max\left\{w_i, h_i\right\} / min\left\{w_i, h_i\right\} \geq 1$.

A layout is defined by the coordinates of the centroid (x_i, y_i) and the horizontal (w_i) and vertical (h_i) dimensions of each facility i. The cost function to be minimized is

$$Cost = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} c_{i,j}\, f_{i,j}\, d_{i,j}$$

where $f_{i,j}$ represents the flow between facilities i and j (we assume that $f_{i,i} = 0$), $c_{i,j}$ is the cost per unit distance between i and j, and $d_{i,j}$ is the distance between the centroids of facilities i and j. This distance $d_{i,j}$ can be measured according to one of the following distance norms:

- Rectilinear distance (R): $d_{i,j} = |x_i - x_j| + |y_i - y_j|$
- Euclidean distance (E): $d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$

- Squared Euclidean distance (SE): $d_{i,j} = (x_i - x_j)^2 + (y_i - y_j)^2$

Since most real-world layout problems make use of the rectilinear distance norm, we will model the problem using this distance norm. In practice the dimensions of the floor space and the area of the facilities are usually not hard constraints and can accommodate small variations. To be able to include this flexibility when searching for a solution we will add to our model the variables $\triangle w_i$ and $\triangle h_i$ to represent, respectively, the amount that each facility i exceeds with respect to the horizontal and vertical dimensions of the floor space dimensions (W, H) and include a term in the objective function to penalize this excess.

The problem can be cast in an intuitive way in the form of a nonlinear mixed integer programming model, *FLP-NMIP*. In the model we will use the following additional notation:

$P_{i,j}, Q_{i,j}$	=	Binary variables used to model the non-overlapping constraints;
M^x, M^y	=	Parameters defining upper bounds on the horizontal and vertical distance between any two facilities, respectively.
$d^x_{i,j}$,$d^x_{i,j}$,	=	Variables representing the distances between the facilities i and j along the X- and Y-axis, respectively.
M	=	Constant used to penalize the solutions for exceeding the floor space dimensions. The value of this constant should be chosen according to the available flexibility to accommodate small variations in the dimensions of the floor space. If no variations are allowed, then $M = \infty$.

A nonlinear integer programming model of the facility layout problem is given in (2)-(15). The objective function (2) minimizes the total cost using the appropriate distance norm. Constraints (3)-(4) define the horizontal and vertical dimensions of the facility according to the area and maximum ratio allowed. Constraints (5)-(8) impose the non-overlapping constraints by forcing the facilities to be separated horizontally and/or vertically. Constraints (9)-(10) force each facility to be within the horizontal and vertical limits of the floor space, respectively. Constraints (11)-(14) represent the distances between all pairs of facilities (i, j) according to the rectilinear norm distance function. Finally, constraints (15) are the domain constraints for the variables.

In the model *FLP-NMIP* the constraints (3) are non-linear resulting in a hard-to-solve model. However, Castillo and Westerlund (2005) developed a linear approximation based on a cutting plane representation of the actual area constraint that guarantees that, at optimality, the final area of each facility is within an $\varepsilon\%$ error of the required area regardless of the aspect ratio of the facilities. To linearize the model and make it easier to solve, we replace the non-convex and hyperbolic area constraints (3) and the aspect ratio constraints 4 with an ε-accurate representation as follows:

$$- h_i - \frac{A_i}{\overline{w}^2_{i,k}} w_i \leq 2\frac{A_i}{\overline{w}_{i,k}}, \qquad k = 0, \ldots, C_i, \quad \forall\, i \tag{1}$$

where $\overline{w}_{i,k}$ corresponds to the tangent points of the cutting planes on the real curve and C_i is the total number of points being used in the approximation according to the chosen $\varepsilon\%$ error value. The resulting model with constraints (1) in place of constraints (3) will be denoted as *FLP-MIP*.

$$(FLP\text{-}NMIP) \quad Minimize\ \ Cost = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} c_{i,j}\, f_{i,j} \left(d^x_{i,j} + d^x_{i,j} \right) + M \sum_{i=1}^{i=N} \left(\triangle w_i + \triangle h_i \right) \tag{2}$$

Subject to:

Facility aspect constraints

$$w_i \times h_i = A_i \qquad \forall\, i \tag{3}$$

$$\frac{1}{R_{max}} \leq \frac{w_i}{h_i} \leq R_{max} \qquad \forall\, i \tag{4}$$

Non-overlapping constraints

$$x_i - x_j + M^x\left(P_{i,j} + Q_{i,j}\right) \geq \frac{w_i + w_j}{2} \qquad \forall\, i, j \,|\, j > i \tag{5}$$

$$x_j - x_i + M^x\left(1 - P_{i,j} + Q_{i,j}\right) \geq \frac{w_i + w_j}{2} \qquad \forall\, i, j \,|\, j > i \tag{6}$$

$$y_i - y_j + M^y\left(1 + P_{i,j} - Q_{i,j}\right) \geq \frac{h_i + h_j}{2} \qquad \forall\, i, j \,|\, j > i \tag{7}$$

$$y_j - y_i + M^y\left(2 - P_{i,j} - Q_{i,j}\right) \geq \frac{h_i + h_j}{2} \qquad \forall\, i, j \,|\, j > i \tag{8}$$

Floor space constraints

$$\frac{w_i}{2} \leq x_i \leq W - \frac{w_i}{2} + \triangle w_i \qquad \forall\, i \tag{9}$$

$$\frac{h_i}{2} \leq y_i \leq H - \frac{h_i}{2} + \triangle h_i \qquad \forall\, i \tag{10}$$

Distance constraints

$$x_i - x_j \leq d^x_{i,j} \qquad \forall\, i, j \,|\, j > i \tag{11}$$

$$x_j - x_i \leq d^x_{i,j} \qquad \forall\, i, j \,|\, j > i \tag{12}$$

$$y_i - y_j \leq d^y_{i,j} \qquad \forall\, i, j \,|\, j > i \tag{13}$$

$$y_j - y_i \leq d^y_{i,j} \qquad \forall\, i, j \,|\, j > i \tag{14}$$

Domain constraints

$$x_i, y_i, w_i, h_i, d^x_{i,j}, d^x_{i,j} \geq 0 \;\; \forall\, i \quad \text{and} \quad P_{i,j},\, Q_{i,j} \in \{0, 1\} \quad \forall\, i, j \,|\, j > i \tag{15}$$

The mixed integer model *FLP-MIP* developed above is still computationally difficult and fails to provide optimal or even near-optimal solutions on real-size problems due to the large number of binary variables. To overcome this problem we developed a new solution methodology, which combines a biased random-key genetic algorithm with a novel placement strategy. In the next section we describe the new methodology.

3 Biased random-key genetic algorithm

We begin this section with an overview of the proposed solution methodology. This is followed by a discussion of the biased random-key genetic algorithm, including detailed descriptions of the solution encoding and decoding, evolutionary process and novel placement strategy.

We will first describe the algorithm for the case where the dimensions of the floor space are unconstrained (i.e., we do not take into account equations 9 and 10). Later in Section 4.2 we extend the approach to the constrained case.

3.1 Overview

The new approach is based on a constructive heuristic algorithm which places the facilities, one at a time, on the rectangular floor space. To avoid overlapping the facilities, we propose a novel placement strategy which uses the concept of *empty maximal-spaces,* as described in Lai and Chan (1997), and a novel, very efficient, placement procedure to position a facility within a given empty maximal space. The new approach combines a biased random-key genetic algorithm and the novel placement strategy.

The role of the genetic algorithm is to evolve the encoded parameters, or *chromosomes*, that represent the facility placement sequence (FPS), the vector of facility aspect ratios (FAR), and the position of the first facility (x_{first}, y_{first}). The vectors of encoded parameters are decoded using a novel placement strategy which results in the placement of each facility. For each chromosome, the following phases are applied to decode the chromosome:

1. *Facility placement sequence decoder*: This first phase decodes part of the chromosome into the FPS, i.e., the sequence in which the facilities are placed on the floor space.

2. *Facility aspect ratio decoder*: The second phase decodes part of the chromosome into the FAR, i.e., the vector of facility aspect ratios.

3. *Position of the first facility decoder*: The third phase decodes part of the chromosome into the coordinates (x_{first}, y_{first}) of the first facility to be placed on the floor space.

4. *Placement strategy*: The fourth phase makes use of FPS, FAR, and (x_{first}, y_{first}), defined in phases 1, 2, and 3, and places all the facilities on the floor space using the novel placement strategy.

5. *Fitness evaluation*: The final phase computes the fitness of the solution obtained in phase 4 (a measure of quality of the facility placement) using equation (2).

Figure 1 illustrates the sequence of steps applied to each chromosome generated by the *BRKGA*.

The remainder of this section describes the genetic algorithm, the decoding procedure, and the placement strategy in detail.

3.2 Biased random-key genetic algorithms

Genetic algorithms with random keys, or *random-key genetic algorithms* (*RKGA*), for solving sequencing problems were introduced in Bean (1994). In a *RKGA*, chromosomes are represented as vectors of randomly-generated real numbers in the interval $[0, 1]$. A *decoder* is a deterministic algorithm that takes as input a chromosome and associates with it a solution of the combinatorial optimization problem for which an objective value or *fitness* can be computed.

A *RKGA* evolves a *population* of random-key vectors over a number of *generations* (iterations). The initial population is made up of p vectors of r random keys. Each component of the solution vector, or random key, is generated independently at random in the real interval $[0, 1]$. After the fitness of each individual is computed by the decoder in generation g, the population is partitioned into two groups of individuals: a small group of p_e *elite* individuals, i.e. those with the best fitness values, and the remaining set of $p - p_e$ *non-elite* individuals. To evolve the population of generation g, a new generation ($g + 1$) of individuals is produced. All elite individuals of the population of generation g are copied without modification to the population of generation $g + 1$. *RKGAs* implement mutation by introducing *mutants* into the population. A mutant is a vector of random keys generated in the same way that an element of the initial population is generated. Its role is similar to that of mutation in other genetic algorithms (Goldberg, 1989), i.e. to introduce noise into the population and avoid convergence of the entire population to a local optimum. At each generation, a small number p_m of mutants is introduced into the population. With $p_e + p_m$ individuals accounted for in

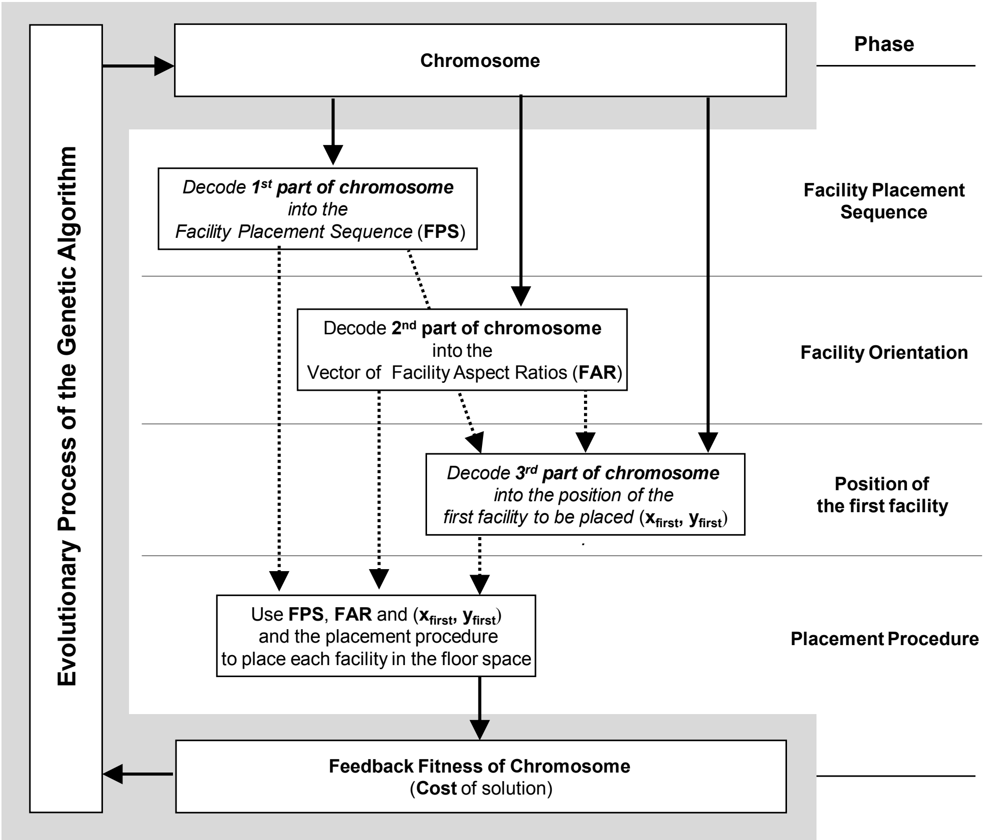

Figure 1: Architecture of the algorithm.

the population $g+1$, $p - p_e - p_m$ additional individuals need to be generated to complete the p individuals that make up population $g+1$. This is done by producing $p - p_e - p_m$ offspring solutions through the process of *mating* or *crossover*.

A *biased random-key genetic algorithm* (Gonçalves and Resende, 2011), or simply *BRKGA*, differs from a *RKGA* in the way parents are selected for mating and how mating is carried out. While in the *RKGA* of Bean (1994) both parents are selected at random from the entire current population, in a *BRKGA* each element is generated combining a parent selected at random from the elite partition in the current population and one from the rest of the population. Repetition in the selection of a mate is allowed and therefore an individual can produce more than one offspring in the same generation. As in a *RKGA*, *parametrized uniform crossover* (Spears and Dejong, 1991) is used to implement mating in a *BRKGA*. Let ρ_e be the probability that an offspring inherits the vector component of its elite parent. Recall that r denotes the number of components in the solution vector of an individual. For $i = 1, \ldots, r$, the i-th component $c(i)$ of the offspring vector c takes on the value of the i-th component $e(i)$ of the elite parent e with probability ρ_e and the value of the i-th component $\bar{e}(i)$ of the non-elite parent $\bar{e}$ with probability $1 - \rho_e$. While in a BRKGA $\rho_e > \frac{1}{2}$, in a RKGA this is not necessarily the case.

When the next population is complete, i.e. when it has p individuals, fitness values are computed for all of the newly created random-key vectors and the population is partitioned into elite and non-elite individuals to start a new generation.

A *BRKGA* searches the solution space of the combinatorial optimization problem indirectly by searching the continuous r-dimensional hypercube, using the decoder to map solutions in the hypercube to solutions in the solution space of the combinatorial optimization problem where the fitness is evaluated.

To specify a biased random-key genetic algorithm, we simply need to specify its parameters, how solutions are encoded and decoded, and how their corresponding fitness values are computed. We specify our algorithm next by first showing how the facility placement problem is encoded and then decoded into a solution and how their fitness evaluation is computed.

3.2.1 Chromosome representation and decoding

A chromosome encodes a solution to the problem as a vector of random keys. In a direct representation, a chromosome represents a solution of the original problem, and is called *genotype*, while in an indirect representation it does not. In an indirect representation, special procedures are needed to extract a solution, or *phenotype*, from it. In the present context, solutions will be represented indirectly by parameters that are later used by a decoding procedure to obtain a solution. To obtain the phenotype we use the decoding procedures described in Section 3.3.3.

Each solution chromosome is made of $2N+2$ genes as depicted in Figure 2, where N is the number of facilities to be laid out. The first N genes are used to obtain the *Facility Placing Sequence* (FPS), genes $N+1$ to $2N$ are used to obtain the *vector of Facility Aspect Ratios* (FAR), and genes $2N+1$ and $2N+2$ are used to obtain the coordinates (x_{first}, y_{first}) of the first facility to be placed. The placement procedure, described in Section 3.3.3, makes use of FPS, FAR, and (x_{first}, y_{first}) to construct a solution from the chromosome.

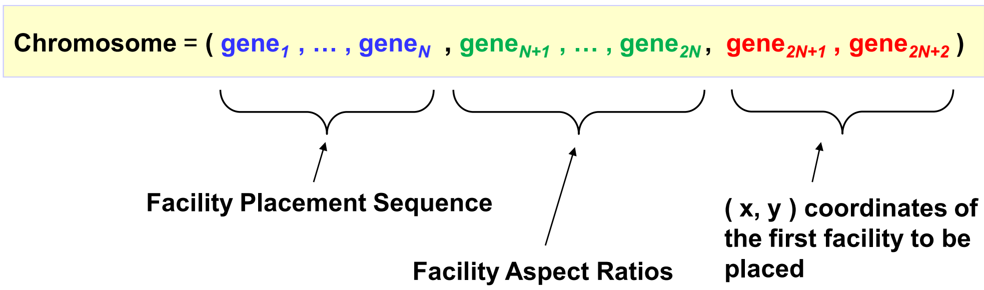

Figure 2: Solution encoding.

The decoding (mapping) of the first N genes of each chromosome into a FPS is accomplished by sorting the key values of the genes in increasing order. The sorted indices correspond to the sequence in which facilities will be laid out. Figure 3 shows an example of the decoding process for the FPS. There are eight facilities in this example. Sorting the eight random keys in increasing order produces the following $FPS = (5, 8, 3, 1, 4, 2, 6, 7)$.

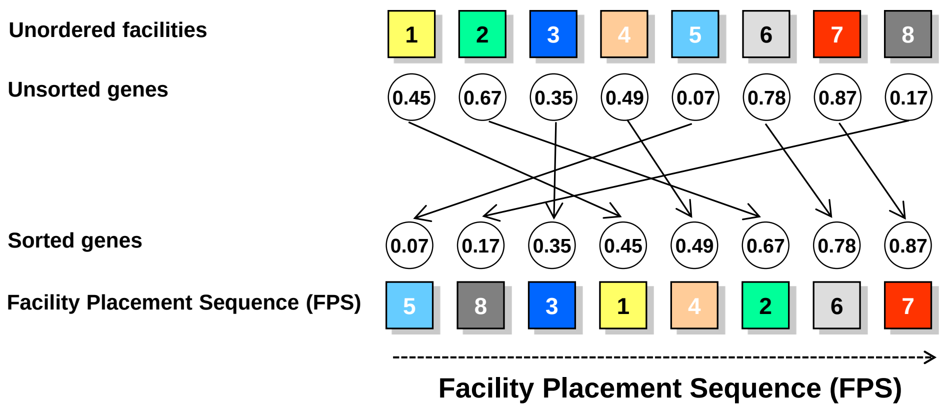

Figure 3: Decoding of the Facility Placement Sequence.

The decoding of the vector of facility aspect ratios (FAR) is accomplished for $i = 1, \ldots, N$, using the following expression:

$$FAR_i = \frac{1}{R_{max}} + gene_i \times \left(R_{max} - \frac{1}{R_{max}}\right). \tag{16}$$

Since $FAR_i = \dfrac{w_i}{h_i}$, the dimensions of each facility i can now be computed as

$$w_i = \sqrt{A_i \times FAR_i}\,, \tag{17}$$

and

$$h_i = A_i / w_i. \tag{18}$$

The *first* facility to be laid out is defined by the FPS and will be denoted as FPS_1. Its coordinates, i.e. (x_{first}, y_{first}) are decoded as follows:

$$x_{first} = \frac{w_i}{2} + gene_{\,2N+1} \times (W - w_i) \tag{19}$$

$$y_{first} = \frac{h_i}{2} + gene_{\,2N+2} \times (H - h_i)\,. \tag{20}$$

FPS, FAR, and (x_{first}, y_{first}) are used later by the placement procedure to construct a layout with all the facilities placed on the floor space.

3.2.2 Fitness function

To evolve the solutions, the evolutionary process needs a measure of the solution fitness, or quality. A natural fitness function for this type of problem is the layout cost defined as

$$Cost = \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} c_{i,j}\, f_{i,j}\, d_{i,j} + M \sum_{i=1}^{i=N} (\triangle w_i + \triangle h_i)\,,$$

where, as defined above, $c_{i,j}$ and $f_{i,j}$ are respectively, the cost per distance unit and the flow between facilities i and j, and $d_{i,j}$ is the distance according to the appropriate norm. The second term in the fitness corresponds to the penalty for exceeding the dimensions of the floor space. Note that when the dimensions of the floor space are unconstrained the value of the penalty term will be equal to zero, i.e., $M = 0$.

3.3 Placement strategy

In the next sections we describe the main components of the placement strategy.

3.3.1 Maximal-spaces and the difference process

While trying to place a facility on the floor space we use a list S of empty maximal-spaces (*EMSs*), i.e., largest empty rectangular spaces available on the floor space. An empty maximal-space s is represented by its vertices with minimum and maximum coordinates (min_x_s, min_y_s and max_x_s, max_y_s, respectively). When searching for a place to position a facility, we need to consider only the available *EMSs* where the facility being placed fits. This way, we guarantee that there will be no overlap between facilities. To generate and keep track of the *EMSs,* we make use of the *difference process* (DP), developed by Lai and Chan (1997). Figure 4 depicts an example of the application of the DP process. In the example, we assume that we have two facilities to be placed on the floor space. Since the floor space is initially empty, we only have the empty maximal-space EMS_0 which is the entire floor space (see Figure 4a). After placing facility 1 in EMS_0 we update the list S of available empty maximal-spaces so we can try to place facility 2. Figure 4b shows the four newly generated *EMSs*. Facility 2 is placed in EMS_2 and the DP process results in the six *EMSs* shown in Figure 4c. Every time a facility is placed on the floor space, we reapply the DP process to update list S before we place the next facility.

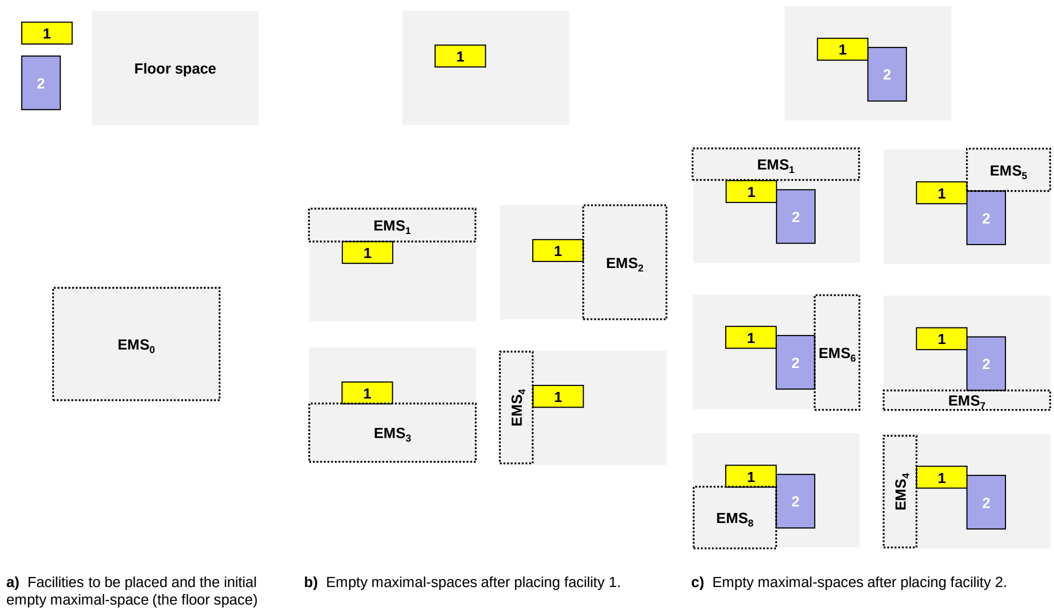

Figure 4: Difference process (DP) example.

In the unconstrained case we assume that the floor space can include all the facilities laid out horizontally or vertically at their maximum horizontal or vertical dimensions, i.e.,

$$EMS_0 = \left(0\,,\,0,\,\sum_i \sqrt{A_i \times R_{max}}\,,\,\sum_i \sqrt{A_i \times R_{max}}\right).$$

3.3.2 Placement of a facility in an empty maximal-space

When attempting to place a facility i on the floor space we want the facility to have the least cost with respect to the facilities already laid out. To achieve this we solve the following problem for every available empty maximal-space s and all facilities $k \in K$ already placed on the floor space:

$$(FLP\_EMS) \qquad \textbf{Minimize} \quad Cost\,(i) = \sum_{k \in K} c_{i,k}\, f_{i,k}\, d_{i,k} \tag{21}$$

Subject to:

$$min\_x_s + \frac{w_i}{2} \le x_i \le max\_x_s - \frac{w_i}{2} \tag{22}$$

$$min\_y_s + \frac{h_i}{2} \le y_i \le min\_y_s - \frac{h_i}{2} \tag{23}$$

Note that we solve FLP_EMS only for empty maximal-spaces in which facility i fits.

FLP_EMS has a non-linear objective function (21) and the variables x_i, y_i are subject only to box constraints (22)-(23). This problem can be solved with the non-monotone spectral projected gradient method proposed in Birgin et al. (2000) (FORTRAN source code in http://www.ime.usp.br/~egbirgin/tango/codes.php#spg). However, we developed a new more efficient surrogate approach. The new approach starts by computing the unconstrained optimal (i.e., without the box constraints), denoted by UO (details on how to compute UO for the three distance norms can be found in Heragu (1997)). Next, facility i is tentatively positioned at the geometric center of EMS s, i.e., at the point

$$\left(\frac{min\_x_s + max\_x_s}{2}, \frac{min\_y_s + max\_y_s}{2}\right).$$

The final position of facility i is obtained by moving its centroid as close as possible to UO without leaving the boundaries of $EMSs$. We first try to get as close as possible to UO by moving vertically and then by moving horizontally (or vice-versa). Figure 5 illustrates this approach for two empty maximal spaces.

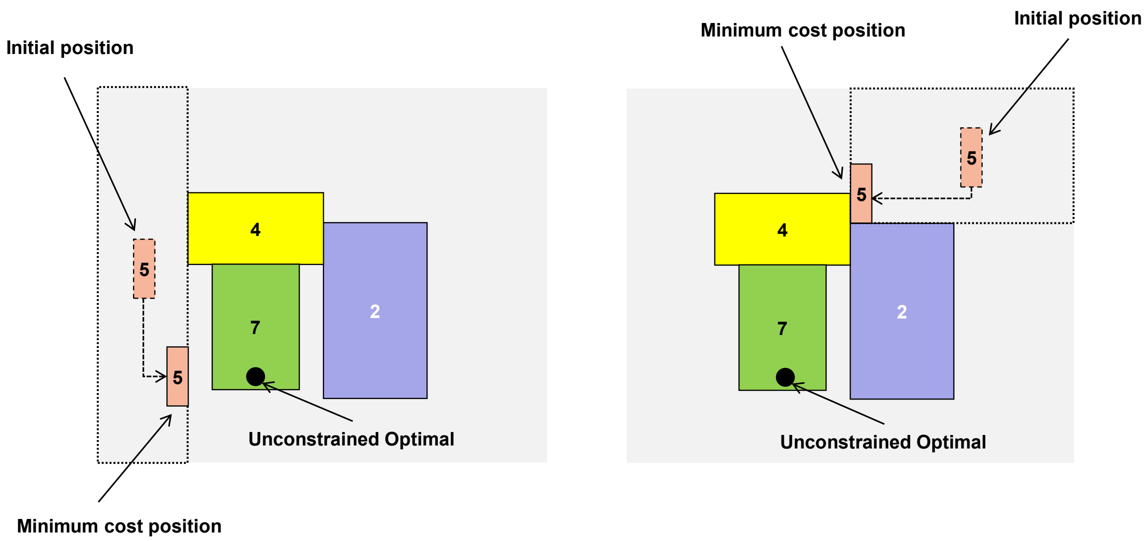

Figure 5: Example of the novel approach used to solve FLP_EMS.

In some cases there is no flow between the facility i being placed and all the other facilities $k \in K$ already placed on the floor space, i.e., $f_{i,k} = 0 \; \forall \, k \in K$. When this occurs we consider UO to be equal to the geometric center of all laid-out facilities, i.e.,

$$(UO_x \, , UO_y) = \left(\frac{1}{|K|} \sum_{k \in K} x_k \, , \, \frac{1}{|K|} \sum_{k \in K} y_k \right).$$

From this point on we will denote by $FLP\_EMS(i, \, s, \, x, \, y)$ the procedure that determines the minimum-cost position $(x, \, y)$ of facility i in EMS s.

3.3.3 Placement procedure

The placement procedure follows a sequential process which places a single facility at each stage. The procedure combines FPS, FAR, and (x_{first}, y_{first}) evolved by the $BRKGA$. Each stage is comprised of four main steps:

1. Facility selection;
2. Computation of the facility aspect ratio and its dimensions;
3. Facility placement;
4. State information update.

Pseudo-code of the placement procedure is given in Figure 6. The facility selection at stage i chooses for placement the facility in the i^{th} position of the *FPS* (line 4 of the pseudo-code). The facility dimensions are defined by the i^{th} position of the *FAR* (line 5). The facility placement is carried out in lines 6-18. The coordinates of the first facility placed are defined in line 16 while the coordinates of the other facilities are defined in lines 7-14 of the pseudo-code. The facility placement is carried out in line_18. The final step, state information update, consists in updating the list of empty maximal spaces, according to the facility being placed and the corresponding coordinates, using the *DP* procedure (line 19 of the pseudo-code).

3.4 Constrained case

Our first approach to extend the algorithm for the case where the dimensions of the floor space are constrained was to use *BRKGA* with a large value assigned to the penalty constant M, i.e., $M = (W + H) \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} f_{i,j}$. This approach works quite well, for all the three distance norms presented above when the area of the floor space exceeds the sum of the areas of the facilities by more than about 25%. However, when the area of the floor space is close to the sum of the areas of the facilities, the quality of the solution worsens significantly and most of the times no feasible solution is found. To overcome this problem, we developed a new hybrid approach, denoted by *BRKGA-LP*, to solve problems with floor space constraints. In the hybrid *BRKGA-LP* approach, the *BRKGA* produces unconstrained solutions defining the relative locations of facilities in the floor space. The relative locations are then used to define the separation constraints of the *FLP-MIP* model transforming it into a linear program, denoted by *FLP-LP*. The solution of the *FLP-LP* determines the new locations and dimensions of all the facilities that may improve the *BRKGA* solution.

The constrained approach, *BRKGA-LP*, can be described for each chromosome by the following steps:

1. Solve the problem with *BRKGA*, the unconstrained approach. In this case and after some experimentation we came to the conclusion that limiting the unconstrained floor space to 1.7 times of the real horizontal and vertical dimensions of the floor space produces better results, when solving in the next step *the FLP-LP,* than if the two dimensions where both unconstrained, i.e., very large. Therefore, when solving the unconstrained problem we use a floor space with dimensions $(1.7 \times W, 1.7 \times H)$, as shown in Figure 7a. Let S^{BRKGA} be the solution produced by the *BRKGA* in this step. Note that sometimes, due to the limitations of the floor dimensions, it is not possible to position all the facilities in the floor space. In that case, we consider the fitness of the solution S^{BRKGA} to be equal to ∞ and skip steps 2 and 3.

2. Based on the solution S^{BRKGA} obtained in the previous step, formulate the corresponding linear model *FLP-LP*(S^{BRKGA}), where the non-overlapping constraints (5), (6), (7), and (8) in the *FLP-MIP* model are replaced by a single constraint defined according to S^{BRKGA}. The single separating constraint that will replace the original separating constraints in the *FLP-MIP* model can be determined as follows. Let $DisW_{i,j} = |x_i - x_j| - \frac{w_i}{2} - \frac{w_j}{2}$ and $DistH_{i,j} = |y_i - y_j| - \frac{h_i}{2} - \frac{h_j}{2}$ be, respectively, the horizontal and vertical distances separating facilities i and j. If $DistW_{i,j} \geq DistH_{i,j}$ then separate facilities i and j horizontally. If $x_i \leq x_j$, use constraint $x_i + \frac{w_i}{2} + \frac{w_j}{2} \leq x_j$. Otherwise, use $x_j + \frac{w_i}{2} + \frac{w_j}{2} \leq x_i$. If $DistW_{i,j} < DistH_{i,j}$ then separate facilities i and j vertically. If $y_i \leq y_j$ use constraint $y_i + \frac{h_i}{2} + \frac{h_j}{2} \leq y_j$. Otherwise, use $y_j + \frac{h_i}{2} + \frac{h_j}{2} \leq y_i$. Note that this way of defining the constraint guarantees that we will always obtain a feasible solution in terms of the relative position and dimensions of the facilities.

3. Solve *FLP-LP*(S^{BRKGA}) to try to improve the solution S^{BRKGA} by using new locations and dimensions for all facilities. Figure 7b depicts a possible improved solution obtained by *FLP-LP*(S^{BRKGA}) for the *BRKGA* solution presented in Figure 7a.

```
procedure PLACEMENT (FPS, FAR, x_first, y_first)
    Let S be the set of EMSs available;

    // ** Initialization
    S ← {FloorSpace};

    for i = 1, ..., N do

        // ** Facility selection
        FacToPlace ← FPS_i // select the facility in the i^th position of FPS;

        // ** Computation of facility aspect ratio and dimensions
        Compute FAR_i, w_i and h_i, using equations 16, 17 and 18, respectively;

        // ** Facility placement
        if i > 1 then // not first facility
            // Place facility FacToPlace in the position having the minimum cost
            BestCost = ∞;
            for all s in EMSs do
                Cost = FLP_EMS(FacToPlace, s, x, y);
                if Cost ≤ BestCost then
                    BestCost = Cost;
                    x* = x, y* = y;
                endif
            end for
        else
            x* = x_first, y* = y_first
        endif

        Place the centroid of FacToPlace at position (x*, y*);

        // ** State information update
        Update the list of S of using
                the DP procedure of Lai and Chan (1997);

    end for
end PLACEMENT;
```

Figure 6: Pseudo-code for the PLACEMENT procedure.

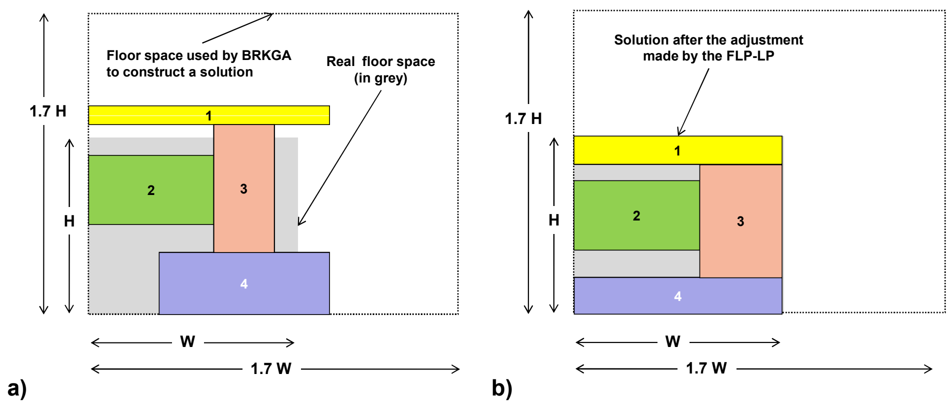

Figure 7: *BRKGA* solution in a) and the corresponding *FLP-LP* improved solution in b).

Since solving *FLP-LP*(S^{BRKGA}) is quite expensive in terms of computational time we only carry on to steps 2 and 3 if the solution S^{BRKGA} seems promising. A solution is considered promising if the following two conditions are satisfied:

1. The area of the facilities outside the real floor space dimensions is smaller than 45% of the real floor space, i.e.,

$$Area\,of\,the\,facilities\,outside\,of\,the\,real\,floor\,space \leq 0.45 \times W \times H.$$

2. The cost of the solution S^{BRKGA} is at most 40% above the cost of the best solution found so far in the solution process. At the beginning of the solution process the cost of the best solution is equal to ∞.

4 Numerical experiments

To evaluate the performance and the capabilities of our *BRKGA* and *BRKGA-LP* approaches, we performed a series of computational experiments. The numerical experiments were conducted on a computer with a Intel Xeon E5-2630 @2.30GHz CPU and 16 GB of physical memory running the Linux operating system with Fedora release 18. The algorithm *BRKGA* and *BRKGA-LP* were coded using the C++ programming language and the linear programs were solved using GUROBI OPTIMIZER version 5.5.

Two different types of FLPs were investigated; in the *unconstrained case*, we consider problems in which the dimensions of the floor space are allowed to be determined by the optimization algorithm; in the *constrained case*, we consider problems with given dimensions for the final layout.

The next subsections report the details of the experiments and the results obtained by approach proposed in this paper.

4.1 BRKGA configuration

The parameters of the *BRKGA* were configured based on our past experience with genetic algorithms based on the same evolutionary strategy (see Gonçalves and Almeida (2002), Gonçalves and Resende (2004), Gonçalves et al. (2005), Mendes et al. (2009), Gonçalves et al. (2009), Gonçalves et al. (2011), Gonçalves and Sousa (2011), Gonçalves and Resende (2012), Fontes and Gonçalves (2013), Gonçalves and Resende (2013), Gonçalves and Resende (2014), Gonçalves et al. (2014a) and Gonçalves et al. (2014b)) has shown that good results can be obtained with the values of p_e, p_m, and Crossover Probability (ρ_e) shown in Table 1.

For the population size we obtained good results by indexing it to the size of the problem, i.e., use small size populations for small problems and larger populations for larger problems.

Table 1: Range of parameters for the evolutionary strategy.

Parameter	**Interval**
p_e	0.10 – 0.25
p_m	0.15 – 0.30
Crossover Probability (ρ_e)	0.70 – 0.85

The configuration shown in Table 2 was held constant for all problem instances in the experiments. The experimental results demonstrate that this configuration not only provides high-quality solutions but it is also very robust.

Table 2: Configuration parameters for the *BRKGA* algorithm.

Parameter	Value
$p =$	$100 \times N$
$p_e =$	$min(0.25 \times p,\, 50)$
$p_m =$	$0.25 \times p$
$\rho_e =$	0.70
Fitness =	Cost of layout given by eq. (2) (to be minimized)
Stopping Criterion =	50 generations for the unconstrained case and 100 generations for the constrained case

4.2 Unconstrained case

As mentioned above in the unconstrained case the dimensions of the floor space are free and are determined by the optimizing algorithm. Additionally, note that the unconstrained approach can be used with any of the distance measures (R-rectilinear, E-Euclidean and SE-Squared Euclidean).

4.2.1 Datasets and benchmark approaches

To compare the performance of the *BRKGA* approach against other approaches for the unconstrained case, we used 16 datasets from the literature and 100 randomly generated datasets which were constructed in such a way that the optimal solution is known. We do that in order to be able to measure, in absolute terms, the deviation from the optimal values. A summary of the datasets used is presented in Table 3.

We compare the unconstrained version of the *BRKGA* with the approaches listed in Table 4.

4.2.2 Experimental results

In Tables 5, 6, and 7 we report the best cost and average times obtained over ten runs of BRKGA for all datasets. For the other approaches we also report the best cost. Even though the computational times reported by the other approaches might not be comparable we report them if they are available.

In Table 5 we evaluate the performance of BRKGA on the datasets TL05-TL30. As can be seen in column %Imp. BRKGA has improved the best solution for six out of the eight datasets. The improvements vary from 1.4% for TL08 to 25% for TL30. For dataset TL06 the approach GA-TSG ranks first and BRKGA ranks second while for dataset TL15 the approach HA-C ranks first and BRKGA second. It is clear that overall the BRKGA approach has the best performance in terms of solution quality.

Table 3: Benchmark datasets used in the unconstrained case.

Dataset	**Distance Measure**	**Description**	**Source**
L020, L028, L050, L075, L100, L125A, L125B	**R** L020, L075, L100, L125A and L125B **SE** L028 **E** L050	Seven datasets with sizes equal to 20, 28, 50, 75, 100, 125, and 125 respectively. There are no constraints in the floor space dimensions.	Imam and Mir (1993), Mir and Imam (1996),Imam and Mir (1998),Mir and Imam (2001) and VIP-PLANOPT (2010) available from from the commercial software Engineering Optimization Software, VIP-PLANOPT 2006 and 2010 at http://www.planopt.com.
Dunker62	**R**	A dataset with 63 facilities. There are no constraints in the floor space dimensions.	Dunker et al. (2003)
TL05-30	**SE**	Eight datasets with sizes equal to 5, 6, 7, 8, 12, 15, 20, and 30 facilities. There are no constraints in the floor space dimensions.	Tam and Li (1991)
RND10-100	**R**	100 random generated datasets. Ten datasets for facility sizes equal to 10, 20, 30, 40, 50, 60, 70, 70, 80, 90, and 100. There are no constraints in the floor space dimensions. The dimensions of each of the facilities are individually given in the datasets. These datasets were generated in such away that the optimal solution is known.	Available from the authors upon request.

Table 4: Benchmark approaches used in the unconstrained case.

Approach	**Type of method**	**Source of approach**
HA-C	Hierarchical approach with clusters.	Tam and Li (1991)
GA-STS	Genetic Algorithm with a Slicing Tree Structure method.	Kado (1996)
GP-STS	Genetic Programming algorithm that generates Slicing Tree Structures later converted to FLP solutions.	Garces-Perez et al. (1996)
GA-TSG	Genetic algorithms with tree-structured genotype representation and hybrid problem-specific operators.	Schnecke and Vornberger (1997)
CA-C	Coevolutionary algorithm with clusters.	Dunker et al. (2003)
TSaST	Tabu search with slicing-tree.	Scholz et al. (2009)
VIP-PLANOPT	VIP-PLANOPT 2010 is a commercial software from Engineering Optimization Software which is based on the algorithms presented in Mir and Imam (1996),Imam and Mir (1998), and Mir and Imam (2001). Since VIP-PLANOPT presents better or equal results than the ones reported in the papers we will be using it for comparisons purposes instead of the above three approaches.	www.planopt.com.
TS-BST	Tabu search with boundary search technique.	McKendall Jr and Hakobyan (2010)
GUROBI	Version 5.5 of commercial software solver from Gurobi Optimization.	http://www.gurobi.com

Table 5: Experimental results: Datasets TL05-TL30.

	HA-C	GA-STS	GP-STS	GA-TSG	TSaST		BRKGA		
Dataset	Cost	Cost	Cost	Cost	Cost	T(s)	Cost	T(s)	%Imp.
TL05	247	228	226	214	213.48	2.3	**210.06**	0.035	1.60
TL06	514	361	384	**327**	348.76	3.0	345.03	0.049	-5.51
TL07	559	596	568	629	562.91	2.5	**549.68**	0.060	1.67
TL08	839	878	878	833	810.43	4.7	**799.09**	0.080	1.40
TL12	3162	3283	3220	3164	3054.23	12.5	**2920.47**	0.162	4.38
TL15	**5862**	7384	7510	6813	6615.81	17.0	6395.43	0.251	-9.10
TL20	–	16393	14033	13190	13198.40	50.0	**9892.38**	0.443	25.00
TL30	–	41095	39018	35358	33721.20	95.4	**31454.23**	1.132	6.72

Best values are in bold.

In Table 6 we evaluate the performance of *BRKGA* on the datasets L020-L125B and Dunker62. The best cost values of VIP-PLANOPT on datasets L020-L125B are obtained from the 2010 demo version while the times are taken from the user manual of the 2006 version since they are not reported elsewhere. The values for the cost and CPU times for dataset Dunker62 are taken from McKendall Jr and Hakobyan (2010). As can be seen in column %Imp., *BRKGA* improved the best solution for all eight datasets. The improvements vary from 1.86% for L020 to 11.05% for L125A. It is clear that, overall, the *BRKGA*-based approach has the best performance in terms of solution quality. In terms of computational times, the *BRKGA* solves all the datasets in less than two minutes.

Table 6: Experimental results: Datasets L020-L125B and Dunker62

	VIP-PLANOPT		TSaST		TS-BST		BRKGA		
Dataset	Cost	T(s)	Cost	T(s)	Cost	T(s)	Cost	T(s)	%Imp.
L20	1157	0.3	-	-	1151.4	10351.86	**1130**	0.48	1.86
L28	6447.25	1.5	-	-	-	-	**6014.07**	0.95	6.72
L50	78224.68	7	-	-	71291.4	7626.52	**69404.64**	6.26	2.65
L75	34396.38	13	-	-			**31482.84**	11.60	8.47
L100	538193.1	14	-	-	496820.4	11397.19	**478910.09**	57.03	3.60
L125A	288774.6	110	-	-			**256860.78**	83.58	11.05
L125B	1084451	70	-	-	1008839	9250.28	**943140.06**	118.65	6.51
Dunker62	3939362	4996	3871510	252.0	3812825	7304.05	**3685136.02**	9.07	3.35

Best values are in bold.

To study the absolute error of layouts produced by *BRKGA* we generated 100 datasets with known optimal solution (ten datasets for a number of facilities equal to 10, 20, 30, 40, 50, 60, 70, 70, 80, 90, and 100). In Table 7 we report on the average and maximum percentage deviation from optimal, % and %, (over all the datasets having the same number of facilities) of the best cost of BRKGA over 10 runs. Additionally, we also include the average deviation from optimal obtained by GUROBI when solving the FLP-MIP model for a maximum of 3600 CPU seconds (1 hour). The times for the BRKGA correspond to the total time for the ten BRKGA runs.

The results in Table 7 show that BRKGA performs quite well in terms of absolute deviation from optimal. From 10 to 40 facilities the % and % equal zero. For datasets having between 50 and 100 facilities, the value of % increases from 0.11% to 7.36% while the value of % increases from 1.12% to 10.97%. The relative performance of *BRKGA* when compared to GUROBI with the *FLP-MIP* model is also good both in terms of CPU time and solution quality. Note that as the number of facilities increases the quality of the solutions found by GUROBI with FLP-MIP decreases, e.g., for datasets with 100 facilities GUROBI has % value equal to 101.78% while for *BRKGA* this value is only 7.36%.

Table 7: Experimental results: Random generated datasets with known optimal solution.

	GUROBI			BRKGA		
Number of Facilities	T(s)	%	%	T(s)	%	%
10	3600	0.21	1.66	1.76	0.00	0.00
20	3600	0.01	0.12	6.13	0.00	0.00
30	3600	0.32	2.14	15.00	0.00	0.00
40	3600	2.37	7.10	28.67	0.00	0.00
50	3600	3.99	9.30	48.30	0.11	1.12
60	3600	16.65	29.73	72.86	0.02	0.15
70	3600	12.21	22.70	102.90	1.44	5.29
80	3600	22.31	50.97	143.37	3.31	7.10
90	3600	36.11	52.99	186.87	6.00	9.09
100	3600	101.78	235.31	235.84	7.36	10.97

4.3 Constrained case

As mentioned above, the dimensions of the floor space in the constrained case are fixed and are given as input. Additionally, note that the constrained approach can be only be used for the rectilinear distance measure (R).

4.3.1 Datasets and benchmark approaches

The performance of the *BRKGA-LP* approach for the constrained case is investigated using a comprehensive set of test problems from the literature. Table 8 summarizes the parameters of the datasets. These datasets were chosen because because of their variety in size (from seven up to 35 facilities) and because they are frequently used in the literature to benchmark alternative approaches for solving the *FLP*.

Table 8: Datasets used in the constrained case.

		Floor dimensions			
Dataset	N	W	H	Facility requirements	Source
O7	7	8.54	13	$R_{max} = 4$	Meller et al. (1998)
O8	8	11.31	13	$R_{max} = 4$	Meller et al. (1998)
O9	9	12	13	$R_{max} = 4$	Meller et al. (1998)
F10	10	90	95	$R_{max} = 3$	Montreuil et al. (2004)
VC10	10	25	51	$w_{min} = h_{min} = 5$	Van Camp et al. (1992)
BA12	12	10	6	$w_{min} = h_{min} = 1$	Bazaraa (1975)
BA14	14	9	7	$w_{min} = h_{min} = 1$	Bazaraa (1975)
AB20	20	30	20	$R_{max} = 4$	Armour and Buffa (1963)
TAM20	20	40	35	$R_{max} = 5$	Tam (1992) and Gau and Meller (1999)
TAM30	30	45	40	$R_{max} = 5$	Tam (1992) and Gau and Meller (1999)
SC30	30	15	12	$R_{max} = 5$	Liu and Meller (2007)
SC35	35	16	15	$R_{max} = 4$	Liu and Meller (2007)

Some authors have relaxed the dimensions of the facilities, or of the floor space, or both, when conducting the computational experiments. Although, in practice, adjusting dataset input parameters to find practical solutions is acceptable, modifying the datasets parameters makes it difficult to benchmark alternative approaches. *BRKGA-LP* finds feasible solutions for all datasets without modifying the original dataset parameters. Therefore, the *BRKGA-LP* will only be compared against previous approaches which use the original dataset parameters. Table 9 lists the approaches used for comparison with *BRKGA-LP*. We included approaches

Table 9: Benchmark approaches used in the unconstrained case.

Approach	Type of method	Source of approach
GA-ST	GA with slicing-tree	Gau and Meller (1999)
ACO-ZB	Ant Colony Optimization with zone-based layout	2004Montreuil et al. (2004)
I-MIP	Improved mixed-integer programming model	Sherali et al. (2003)
MIP-ε	MIP ε-accurate	Castillo and Westerlund (2005)
MIP-MINLP	Mixed-integer linear and mixed-integer nonlinear optimization methods	Castillo et al. (2005)
GA-SP-MIP	GA with sequence pair and MIP	Liu and Meller (2007)
STaTS	Tabu search with slicing-tree	Scholz et al. (2009)
ACO-ST	Ant colony optimization with slicing tree	Komarudin and Wong (2010)
AS-FBS	Ant system with flexible bay structure	Wong and Komarudin (2010)
ACO-LS-FBS	Ant colony optimization with local search and flexible bay structure	Kulturel-Konak and Konak (2011b)
PSO-RFBS	Particle swarm optimization with relaxed flexible bay structure	Kulturel-Konak and Konak (2011a)
GA-LP	Linear programming based genetic algorithm	Kulturel-Konak and Konak (2013)

which do not impose any additional constraints in the solutions sought (Castillo and Westerlund (2005), Castillo et al. (2005), Liu and Meller (2007) and Kulturel-Konak and Konak (2013)) and the other approaches based on zone-based layouts, slicing-tree representation, and flexible bay structure representation which impose additional limitations in the search domain. The former approaches require longer computational times since they have a larger search domain but usually find better solutions. The latter take less computing times due to the reduced search domain but usually generate worst solutions (with higher cost).

4.3.2 Experimental results

The *BRKGA-LP* approach approximates the facility areas by tangential supports (see eq.1) which tends to produce solutions with smaller facility areas than the actual area requirements. However, the area approximation quality can be increased by increasing the number of tangential supports ($\triangle$) at the expense of an increase in computational effort. To evaluate the area approximation error incurred for each facility, we use the average percent area approximation error ($\%E_{avg}$) and the maximum percent area approximation error ($\%E_{max}$) metrics (as proposed in Castillo and Westerlund (2005) and Kulturel-Konak and Konak (2013)). These are defined, respectively, as:

$$\%E_{avg} = \frac{100\%}{N} \sum_{1}^{N} \frac{|A_i - w_i h_i|}{A_i},$$

$$\%E_{max} = 100\% \max_{i=1,\ldots,N} \left\{ \frac{|A_i - w_i h_i|}{A_i} \right\}.$$

The solution process of the *BRKGA-LP* uses a strategy equal to the one implemented by Kulturel-Konak and Konak (2013) in their *GA-LP* approach, i.e., we used $\Delta = 25$ to obtain a solution and after the last generation is complete we solve the best solution found again using $\Delta = 100$ to ensure that area approximation error is negligible. Consequently, the best solutions reported in this paper for the *BRKGA-LP* approach have a facility area approximation error either equal to 0% or very close to 0%. Tables 10 and 12 present the experimental results, respectively, for the datasets O7-O9 and F10-SC35. For all the best found solutions we report $\%E_{avg}$ and $\%E_{max}$. As can be observed, the largest value of $\%E_{max}$ was 0.0095 %, which for a facility with an area of 100 units corresponds to a negligible reduction of only 0.0095 units.

Table 10 reports the best cost solutions found by *BRKGA-LP*, over 10 runs, and six other approaches from the literature for datasets O7-O9. Even though the computational times reported by the other approaches might not be comparable we report them, if available, in Table 11.

The best costs obtained by *BRKGA-LP* are very similar to those of the other approaches. The best solutions reported in Table 10 vary only slightly; this may be due to different approximations of the area of facilities (e.g., Castillo and Westerlund (2005) reports the best values but has a $\%E_{max}$ equal to 0.05, 0.15 and 0.30, respectively, for O7, O8 and O9, while *BRKGA-LP* has $\%E_{max}$ values equal to 0.0030, 0.0012, and 0040 which are at least one order of magnitude smaller).

Table 10: Experimental results: Layout costs for datasets 07-09.

Dataset	I-MIP	MIP-ε	MIP-MINLP	GA-SP-MIP	STaTS	AS-FBS	*BRKGA-LP*	*BRKGA-LP* $\%E_{avg}$	*BRKGA-LP* $\%E_{max}$
07	131.64	131.57	131.64	131.63	132	131.68	131.56	0.0019	0.0030
08	242.89	242.77	242.73	242.88	243.16	243.12	242.92	0.0004	0.0012
09	235.95	235.87	236.14	235.95	239.07	236.14	236.57	0.0013	0.0040

As can be observed in Table 11 the computational times of *BRKGA-LP* and *STaST* are small and similar and vary from 4.3s to 12.77s. However, the other approaches run in computational times that are between 115 and 6676 times greater than those of *BRKGA-LP*.

The final solutions generated by *BRKGA-LP* for datasets O7-O9 are shown in the appendix in Figures 16 and 17.

Table 11: Experimental results: Computing times for datasets 07-09.

Dataset	I-MIP	MIP-ε	MIP-MINLP	GA-SP-MIP	STaTS	AS-FBS	*BRKGA-LP*
07	7700	2301	1195	790	4.3	4032	6.85
08	43000	54443	18392	3860	6.2	4248	10.18
09	60000	85255	83211	5384	8.9	5184	12.77

Table 12 reports the cost for the best solutions found by *BRKGA-LP* , over 10 runs, and ten other approaches from the literature for datasets F10-SC35. Even though the computational times reported by the other approaches might not be comparable we report them, if available, in Table 13.

As can be observed in Table 12 the layout costs obtained by *BRKGA-LP*, with the exception of dataset VC10, are better than or equal to the best found costs reported by any other approach in the study. *BRKGA-LP* improved the best known solution costs for datasets BA14, AB20, TAM20, TAM30, and SC35 by, respectively, 1.24%, 1.04%, 0.89%, 1.42%, and 2.03%. Given that these datasets have been extensively studied in the literature, an improvement greater then 0.89% over the previously best-known solutions is significant. Also, given the small area approximation errors for these datasets, we believe that the achieved improvements cannot be attributed to the area approximation.

The computational times reported in Table 13 show that *BRKGA-LP* takes significantly less computing time when compared with the similar approaches *MIP-MINLP*, *GA-SP-MIP*, and *GA-LP*. However, the approaches using a smaller search domain with slicing trees or

Table 12: Experimental results: Layout costs for datasets F10-SC35.

	Datasets								
Approaches	F10	VC10	BA12	BA14	AB20	TAM20	TAM30	SC30	SC35
GA-ST			8485.4	4804.1		9513.5	20658		
ACO-ZB	8567								
MIP-MINLP		21297.98	8180						
GA-SP-MIP		1997	8702	5004	8180.00			3707	3604
STaTS		19994.1	8264	4712.33	5225.96				
ACO-ST		19967	8252.67	4724.68	5073.82			3868.55	4132.36
AS-FBS			8299.5	4913.22				3679.85	3962.72
ACO-LS-FSB	9020.75		8801.33	4904.67	5360.8	9003.82	19667.45	3794.82	4011.31
PSO-RFBS	9020.75	**18823.74**	8129	4780.91	5336.36	8753.57	19462.41	3443.34	3700.75
GA-LP	7651.28		**8021**	4686.81	5196.38	8058.06	19009.9	**3370.98**	3385.48
BRKGA-LP	**7650.95**	19951.17	**8020.97**	**4628.79**	**5021.17**	**7986.48**	**18740.3**	**3367.87**	**3316.77**
%Imp	0.00	-5.99	0.00	1.24	1.04	0.89	1.42	0.09	2.03
$\%E_{avg}$	0.0016	0.0054	0	0.0016	0.0019	0.0011	0.0016	0.0012	0.0010
$\%E_{max}$	0.0054	0.0016	0	0.0095	0.0046	0.0065	0.0061	0.0052	0.0048

Best values are in bold.

flexible bays, like PSO-RFBS (from Kulturel-Konak and Konak (2011a)) and *ACO-LS-FSB* (from Kulturel-Konak and Konak (2011b)) use significantly less computing time but generate solutions with higher layout costs.

The final solutions generated by *BRKGA-LP* for datasets F10-SC35 are shown in the appendix in Figures and 17, 18, 19, 20, and 21.

Table 13: Experimental results: Computing times for datasets F10-SC35.

	Datasets								
Approaches	F10	VC10	BA12	BA14	AB20	TAM20	TAM30	SC30	SC35
GA-ST			412	396		499	1397		
ACO-ZB	26300								
STaTS		8.9	13.6	16.1	13.6				
ACO-ST		1008	4104	8568	17820			23544	83988
AS-FBS			164	292	919			19135	28671
ACO-LS-FSB	51		146	131	106	226	623	902	1185
PSO-RFBS	2	3	10	19	85	104	924	873	1842
MIP-MINLP		43200	43200						
GA-LP	3000		6000	7500	22500	22500	73500	22038.3	29538.9
GA-SP-MIP		6660	2988	2880	4919.47			14652	57744
BRKGA-LP	49.15	46.40	113.81	160.46	426.09	686.00	3004.55	998.74	1556.03

5 Concluding remarks

In this paper we present a biased random key genetic algorithm (*BRKGA*) for the unequal area facility layout problem where a set of rectangular facilities with a given area requirements have to be placed, without overlapping, on a rectangular floor space, so as to minimize the quadratic cost of products of inter-facility flows and inter-facility distances. The hybrid approach combines a *BRKGA*, to determine the order of placement and the dimensions of each facility, a novel placement strategy, to position each facility, and a linear programming model, to fine-tune the solutions. The unconstrained version of the approach generates high-quality solutions in relatively small computing times. The constrained version of the algorithm, *BRKGA-LP*, uses the solutions generated by *BRKGA* and tries to improves them in terms of cost and feasibility using a linear programming model. The approach is tested on 100 random datasets and 28 of benchmark datasets taken from the literature and compared against 21 other benchmark approaches proposed in the literature. The unconstrained version *BRKGA* improved the best known solutions for 14 of the 16 benchmark datasets while the constrained version *BRKGA-LP* improved the best known solutions for 5 of the 8 extensively studied benchmark datasets.

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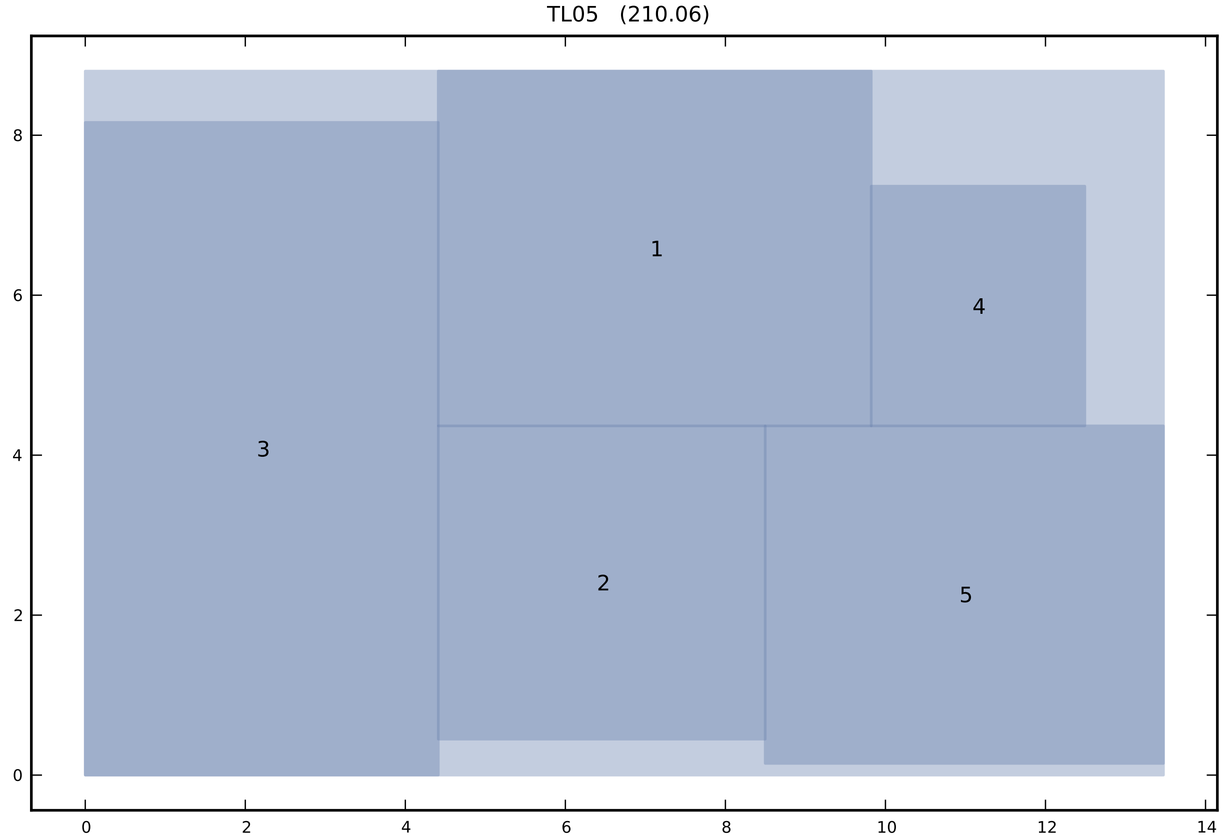
Appendix



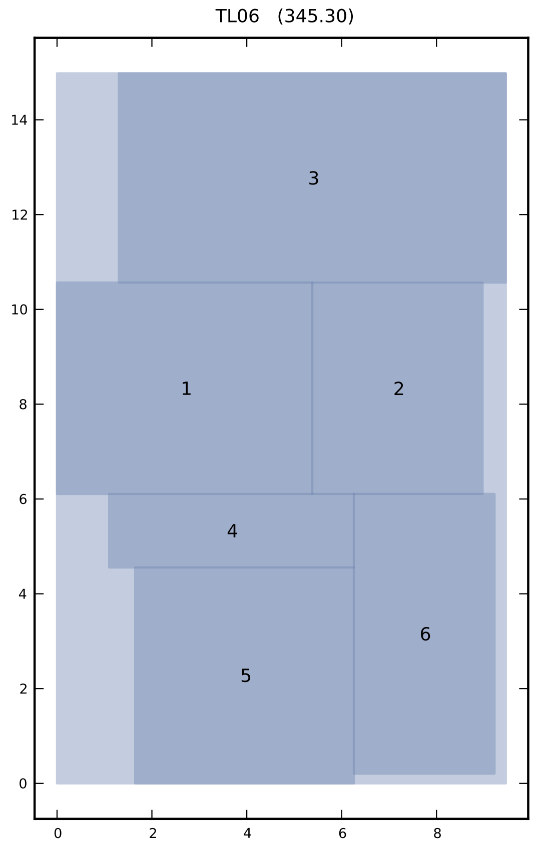


Figure 8: *BRKGA* solutions for datasets TL05 and TL06.

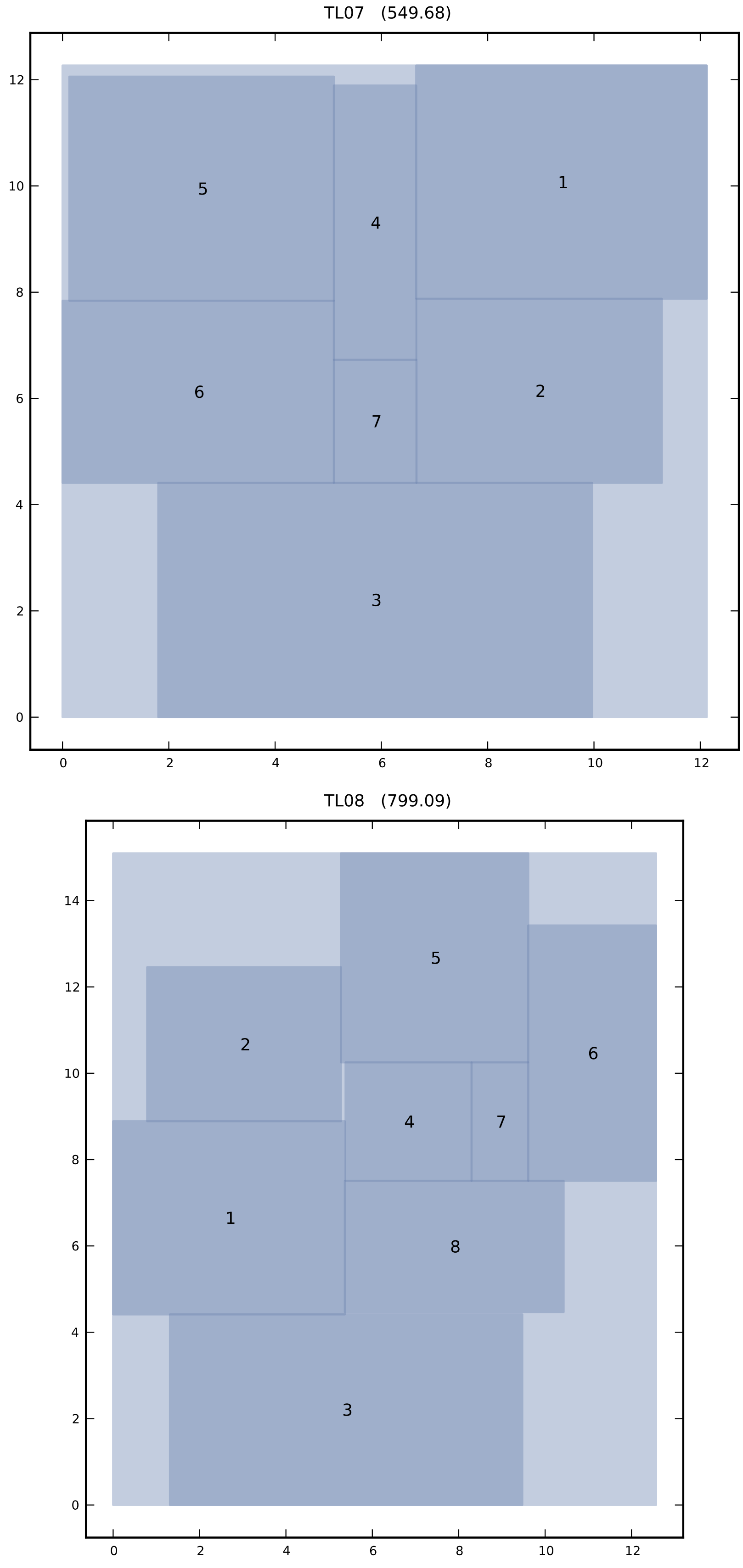


Figure 9: *BRKGA* solutions for datasets TL07 and TL08.

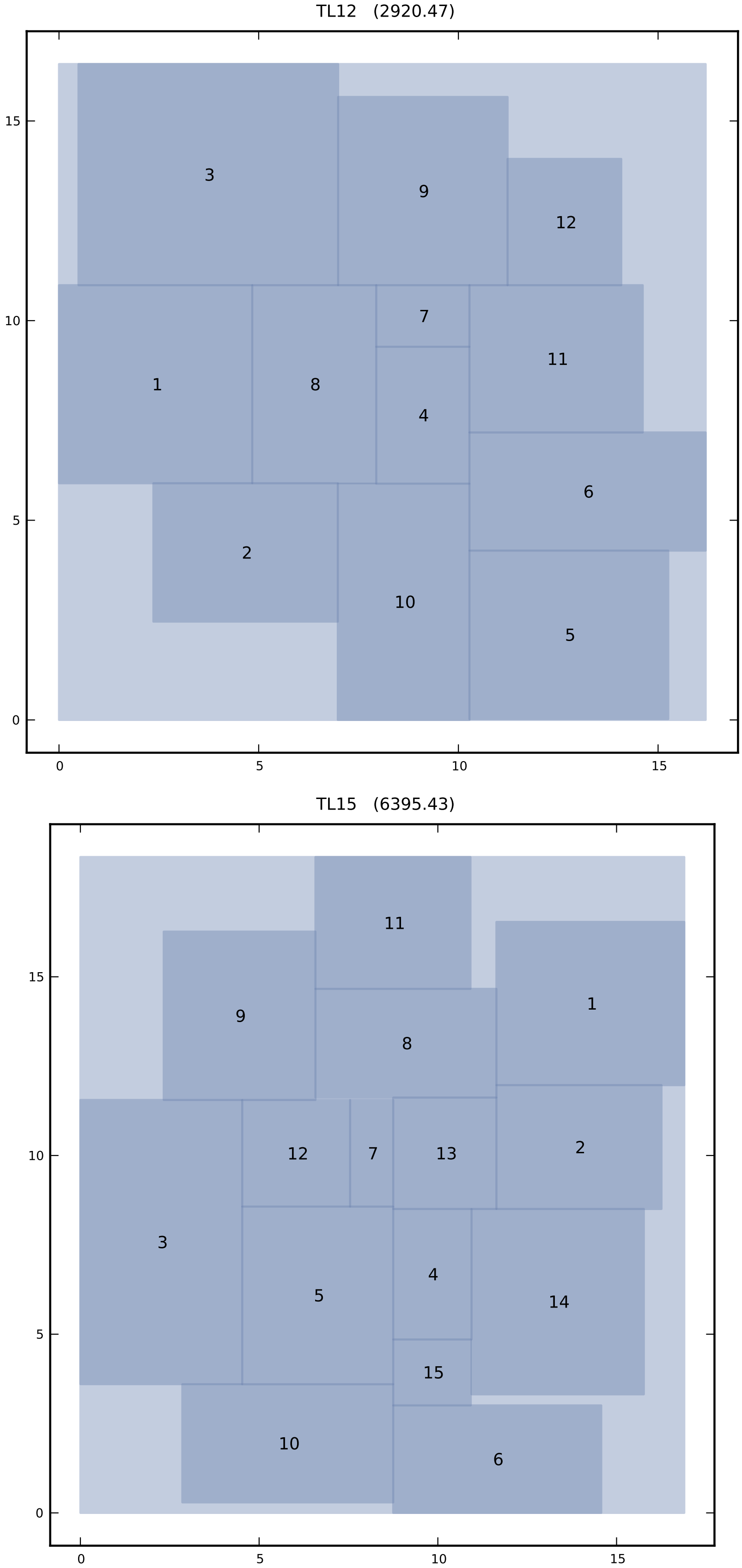


Figure 10: *BRKGA* solutions for datasets TL12 and TL15.

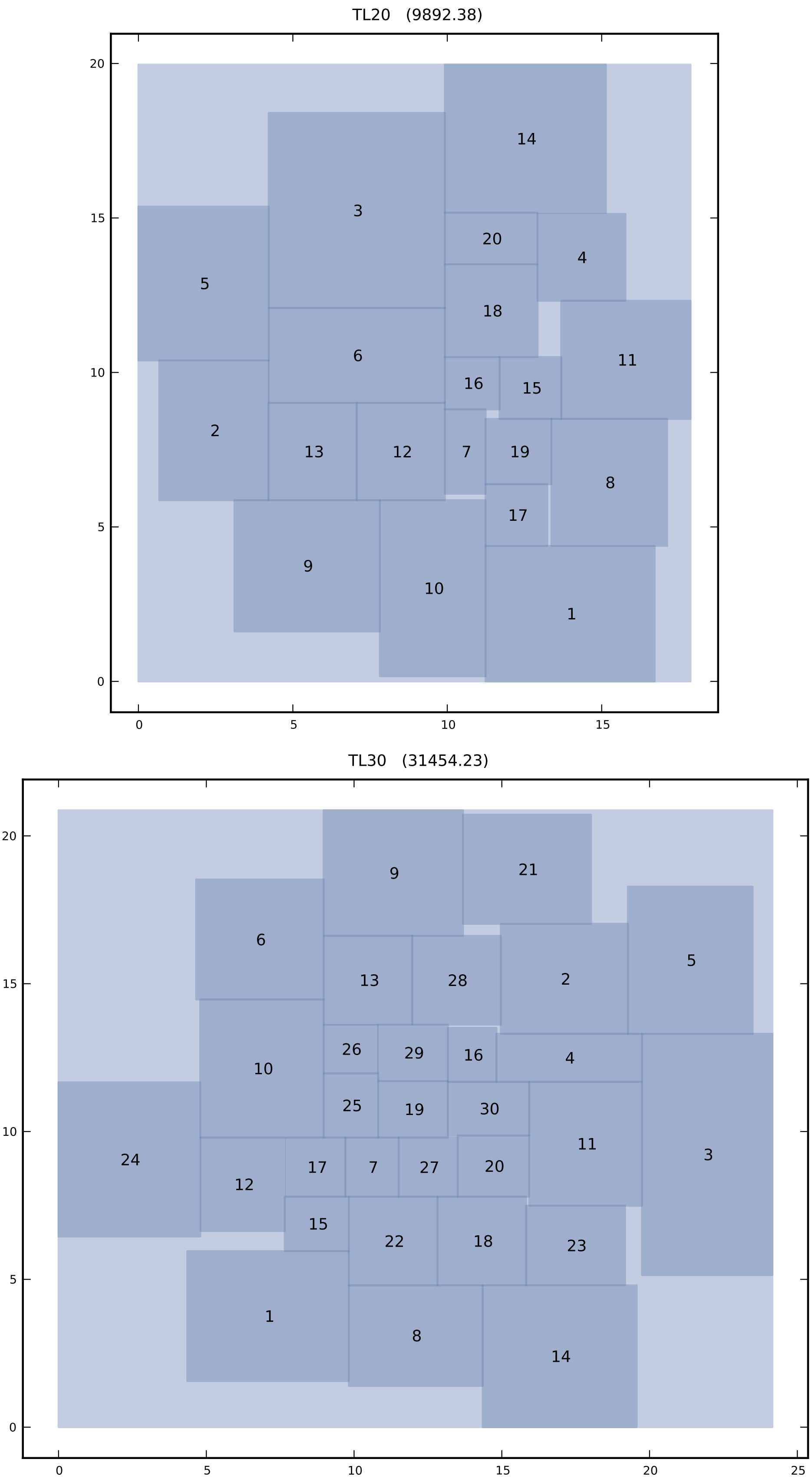


Figure 11: *BRKGA* solutions for datasets TL20 and TL30.

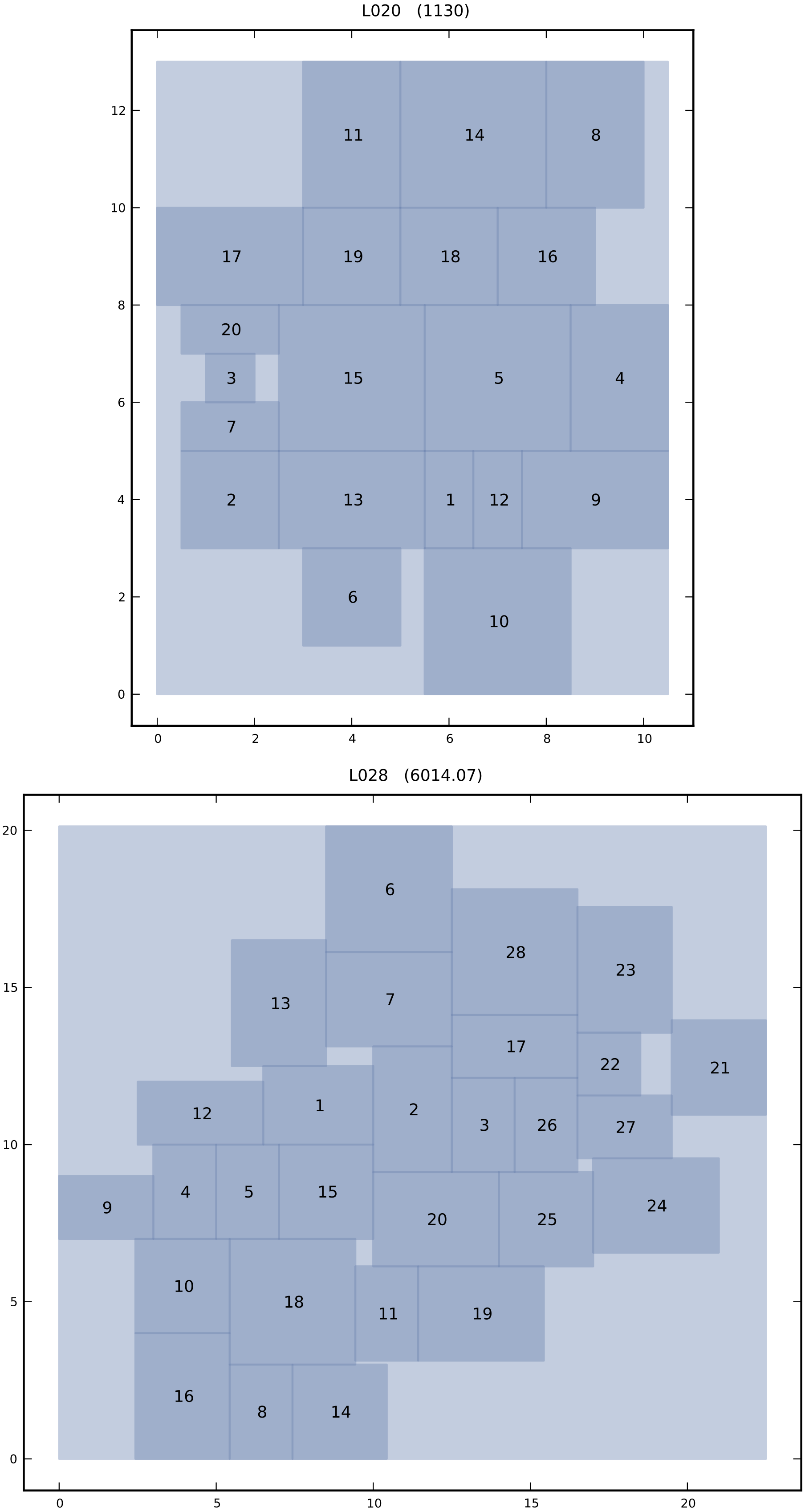


Figure 12: *BRKGA* solutions for datasets L020 and L028.

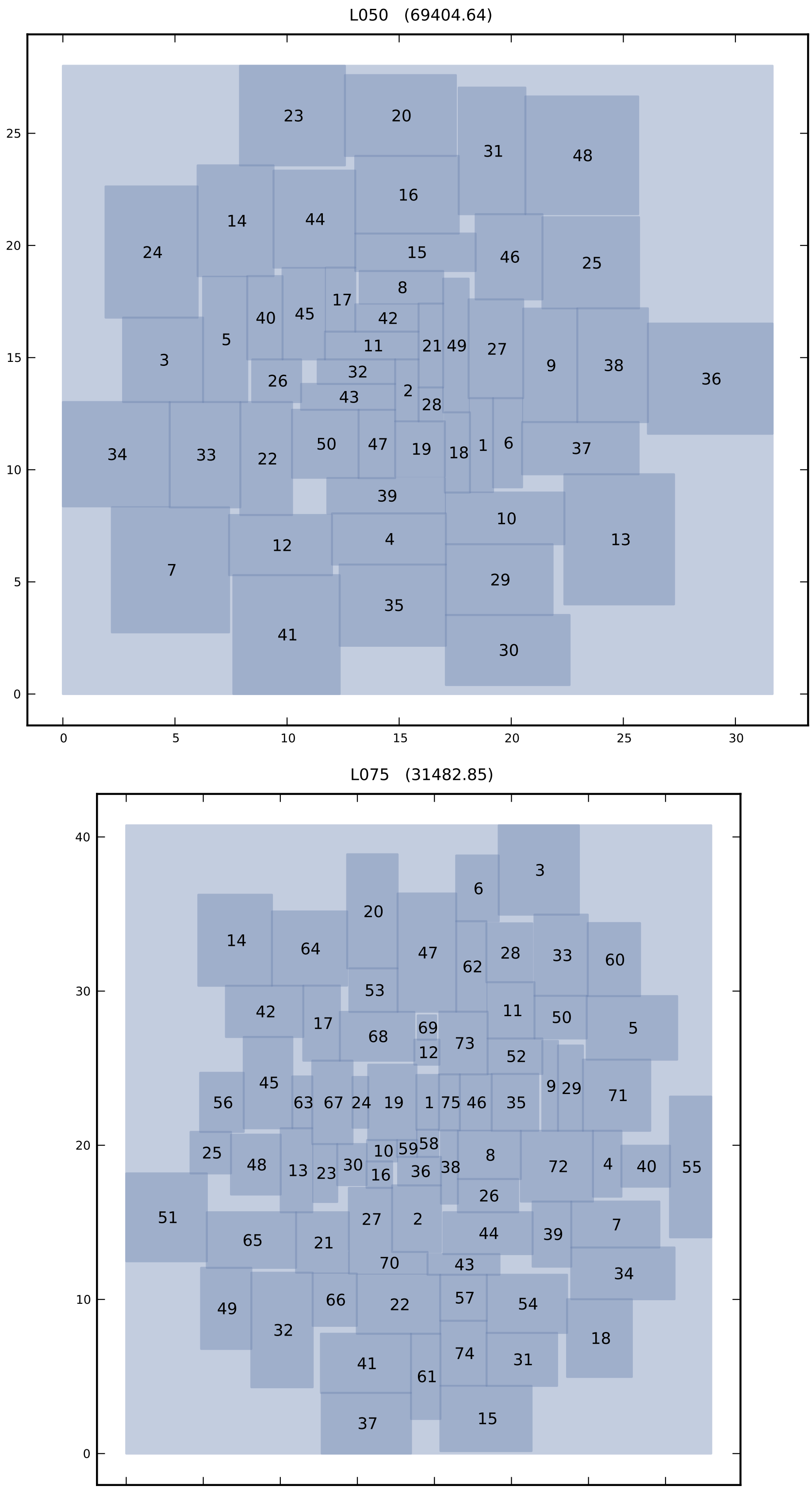


Figure 13: *BRKGA* solutions for datasets L050 and L075.

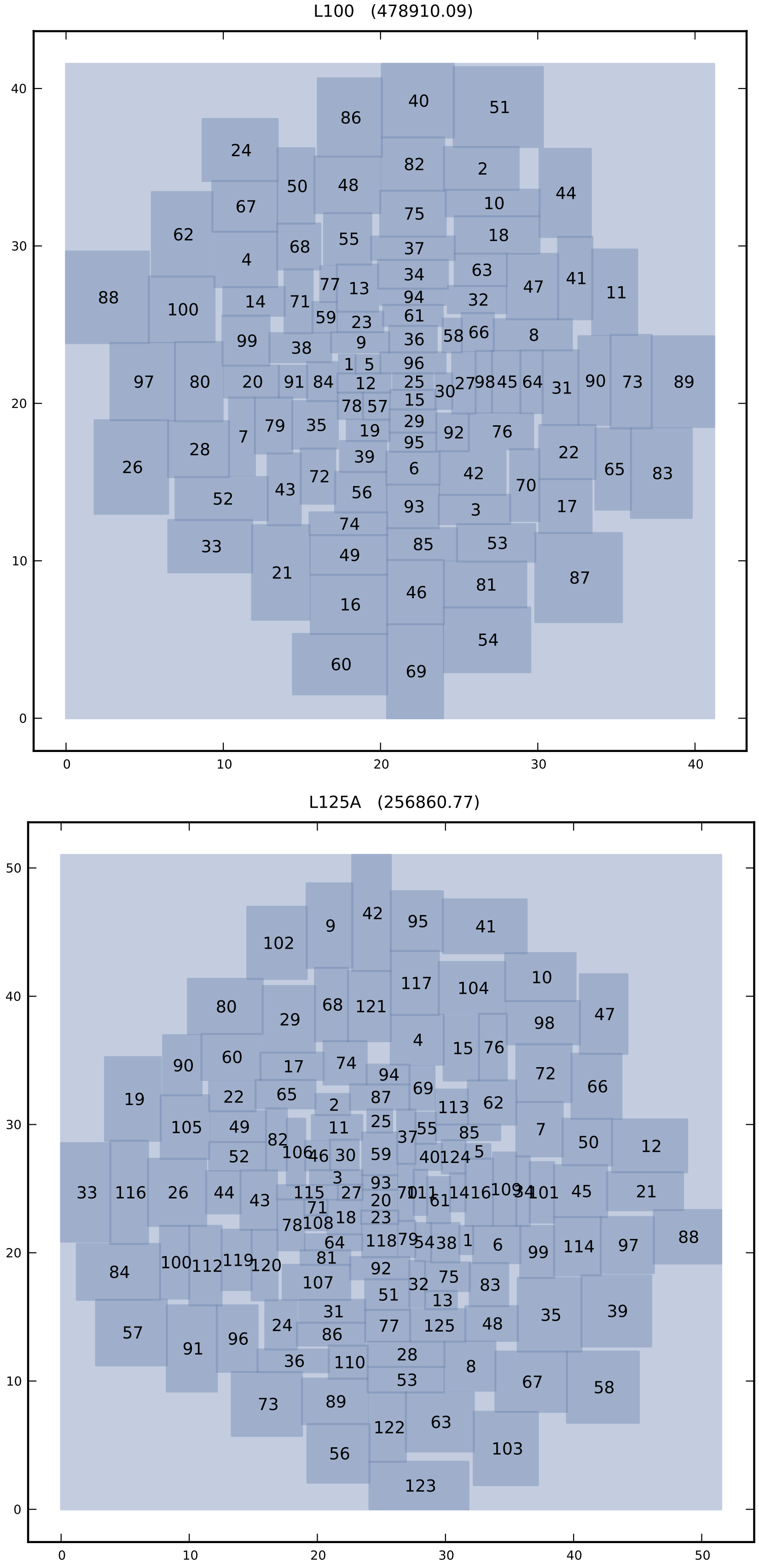


Figure 14: *BRKGA* solutions for datasets L100 and L125A.

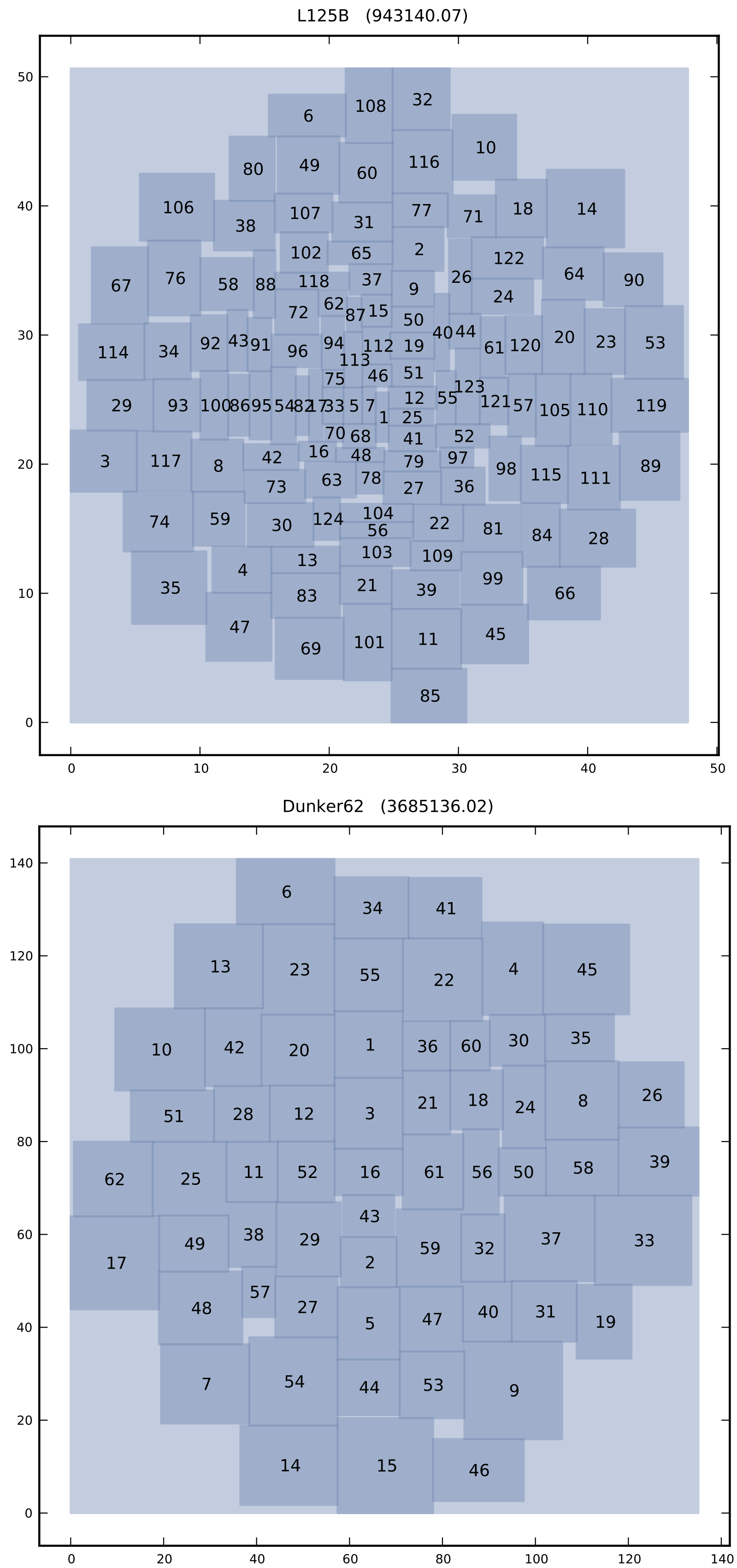
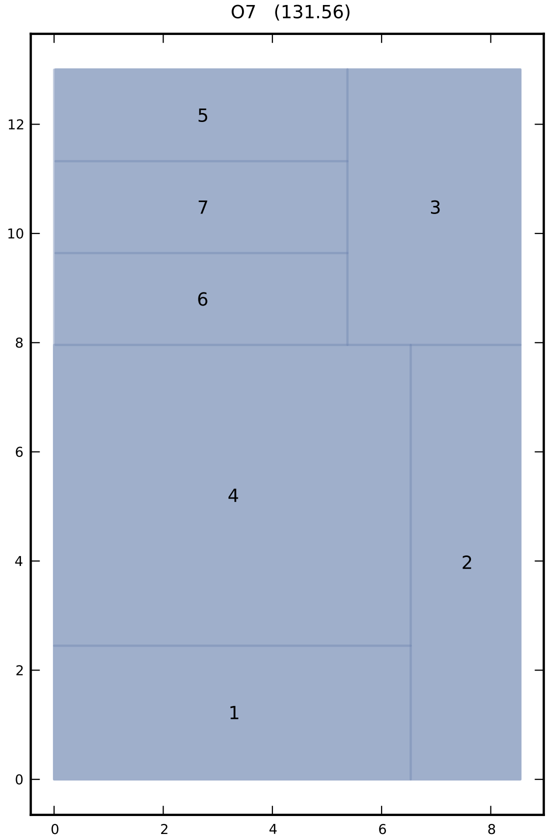


Figure 15: *BRKGA* solutions for datasets L125B and Dunker62.



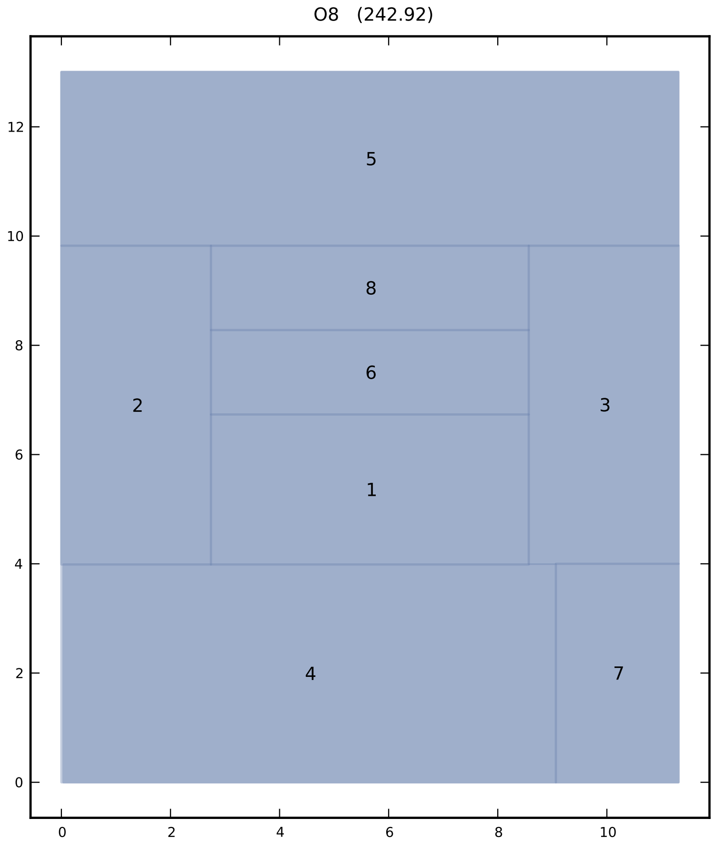


Figure 16: *BRKGA-LP* solutions for datasets O7 and O8.

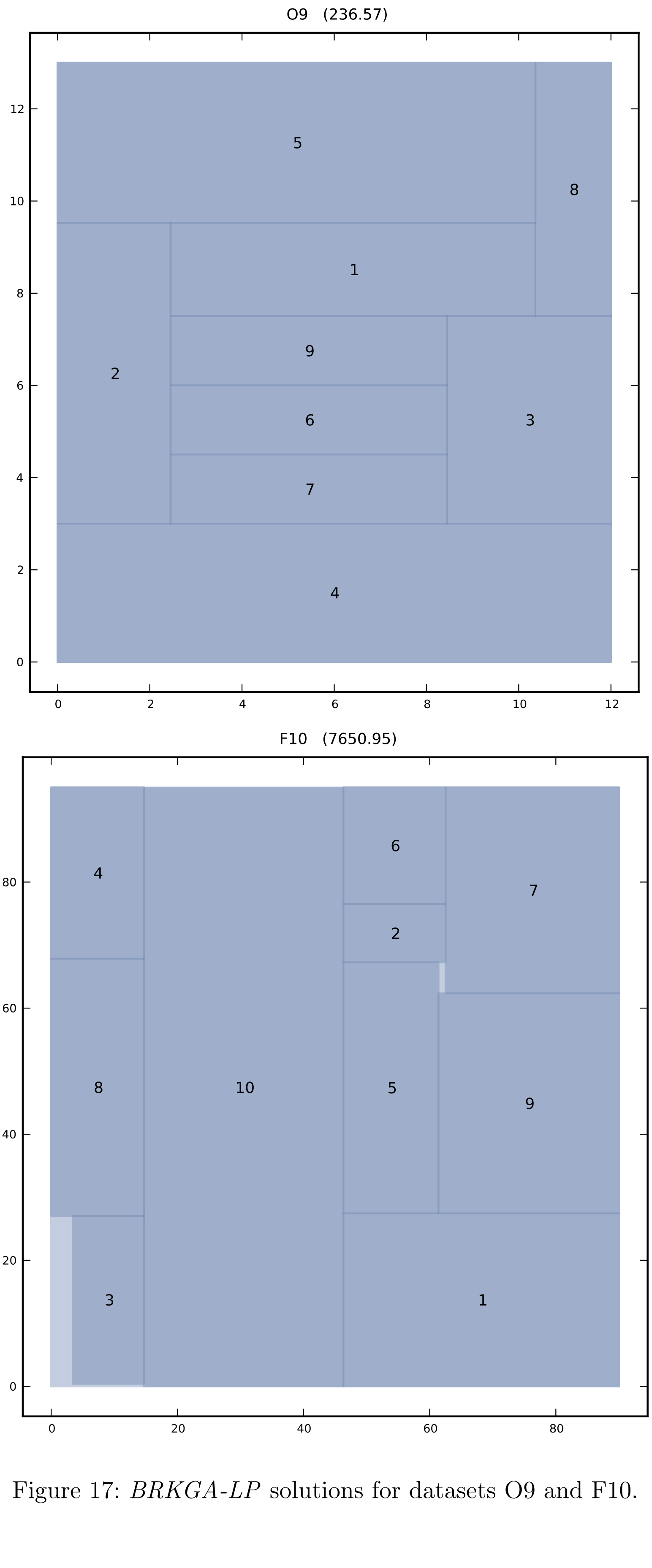


Figure 17: *BRKGA-LP* solutions for datasets O9 and F10.

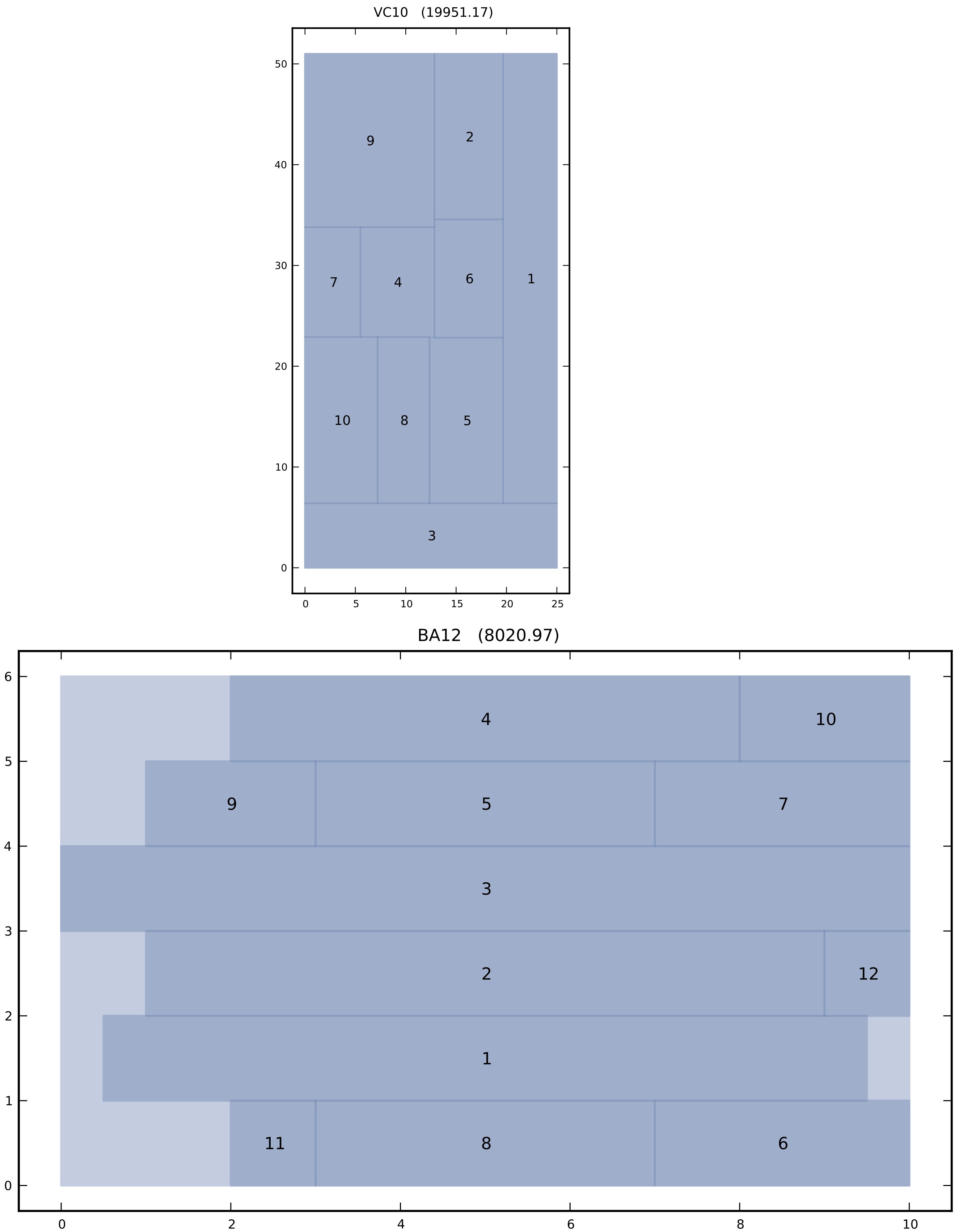


Figure 18: *BRKGA-LP* solutions for datasets VC10 and BA12.

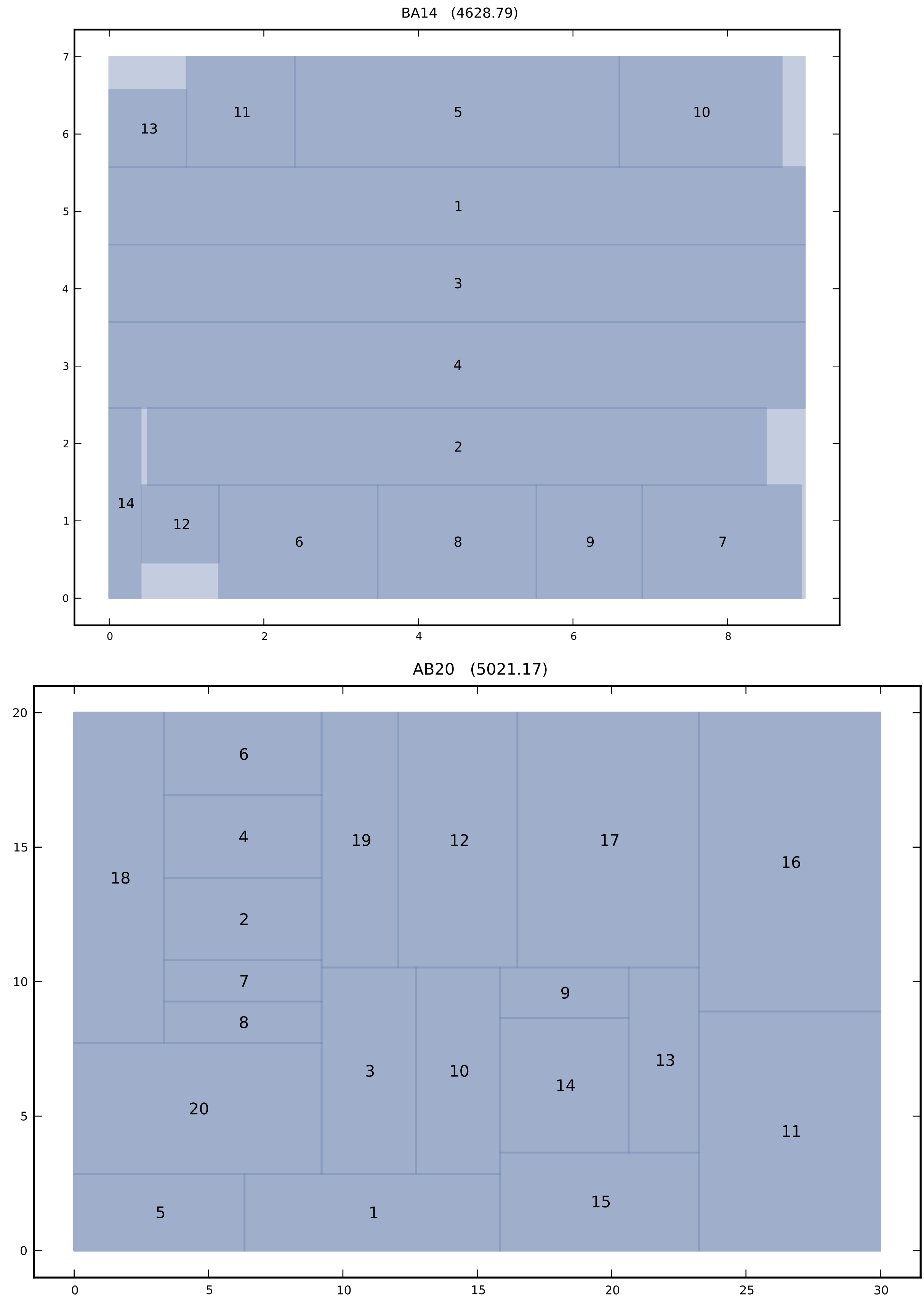
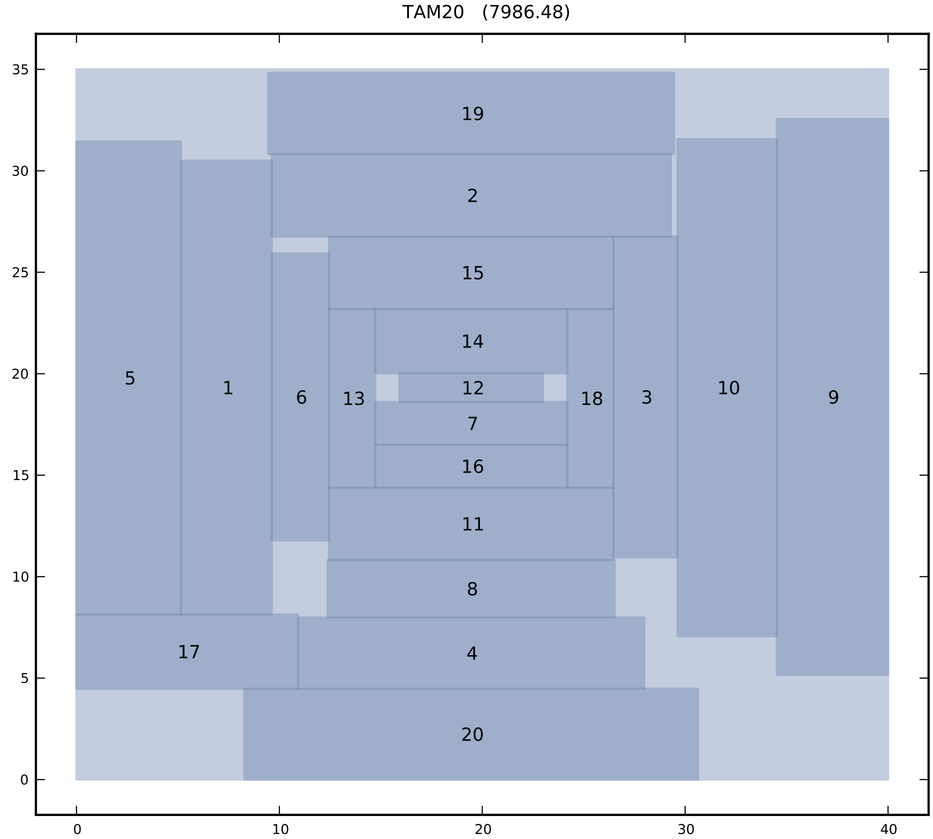


Figure 19: *BRKGA-LP* solutions for datasets BA14 and AB20.



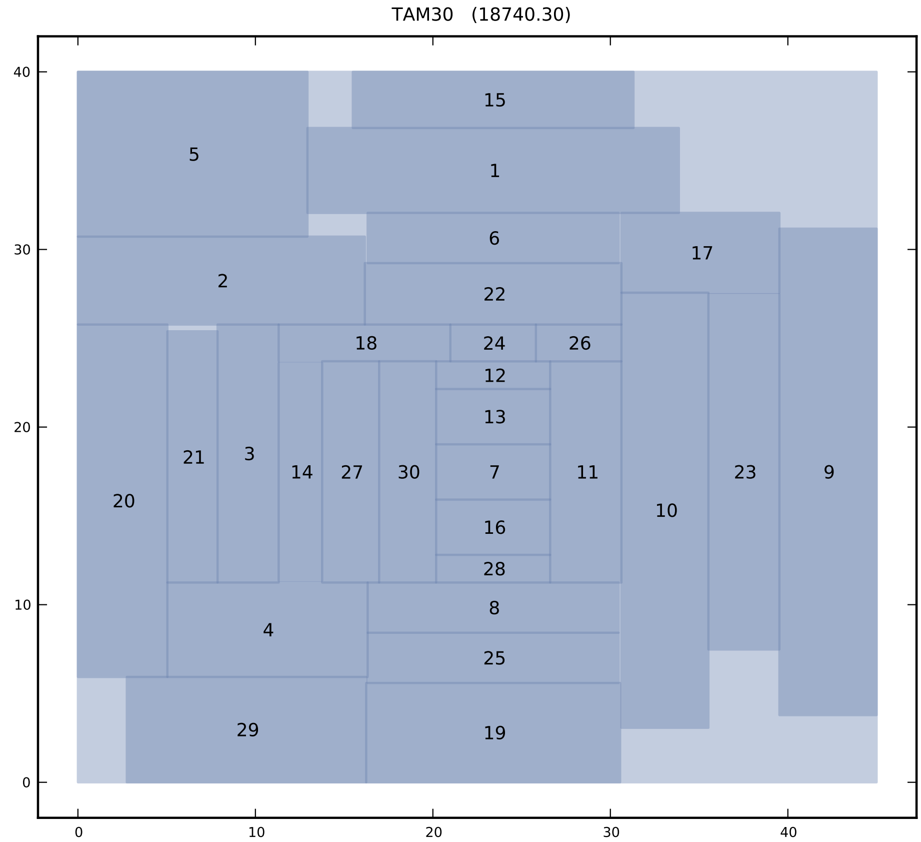


Figure 20: *BRKGA-LP* solutions for datasets TAM20 and TAM30.

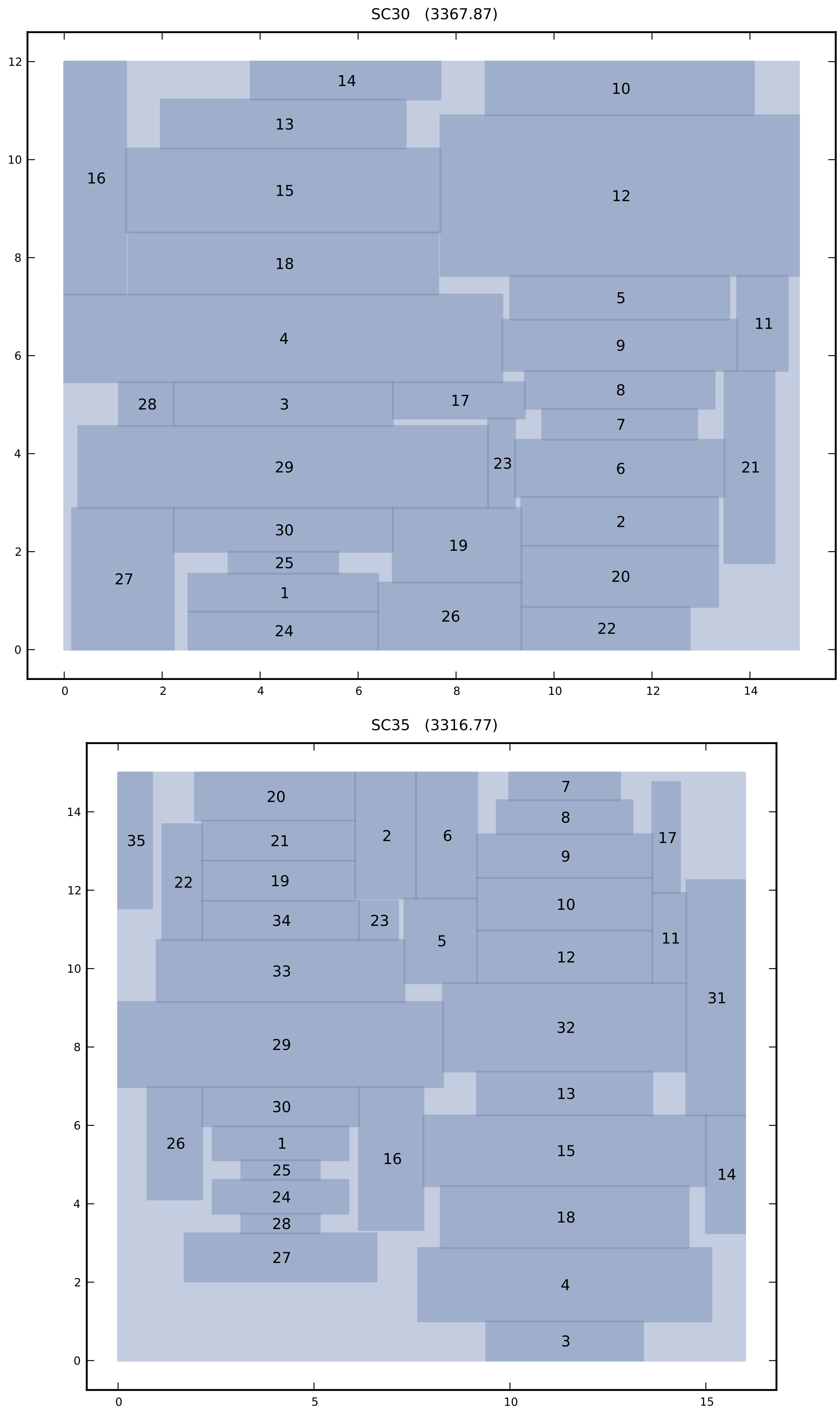


Figure 21: *BRKGA-LP* solutions for datasets SC30 and SC35.